Research Article

Dynamics of Mosquito Population Models with Spatial Diffusion

Urbain Traoré

Laboratoire de Mathématiques et Informatique (LAMI), Université Joseph KI-ZERBO, 03 BP 7021, Ouagadougou 03, Burkina Faso

Correspondence should be addressed to Urbain Traoré; urbain.traore@yahoo.fr

Received 27 August 2021; Revised 12 November 2021; Accepted 8 December 2021; Published 29 December 2021

Academic Editor: Jianshe Yu

Copyright © 2021 Urbain Traoré. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In this paper, we study some reaction-diffusion models of interactive dynamics of the wild and sterile mosquitoes. The well-posedness of the concerned model is proved. The stability of the steady states is discussed. Numerical simulations are presented to illustrate our theoretical results.

1. Introduction

Mosquito-borne diseases (e.g., malaria, dengue fever, and West Nile virus) remain a public health problem in developing countries despite many efforts of controls. The mortality and morbidity associated to this disease are still important for these countries. Many control strategies have been explored to control this disease among which are the development of effective insecticides and drug treatments, vaccine, and insecticide-treated bed nets. Nowadays, biological control of wild mosquito population by using genetically modified or transgenic mosquitoes is also used (see [1, 2]). This strategy control releases male mosquitoes (sterilized by radiological or chemical means) into the environment to mate with wild mosquitoes that are present in the environment. Female mosquitoes successfully mating with a sterile male will not breed or produce eggs that will not hatch. Repetitive releases of sterile mosquitoes or the release of a significantly large number of sterile mosquitoes can possibly wipe out or suppress a population of wild mosquitoes (see [3, 4]). Recently, several mathematical models have been proposed to measure the effectiveness of this control strategy (see [5–11]).

The model that we are studying by integrating the diffusion is obtained by assuming that the dynamics of the two

mosquito populations follow a logistic growth, the birth rate of sterile mosquitoes being their release rate (see [5]):

$$\begin{cases} \frac{dw(t)}{dt} = \frac{aC(N)w^2(t)}{w(t) + g(t)} - (\mu_1 + \xi_1(w(t) + g(t)))w(t), \\ \frac{dg(t)}{dt} = B(\cdot) - (\mu_2 + \xi_2(w(t) + g(t)))g(t). \end{cases} \quad (1)$$

The terms $w(t)$ and $g(t)$ are the numbers of wild and sterile mosquitoes consecutively, μ_i and ξ_i , $i = 1, 2$ are the density independent and dependent death rates of the wild and sterile mosquitoes, respectively. The parameter $C(N)$ is the number of matings per individual, per unit of time, with $N = w + g$; a is the number of wild offspring produced per mate. $B(\cdot)$ designs the releasing rate of the sterile mosquitoes. Furthermore, it is well known that, among a number of factors that contribute to the upsurge of malaria, we have vector immigration (see [12]). Mobility is well known to play an important role in the study of population dynamics (see [13–16] and the references therein). Thus, it becomes important to take into account this aspect on the propagation of the disease. However, in the works cited above, we noticed that their authors do not take into account population mobility in space. Thus, motivated by the works in [5], we

present a model that includes spatial diffusion of wild and sterile mosquitoes. This model is given by

$$\begin{cases} \frac{\partial w}{\partial t} = d_w \Delta w + \frac{aC(N)w^2(x,t)}{w(x,t) + g(x,t)} - (\mu_1 + \xi_1(w(x,t) + g(x,t)))w(x,t), \\ \frac{\partial g}{\partial t} = d_g \Delta g + B(\cdot) - (\mu_2 + \xi_2(w(x,t) + g(x,t)))g(x,t). \end{cases} \quad (2)$$

