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Research Article

Applications of Two Methods in Exact Wave Solutions in the Space-Time Fractional Drinfeld–Sokolov–Wilson System

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The fractional differential equations (FDEs) are ubiquitous in mathematically oriented scientific fields, such as physics and engineering. Therefore, FDEs have been the focus of many studies due to their frequent appearance in several applications such as physics, engineering, signal processing, systems identification, sound, heat, diffusion, electrostatics and fluid mechanics, and other sciences. The perusal of these nonlinear physical models through wave solutions analysis, corresponding to their FDEs, has a dynamic role in applied sciences. In this paper, the exp-function method and the rational (G'/G)-expansion method are presented to establish the exact wave solutions of the space-time fractional Drinfeld–Sokolov–Wilson system in the sense of the conformable fractional derivative. The fractional Drinfeld–Sokolov–Wilson system contains fractional derivatives of the unknown function in terms of all independent variables. This system describes the shallow water wave models in fluid mechanics. These presented methods are a powerful mathematical tool for solving nonlinear conformable fractional evolution equations in various fields of applied sciences, especially in physics.

1. Introduction

Nowadays, the nonlinear differential equations play a pivotal role in the mathematically-oriented scientific fields, such as physics, and many research studies have discussed the study of nonlinear differential equations [1–4]. Also, the mathematical modeling of fractional orders has attracted more and more attention, and the study of mathematical models involving fractional-order has become a hot topic. Hence, fractional differential equations have been the focus of many studies [5–8]. In recent years, a variety of fractional partial differential equations have been used in many studies, which has increased the incentive to use and develop numerical and approximate methods for such problems. Several numerical methods have been used to solve fractional differential equations for example the PicardChelyshkov polynomial method (PCPM), the Chelyshkov polynomial method (CPM) that further coupled with a finite difference scheme (FDM) and named as a semispectral method (SSM), and a hybrid method based on operational matrices of derivative has been provided by Muhammad Hamid et al. [9–11]. In mathematically-oriented scientific fields, obtaining exact solutions of nonlinear fractional differential equations (NLFDEs) has become one of the most exciting and many active areas of research investigation. Therefore, methods have been presented to solve of FDEs in the past few years, so after introducing and presenting various types of fractional derivatives such as the Riemann–Liouville derivative, the Caputos fractional derivative, the Riesz fractional derivatives, and the conformable fractional derivative and using complex fraction transformations, analytical methods have been used to obtain exact solutions to nonlinear fractional equations. Including several investigators who have successfully used reliable and algebraic methods for this intention [12–17].

In the year 2006, the exp-function method for nonlinear differential equations was presented by He and Wu [18]. In the year 2015, Islam, Akbar, and Azad proposed the rational (G'/G)-expansion method as a new method of exact solutions [19], which has been applied to solve nonlinear

development equations [20]. In this paper, the exp-function method and the rational (G'/G)-expansion method are applied to acquire new, exact solutions to a nonlinear, fractional-order partial differential system arising in mathematical physics. The considered system comprises the space-time fractional Drinfeld–Sokolov–Wilson (DSW) system in the sense of the conformable fractional derivative, such that the nonlinear Drinfeld–Sokolov–Wilson system naturally occurs in dispersive water waves. Applying traveling wave transformations to the equations, we obtain the corresponding ordinary differential equations, in which each of them provides a system of nonlinear algebraic equations when the method is used. As a result, some analytical exact solutions obtained from these equations are presented. The classical (DSW) equation originates from the shallow water wave models originally proposed by Drinfeld and Sokolov (1981, 1985) and later developed by Wilson (1982) in its explicit form. The equation is used to describe nonlinear surface gravity waves propagating over the horizontal seabed. Recently, due to the significant and crucial applications in physical sciences, some researchers have done considerable methods to get exact and approximate solutions of (DSW) system, for example, the sine-Gordon expansion method [21], the modified Kudryashov method [22], the homotopy analysis method [23], the stability analysis of Drinfeld–Sokolov–Wilson system [24], the complete discrimination system for polynomial method [25], the first integral method [26], the generalized complex method and the (G'/G)-expansion method [27], the q-homotopy analysis method, and the Lie symmetry method [28]. In the general form, the nonlinear fractional DSW equation is expressed as follows [25–27]:

$$\begin{cases} D_t^\alpha u + pvD_x^\alpha v = 0; \qquad 0 < \alpha \le 1. \\ D_t^\alpha v + qD_x^{3\alpha} v + ruD_x^\alpha v + svD_x^\alpha u = 0 \end{cases} \tag{1}$$

Here, p, q, r, and s are nonzero parameters. So that u and v are the functions of (x, t). The exp-function method and the rational (G'/G) -expansion method are described in Section 3. In Section 4 of the paper, we present exact wave solutions to the fractional (DSW) system by the methods mentioned. Also, we used the conformable fractional derivative to convert fractional nonlinear partial differential equations to nonlinear ordinary differential equations. In the end, the general conclusion is given in section five.

2. The Conformable Fractional Derivative

There are several definitions for fractional differential equations. These definitions include Grunwald–Letnikov, Riemann–Liouville, Caputo, Weyl, Marchaud, and Riesz fractional derivatives [5]. Recently, a new modification of the Riemann–Liouville derivative is proposed by Jumarie,

$$D_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-\xi)^{-\alpha} (f(\xi) - f(0)) d\xi, \quad 0 < \alpha \le 1. \tag{2}$$

And, some basic fractional calculus formulae are given, for example, [29, 30]

$$D_t^\alpha (f(t)g(t)) = g(t)D_t^\alpha f(t) + f(t)D_t^\alpha g(t), \tag{3}$$

$$D_t^\alpha f[g(t)] = f_g' D_t^\alpha g(t) = D_t^\alpha f[g(t)] g'(t)^\alpha. \tag{4}$$

Formula (3) has been applied to solve the exact solutions to some nonlinear fractional-order differential equations. If this formula were true, then we could take the transformation $\xi = (kx^\alpha + ct^\alpha/\Gamma(1+\alpha))$ and reduce the partial derivative $(\partial^\alpha u(x,t)/\partial t^\alpha)$ = to $u'(\xi)$. Therefore, the corresponding fractional differential equations become the ordinary differential equations which are easy to study. But we must point out that Jumarie's basic formulae (3) and (4) are not correct, and therefore the corresponding results on differential equations are not true [31].

