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Research Article

Improvement of Image Compression by Changing the Mathematical Equation Style in Communication Systems

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Compression is an essential process to reduce the amount of information by reducing the number of bits; this process is necessary for uploading images, audio, video, storage services, and TV transmission. In this paper, image compressions with losses from this action will be shown for some common patterns. The compression process uses different mathematical equations that have different methods and efficiencies, so some common mathematical methods for each style are presented taking into consideration the pros and cons of each method. In this paper, it is demonstrated that there is a quality improvement by applying anisotropic interpolation to edge enhancement for its ability to satisfy the dispersed data of the propagation process, which leads to faster compression due to concern for optimum quality rather than fast algorithms. The test images for these patterns showed a discrepancy in the image resolution when the compression coefficient was increased, as the results using three types of image compression methods proved a clear superiority when using "partial differential equations (PDE)".

1. Introduction

Compression is an important area in high-performance digital image processing, as there are standard methods for reducing the amount of information in images. Compression allows storing multiple images in one place, speeds up the transfer of images from one medium to another, or shortens the image upload time to the Internet [1], but some obstacles occur due to compression, where the problem of compression operations lies in the loss of a part of the information commensurate with the increase in compression factor; that is, when reducing the size of the information to increase the download speed as well as the transfer speed, this reduces the purity of the image, sometimes up to distortion, so the best methods should be used to preserve image quality during compression. Several different methods of compressing images have been followed, depending on the purpose and methods of compression [2]. "JPEG" (joint photographic experts group), "GIF" (Graphics Interchange Format), and "PNG" (Portable Network Graphics) files are mainly used for Internet use, while GIF is used for graphics and animation. In compressed images, in most cases, a small amount of data is lost, and the states are not visible, so when compressing images, care is mainly taken to withstand the amount of data that can be lost, and the resolution of the bitmap (uncompressed) is usually 24 bits per image element and for image resolution (1024×786 bits) you need 2.3 MB, so most formats use some compression method to reduce memory usage. [1–3] The JPEG 2000 standard, which relies heavily on Discrete Wavelength Transformation (DWT) rather than DCT conversion, provides better quality than JPEG formats at high compression levels, and the PDE chosen is Edge Enhancement Spread (EED) which uses a spread tensor that smoothens opponents while preventing smoothing over them.

A characteristic of image compression is the loss of part of the image information due to the compression and decompression algorithms, but this does not greatly affect

human vision due to the limitations of noticing small differences, such as in animated films where one does not notice their sequence [3].

Developers and users of lossy image compression methods need a standard metrics method to measure the quality of the reconstructed images compared to the original image, the higher the standard value of the selected image. This value system should produce dimensionless numbers and these numbers should not be sensitive to small changes in the image reconstruction. A common scale used for this purpose is PSNR (Peak Signal-to-Noise Ratio). [1, 2]

2. Related Work

There are a lot of research and articles about image and information compression processes, but most of them dealt with one way to discuss it with a statement of the advantages and disadvantages of each type; as these articles can be benefited from previous experiences to reach continuous improvement that allows sufficient pressure to reduce the size of the information while preserving as much as possible its original size natural precompress, from these articles [3] and [4], and many other articles in this specialty have been used to gather as much information about image compression. So in this research, several types of compression methods were reviewed with a comparison in performance between them to reach scientific results that contribute to increasing the speed of information by reducing its size without affecting its quality.

3. Method

Compression is an important area in 'digital image processing' where there are standard methods for high-performance still and moving images. By various tests of the compressed image and comparison of the obtained results with the existing methods, it was concluded that the existing methods do not meet the expected quality; for this reason, a new compression procedure using different equations was applied and it was observed after a series of experiments that the test results improved, quality improvement was achieved by applying interpolation related to the anisotropic diffusion of edge improvement. Although this anisotropic diffusion equation with a diffusion tensor was originally proposed to remove the noise from the image, it will be shown to outperform many other partial differential equations when it is necessary to interpolate a very small amount of scattered data. The paper deals with the different methods of image compression and provides an overview of the different types of mathematical equations for image compression and video signal compression so that the development of compression processes that lead to fewer bits can be compared with the least possible losses in the amount of information.