Here, the parameters d_w and d_g represent the diffusion coefficients of wild and sterile mosquitoes, respectively. Symbol Δ is the Laplacian operator, and Ω is a connected and bounded spatial domain in $\mathbf{R}^n$ with a smooth boundary $\partial\Omega$. For model 2, we consider homogeneous Neumann boundary conditions

$$\begin{aligned} \frac{\partial w}{\partial \nu} &= 0, \\ \frac{\partial g}{\partial \nu} &= 0, \quad \text{on } \partial\Omega \times (0, +\infty), \end{aligned} \quad (3)$$

and initial conditions

$$\begin{aligned} w(x, 0) &= \phi_1(x) > 0, \\ g(x, 0) &= \phi_2(x) > 0, \\ \forall (x, 0) &\in \overline{\Omega}. \end{aligned} \quad (4)$$

The functions ϕ_1 and ϕ_2 are nonnegative continuous functions in $\overline{\Omega}$ and denotes the outward normal derivative on the boundary $\partial\Omega$.

In this paper, we proposed a complete mathematical analysis of model (2). The analysis of model (2) in the case of constant releasing rate of sterile mosquitoes $B(\cdot)$ is studied Section 2. The well-posedness is established, and the stability of the equilibria is derived. Section 3 is devoted to the analysis of model (2) in the case of proportional releasing rate of sterile mosquitoes. In Section 4, numerical simulations are carried out to illustrate the theoretical results.

2. Existence and Positivity

In this paper, we study model (2) with a constant release rate of sterile mosquitoes, i.e., $B(\cdot) = b > 0$. We assume that the interaction rate $C(N)$ is constant, and in order to simplify the writing, we always write a instead of $C(N)a$ with $a > \mu_1$; then, system (2) becomes

$$\begin{cases} \frac{\partial w}{\partial t} = d_w \Delta w + \frac{aw^2(x,t)}{w(x,t) + g(x,t)} - (\mu_1 + \xi_1(w(x,t) + g(x,t)))w(x,t) \\ \frac{\partial g}{\partial t} = d_g \Delta g + b - (\mu_2 + \xi_2(w(x,t) + g(x,t)))g(x,t) \end{cases}, \quad (5)$$

with initial condition (4) and Neumann boundary condition (3).

We now derive the result about the existence and the positivity of the solution of system (5).

In this section, we establish the global existence, positivity, and boundedness of solutions of problem (5) under the conditions (3) and (4).

We state the following result.

Theorem 1. *For any given initial data satisfying condition (4), there exists a unique solution of system (5) defined on $[0, +\infty)$, and this solution remains nonnegative and bounded for all $t \geq 0$. Proof. Let $\phi = (\phi_1, \phi_2)^T \in C(\overline{\Omega}) \times C(\overline{\Omega})$. System (5) can be rewritten abstractly in the Banach space X as follows:*

$$\begin{cases} u'(t) = Au(t) + F(u(t)), & t > 0 \\ u(0) = \phi \in X \end{cases}, \quad (6)$$

where $u = (w, g)^T$, $u_0 = (\phi_1, \phi_2)^T$, and $Au(t) = (d_w \Delta w, d_g \Delta g)^T$ and

$$F(u(t)) = \begin{pmatrix} \frac{aw^2}{w+g} - (\mu_1 + \xi_1(w+g))w \\ b - (\mu_2 + \xi_2(w+g))g \end{pmatrix}. \quad (7)$$

Since F is locally Lipschitzian in X , thus there exists a unique local solution of system (5) on the interval $[0, T_{\max})$, where $T_{\max}$ denotes the maximal existence time for solution of system (6) (see [7, 14, 15, 17]).

To achieve the global existence in $[0, +\infty)$, we need to prove the positiveness and boundedness of this solution.