The authors introduced a new definition of the fractional derivative as follows [32]:

$$D_t^\alpha f(t) = \lim_{\varepsilon \longrightarrow 0} \frac{f\left(t + \varepsilon t^{1-\alpha}\right) - f(t)}{\varepsilon}. \tag{5}$$

Here, for $t > 0$, $\alpha \in [0, 1)$ and f: $[0, \infty) \longrightarrow R$. D_t^α is called the conformable fractional derivative of f of order α [33, 34].

Using this kind of fractional derivative and some useful formulae, we can convert differential equations into integer-order differential equations.

Some properties for the suggested conformable fractional derivative given in [27] are as follows:

$$\begin{aligned} D_t^\alpha t^\beta &= \beta t^{\beta-\alpha}, \\ D_t^\alpha (f(t)g(t)) &= g(t)D_t^\alpha f(t) + f(t)D_t^\alpha g(t), \\ D_t^\alpha f[g(t)] &= f_g'[g(t)]D_t^\alpha g(t) = D_t^\alpha f[g(t)] (g'(t))^\alpha. \end{aligned} \tag{6}$$

We consider the fractional partial differential equation, with independent variables $t, x_1, x_2, \ldots, x_n$ and dependent variable v.

$$P\left(v, D_t^\alpha v, D_{x_1}^\alpha v, \ldots, D_t^{2\alpha} v, \ldots\right) = 0, 0 < \alpha \le 1, t > 0. \tag{7}$$

By using the fractional variable transformation,

$$v(t, x_1, \ldots, x_n) = v(\xi), \xi = \frac{ct^\alpha}{\alpha} + \frac{k_1 x_1^\alpha}{\alpha} + \cdots + \frac{k_n x_n^\alpha}{\alpha}, \tag{8}$$

where c and k_i, $(i = 1, \ldots, n)$ are non-zero arbitrary constants.

The fractional differential (7) reduced to a nonlinear ordinary differential equation.

$$F(v, v', v'', \ldots) = 0, \tag{9}$$

where the prime denotes the derivation with respect to ξ and $v' = (\mathrm{d}u/\mathrm{d}\xi)$.

3. Description of the Methods Provided

As mentioned before, there are several powerful methods to obtain approximate and precise solutions of fractional differential equations system; in this section, we introduce the exp-function and the rational (G'/G)-expansion method.

3.1. The Exp-Function Method

Step 1. In the exp-function method, we assume that the solution is a function.

$$v(\xi) = \frac{\sum_{p=-m_1}^{m_2} a_p \exp[p\xi]}{\sum_{q=-n_1}^{n_2} a_q \exp[q\xi]}, \tag{10}$$

where m_1, m_2, n_1, and n_2 are unknown positive integers that will be determined by using the balance method, for construct the relations between m_2, m_1, n_2, and n_1, we consider the highest degree and the highest derivative order of function $v(\xi)$ in the nonlinear (ODE) (6).

Step 2. For the relation between m_2 and n_2 by creating a balance between the first sentence of the highest degree and the first sentence of the highest derivative order.

Similarly, the relation between m_1 and n_1 is obtained by balancing the last sentence of the highest degree and the last sentence of the highest derivative order.

In the year 2012, Ebaid proved in Theorem 1 that $m_2 = n_2$ and $m_1 = n_1$ are the only relations that can be obtained by applying the balancing method in nonlinear (ODE) (6) [35].

Step 3. Substituting the function obtained in step 3 in (ODE) (6) and with the help of Maple, we obtain exact solutions of nonlinear fractional differential equations (10).

In the exp-function method, additional calculations of balancing the highest order linear term with the highest order nonlinear term are not longer required in the future. Because we only use their first and last sentences. Hence, the method becomes more straightforward.

3.2. The Rational (G'/G)-Expansion Method

Step 4. We consider the solution of equation (12) in terms of (G'/G) as follows:

$$v(\xi) = \frac{\alpha_0 + \alpha_1 (G'/G) + \cdots + \alpha_m (G'/G)^m}{\beta_0 + \beta_1 (G'/G) + \cdots + \beta_m (G'/G)^m} = \frac{\sum_{i=0}^{m} \alpha_i (G'/G)^i}{\sum_{i=0}^{m} \beta_i (G'/G)^i}, \tag{11}$$

where α_m and β_m are nonzero constants to be determined later while $G(\xi)$ satisfies in the second-order linear ordinary differential equation (LODE).

$$G'' + \lambda G' + \mu G = 0, \tag{12}$$

where λ and μ being constants.

We determine m by using balance between the highest order linear term with the highest order derivative nonlinear term in (ODE) (6).

By the generalized solution of second-order LODE $G'' + \lambda G' + \mu G = 0$, we have the following equation:

$$\left(\frac{G'}{G}\right) = \begin{cases} -\dfrac{\lambda}{2} + \dfrac{\sqrt{\lambda^2 - 4\mu}}{2}\left(\dfrac{A\sinh\left(1/2\sqrt{\lambda^2 - 4\mu}\right)\xi + B\cosh\left(1/2\sqrt{\lambda^2 - 4\mu}\right)\xi}{A\cosh\left(1/2\sqrt{\lambda^2 - 4\mu}\right)\xi + B\sinh\left(1/2\sqrt{\lambda^2 - 4\mu}\right)\xi}\right), \lambda^2 - 4\mu > 0, \\ -\dfrac{\lambda}{2} + \dfrac{\sqrt{4\mu - \lambda^2}}{2}\left(\dfrac{-A\sin\left(1/2\sqrt{4\mu - \lambda^2}\right)\xi + B\cos\left(1/2\sqrt{4\mu - \lambda^2}\right)\xi}{A\cos\left(1/2\sqrt{4\mu - \lambda^2}\right)\xi + B\sin\left(1/2\sqrt{4\mu - \lambda^2}\right)\xi}\right), \lambda^2 - 4\mu < 0, \\ -\dfrac{\lambda}{2} + \dfrac{B}{A + B\xi}, \lambda^2 - 4\mu = 0, \end{cases} \tag{13}$$

And, $G = G(\xi)$ satisfies the second-order ODE (12), also whose general solutions are given by (13).