Furthermore, the interpolation techniques based on partial differential equations are analyzed and it is shown that interpolation of scattered data with anisotropic diffusion has particularly good characteristics. Experiments show that for a very high degree of compression, this approach based on partial differential equations not only gives much better results than the widely-used JPEG standard but even the highly optimized JPEG 2000 codec.

4. Theory

Image and video compression refer to the reduction of information required for image display. It is achieved by removing one of the three basic redundancies:

(i) Code redundancy

(ii) Spatial and/or temporal redundancy (image sequences)

(iii) Irrelevant information, i.e., information that the human eye itself rejects because of imperfections [1, 5]

Image compression techniques (coding) reduce the number of bits required to represent the image by taking advantage of this redundancy; this is done at the transmitter, and a reverse process called decompression (decoding) is applied to the compressed data to obtain the reconstructed image at the receiving side as in Figure 1. The goal of compression is to reduce the number of bits as much as possible while maintaining the resolution and visual quality of the reconstructed image as close as possible to the original image [4, 6, 7].

If $N1$ and $N2$ denote the information carrier units (bits), u original and coded images, respectively, to describe the degree of compression, with the use of the term compression factor, then the compression can be expressed by the ratio:

Compression factor = number of bits of the compressed image/ the number of bits of the original image

$$\mathrm{Cr} = \frac{N1}{N2}. \tag{1}$$

In this case, a value of 0.02 means that the image data occupies 2% of its original size after summing up 2%. Values greater than 1 mean that the compressed image is larger than the original image [4].

5. Image Compression Standards

Current image compression methods are dominated by concepts involving discrete cosine transformation, such as the widely-used JPEG (joint photographic experts group) standard or discrete wavelet transformation in JPEG 2000. [5]

5.1. The Joint Photographic Experts Group (JPEG). JPEG is an image compression standard that was developed in 1992, it was accepted as an international standard, and it uses the DCT (discrete cosine transform) method for coding transformation. There are four modes of operation of the JPEG standard: sequential coding, progressive coding, lossless coding, and hierarchical coding. Sequential coding is a basic model based on DCT (discrete cosine transform) transformation on 8×8 blocks, where each part of the image is coded in one pass from left to right and from top to bottom. Progressive coding is also based on DCT transformation on 8×8

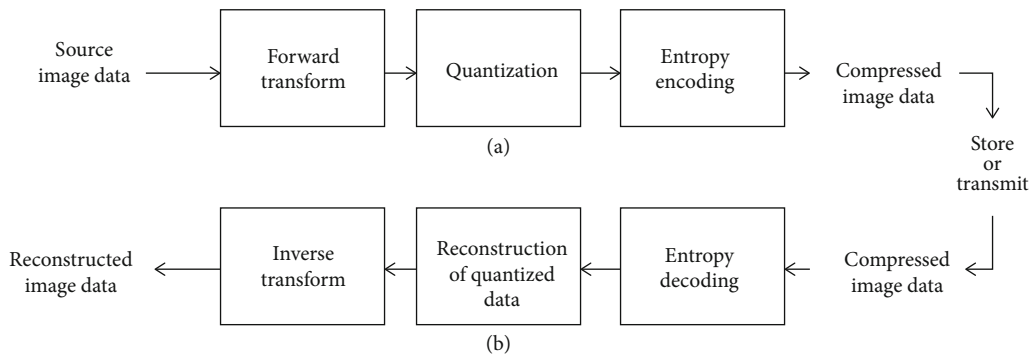


Figure 1: A general block diagram of image compression systems: (a) encoder and (b) decoder.