Since $w(x, 0) = \phi_1(x) > 0$ and $g(0) = \phi_2(x) > 0$, for $t \in [0, T_{\max})$,

$$w(x, t) - d_w \int_0^t \Delta w(x, s) ds = e^{-\int_0^t [\mu_1 + \xi_1(w(x, s) + g(x, s))] ds} \left(w(x, 0) + \int_0^t \frac{\alpha w^2(x, s) e^{-\int_0^s [\mu_1 + \xi_1(w(x, \eta) + g(x, \eta))] d\eta} ds}{w(x, s) + g(x, s)} \right) \quad (8)$$

and

$$g(x, t) - d_g \int_0^t \Delta g(x, s) ds = e^{-\int_0^t [\mu_2 + \xi_2(w(x, s) + g(x, s))] ds} \left(g(x, 0) + b \int_0^t e^{-\int_0^s [\mu_2 + \xi_2(w(x, \eta) + g(x, \eta))] d\eta} ds \right). \quad (9)$$

Thus,

$$w(t, x) - d_w \int_0^t \Delta w(x, s) ds > 0 \text{ and } g(x, t) - d_g \int_0^t \Delta g(x, s) ds > 0, \quad \forall x \in \Omega. \quad (10)$$

Then, integrating (10) over Ω and using the fact that $(\partial w / \partial \nu) = (\partial g / \partial \nu) = 0$, we obtain

$$\int_{\Omega} w(x, t) dx > 0 \text{ and } \int_{\Omega} g(x, t) dx > 0, \quad \forall t \in [0, T_{\max}). \quad (11)$$

Hence, it follows that

$$w(x, t) > 0 \text{ a.e. } \Omega \text{ and } g(x, t) > 0 \text{ a.e. } \Omega, \quad \forall t \in [0, T_{\max}), \quad (12)$$

where a.e. Ω defines almost everywhere in Ω .

Furthermore, from (5), we deduce that

$$\begin{aligned} \frac{\partial g}{\partial t} - d_g \Delta g &\leq b - \mu_2 g(x, t), \\ \frac{\partial g}{\partial \nu} &= 0, \end{aligned} \quad (13)$$

$$g(x, 0) = \phi_2(x, 0) \leq \|\phi_2\|_{\infty} = \max_{x \in \overline{\Omega}} (\phi_2(x)).$$

By the principle comparison [18], it follows that $g(x, t) \leq g_1(t)$, where $g_1(t) = \phi_2(x, 0)e^{-\mu_2 t} + (b/\mu_2)(1 - e^{-\mu_2 t})$ is the solution of

$$\frac{dg_1}{dt} = b - \mu_2 g_1, \quad (14)$$

$$g_1(0) = \|\phi_2\|_{\infty}.$$

Since $g_1(t) \leq \max\{(b/\mu_2), \|\phi_2\|_{\infty}\}$ for all $t \in [0, +\infty)$, thus

$$g(x, t) \leq \max\{(b/\mu_2), \|\phi_2\|_{\infty}\}, \quad \forall (x, t) \in \overline{\Omega} \times [0, T_{\max}). \quad (15)$$

In order to prove L^{∞} uniformly boundedness of $w(x, t)$, it is sufficient to show that $w(x, t)$ is L^1 uniformly bounded (see [1], Theorem 2).

Let

$$\begin{aligned} u(x, t) &= \xi_2 \gamma w(x, t) + a g(x, t), \text{ with } \gamma \\ &= \min_{t \in [0, T_{\max}) \times \overline{\Omega}} g(x, t). \end{aligned} \quad (16)$$

Thus,

$$\begin{aligned} &\frac{\partial u}{\partial t}(x, t) - \Delta(\xi_2 \gamma d_w w(x, t) + a d_g g(x, t)), \\ &= \xi_2 \gamma \frac{\partial w}{\partial t} + a \frac{\partial g}{\partial t} - \xi_2 \gamma d_w \Delta w - a d_g \Delta g(x, t), \\ &= \frac{\xi_2 \gamma a w^2(x, t)}{w(x, t) + g(x, t)} - \xi_2 \gamma (\mu_1 + \xi_1(w(x, t) + g(x, t)))w(x, t) + ab - a(\mu_2 + \xi_2(w(x, t) + g(x, t)))g(x, t), \end{aligned}$$

$$\begin{aligned}
&\leq \xi_2 \gamma a w(x, t) - \xi_2 \gamma \mu_1 w(x, t) + \alpha b - a \mu_2 g(x, t) - a \xi_2 w(x, t) g(x, t), \\
&\leq ab + \xi_2 a w(x, t) (\gamma - g(x, t)) - \xi_2 \gamma \mu_1 w(x, t) - \mu_2 a g(x, t), \\
&\leq ab - \min\{\mu_1, \mu_2\} (\xi_2 \gamma w(x, t) + \alpha g(x, t)), \\
&\leq ab - \min\{\mu_1, \mu_2\} u(x, t).
\end{aligned} \tag{17}$$

