Hindawi
International Journal of Differential Equations
Volume 2021, Article ID 1506485, 10 pages
https://doi.org/10.1155/2021/1506485

Research Article

Electric Transverse Emissivity of Sinusoidal Surfaces Determined by a Differential Method: Comparison with Approximation of Geometric Optics

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Received 11 September 2021; Revised 22 October 2021; Accepted 6 November 2021; Published 29 November 2021

Academic Editor: Davood D. Ganji

We present in this paper a numerical study of the validity limit of the optics geometrical approximation in comparison with a differential method which is established according to rigorous formalisms based on the electromagnetic theory. The precedent studies show that this method is adopted to the study of diffraction by periodic rough surfaces. For periods much larger than the wavelength, the mechanism is analog to what happens in a cavity where a ray is trapped and undergoes a large number of reflections. For gratings with a period much smaller than the wavelength, the roughness essentially behaves as a transition layer with a gradient of the optical index. Such a layer reduces the reflection there by increasing the absorption. The code has been implemented for TE polarization. We determine by the two methods such as differential method and the optics geometrical approximation the emissivity of gold and tungsten cylindrical surfaces with a sinusoidal profile, for a wavelength equal to 0.55 microns. The obtained results for a fixed height of the grating allowed us to delimit the validity domain of the optic geometrical approximation for the treated cases. The emissivity calculated by the differential method and that given on the basis of the homogenization theory are satisfactory when the period is much smaller than the wavelength.

1. Introduction

The modeling of the monochromatic directional emissivity of a rough surface remains a topical problem both on a purely fundamental level and on those of numerical simulation or experimental determination [1–3]. More generally, the interest given to the radiative properties of surfaces finds its justification through the fields of advanced technology, for which complete knowledge of these quantities is essential. We cite as examples the semiconductor industry [4], solar energy, or computer graphics [5]. Research work brings out two approaches in this context: one is exact and based on electromagnetic theory and the other is approximated and based on geometric optics. The classical problem of the diffraction of a plane wave by a grating (periodic surface) has long been dealt with using the Rayleigh development. The validity of this formalism has been controversial by Lipmann [6] and discussed by various authors [7–10]. Petit and Cadilhac [11] were able to show in the mid-sixties that this approach is not always valid and gave an example proving its invalidity in certain cases. Similar work was carried out by Nevière in 1970 [12]. The validity of Rayleigh's theory has also been studied by Wirgin [13].

Differential methods are better known in the literature as coupled wave analysis. Among the authors who have been interested in studying the problem of scattering by the coupled wave analysis approach, we first mention Moharam and Gaylord [14, 15]. They applied this approach, first, to plane arrays [16, 17] in the case of TE polarization, and then they extended it to the case of TM polarization [18].

In our work, we have used this method of coupled wave analysis [19, 20] in order to determine the emissivity or hemispherical directional reflectivity of periodic rough surfaces. Indeed, this method makes it possible to determine the complex amplitudes of the reflected and transmitted fields; we deduce in particular the reflected efficiency in each order. This is none other than the ratio of the reflected flux by the incident one. The sum of these efficiencies leads to the essential radiative property, making it possible to go back to the other properties.

At the same time, different versions of algorithms have been proposed. They aim to reduce the numerical instability that appears in matrix inversion [21–23]. The algorithm associated with the multilayer model method proposed by Chateau and Hugonin [21] plays an important step in the resolution of this instability problem. This method consists first of subdividing the region of the grating into sublayers and then of representing the dielectric function as well as the fields by Fourier series expansions. Maxwell's equations thus lead to a system of coupled differential equations. The fields are then represented on either side of the grating region by the complex amplitudes of the rising and falling waves. A recursive resolution, in terms of these latter amplitudes, makes it possible to determine the reflected and transmitted efficiencies.

Another approach to determine the radiative properties of rough surfaces is approximate; it is based on geometric optics. It uses the notion of light ray, the laws of Snell–Descartes as well as notions of classical or analytical geometry. It is about following an incident ray, in a cavity, from its entrance and until it emerges from it, after having undergone a certain number of reflections. Fresnel's laws giving the reflectivity of smooth flat surfaces lead, with the help of a physical study, to determine the desired radiative properties. In this context, several authors have dealt with this problem for different forms of surface roughness [24–27]. The geometric optics approximation is only valid when the dimensions of the surface roughness are very large in relation to the wavelength. Some studies have been interested in determining the limit of validity of this approximation in comparison with the integral method [28, 29].

We also approach the study of the radiative behavior of these rough surfaces when the period is very small compared with the wavelength. When the grating has a period much less than the wavelength, it no longer diffracts light. In other words, only the order 0, which corresponds to reflection or specular transmission, propagates. It therefore behaves exactly like a smooth surface. However, its structure gives it radiative properties that are very different from those of the homogeneous material. Bouchitté and Petit [30] have shown that a sufficiently small period lattice is comparable to an anisotropic layer with an index gradient. This equivalence between grating and layer makes it possible to link the "multilayer" process with the "roughness" approach. It has been used by Gaylord and Baird [31] and Southwell [32] to replace the classic antireflective coating by a grating. This technique was then extended to the creation of polarizers and notch filters (Glytsis and Gaylord, 1992 [33]).