Step 5. To determine the positive integer m, we substitute (8) along with (12) into (9) and balance between the highest order derivatives and the highest order nonlinear terms appearing in (9). Furthermore, if the degree of $v(\xi)$ is defined as $\deg[v(\xi)] = m$, the degree of the other expressions are as follows:

$$\deg\left[\frac{d^n v(\xi)}{d\xi^n}\right] = m + n, \deg\left[v^n\left(\frac{d^l v(\xi)}{d\xi^l}\right)^p\right] = nm + p(m + l). \tag{14}$$

Step 6. Substituting equation (8) together with equation (12) into equation (12), we obtain a polynomial equation with indeterminate (G'/G). Setting each coefficient of (G'/G) to zero gives a system of algebraic equations. This system of equations is solved for α_i, β_i, λ, and μ by means of the symbolic computation software, such as Maple.

Step 7. We use the values of α_i, β_i, λ, and μ together with equation (13) into equation (11) to obtain the closed form traveling wave solutions of the nonlinear fractional partial differential equation (7).

4. The Fractional Drinfeld–Sokolov–Wilson System

We consider the fractional Drinfeld–Sokolov–Wilson system and we obtain exact wave solutions by the exp-function method and the rational (G'/G)-expansion method respectively. This system has the formula (1), by using variable transformations:

$$u(x,t) = u(\xi), v(x,t) = v(\xi), \xi = \frac{kx^{\alpha}}{\alpha} + \frac{ct^{\alpha}}{\alpha}, \tag{15}$$

where k and c are constants, system (1) is reduced to an ordinary differential system.

$$\begin{aligned} cu' + ppkvv' &= 0, \\ cv' + qk^3 v''' + rkuv' + skvu' &= 0. \end{aligned} \tag{16}$$

Also, we have the following equation:

$$\begin{aligned} u' &= \left(\frac{-pk}{c}\right) vv', \\ u &= \left(\frac{-pk}{2c}\right) v^2, \end{aligned} \tag{17}$$

where $u' = (du/d\xi)$ and $v' = (dv/d\xi)$.

By equations (6) and equation (17) we have the following equation:

$$2cqk^3 v''' - pk^2 (r + 2s) v^2 v' + 2c^2 v' = 0. \tag{18}$$

Integrating equation (6) with respect to ξ yields,

$$2cqk^3 v'' - \frac{1}{3} pk^2 (r + 2s) v^3 + 2c^2 v = 0. \tag{19}$$

4.1. Application of the Exp-Function Method. We consider (10) is solution of (19)) we have the following equation:

$$\begin{aligned} v''(\xi) &= \frac{A \exp[(3n_2 + m_2)\xi] + \cdots + B \exp[-(3n_1 + m_1)\xi]}{C \exp[4n_2\xi] + \cdots + D \exp[-4n_1\xi]}, \\ v^3(\xi) &= \frac{E \exp[3m_2\xi] + \cdots + F \exp[-3m_1\xi]}{G \exp[3n_2\xi] + \cdots + H \exp[-3n_1\xi]}, \end{aligned} \tag{20}$$

where A, B, C, D, E, F, G, and H are coefficients determined by $a_{m_2}, a_{-m_1}, b_{n_2}$, and b_{-n_1}.

By balancing the highest order exp-function in v'' and v^3 in (19), we have $3n_2 + m_2 = 3m_2 + n_2$, which implies $m_2 = n_2$. Similarly, balancing the lowest order exp-function in v''' and v^3 , we have $3n_1 + m_1 = 3m_1 + n_1$, which implies $m_1 = n_1$.

For simplicity, we set $m_2 = n_2 = 1$ and $m_1 = n_1 = 1$ so (10) reduces to

$$v(\xi) = \frac{a_1 \exp[\xi] + a_0 + a_{-1} \exp[-\xi]}{b_1 \exp[\xi] + b_0 + b_{-1} \exp[-\xi]}. \tag{21}$$

Substituting (21) into equation (19) and with the help of Maple, we have the following equation:

$$\begin{aligned} &\frac{1}{A}\left(R_1 \exp[3\xi] + R_2 \exp[2\xi] + R_3 \exp[\xi] + R_4\right. \\ &\left. + R_5 \exp[-\xi] + R_6 \exp[-2\xi] + R_7 \exp[-3\xi]\right) = 0, \end{aligned} \tag{22}$$

where

$$\begin{aligned} a_1 &= \frac{1}{4} \frac{a_0^2 b_{-1}^2 - b_0^2 a_{-1}^2}{a_{-1} b_{-1}^2}, b_1 = -\frac{1}{4} \frac{a_0^2 b_{-1}^2 - b_0^2 a_{-1}^2}{b_{-1} a_{-1}^2}, q = \frac{2c}{k^3}, \\ r &= \frac{2\left(3c^2 b_{-1}^2 - pk^2 s a_{-1}^2\right)}{pk^2 a_{-1}^2}. \end{aligned} \tag{23}$$

Solving this system of algebraic equation (23) by using Maple, we obtain the following results.

Result 1

$$\begin{aligned} a_1 &= \frac{1}{4} \frac{a_0^2 b_{-1}^2 - b_0^2 a_{-1}^2}{a_{-1} b_{-1}^2}, b_1 = -\frac{1}{4} \frac{a_0^2 b_{-1}^2 - b_0^2 a_{-1}^2}{b_{-1} a_{-1}^2}, q = \frac{2c}{k^3} \\ r &= \frac{2\left(3c^2 b_{-1}^2 - pk^2 s a_{-1}^2\right)}{pk^2 a_{-1}^2}, \end{aligned} \tag{24}$$

where a_0, a_{-1}, b_0, b_{-1} , k, c, s, and p are arbitrary constants. Inserting the value provided in (24) into solution (21), gives the following equation:

$$v_1(\xi) = -\frac{a_{-1}\left(\left(b_0^2 a_{-1}^2 - a_0^2 b_{-1}^2\right) e^{\xi} - 4a_0 a_{-1} b_{-1}^2 - 4b_{-1}^2 a_{-1}^2 e^{-\xi}\right)}{b_{-1}\left(\left(b_0^2 a_{-1}^2 - a_0^2 b_{-1}^2\right) e^{\xi} + 4b_0 b_{-1} a_{-1}^2 + 4b_{-1}^2 a_{-1}^2 e^{-\xi}\right)}. \tag{25}$$