Since $(\partial w / \partial \nu) = (\partial g / \partial \nu) = 0$, it follows that

$$\frac{d}{dt} \left(\int_{\Omega} u(x, t) dx \right) \leq \text{meas}(\Omega) ab - \min\{\mu_1, \mu_2\} \int_{\Omega} u(x, t) dx. \tag{18}$$

Hence,

$$\int_{\Omega} u(x, t) dx \leq \text{meas}(\Omega) \max \left(\frac{ab}{\min\{\mu_1, \mu_2\}}, \|\xi_2 \gamma \phi_1 + \alpha \phi_2\|_{\infty} \right), \tag{19}$$

which implies that

$$\sup_{t \geq 0} \int_{\Omega} w(x, t) dx \leq K, \tag{20}$$

where

$$K = \left(\frac{1}{\xi_2 \gamma} \right) \text{means}(\Omega) \max \left(\frac{ab}{\min\{\mu_1, \mu_2\}}, \|\xi_2 \gamma \phi_1 + \alpha \phi_2\|_{\infty} \right). \tag{21}$$

Therefore, from [13], we deduce that there exists a positive constant K^* that depends on K such that

$$\sup_{t \geq 0} \|w(\cdot, t)\|_{\infty} \leq K^*, \tag{22}$$

that is, $w(x, t)$ is L^{∞} .

Since $w(x, t)$ and $g(x, t)$ are L^{∞} bounded on $[0, T_{\max})$, it follows from the standard theory for semilinear parabolic systems [19] that $T_{\max} = +\infty$. Thus, the global existence follows. This completes the proof.

3. Analysis of the Equilibrium Points

Here, we are interested in the equilibrium points of model (5). Clearly, the presence of the reaction-diffusion term does not affect the number of the equilibrium. As in [5], for model (5), there is only one equilibrium point whose number of wild mosquitoes is zero, $E_0(0, g_0)$, where

$$g^0 = \frac{\sqrt{\mu_2^2 + 4b\xi_2} - \mu_2}{2\xi_2} > 0. \tag{23}$$

Concerning the positive equilibrium of system (5), its components verify

$$\begin{aligned}
\frac{aw}{w+g} &= \mu_1 + \xi_1(w+g), \\
\frac{b}{g} &= \mu_2 + \xi_2(w+g),
\end{aligned} \tag{24}$$

which implies

$$\begin{aligned}
aw &= N(\mu_1 + \xi_1 N), \\
ag &= \frac{ab}{\mu_2 + \xi_2 N}.
\end{aligned} \tag{25}$$

From (23), we deduce that

$$aN = N(\mu_1 + \xi_1 N) + \frac{ab}{\mu_2 + \xi_2 N}. \tag{26}$$

Hence, we introduce the following function:

$$\begin{aligned}
P(N) &:= N(\mu_1 + \xi_1 N)(\mu_2 + \xi_2 N) - aN(\mu_2 + \xi_2 N) + ab, \\
&= \xi_1 \xi_2 N^3 - (\xi_2(a - \mu_1) - \xi_1 \mu_2)N^2 - (a - \mu_1)\mu_2 N + ab.
\end{aligned} \tag{27}$$

Consequently, there is an equivalence between the fact that (w, g) is an equilibrium point for system (5) and the fact that $N = w + g$ is the solution of the problem $P(N) = 0$.