In this paper, we report a systematic study of the emissivity of a sinusoidal grating. As a reference solution, we will use a numerical simulation to derive the absorptivity of a surface illuminated by a monochromatic plane wave. We use an algorithm proposed by Chateau and Hugonin [21] to obtain the emissivity of the grating. Using geometric optics approximation (GOA), we know the apparent monochromatic directional emissivity of the cavity. In this work, we established the region of validity between the geometric optics approximation (GOA) and differential method (MD) especially in the cases of sinusoidal surfaces of finished conductivity (gold and tungsten). We have analyzed the different physical phenomena depending on the period and the angles of incidence. We have found a satisfactory agreement between the emissivity calculated by differential method and that given on the basis of homogenization regime when the period is much smaller than the wave length.

2. Prensentation of Formalism

We consider a grating of height h whose directrix, located in the plane (xOz), is a sinusoidal function of period d. A monochromatic plane wave of vector $\overset{0}{\vec{k}}$ contained in this plane illuminates this grating under the angle of incidence θ, the time dependence is in $\exp(-j\omega t)$. We assimilate the surface of this grating to a periodic rough surface, and we propose to determine its directional emissivity in electric transverse mode. For this, we first adopt a differential method. The problem comes down to calculate the amplitudes of the reflected and transmitted fields.

In the region of the lattice ($z_{\min} \leq z \leq z_{\max}$, $z_{\max} - z_{\min} = h$), the complex refractive index of the medium is a function of the space variables x and z: $n = n\,(x, z)$, periodic along x. The dielectric function is represented by the development in Fourier series [21]:

$$n_2(x,z) = \sum_{i=-\infty}^{+\infty} \tilde{n}_i(z)\exp(jiKx)\text{avec}K = \frac{2\pi}{d}. \qquad (1)$$

We use the tangential components $E_y(x,z)$ of the electric field and associated with the magnetic field $h_x = \mu c H_x$. These components of the electromagnetic field are continuous at the interface; they are represented by the Fourier series expansions, defined in the lattice region, as follows [21]:

$$\begin{aligned} E_y(x,z) &= \sum_{i=-\infty}^{+\infty} E_y^{(i)}(z)\exp\left(jk_x^{(i)}x\right), \\ h_x(x,z) &= \sum_{i=-\infty}^{+\infty} h_x^{(i)}(z)\exp\left(jk_x^{(i)}x\right), \end{aligned} \qquad (2)$$

$$\text{avec: } k_x^{(0)} = k_0 \cdot n_0 \sin\theta \, et \, k_x^{(i)} = k_x^{(0)} + iK,\ i \in Z.$$

In TE polarization, Maxwell's equations are written as follows:

$$j\omega\mu H_x(x,z) = -\frac{\partial E_y}{\partial z}\, et\, \frac{\partial h_x}{\partial z} = -jk_0 n^2 E_y + \frac{1}{jk_0}\frac{\partial^2 E_y}{\partial x^2}. \quad (3)$$

The region of the grating was subdivided into M sublayers perpendicular to the axis (Oz), by adding the previous developments in (3) [16]:

$$\frac{\mathrm{d}E_y^{(i)}(z)}{\mathrm{d}z} = -jk_0 h_x^{(i)}(z),$$
$$\frac{\mathrm{d}h_x^{(i)}(z)}{\mathrm{d}z} = -j\left\{\frac{k_{z,k}^{(i)2}}{k_0}E_y^{(i)}(z) + k_0\sum_{l\neq i}\widetilde{n}_{k,i-l}E_y^{(l)}(z)\right\}, \quad i \in Z, \quad (4)$$

avec: $[k_z^{(i)}]^2 = k_0^2\widetilde{n}_0 - k_x^{(i)2}$, $i \in Z$; $[k_{z,k}^{(i)}]^2 = k_0^2\widetilde{n}_{k,0} - k_x^{(i)2}$, $i \in Z$.

2.1. Algebric Resolution of the Differential System. System (4) can be in the form as follows:

$$\frac{\mathrm{d}U(z)}{\mathrm{d}z} = [M_k]U(z). \quad (5)$$

Algebraic resolution using the matrix exponential leads to the following:

$$U(z_k) = \exp\{-(z_{k+1}-z_k)[M_k]\}U(z_{k+1}). \quad (6)$$

We can express the matrix exponential in terms of eigenvectors $[M_k]$. The diagonalization of this leads to the following:

$$[M_k] = [P_k][D_k][P_k]^{-1}, \quad (7)$$

with $[P_k]$ is the matrix of passage formed the eigenvectors of $[M_k]$ and $[D_k]$ is a diagonal matrix composed by the eigenvalues of $[M_k]$.

Solution (5) is then written in the form as follows:

$$U(z_k) = [P(z_k)]\begin{bmatrix} \exp[(-e_0(z_{k+1}-z_k))] & & 0 \\ & \ddots & \\ 0 & & \exp[(-e_{2N-1}(z_{k+1}-z_k))] \end{bmatrix}[P(z_{k+1})]^{-1}U(z_{k+1}). \quad (8)$$

Numerical resolution reveals instability problems related to matrix inversion [1, 2]. To overcome these problems, we adopt the algorithm of Château and Hugonin [21]. It is a question of rearranging the positions of the eigenvectors of the matrix in relation (7), so as to be able to write the eigenvalues in ascending order in the diagonal matrix $[D]$.