Substituting (25) into (17), we have the following equation:w

$$u_1(\xi) = \left(\frac{-pk}{2c}\right)\left[-\frac{a_{-1}\left(\left(b_0^2 a_{-1}^2 - a_0^2 b_{-1}^2\right) e^{\xi} - 4a_0 a_{-1} b_{-1}^2 - 4b_{-1}^2 a_{-1}^2 e^{-\xi}\right)}{b_{-1}\left(\left(b_0^2 a_{-1}^2 - a_0^2 b_{-1}^2\right) e^{\xi} + 4b_0 b_{-1} a_{-1}^2 + 4b_{-1}^2 a_{-1}^2 e^{-\xi}\right)}\right]^2. \tag{26}$$

Result 2

$$a_1 = -\frac{a_{-1}b_1}{b_{-1}}, a_0 = 0, b_0 = 0, q = \frac{1}{2}\frac{c}{k^3},$$
$$r = \frac{2\left(3c^2b_{-1}^2 - pk^2sa_{-1}^2\right)}{pk^2a_{-1}^2}, \tag{27}$$

where a_{-1}, b_1, b_{-1} , k, c, s, and p are arbitrary constants. Inserting the value provided in (27) into solution (21), gives the following equation:

$$v_2(\xi) = -\frac{a_{-1}\left(b_1e^{\xi} - b_{-1}e^{-\xi}\right)}{b_{-1}\left(b_1e^{\xi} + b_{-1}e^{-\xi}\right)}, u_2(\xi) = \left(\frac{-pk}{2c}\right)\left[-\frac{a_{-1}\left(b_1e^{\xi} - b_{-1}e^{-\xi}\right)}{b_{-1}\left(b_1e^{\xi} + b_{-1}e^{-\xi}\right)}\right]^2. \tag{28}$$

Result 3

$$a_0 = 0, b_1 = 0, b_{-1} = 0, r = -2s, c = -k^3q, \tag{29}$$

where a_1, a_{-1}, b_0 , k, s, q, and p are arbitrary constants. Inserting the value provided in (29) into solution (21), gives the following equation:

$$v_3(\xi) = \frac{a_1e^{\xi} + a_{-1}e^{-\xi}}{b_0}, 1cmu_3(\xi) = \left(\frac{-pk}{2c}\right)\left[\frac{a_1e^{\xi} + a_{-1}e^{-\xi}}{b_0}\right]^2. \tag{30}$$

Result 4

$$a_{-1} = 0, b_1 = 0, b_{-1} = \frac{a_0b_0}{a_1}, r = -2s, c = -k^3q, \tag{31}$$

where a_1, a_0, b_0 , k, s, q, and p are arbitrary constants. Inserting the value provided in (31) into solution (21), gives the following equation:

$$v_4(\xi) = \frac{a_1\left(a_1e^{\xi} + a_0\right)}{b_0\left(a_1 + a_0e^{-\xi}\right)}, u_4(\xi) = \left(\frac{-pk}{2c}\right)\left[\frac{a_1\left(a_1e^{\xi} + a_0\right)}{b_0\left(a_1 + a_0e^{-\xi}\right)}\right]^2. \tag{32}$$

Result 5

$$a_1 = -\frac{a_0b_1}{b_0}, a_{-1} = 0, b_{-1} = 0, q = \frac{2c}{k^3}$$
$$r = \frac{2\left(-pk^2sa_0^2 + 3b_0^2c^2\right)}{pk^2a_0^2}, \tag{33}$$

where a_0, b_1, b_0 , k, c, s, and p are arbitrary constants. Inserting the value provided in (33) into solution (21), gives the following equation:

$$v_5(\xi) = -\frac{a_0\left(b_1e^{\xi} - b_0\right)}{b_0\left(b_1e^{\xi} + b_0\right)}, u_5(\xi) = \left(\frac{-pk}{2c}\right)\left[-\frac{a_0\left(b_1e^{\xi} - b_0\right)}{b_0\left(b_1e^{\xi} + b_0\right)}\right]^2. \tag{34}$$

Result 6

$$a_0 = 0, a_{-1} = 0, b_1 = 0, b_0 = 0, r = -2s, c = -4k^3q, \tag{35}$$

where a_1, b_{-1}, k, s, q, and p are arbitrary constants. Inserting the value provided in (35) into solution (21), gives the following equation:

$$v_6(\xi) = \frac{a_1e^{\xi}}{b_{-1}e^{-\xi}} = \frac{a_1}{b_{-1}}e^{2\xi}, u_6(\xi) = \left(\frac{-pk}{2c}\right)\left[\frac{a_1e^{\xi}}{b_{-1}e^{-\xi}} = \frac{a_1}{b_{-1}}e^{2\xi}\right]^2. \tag{36}$$

Result 7

$$a_1 = 0, b_1 = \frac{a_0b_0}{a_{-1}}, b_{-1} = 0, r = -2s, c = -k^3q, \tag{37}$$

where a_0, a_{-1}, b_0, k, s, q, and p are arbitrary constants. Inserting the value provided in (37) into solution (21), gives the following equation:

$$v_7(\xi) = \frac{a_{-1}\left(a_0 + a_{-1}e^{-\xi}\right)}{b_0\left(a_0e^{\xi} + a_{-1}\right)}, u_7(\xi) = \left(\frac{-pk}{2c}\right)\left[\frac{a_{-1}\left(a_0 + a_{-1}e^{-\xi}\right)}{b_0\left(a_0e^{\xi} + a_{-1}\right)}\right]^2. \tag{38}$$

Result 8

$$a_1 = 0, a_{-1} = 0, b_0 = 0, b_{-1} = \frac{1}{48}\frac{pa_0^2(r + 2s)}{k^4b_1q^2}, c = -k^3q, \tag{39}$$

where a_0, b_1, k, r, s, q, and p are arbitrary constants. Inserting the value provided in (39) into solution (21), gives the following equation:

$$v_8(\xi) = \frac{48a_0k^4b_1q^2}{48b_1^2k^4q^2e^{\xi} + pa_0^2(r + 2s)e^{-\xi}},$$
$$u_8(\xi) = \left(\frac{-pk}{2c}\right)\left[\frac{48a_0k^4b_1q^2}{48b_1^2k^4q^2e^{\xi} + pa_0^2(r + 2s)e^{-\xi}}\right]^2. \tag{40}$$