We have

$$P'(N) = 3\xi_1 \xi_2 N^2 - 2(\xi_2(a - \mu_1) - \xi_1 \mu_2)N - (a - \mu_1)\mu_2, \tag{28}$$

that if $\bar{N} > 0$ is an extrema of $P(N)$ with $P'(\bar{N}) = 0$, then

$$\begin{aligned}\bar{N} &= \bar{N}(\tau): \\ &= \frac{1}{3\xi_1\xi_2} \left(\sqrt{(\xi_2(\alpha - \mu_1) - \xi_1\mu_2)^2 + 3\xi_1\xi_2(\alpha - \mu_1)\mu_2} + \xi_2(\alpha - \mu_1) - \xi_1\mu_2 \right).\end{aligned}\quad (29)$$

Let us observe that

$$2P(\bar{N}) = 2P(\bar{N}) - P'(\bar{N})\bar{N} = -\xi_1\xi_2\bar{N}^3 - (a - \mu_1)\mu_2\bar{N} + 2ab. \quad (30)$$

Thus, by defining the release threshold value of sterile mosquitoes as follows:

$$b_0(\tau) := \frac{1}{2\alpha} (\xi_1\xi_2\bar{N}^2 + (\alpha - \mu_1)\mu_2)\bar{N}, \quad (31)$$

then $P(\bar{N}) < 0$ is equivalent to $b < b_0$. Moreover, since $P(0) = ab > 0$ and $\lim_{N \rightarrow +\infty} P(N) = +\infty$, then problem (26) admits one and only one positive solution $N^* = \bar{N}$ if and only if $b = b_0$, and two positive solutions, N_1^* and N_2^* where $N_1^* < \bar{N} < N_2^*$ if and only if $b < b_0$.

From the above discussion as in [5], we have the following result.

Theorem 2. Boundary equilibrium $E_0(0, g_0)$, see (22), is the unique equilibrium of (5) if and only if $b > b_0$.

Model (5) admits a unique positive equilibrium $E^*(w^*, g^*)$ with

$$\begin{aligned}w^* &= \frac{\bar{N}(\mu_1 + \xi_1\bar{N})}{a}, \\ g^* &= \frac{b}{\mu_2 + \xi_2\bar{N}},\end{aligned}\quad (32)$$

where $\bar{N}$ given in (27), if and only if $b = b_0$. For $b < b_0$, we have two positive equilibria $E_1^*(w_1^*, g_1^*)$ and $E_2^*(w_2^*, g_2^*)$ for the model with

$$\begin{aligned}w_1^* &= \frac{N_1^*(\mu_1 + \xi_1N_1^*)}{a} < w_2^* = \frac{N_2^*(\mu_1 + \xi_1N_2^*)}{a}, \\ g_2^* &= \frac{b}{\mu_2 + \xi_2N_2^*} < g_1^* = \frac{b}{\mu_2 + \xi_2N_1^*}.\end{aligned}\quad (33)$$

Using (27), we verify that $\bar{N}$ does not depend on b ; therefore, b_0 is also independent of b . As soon as the mosquito release rate is higher than this rate, wild mosquitoes end up disappearing.

We now discuss the stability of the corresponding steady states of the model. Let us assume that $0 = \lambda_1 < \lambda_2 < \dots < \lambda_n < \dots$ be the eigenvalues of the operator $-\Delta$ on Ω with the homogeneous Neumann boundary conditions and $E(\lambda_i)$ be the eigenfunction space corresponding to λ_i in $C^1(\Omega)$. Let $\mathcal{Y} = [C^1(\Omega)]^2$, $\{\psi_{i,j}: j = 1, 2, \dots, \dim E(\lambda_i)\}$ be an orthonormal basis of $E(\lambda_i)$ and $\mathcal{Y}_{ij} = \{d\psi_{i,j}: d \in \mathbb{R}^2\}$. Then,

$$\mathcal{Y} = \oplus_{i=1}^{\infty} \mathcal{Y}_i \text{ and } \mathcal{Y}_i = \oplus_{j=1}^{\dim E(\lambda_i)} \mathcal{Y}_{ij}. \quad (34)$$