2.2. Recursive Resolution of the Problem. The Rayleigh developments of the electromagnetic field in the first half-space ($z \le z_{\min} = z_0$) [16] are introduced:

$$E_y(x,z) = \sum_{i=-\infty}^{+\infty} f_F^{(i)} \exp\left\{j\left[k_x^{(i)}x + k_{Fz}^{(i)}z\right]\right\} + \sum_{i=-\infty}^{+\infty} b_F^{(i)} \exp\left\{j\left[k_x^{(i)}x - k_{Fz}^{(i)}z\right]\right\}, \quad (9)$$

$$h_x(x,z) = -\frac{1}{k_0}\sum_{i=-\infty}^{+\infty} k_{Fz}^{(i)} f_F^{(i)} \exp\left\{j\left[k_x^{(i)}x + k_{Fz}^{(i)}z\right]\right\} + \frac{1}{k_0}\sum_{i=-\infty}^{+\infty} k_{Fz}^{(i)} b_F^{(i)} \exp\left\{j\left[k_x^{(i)}x - k_{Fz}^{(i)}z\right]\right\}, \quad (10)$$

where $f_F^{(i)}$ and $b_F^{(i)}$ denote, respectively, the amplitudes of the electric field of the incident and reflected waves, with $k_{Fz}^{(i)} = [k_0^2 n^2 - k_x^{(i)2}]^{1/2}$.

$n_F = 1$; n_F and n_L are, respectively, the indices of the first and the second half-space.

We can then write that in $z = z_{\min} = z_1$,

$$U(z_1) = \begin{bmatrix} \vdots \\ E_y^{(i-\gamma)}(z_1) \\ \vdots \\ h_x^{(i-\gamma)}(z_1) \\ \vdots \end{bmatrix} = [C(z_1)]\begin{bmatrix} \vdots \\ f_F^{(i-\gamma)} \\ \vdots \\ b_F^{(i-\gamma)} \\ \vdots \end{bmatrix}, \quad (11)$$

with $[C(z_1)]$ is the order matrix ($2N$), formed by four diagonal blocks of order (N) each.

We can also write at the interface $z = z_{M+1}$ analogous relations involving amplitudes, fields in the transmission medium, and the matrix $[C(z_{M+1})]$ [21].

Finally, the fields in each half-space delimited by the grating are connected by

$$\begin{bmatrix} \vdots \\ f_F^{(i-\gamma)} \\ \vdots \\ b_F^{(i-\gamma)} \\ \vdots \end{bmatrix} = \underbrace{[C(z_1)]^{-1} \prod_{l=1}^{M} \left([P_k][\Delta_k][P_k]^{-1} \right) [C(z_{M+1})]}\ \prod_{l=0}^{3M+1} [A_k] \begin{bmatrix} \vdots \\ f_L^{(i-\gamma)} \\ \vdots \\ b_L^{(i-\gamma)} \\ \vdots \end{bmatrix}. \tag{12}$$

However, only the reflected b_F and transmitted fields f_L interest us knowing that [21]

$$b_L = \begin{bmatrix} \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} et\ f_F = \begin{bmatrix} \vdots \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \longleftarrow \text{indice}_0. \tag{13}$$

We then write $[A_k]$ in the form as follows: $[A_k] = \begin{bmatrix} [A_k^{00}] & [A_k^{01}] \\ [A_k^{10}] & [A_k^{11}] \end{bmatrix}$ for $0 \leq k \leq 3M + 1$.

The descent is initialized using vectors $[V_{3M+2}]$ and $[W_{3M+2}]$ as follows:

$$\begin{aligned} [V_{3M+2}] &= [0], \\ [W_{3M+2}] &= I_N. \end{aligned} \tag{14}$$

The loop is written, for k going from $3M+2$ to 1, $[V_{k-1}] = ([A_{k-1}^{10}] + [A_{k-1}^{11}][V_k])([A_{k-1}^{00}] + [A_{k-1}^{01}][{}^{k}_{V}])^{-1}$; $[W_{k-1}] = [W_k]([A_{k-1}^{00}] + [A_{k-1}^{01}][V_k])^{-1}$.

Finally, this leads to the reflected fields R and transmitted T by the grating:

$$\begin{aligned} R &= \begin{bmatrix} \vdots \\ \\ b_F^{(i-\gamma)} \\ \\ \vdots \end{bmatrix} = [V_0] I, \\ T &= \begin{bmatrix} \vdots \\ f_L^{(i-\gamma)} \\ \\ \\ \vdots \end{bmatrix} = [W_0] I. \end{aligned} \tag{15}$$

2.3. Calculation of Emissivity by the Differential Method. The determination of the complex reflected $b_f^{(i-\gamma)}$ and transmitted $f_L^{(i-\gamma)}$ amplitudes makes it possible to determine the reflected and transmitted efficiencies:

$$\begin{aligned} \eta_B^{(i-\gamma)} &= \frac{k_{Fz}^{(i-\gamma)}}{k_{Fz}^{(0)}} \left| b_F^{(i-\gamma)} \right|^2, \\ \eta_F^{(i-\gamma)} &= \frac{k_{Lz}^{(i-\gamma)}}{k_{Fz}^{(0)}} \left| f_L^{(i-\gamma)} \right|^2. \end{aligned} \tag{16}$$

We then go back to the emissivity defined for each angle of incidence θ by the following relation:

$$\varepsilon_\lambda(\theta) = 1 - \sum_{i=-N}^{i=+N} \eta_B^{(i-\gamma)}. \tag{17}$$

3. Determination of Emissivity by the Geometric Optics Approximation

The geometric optics method is one of the approximate methods of calculating the emissivity of rough surfaces. It is based on the notion of light ray, and we use notions of classical or analytical geometry associated with Snell–Descartes laws to determine the path followed by the incident ray inside the cavity [24].