Result 9

$$a_1 = 0, a_{-1} = 0, b_1 = 0, b_0 = 0, r = -2s, c = -k^3q, \tag{41}$$

where a_0, b_{-1}, k, s, q, and p are arbitrary constants. Inserting the value provided in (41) into solution (21), gives the following equation:

$$v_9(\xi) = \frac{a_0}{b_{-1}} e^{\xi}, u_9(\xi) = \left(\frac{-pk}{2c}\right)\left[\frac{a_0}{b_{-1}} e^{\xi}\right]^2. \tag{42}$$

Result 10

$$a_1 = 0, a_0 = 0, b_0 = 0, b_{-1} = 0, r = -2s, c = -4k^3 q, \tag{43}$$

where a_{-1}, b_1, k, s, q, and p are arbitrary constants. Inserting the value provided in (43) into solution (21), gives the following equation:

$$v_{10}(\xi) = \frac{a_{-1}}{b_1} e^{-2\xi}, u_{10}(\xi) = \left(\frac{-pk}{2c}\right)\left[\frac{a_{-1}}{b_1} e^{-2\xi}\right]^2. \tag{44}$$

4.2. Application of the Rational (G'/G)-Expansion Method. We consider (11) to be the solution of (19)) and balancing the highest order derivative and the nonlinear term appearing in (19), we deduce that $m = 1$. Thus, the solution (11) reduces to the form.

$$v(\xi) = \frac{\alpha_0 + \alpha_1 (G'/G)}{\beta_0 + \beta_1 (G'/G)}, \alpha_1, \beta_1 \neq 0. \tag{45}$$

Substituting (45) into (19), the left hand side becomes a polynomial in (G'/G) . Setting each coefficient of this polynomial to zero,

$$\begin{aligned}
&3:\ 12cqk^3\alpha_1\beta_0^2 + 6c^2\alpha_1\beta_1^2 - pk^2 r\alpha_1^3 - 2pk^2 s\alpha_1^3 - 12cqk^3\beta_1\alpha_0\beta_0 + 6cqk^3\beta_1^2\alpha_0\lambda - 6cqk^3\alpha_1\beta_0\lambda\beta_1 \\
&2:\ 12cqk^3\alpha_0\beta_1^2\mu + 18cqk^3\beta_0^2\alpha_1\lambda + 12c^2\alpha_1\beta_1\beta_0 + 6cqk^3\alpha_0\beta_1^2\lambda^2 - 6pk^2 s\alpha_1^2\alpha_0 \\
&\quad - 18cqk^3\alpha_0\beta_1\lambda\beta_0 - 3pk^2 r\alpha_{12}\alpha_0 + 6c^2\alpha_0\beta_1^2 - 12cqk^3\beta_0\alpha_1\mu\beta_1 - 6cqk^3\beta_0\alpha_1\lambda^2\beta_1 \\
&1:\ -6pk^2 s\alpha_1\alpha_0^2 + 12c^2\alpha_0\beta_1\beta_0 + 6cqk^3\beta_0^2\alpha_1\lambda^2 + 12cqk^3\beta_0^2\alpha_1\mu - 3pk^2 r\alpha_1\alpha_0^2 \\
&\quad - 6cqk^3\alpha_0\beta_1\lambda^2\beta_0 - 12cqk^3\alpha_0\beta_1\mu\beta_0 + 18cqk^3\alpha_0\beta_1^2\lambda\mu - 18cqk^3\beta_0\alpha_1\lambda\mu\beta_1 + 6c^2\alpha_1\beta_0^2 \\
&0:\ 12cqk^3\alpha_0\beta_1^2\mu^2 - 12cqk^3\beta_0\alpha_1\beta_1\mu^2 + 6c^2\alpha_0\beta_0^2 - pk^2 r\alpha_0^3 - 2pk^2 s\alpha_0^3 - 6cqk^3\alpha_0\beta_1\lambda\mu\beta_0 + 6cqk^3\beta_0^2\alpha_1\lambda\mu.
\end{aligned} \tag{46}$$

Solving this system of algebraic equation (46) by Maple provides the following results.

Result 11

$$\begin{aligned}
&r = -2s, \lambda = \frac{\beta_0\alpha_1 + \alpha_0\beta_1}{\alpha_1\beta_1}, \mu = \frac{\alpha_0\beta_0}{\alpha_1\beta_1}, \\
&c = -\frac{qk^3\left(-2\alpha_0\alpha_1\beta_0\beta_1 + \beta_0^2\alpha_1^2 + \alpha_0^2\beta_1^2\right)}{\beta_1^2\alpha_1^2},
\end{aligned} \tag{47}$$

where α_1, α_0, β_1, β_0 , k, q, s, and p are arbitrary constants.

$$\omega = \lambda^2 - 4\mu = \frac{(\alpha_1\beta_0 - \alpha_0\beta_1)^2}{\alpha_1^2\beta_1^2} \geq 0. \tag{48}$$

Here, ω is always positive $(\omega > 0)$. Because if we consider $\omega = 0$, then parameter c in the variable transformations equation (15) becomes $c = 0$. This contradicts the definition of the parameter c.

When $\omega = \lambda^2 - 4\mu > 0$, the hyperbolic function solution becomes

$$v(\xi) = \frac{2\alpha_0 - \lambda\alpha_1 + \sqrt{\omega}\alpha_1 \left(A\sinh[(\sqrt{\omega}/2)\xi] + B\cosh[(\sqrt{\omega}/2)\xi]/A\cosh[(\sqrt{\omega}/2)\xi] + B\sinh[(\sqrt{\omega}/2)\xi]\right)}{2\beta_0 - \lambda\beta_1 + \sqrt{\omega}\beta_1 \left(A\sinh[(\sqrt{\omega}/2)\xi] + B\cosh[(\sqrt{\omega}/2)\xi]/A\cosh[(\sqrt{\omega}/2)\xi] + B\sinh[(\sqrt{\omega}/2)\xi]\right)}. \tag{49}$$