Let $E^*(w^*, g^*)$ be an arbitrary steady state of system (5). We define the following perturbation about $E^*(w^*, g^*)$:

$$U_1(x, t) = w(x, t) - w^* \text{ and } U_2(x, t) = g(x, t) - g^*. \quad (35)$$

By linearizing system (5) at $E^*(w^*, g^*)$, we obtain

$$\frac{\partial U}{\partial t} = \mathcal{D}\Delta U + AU(x, t), \quad (36)$$

where

$$A = \begin{pmatrix} \left(\frac{ag}{(w^* + g^*)^2} - \xi_1 \right) w^* + \frac{aw^*}{w^* + g^*} - (\mu_1 + \xi_1(w^* + g^*)) - \left(\frac{aw^*}{(w^* + g^*)^2} + \xi_1 \right) w^* & \\ -\xi_2 g^* & -\mu_2 - \xi_2(w^* + 2g^*) \end{pmatrix} \quad (37)$$

and

$$\mathcal{D} = \text{diag}(d_w, d_g) \text{ and } U = (U_1, U_2)^T. \quad (38)$$

We define the operator L by

$$LU = \mathcal{D}\Delta U + A * U(x, t). \quad (39)$$

$\mathcal{Y}_i$ is invariant under the operator L for all $i \geq 1$. Furthermore, η is an eigenvalue of L if and only if it is a root of the following equation:

$$\det(-\lambda_i \mathcal{D} + A - \eta \mathcal{F}) = 0 \quad (40)$$

for some $i \geq 1$, and in this case, there exists an eigenvector in $\mathcal{Y}_i$.

At the boundary E_0 , matrix A becomes

$$\begin{pmatrix} -(\mu_1 + \xi_1 g^0) & 0 \\ -\xi_2 g^0 & -(\mu_2 + 2\xi_2 g^0) \end{pmatrix}, \quad (41)$$

$$\begin{aligned} B_1 &= \left(\mu_2 + \xi_2 (N + g) + \lambda_i (d_w + d_g) - \frac{awg}{N^2} + \xi_1 w \right), \\ B_2 &= \lambda_i d_w (\lambda_i d_g + \mu_2 + \xi_2 (N + g)) - \lambda_i \left(\frac{agw}{N^2} - \xi_1 w \right) + \left(-\frac{agw}{N^2} + \xi_1 w \right) (\mu_2 + \xi_2 (N + g)) - \left(\frac{aw}{N^2} + \xi_1 \right) \xi_2 wg. \end{aligned} \quad (43)$$

So,

$$B_2 = \frac{w}{N} P'(N) + \lambda_i d_w (\mu_2 + \xi_2 (g + N) + \lambda_i d_g) - \lambda_i \left(\frac{agw}{N^2} - \xi_1 w \right). \quad (44)$$

Since $P'(N_1^*) < 0$ (Cai et al., 2014), then for $\lambda_i = 0$, we get $L(0) = w/NP'(N) - \lambda_i d_g < 0$, since $L(\eta) > 0$ for sufficiently large $\eta > 0$. Therefore, there is at least one positive root for $L(\eta) = 0$. This yields that E_1^* is unstable.

At the boundary E_2 , we have

$$\begin{aligned} B_2 &= \frac{w}{N_2^*} P'(N_2^*) + \lambda_i d_w (\mu_2 + \xi_2 (g + N_2^*)) \\ &\quad + \lambda_i (d_w \mu_2 - d_g). \end{aligned} \quad (45)$$

Thanks to Cai et al., $P'(N_2^*) > 0$ and choosing d_w and d_g such as $d_g < d_w \mu_2$; it follows that $B_2 > 0$. Moreover, using the fact that $aw = N(\mu_1 + \xi_1 N)$, we have

$$\begin{aligned} \frac{awg}{N^2} &= \frac{(\mu_1 + \xi_1 N)g}{N} < \mu_1 + \xi_1 N \leq \mu_2 \\ &\quad + \xi_2 N \text{ if } \mu_1 \leq \mu_2, \quad \xi_1 \leq \xi_2. \end{aligned} \quad (46)$$

Therefore this choice of the parameters μ_1 , μ_2 and ξ_1 , ξ_2 allowed us to deduce that $B_1 > 0$. Since $B_1 > 0$ and $B_2 > 0$, then (40) has no positives roots; therefore, E_2^* is locally asymptotically stable. We summarize the previous discussion in the following results.