We adopt the approach whose principle consists in considering a beam of parallel rays incident on the cavity under an angle of incidence θ and to determine the absorbed part. To do this, we proceed as follows: we consider M equidistant points belonging to the opening of the cavity. An incident ray enters the cavity through an abscissa point X_{s_j}, j ranging from 1 to M, and undergoes Nj reflections inside the cavity.

The portion absorbed by the cavity of the power transported by this ray is equal to the following:

$$\begin{gathered} 1 - \rho_\lambda(\phi_1)\rho_\lambda(\phi_2) \times \cdots \times \rho_\lambda(\phi_{Nj}) \\ \text{oùl'on désigne par } \phi_1, \phi_2, \ldots, \phi_{\mathrm{Nj}}. \end{gathered} \tag{18}$$

The local reflection angles at the different impact points are associated with the incident ray and the monochromatic directional reflectivity ρ_λ. The latter is calculated using Fresnel relations deduced from Maxwell's electromagnetic theory. The fraction of power absorbed is none other than the apparent local emissivity of the cavity at the abscissa X_{s_j}

point in the direction of the angle θ; it is expressed by the following [29]:

$$\varepsilon_\lambda\left(\theta, X_{s_j}\right) = 1 - \prod_{i=1}^{n} \rho_\lambda(\phi_i). \tag{19}$$

The apparent monochromatic directional emissivity of the cavity is none other than the average of $\varepsilon_\lambda(\theta, X_{s_j})$ over all positions X_{s_j} and can be expressed in the form as follows [27]:

$$\varepsilon_\lambda(\theta) = \frac{1}{d}\int_0^d \varepsilon_\lambda\left(\theta, X_{s_j}\right)\mathrm{d}X, \tag{20}$$

where d being the period of the profile considered.

In this study, we have considered cylindrical surfaces with a crenellated profile. For an incident ray, the determination of the number of points of impact inside the cavity and of the corresponding local angles of incidence or reflection is carried out using a geometric study making it possible to easily show that N_j is related and has the whole part of the quantity:

$$2\frac{h}{d}\tan\theta - 2\left(1 - \frac{X_{s_j}}{d}\right), \quad X_{s_j} \in \left]\frac{d}{2}, d\right[. \tag{21}$$

The product appears in the expression of

$$\varepsilon_\lambda\left(\theta, X_{s_j}\right) = \rho(\theta)\left[\rho\left(\frac{\pi}{2} - \theta\right)\right]^{N_j-1}. \tag{22}$$

4. The Notion of Homogenization

We now deal with the radiative behavior of the rough surface when its period is very small compared with the wavelength [30]. Under these conditions, the wavy part is equivalent to a superposition of layers of effective indices determined using the theory of effective media [31–33]. The dielectric constant of the k layer is given by the following:

$$\varepsilon_{eff,k} = f_k \cdot \varepsilon_3 + (1 - f_k) \equiv \left(n_{eff,k+1}\right)^2. \tag{23}$$

It is established that the shape of the field in the superstrate is deduced from the shape of the field in the substrate by simple matrix products [31]. We write

$$\vec{U}_1(z_1) = M \cdot \vec{U}_{M+2}(z_{M+1}). \tag{24}$$

The matrix $M = T_1 \cdot \prod_{j=2}^{M+1} C_j T_j$ characterizes the studied system, and it is obtained from the transition matrices T_j and the C_j layer matrices [31]. These matrices are given by the following:

$$C_j = \begin{pmatrix} \exp\left(-i\beta_j e\right) & 0 \\ 0 & \exp\left(i\beta_j e\right) \end{pmatrix}, \quad e = \frac{h}{M}, \tag{25}$$

$j = 2, 3, \ldots, M+1$, with

$$\begin{cases} S_j = \dfrac{\beta_j + \beta_{j+1}}{2\beta_j}, \\ \\ d_j = \dfrac{\beta_j - \beta_{j+1}}{2\beta_j}, \end{cases} \quad j = 1, 2, \ldots, M+1,$$

$$\text{où: } \beta_j^2 = \mathrm{k}_j^2 - \mathrm{k}_1^2 \sin\theta^2, \mathrm{etn}_j = \mathrm{n}_{\mathrm{eff},j-1}, \ j = 2, 3, \ldots, \mathrm{M}+1. \tag{26}$$

We will take $n_1 = 1 et n_{M+2} = n$. The final form of M is as follows:

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}. \tag{27}$$

We define the reflection coefficient by the following:

$$r = \frac{\text{complex amplitude of the reflected field}}{\text{complex amplitude of the incident field}} = \frac{M_{21}}{M_{11}}. \tag{28}$$

The emissivity is given by the following relation:

$$\varepsilon_\lambda(\theta) = 1 - |r|^2. \tag{29}$$

5. Discussion of the Numerical Results

We have developed a code for calculating emissivity in TE mode by the differential method; its validation is based on the criteria of conservation of energy and verification of the theorem of reciprocity and convergence in addition to the comparison with results of the bibliography [1, 3]. We have also implemented calculation programs by the other two methods.