Substituting (47) into (49), we have the following equation:

$$v_{11}(\xi) = \frac{\alpha_0 + \alpha_1\left(-\beta_0/\beta_1 + \left(B\cosh\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\right] + A\sinh\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\right]\right)\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}/A\cosh\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\right] + B\sinh\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\right]\right)}{\beta_0 + \beta\left({}_1-\alpha_0/\alpha_1 + \left(B\cosh\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\right] + A\sinh\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\right]\right)\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}/A\cosh\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\right] + B\sinh\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\right]\right)}. \tag{50}$$

I: if we choose $A \neq 0$ and $B = 0$, then

$$v_{12}(\xi) = \frac{\alpha_1\left(\alpha_0\beta_1 - \alpha_1\left(\beta_0 - \beta_1\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\mathrm{Tanh}\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\right]\right)\right)}{\beta_1\left(-\alpha_0\beta_1 + \alpha_1\left(\beta_0 + \beta_1\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\mathrm{Tanh}\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\right]\right)\right)},$$

$$u_{12}(\xi) = \left(\frac{-pk}{2c}\right)\left[\frac{\alpha_1\left(\alpha_0\beta_1 - \alpha_1\left(\beta_0 - \beta_1\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\mathrm{Tanh}\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\right]\right)\right)}{\beta_1\left(-\alpha_0\beta_1 + \alpha_1\left(\beta_0 + \beta_1\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\mathrm{Tanh}\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\right]\right)\right)}\right]^2, \tag{51}$$

$$u_{12}(\xi) = \left(\frac{-pk}{2c}\right)\left[\frac{\alpha_1\left(\alpha_0\beta_1 - \alpha_1\left(\beta_0 - \beta_1\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\mathrm{Tanh}\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\right]\right)\right)}{\beta_1\left(-\alpha_0\beta_1 + \alpha_1\left(\beta_0 + \beta_1\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\mathrm{Tanh}\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\right]\right)\right)}\right]^2.$$

II: if we choose $B \neq 0$ and $A = 0$, then

$$v_{13}(\xi) = \frac{\alpha_1\left(\alpha_0\beta_1 - \alpha_1\left(\beta_0 - \mathrm{Coth}\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\right]\beta_1\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\right)\right)}{\beta_1\left\{-\alpha_0\beta_1 + \alpha_1\left(\beta_0 + \mathrm{Coth}\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\right]\beta_1\sqrt{\left((\alpha_1\beta_0 - \alpha_0\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\right)\right\}},$$

$$u_{13}(\xi) = \left(\frac{-pk}{2c}\right)\left[\frac{\alpha_1\left(\alpha_\theta\beta_1 - \alpha_1\left(\beta_\theta - \mathrm{Coth}\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_\theta - \alpha_\theta\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\right]\beta_1\sqrt{\left((\alpha_1\beta_\theta - \alpha_\theta\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\right)\right)}{\beta_1\left(-\alpha_\theta\beta_1 + \alpha_1\left(\beta_\theta + \mathrm{Coth}\left[(1/2)\xi\sqrt{\left((\alpha_1\beta_\theta - \alpha_\theta\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\right]\beta_1\sqrt{\left((\alpha_1\beta_\theta - \alpha_\theta\beta_1)^2/\alpha_1^2\beta_1^2\right)}\,\right)\right)}\right]^2, \tag{52}$$

where

$$\xi = \frac{kx^\alpha}{\alpha} + \frac{\left(-qk^3\left(-2\alpha_0\alpha_1\beta_0\beta_1 + \beta_0^2\alpha_1^2 + \alpha_0^2\beta_1^2\right)/\beta_1^2\alpha_1^2\right)t^\alpha}{\alpha}. \tag{53}$$

Result 12

$$\begin{aligned}
\mu &= \frac{1}{2}\,\frac{\beta_0\alpha_1\lambda + \beta_1\alpha_0\lambda - 2\alpha_0\beta_0}{\alpha_1\beta_1},\\
c &= \frac{1}{2}\,\frac{qk^3\left(-2\beta_0\alpha_1\lambda - 2\alpha_0\beta_1\lambda + 4\alpha_0\beta_0 + \alpha_1\lambda^2\beta_1\right)}{\beta_1\alpha_1},\\
r &= \frac{1}{2}\,\frac{1}{\alpha_1^3\beta_1 p}\left(-6q^2k^4\beta_1^3\alpha_0\lambda^3 + 36q^2k^4\beta_0^2\alpha_1\lambda^2\beta_1 - 18q^2k^4\beta_0\lambda^3\beta_1^2\alpha_1 - 4ps\alpha_1^3\beta_1 - 24q^2k^4\beta_0^3\alpha_1\lambda + 48q^2k^4\beta_0^3\alpha_0\right.\\
&\quad \left. + 3q^2k^4\beta_1^3\alpha_1\lambda^4 - 72q^2k^4\alpha_0\beta_0^2\beta_1\lambda + 36q^2k^4\alpha_0\beta_0\beta_1^2\lambda^2\right),
\end{aligned} \tag{54}$$

where α_1, α_0, β_1, β_0, , λ, k, q, s and p are arbitrary constants.

$$\omega = \lambda^2 - 4\mu = \frac{(-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)}{\alpha_1\beta_1}. \tag{55}$$

Case 1. When $\omega = \lambda^2 - 4\mu > 0$, the hyperbolic function solution becomes

$$v(\xi) = \frac{2\alpha_0 - \lambda\alpha_1 + \sqrt{\omega}\alpha_1 (A\sinh[(\sqrt{\omega}/2)\xi] + B\cosh[(\sqrt{\omega}/2)\xi]/A\cosh[(\sqrt{\omega}/2)\xi] + B\sinh[(\sqrt{\omega}/2)\xi])}{2\beta_0 - \lambda\beta_1 + \sqrt{\omega}\beta_1 (A\sinh[(\sqrt{\omega}/2)\xi] + B\cosh[(\sqrt{\omega}/2)\xi]/A\cosh[(\sqrt{\omega}/2)\xi] + B\sinh[(\sqrt{\omega}/2)\xi])}. \tag{56}$$