Theorem 3

- (i) For $b > b_0$ such that the boundary equilibrium E_0 is the only equilibrium for system (5), it is locally asymptotically stable

and in this case, characteristic (33) has only two roots $\eta_1 = -(\mu_1 + \xi_1 g^0 + \lambda_i d_w) < 0$, $\eta_2 = -(\mu_2 + 2\xi_2 g^0 + \lambda_i d_g) < 0$. So, E_0 is locally asymptotically stable.

It is easy to verify that at the equilibria E_1^* and E_2^* (when $0 < b < b_0$), characteristic (33) takes the form

$$L(\eta) = \eta^2 + B_1 \eta + B_2 = 0, \quad (42)$$

where with $N = N_1^*$ or N_2^*

- (ii) For $b < b_0$, the boundary equilibrium E_0 is a locally asymptotically stable node, E_1^* is a unstable and moreover if $\mu_1 \leq \mu_2$, $\xi_1 \leq \xi_2$, then E_2^* is stable

- (iii) For $b = b_0$, the boundary equilibrium E^* , see (26) is an unstable saddle node

4. Numerical Results

In this section, we give some numerical simulations to illustrate the theoretical results obtained in Section 3, we take one-dimensional spatial domain. $\Omega = [0, 50]$ and the grid sizes are $dx = 0.1$ and $dt = 0.01$. Figure 1 presents the evolution of the population size of sterile and wild mosquito in system (5).

The threshold release value is $b_0 = 21.935161$. For $b = 21 < b_0$, $E_0 = (0, 6.7629)$ is a locally asymptotically stable node. We have two positive equilibria $E_1 = (7.84, 4.07)$ which is a saddle point an equilibrium $E_2 = (8.70, 3.87)$ which is a locally asymptotically stable. Solutions with different initial values approach either E_0 or E_2^* as shown in the lower left and right figures in Figure 1, respectively. From these numerical simulations, we can, therefore, say that wild and sterile mosquitoes can coexist or although wild mosquitoes can disappear, depending on the initial quantity of the two populations.

Next, we take values $b = 30 > b_0$ and the other parameters remain unchanged. For this set of parameter values, the boundary equilibrium E_0 is the only equilibrium for system (5), and it is globally asymptotically stable and this is evident from Figure 2.

As shown by the results of the numerical simulations, the introduction of the diffusion in the model does not suppress the dynamics of wild mosquitoes or genetically modify, and one obtains globally the configurations that in the absence of the diffusion. However, the mathematical analysis becomes more complicated.

FIGURE 1: Parameter values are chosen to be $a = 10$, $\mu_0 = 0.1$, $\mu_1 = 0.5$, $\mu_2 = 0.4$, $b = 21$, $\xi_1 = 0.5$, $\xi_2 = 0.4$, $d_g = 10$, and $d_w = 10$.

FIGURE 2: Parameter values are chosen to be $a = 10$, $\mu_0 = 0.1$, $\mu_1 = 0.5$, $\mu_2 = 0.4$, $b = 30$, $\xi_1 = 0.5$, $\xi_2 = 0.4$, $d_g = 10$, and $d_w = 10$.

5. Discussion

In this work, we carried out a theoretical and numerical study of reaction-diffusion model describing the dynamic of the interaction between wild and sterile mosquitoes. We have established the theoretical and numerical effects of the introduction of the population diffusion on the stability of

equilibrium states. On the other hand, when the number of sterile mosquitoes rejections is proportional to the population of wild mosquitoes, the results show that the solutions of the model exhibit a complex dynamic behavior. However, in order to have more realistic and more economical models for the release of sterile mosquitoes, it would be interesting to have a variable release rate that is

proportional to the size of the wild mosquitoes while taking into account the mobility of the mosquitoes. This approach could be the subject of future studies [20].

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares that there are no conflicts of interest.