We present in what follows results obtained in the context of transverse electrical polarization (TE) concerning the variation of the emissivity of sinusoidal surfaces as a function of the geometric period d for heights h fixed at 0.1λ and 1λ. The materials used are gold and tungsten for a wavelength equal to 0.55μ, and the angles of incidence are, respectively, 10° and 50°. We associate with a sinusoidal cavity defined by d and h the angle defined by $\tan\beta = (2h/d)$ or the angle $\alpha = 90^\circ - \beta$. Note that when d/λ varies from 0.05 to 20.5, the angle β varies from 75.96° to 0.55° for $h = 0.1\lambda$ and from 88.56° to 5.57° for $h = 1\lambda$.

For an angle of incidence equal to 10°, there is agreement between the differential method (MD) and the geometric optics method (MOG) in the following cases.

For gold, the ratio d/λ equals to

(i) 0.4 when the height h of the grating is 0.1λ (Figure 1(a)),

(ii) 3.2 when the height h of the grating is 1λ (Figure 1(b));

These two limits can result, respectively, in angles β less than 26.56° and then 32°.

The ratio d/λ for tungsten was equal to

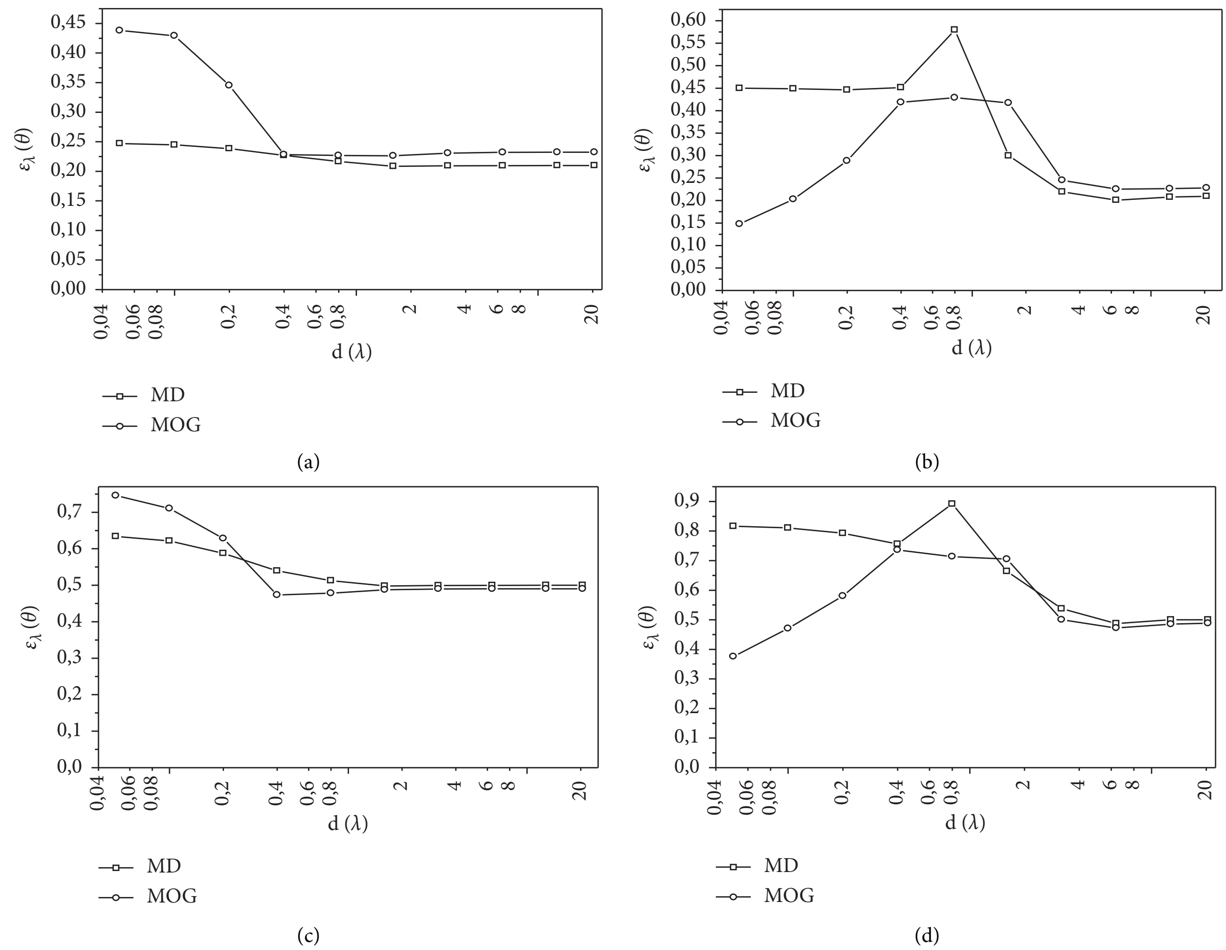

Figure 1: Emissivity as a function of d/λ parameters: (a) Or: $h = 0.1\lambda$ and $\theta = 10^\circ$; (b) gold: $h = 1\lambda$ and $\theta = 10^\circ$; (c) tungsten: $h = 0.1\lambda$ and $\theta = 10^\circ$; (d) tungsten: $h = 1\lambda$ and $\theta = 10^\circ$.