Substituting (54) into (56), we have the following equation:

$$v_{18}(\xi) = \frac{2\alpha_0 - \lambda\alpha_1 + \left(B\cosh\left[(1/2)\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right] + A\sinh\left[(1/2)\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]\right)\alpha_1\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}/A\cosh\left[(1/2)\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right] + B\sinh\left[1/2\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]}{2\beta_0 - \lambda\beta_1 + \left(B\cosh\left[(1/2)\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right] + A\sinh\left[(1/2)\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]\right)\beta_1\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}/A\cosh\left[(1/2)\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right] + B\sinh\left[(1/2)\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]}. \tag{57}$$

I: if we choose $A \neq 0$, $B = 0$ then,

$$v_{19}(\xi) = \frac{-2\alpha_0 + \alpha_1\left(\lambda - \sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\,\mathrm{Tanh}\left[(1/2)\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]\right)}{-2\beta_0 + \beta_1\left(\lambda - \sqrt{(-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1}\,\mathrm{Tanh}\left[(1/2)\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]\right)},$$

$$u_{19}(\xi) = \left(\frac{-pk}{2c}\right)\left[\frac{-2\alpha_0 + \alpha_1\left(\lambda - \sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\,\mathrm{Tanh}\left[(1/2)\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]\right)}{-2\beta_0 + \beta_1\left(\lambda - \sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\,\mathrm{Tanh}\left[(1/2)\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]\right)}\right]^2. \tag{58}$$

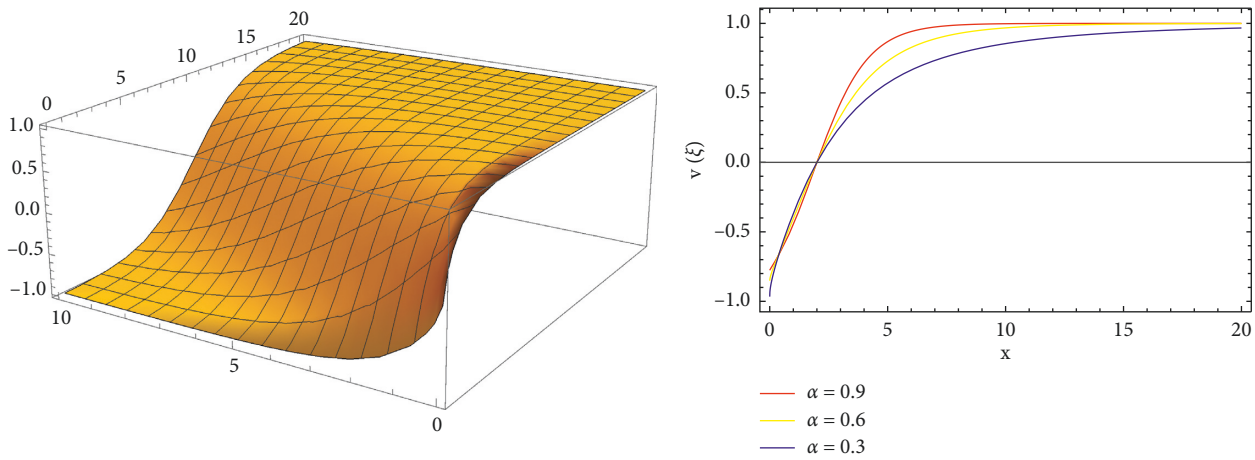
II: if we choose $B \neq 0$, $A = 0$ then,

$$\begin{aligned} v_{20}(\xi) &= \frac{-2\alpha_0 + \alpha_1\left(\lambda - \sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\,\mathrm{Coth}\left[(1/2)\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]\right)}{-2\beta_0 + \beta_1\left(\lambda - \sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\,\mathrm{Coth}\left[(1/2)\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1, \beta_1)}\right]\right)}, \\ u_{20}(\xi) &= \left(\frac{-pk}{2c}\right)\left[\frac{-2\alpha_0 + \alpha_1\left(\lambda - \sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\,\mathrm{Coth}\left[(1/2)\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]\right)}{-2\beta_0 + \beta_1\left(\lambda - \sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\,\mathrm{Coth}\left[(1/2)\xi\sqrt{((-2\alpha_0 + \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]\right)}\right]^2. \end{aligned} \tag{59}$$

Case 2. When $\omega = \lambda^2 - 4\mu < 0$, the trigonometric function solution becomes

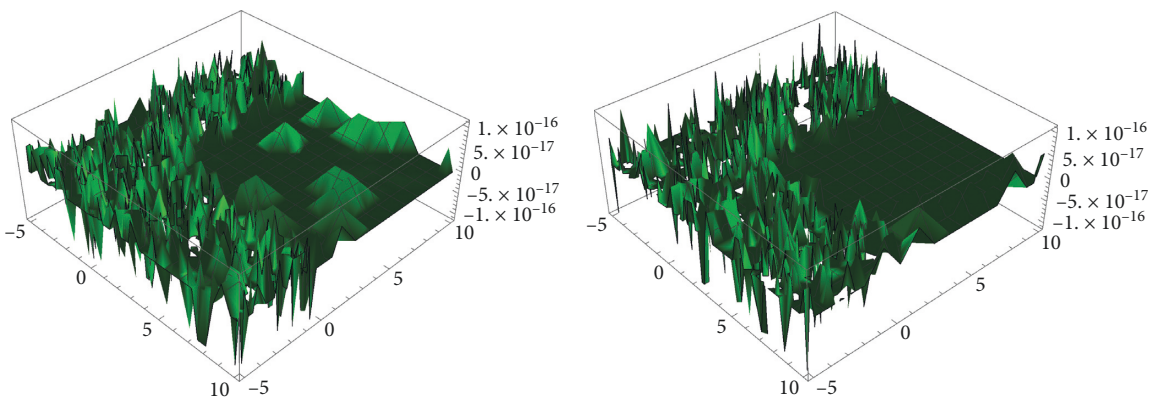
$$v(\xi) = \frac{2\alpha_0 - \lambda\alpha_1 + \sqrt{\Psi}\alpha_1 (-A\sin[(\sqrt{\Psi}/2)\xi] + B\cos[(\sqrt{\Psi}/2)\xi]/A\cos[(\sqrt{\Psi}/2)\xi] + B\sin[(\sqrt{\Psi}/2)\xi])}{2\beta_0 - \lambda\beta_1 + \sqrt{\Psi}\beta_1 (-A\sin[(\sqrt{\Psi}/2)\xi] + B\cos[(\sqrt{\Psi}/2)\xi]/A\cos[(\sqrt{\Psi}/2)\xi] + B\sin[(\sqrt{\Psi}/2)\xi])}, \tag{60}$$

where $\Psi = 4\mu - \lambda^2 > 0$, substituting (54) into (60), we have the following equation:



(a) (b)

Figure 1: Exact wave solutions $v_1(\xi)$ of fractional (DSW) system at $a_0 = 1, a_{-1} = -1, b_0 = 1, b_{-1} = 1, c = -1, k = 1$, and $\alpha = 0.7$, (a) 3D plot and (b) 2D plot at $t = 2$.