References

- [1] V. A. Dyck, J. Hendrichs, and A. S. Robinson, *Sterile Insect Technique: Principles and Practice in Area-wide Integrated Pest Management*, Taylor & Francis, New York, NY, USA, 2005.
- [2] J. Ito, A. Ghosh, L. A. Moreira, E. A. Wimmer, and M. Jacobs-Lorena, "Transgenic anopheline mosquitoes impaired in transmission of a malaria parasite," *Nature*, vol. 417, no. 6887, pp. 452–455, 2002.
- [3] L. Alphey, M. Benedict, R. Bellini et al., "Sterile-insect methods for control of mosquito-borne diseases: an analysis," *Vector Borne and Zoonotic Diseases*, vol. 10, no. 3, pp. 295–311, 2010.
- [4] M. Benedict and A. S. Robinson, "The first releases of transgenic mosquitoes: an argument for the sterile insect technique," *Trends in Parasitology*, vol. 19, no. 8, pp. 349–355, 2003.
- [5] L. Cai, S. Ai, and J. Li, "Dynamics of mosquitoes populations with different strategies for releasing sterile mosquitoes," *SIAM Journal on Applied Mathematics*, vol. 74, no. 6, pp. 1786–1809, 2014.
- [6] L. Esteva and H. Mo Yang, "Mathematical model to assess the control of aedes aegypti mosquitoes by the sterile insect technique," *Mathematical Biosciences*, vol. 198, no. 2, pp. 132–147, 2005.
- [7] J. E. Gentile, S. S. Rund, and G. R. Madey, "Modelling sterile insect technique to control the population of anopheles gambiae," *Malaria Journal*, vol. 14, no. 1, pp. 92–12, 2015.
- [8] M. A. Lewis and P. Van Den Driessche, "Waves of extinction from sterile insect release," *Mathematical Biosciences*, vol. 116, no. 2, pp. 221–247, 1993.
- [9] J. Li, "New revised simple models for interactive wild and sterile mosquito populations and their dynamics," *Journal of Biological Dynamics*, vol. 11, no. 2, pp. 316–333, 2017.
- [10] B. Zheng, M. Tang, and J. Yu, "Modeling wolbachia spread in mosquitoes through delay differential equations," *SIAM Journal on Applied Mathematics*, vol. 74, no. 3, pp. 743–770, 2014.
- [11] B. Zheng, M. Tang, J. Yu, and J. Qiu, "Wolbachia spreading dynamics in mosquitoes with imperfect maternal transmission," *Journal of Mathematical Biology*, vol. 76, no. 1, pp. 235–263, 2018.
- [12] A. Traoré, "Analysis of a vector-borne disease model with human and vectors immigration," *Journal of Applied Mathematics and Computing*, vol. 64, no. 1-2, pp. 411–428, 2020.
- [13] N. D. Alikakos, "An application of the invariance principle to reaction-diffusion equations," *Journal of Differential Equations*, vol. 33, no. 2, pp. 201–225, 1979.
- [14] K. Hattaf and N. Yousfi, "A generalized hbv model with diffusion and two delays," *Computers & Mathematics with Applications*, vol. 69, no. 1, pp. 31–40, 2015.
- [15] K. Manna, "Dynamics of a delayed diffusive HBV infection model with capsids and CTL immune response," *International Journal of Applied and Computational Mathematics*, vol. 4, no. 116, 2018.
- [16] J. Yang, S. Liang, and Y. Zhang, "Travelling waves of a delayed sir epidemic model with nonlinear incidence rate and spatial diffusion," *PLoS One*, vol. 6, no. 6, Article ID e21128, 2011.
- [17] A. Pazy, "Semigroups of linear operators and applications to partial differential equations," *Springer Science & Business Media*, vol. 44, 2012.
- [18] M. H. Protter and H. F. Weinberger, *Maximum Principles in Differential Equations*, Springer Science & Business Media, Berlin, Germany, 2012.
- [19] D. Henry, "Geometric theory of semilinear parabolic equations," *Springer*, vol. 840, 2006.