(i) 0.8 when the height h of the grating is 0.1λ (Figure 1(c)),

(ii) 6.4 when the height h of the grating is 1λ (Figure 1(d));

These two limits can result, respectively, in angles β less than 14.036° and then 17.35°.

In the case of gold, the agreement between the two methods for low angles β of values less than approximately 30° corresponds to sinusoidal cavities within which an incident ray at the angle of 10° only undergoes a single reflection at a local angle substantially equal to β. The value of the emissivity should be that of the smooth flat surface in that direction. For a fixed height h of the grating, by increasing the geometric period so as to make the emissivity tends β towards zero, the emissivity should tend towards that of the smooth flat surface in the direction 10°. This is of the order of 0.23 as shown by the calculation using Fresnel formulas [34–42]. This appears clearly in Figure 1 where the angle β reaches 0.5° for d/λ equal to 20.5 and $h = 0.1\lambda$ On the other hand, for h equal to 1λ (Figure 1(b)) only the last four values of (32°, 17.35°, 8.88°, and 5.57°), translate this agreement is translated.

In the case of tungsten, the agreement between the two methods is similar to that of the case of gold. The asymptotic limit of the emissivity, corresponding to values of β less than about 14°, is indeed that of the smooth plane surface in the direction of 10°, and it is equal to 0.5 [34].

In the case of the sinusoidal grating of gold or tungsten and for an angle of incidence equal to 50°, the agreement between the two methods is achieved as soon as the ratio d/λ exceeds 1.6 when the height h of the grating is equal to 0.1λ or to 1λ (Figures 2(a)–2(d)). This limit resulting in slopes of less than approximately 10° corresponds in these cases also to a single point of impact inside the cavity at a local angle equal to $(60^\circ \pm \beta)$.

The asymptotic limit of the emissivity when β tends towards zero is indeed that of the smooth plane surface according to the angle direction 50°; this limit is equal to 0.162 for gold and 0.363 for tungsten.

5.1. Limit Slope $(2h/d)_{lim}$ Depending on the Angle of Incidence θ. We resume the previous study in the case of gold and the heights $h = 0.1\lambda$ and $h = 1\lambda$ for the angles of incidence θ ranging from 10° to 80° with a step of 10°. The two emissivity