(a) (b)

Figure 2: Exact wave solutions $v_{11}(\xi)$ of fractional (DSW) system at $\alpha_1 = 1, \alpha_0 = 0, \beta_1 = -1, \beta_0 = 1, c = -0.125$, and $k = 0.5$, (a) 3D plot at $\alpha = 0.7$, and (b) 3D plot at $\alpha = 0.02$.

$$v_{21}(\xi) = \frac{2\alpha_0 - \lambda\alpha_1 + \left(B\cos\left[(1/2)\xi\sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right] - A\sin\left[(1/2)\xi\sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]\right)\alpha_1\sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}/A\cos\left[(1/2)\xi\sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right] + B\sinh\left[(1/2)\xi\sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]}{2\beta_0 - \lambda\beta_1 + \left(B\cos\left[(1/2)\xi\sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right] - A\sin\left[(1/2)\xi\sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]\right)\beta_1\sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}/A\cos\left[(1/2)\xi\sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right] + B\sinh\left[(1/2)\xi\sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]}. \quad (61)$$

I: if we choose $A \neq 0$ and $B = 0$, then

$$v_{21}(\xi) = \frac{-2\alpha_0 + \alpha_1\left(\lambda + \sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\,\mathrm{Tan}\left[(1/2)\xi\sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]\right)}{-2\beta_0 + \beta_1\left(\lambda + \sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\,\mathrm{Tan}\left[(1/2)\xi\sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]\right)},$$

$$u_{21}(\xi) = \left(\frac{-pk}{2c}\right)\left[\frac{-2\alpha_0 + \alpha_1\left(\lambda + \sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\,\mathrm{Tan}\left[(1/2)\xi\sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]\right)}{-2\beta_0 + \beta_1\left(\lambda + \sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\,\mathrm{Tan}\left[(1/2)\xi\sqrt{((2\alpha_0 - \lambda\alpha_1)(-2\beta_0 + \lambda\beta_1)/\alpha_1\beta_1)}\right]\right)}\right]^2. \quad (62)$$

II: if we choose $B \neq 0$ and $A = 0$, then

$$v_{22}(\xi)=\frac{\left(-2\alpha_0+\alpha_1\left(\lambda-\sqrt{((2\alpha_0-\lambda\alpha_1)(-2\beta_0+\lambda\beta_1)/\alpha_1\beta_1)}\,\mathrm{Cot}\left[(1/2)\xi\sqrt{((2\alpha_0-\lambda\alpha_1)(-2\beta_0+\lambda\beta_1)/\alpha_1\beta_1)}\right]\right)\right)}{-2\beta_0+\beta_1\left(\lambda-\sqrt{((2\alpha_0-\lambda\alpha_1)(-2\beta_0+\lambda\beta_1)/\alpha_1\beta_1)}\,\mathrm{Cot}\left[(1/2)\xi\sqrt{((2\alpha_0-\lambda\alpha_1)(-2\beta_0+\lambda\beta_1)/\alpha_1\beta_1)}\right]\right)},$$

$$u_{22}(\xi)=\left(\frac{-pk}{2c}\right)\left[\frac{-2\alpha_0+\alpha_1\left(\lambda-\sqrt{((2\alpha_0-\lambda\alpha_1)(-2\beta_0+\lambda\beta_1)/\alpha_1\beta_1)}\,\mathrm{Cot}\left[(1/2)\xi\sqrt{((2\alpha_0-\lambda\alpha_1)(-2\beta_0+\lambda\beta_1)/\alpha_1\beta_1)}\right]\right)}{-2\beta_0+\beta_1\left(\lambda-\sqrt{((2\alpha_0-\lambda\alpha_1)(-2\beta_0+\lambda\beta_1)/\alpha_1\beta_1)}\,\mathrm{Cot}\left[(1/2)\xi\sqrt{((2\alpha_0-\lambda\alpha_1)(-2\beta_0+\lambda\beta_1)/\alpha_1\beta_1)}\right]\right)}\right]^2, \tag{63}$$

where

$$\xi=\frac{kx^{\alpha}}{\alpha}+\frac{\left((1/2)\left(qk^3\left(-2\beta_0\alpha_1\lambda-2\alpha_0\beta_1\lambda+4\alpha_0\beta_0+\alpha_1\lambda^2\beta_1\right)/\beta_1\alpha_1\right)\right)t^{\alpha}}{\alpha}. \tag{64}$$

If we consider $\omega = 0$, then parameter c in the variable transformations equation (15) becomes $c = 0$. This contradicts the definition of the parameter c (Figure 1 and Figure 2).

5. Conclusion

In the study, we presented the exp-function method and the rational (G'/G)-expansion method to obtain the exact wave solution of the space-time fractional (DSW) system. Both methods are direct, concise, and powerful application in the fractional nonlinear partial differential equations arising in mathematical physics. Whereas the exp-function method gives more solutions than the rational (G'/G)-expansion method. In section (4.2), result.1, according to the value of parameter ω, we have only analytical exact wave solutions which include hyperbolic functions, because the value of ω can not be negative or zero. Also, in the result 2, we have hyperbolic and trigonometric function solutions of the space-time fractional (DSW) system, the parameter ω can not take the value of zero. However, these methods can be useful for the application of other nonlinear FDEs in mathematical physics. The solutions obtained from these two methods are inconvertible to each other. Both methods present new solutions to the space-time fractional (DSW) system. The figures of some obtained solutions are demonstrated to better understand their physical features, including wave solutions 3D. In conclusion, these presented methods are a powerful mathematical tool for solving nonlinear conformable fractional evolution equations in various fields of applied sciences, especially in physics.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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