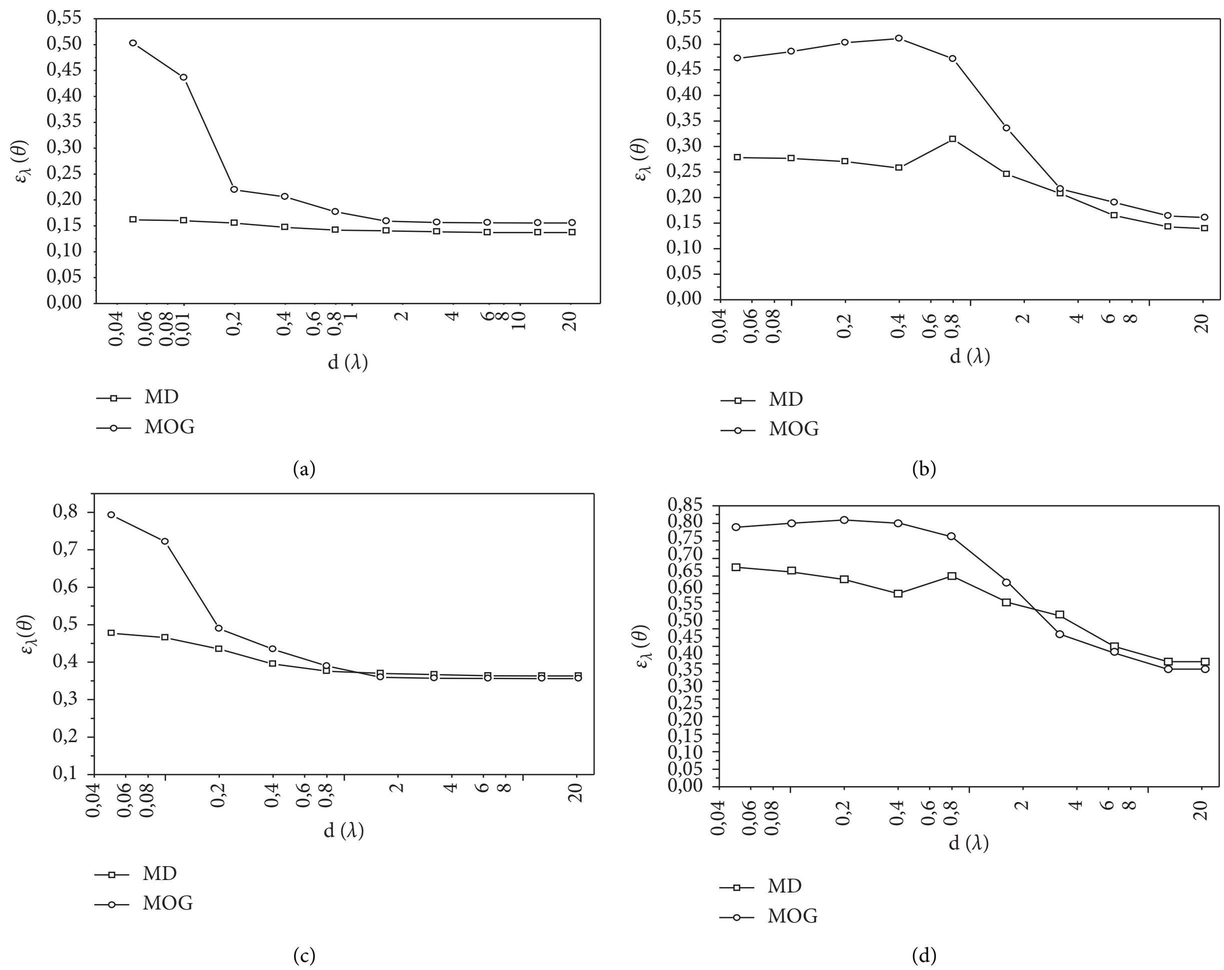

Figure 2: Emissivity as a function of d/λ parameters: (a) Or, $h = 0.1\lambda$ and $\theta = 50^\circ$; (b) gold: $h = 1\lambda$ and $\theta = 50^\circ$; (c) tungsten: $h = 0.1\lambda$ and $\theta = 50^\circ$; (d) tungsten: $h = 1\lambda$ and $\theta = 50^\circ$.

curves ε_λ' (d/λ), for h and θ fixed, calculated by the two methods and having shapes similar to those of the curves shown in Figures 1(a) and 1(b), make it possible to determine the value of the limit ratio $(d/\lambda)_{\lim}$ from which the MOG is valid. With this report, one associates, for fixed h, the limiting slope $(2h/d)$lim.

A curve in Figure 3 representative of this slope as a function of the cosine of the angle of incidence or of emission, for a fixed height h, delimits two regions. For the first, located above this curve, the use of MD or some other exact method is necessary since the MOG is not valid. This is very satisfactory at all points of the second region located below the same curve.

5.2. Limit Ratio $(2h/d)_{lim}$ Function of $(h/\lambda)cos\theta$. For the angles of incidence 1° and 60° and for the ratios $h/$going up to the value 10 with a step equal to 2, we first determine the limit of validity of the MOG, by varying the ratio $d/$for $h/$and fixed. We then include the points determined for the representation of the curves in Figure 3.

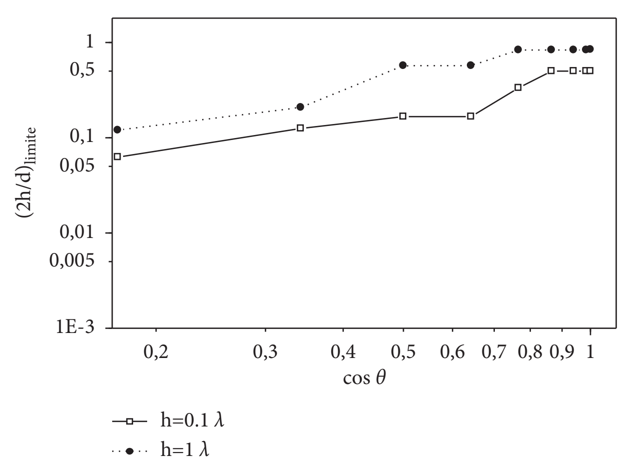

Figure 3: Limit slope $(2h/d)_{\lim}$ as a function of the angle of incidence θ.

Thus, for sinusoidal surfaces, Figure 4 gives a fairly precise idea concerning the domain of validity of the geometric optics approximation.

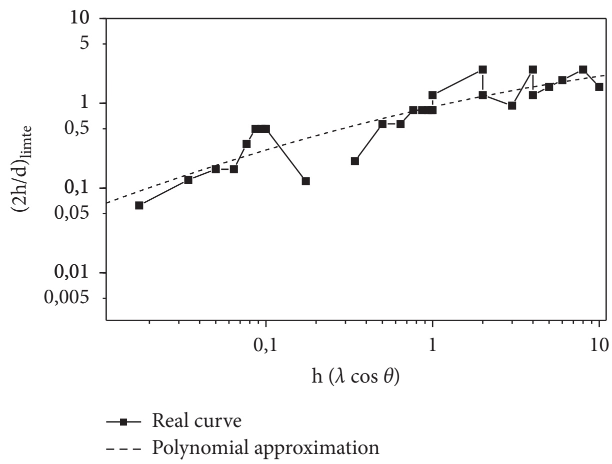

FIGURE 4: Limit ratio $(2h/d)_{\text{lim}}$ function of $(h/\lambda)\cos\theta$.

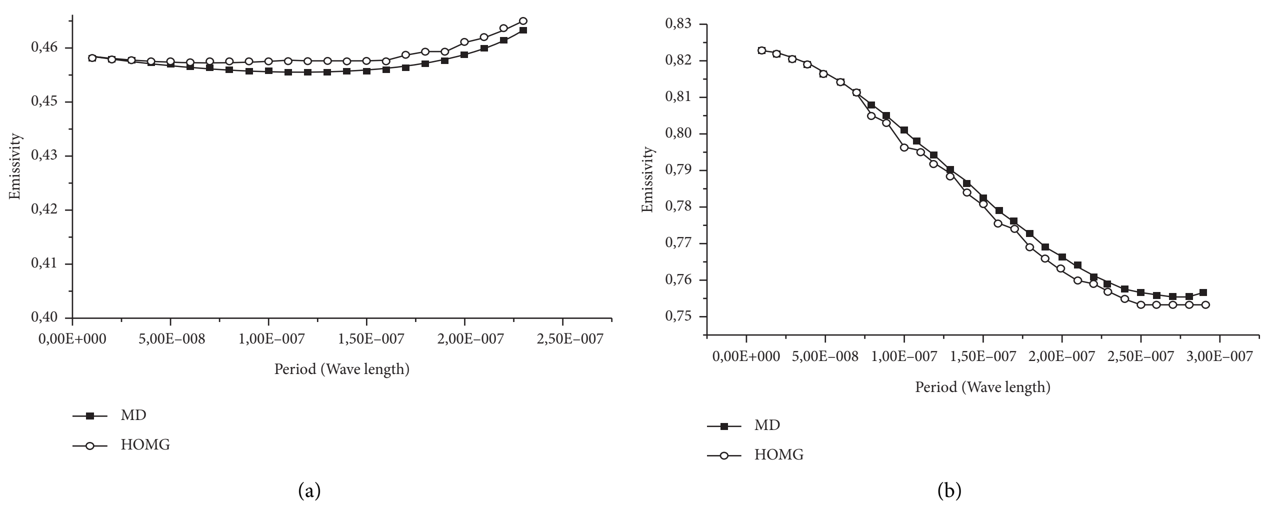

FIGURE 5: Emissivity following the normal of a sinusoidal grating, determined by MD, compared with that determined by the homogenization process: (a) case of gold; (b) case of tungsten.

TABLE 1: Comparison between the emissivity calculated by the homogenization process and those calculated by the differential and geometric optics methods for λ equal to 0.05, in the case of TE polarization. Case of the sinusoidal-shaped surface.

		Au			W		
h	θ	Differential method (MD)	Homogenization method	Geometric optics method	Differential method (MD)	Homogenization method	Geometric optics method
$h = 0.1\ \lambda$	1°	0,2509	0,2520	0,4712	0,6396	0.6449	0,7624
	10°	0,2470	0,2481	0,5773	0,6338	0,6391	0,8446
	50°	0,1618	0,1625	0,5046	0,4771	0,4720	0,7849
	60°	0,1267	0,1273	0,3846	0,3953	0,3995	0,6747
$h = 1\ \lambda$	1°	0,4586	0,4620	0,7650	0,8228	0.8201	0,9319
	10°	0,4480	0,4534	0,9607	0,8161	0,8151	0,9430
	50°	0,2783	0,2810	0,8577	0,6599	0,6604	0,8791
	60°	0,2156	0,2177	0,8547	0,5670	0,5754	0,9351

From Figure 4, under the cloud, the geometric optics approximation is valid.

5.3. Homogenization. We compare the reflected and transmitted efficiencies of a grating using the differential method (MD), with those of the equivalent superposition of layers whose dielectric constants are given by relation (23). We consider sinusoidal arrays illuminated at normal incidence by monochromatic wavelength light $\lambda = 0.55\,\mu$m. The period d varies from $\lambda/80$ to $\lambda/2$. These gratings are successively made of two materials, gold with index $n = (0.429, 2.454)$ and tungsten with index $n = (3.5, 2.73)$. In Figures 5(a) and 5(b), the emissivities of a sinusoidal grating and that of the equivalent layer have been shown. We notice that in the case of TE polarization, for both materials, the relative error is less than 2% when the period is less than $\lambda/10$.

We have also dealt with the behavior of emissivity curves when the period is very small compared with the wavelength. Under these conditions, the wavy part is equivalent to a superposition of layers of effective indices determined using the theory of effective media. We compare the emissivities calculated by this homogenization process, for d/λ equal to 0.05, in the cases discussed above with that calculated by the MD and the MOG. We note a very good agreement with the differential method in all these cases and on the other hand no agreement with the geometrical optics method to report (Table 1), in accordance with what we should expect.

6. Conclusion

In this work, we were able to determine the emissivity of periodic rough surfaces in sinusoidal form. One of the objectives pursued is the treatment by an exact method of the cases of metal surfaces of finite conductivity. Another objective of our work is to determine the emissivity by the geometric optics approximation and to seek its limit of validity in comparison with the exact method. The results obtained, by using the numerical calculation programs that we have developed and validated, have enabled us to delimit the regions of validity of the geometric optics approximation in comparison with the differential method. We retain in particular that, in certain cases where the dimensions of the asperities are of the order of magnitude of the wavelength, the MOG is indeed valid. In addition, we have treated the borderline cases, of surfaces of very small period compared with the incident wavelength, by the homogenization process. A perfect agreement between the results obtained by this process and those of the exact method is obtained. An extension of this work to rough two-dimensional surfaces is to be considered, as well as an exploitation of the models, elaborated in this work, for other materials and wavelengths.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that there are no conflicts of interest.

Acknowledgments

The researchers would like to thank the Deanship of Scientific Research, Qassim University, for funding the publication of this project.

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