blocks. Encode images in multiple passes to quickly obtain a coarse decoded image when the transmission time is long. The basic JPEG mode is a data loss mode it uses (DCT), which is lossless coding. Quantization reduces the number of bits required to transmit coefficients that lead to losses. It also uses zigzag scanning (Figure 2), where the quantized DCT coefficients have values of 0 in the lower right part of the matrix. Zigzag maps the 8×8 vector of DCT coefficients to a 1×64 vector; smaller coefficients (0) are located at the end of the vector. [4–6]

5.1.1. Discrete Cosine Transform (DCT). The main advantage of DCT over other transfer-coding procedures is the static computation algorithm, so the essence of the transformation is the same for all blocks where it is possible to perform in real-time, i.e., the compressed data is rapidly generated as input data.

DCT is applied to $M \times N$ blocks of the image, and the result is $M \times N$ blocks of DCT coefficients:

$$f^{\sim}(u,v) = \alpha(u)\alpha(v) \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(x,y) \cos \frac{(2x+1)u\pi}{2M} \cos \frac{(2y+1)v\pi}{2N}, \tag{2}$$

where $f(x,y)$ is the image element at position (i,j), $f \sim (u,v)$ is the DCT coefficient at position (u,v) for $0 \le u < M$ and $0 \le v < N$, and $\alpha(u)$ and $\alpha(v)$ scaling functions given by the expression:

$$\alpha(u) = \begin{cases} \sqrt{\dfrac{1}{M}} & u = 0, \\ \sqrt{\dfrac{2}{M}} & \text{otherwise}, \end{cases}$$
$$\alpha(v) = \begin{cases} \sqrt{\dfrac{1}{N}} & v = 0, \\ \sqrt{\dfrac{2}{N}} & \text{otherwise}. \end{cases} \tag{3}$$

The inverse discrete cosine transform returns data from the frequency to the spatial domain. The inverse transformation agrees with a two-dimensional by combining the basic waveforms as determined by the DCT coefficient of the team $f^{\sim}(u,v)$ [8, 9].

$$f(u,v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \alpha(u)\alpha(v) f^{\sim}(u,v) \cos \frac{(2x+1)u\pi}{2M} \cos \frac{(2y+1)v\pi}{2N}. \tag{4}$$

For the DCT transformation for a block of 8×8 image elements, the equation holds

$$f^{\sim}(u,v) = \alpha(u)\alpha(v) \sum_{m=0}^{7} \sum_{n=0}^{7} f(x,y) \cos \frac{(2x+1)u\pi}{16} \cos \frac{(2y+1)v\pi}{16}. \tag{5}$$

Each 8×8 blocks of image elements can be fully represented by a linear combination of 64 basic waveforms. The basic waveforms of the DCT transformation for a block size of 8×8 image elements are shown in Figure 3.

The inverse DCT for a block of 8×8 image elements is then

$$f(u,v) = \sum_{m=0}^{7} \sum_{n=0}^{7} \alpha(u)\alpha(v) f^{\sim}(u,v) \cos \frac{(2x+1)u\pi}{16} \cos \frac{(2y+1)v\pi}{16}. \tag{6}$$

5.2. JPEG 2000. The official name of the standard is ISO/IEC 15444-1: 2000. ITU-T Recommendation BT. The T.800 also includes the JPEG 2000 standard, which relies heavily on DWT and has additional features (metadata, progressive image transfer). The most obvious is the removal of the visibility of the block boundaries in the image compared to the JPEG standard.

JPEG 2000 is designed to meet the requirements of various applications, for example, Internet, color faxing, printing, scanning, digital photography, remote reading, mobile phone applications, medical imaging, and digital libraries.

JPEG 2000 is based on DWT, numerical quantization, contextual modeling, adaptive computer coding, and post-compression rate assignment. The main purpose of wave analysis is to obtain different approximate estimates of images of different levels of resolution.

0	1	5	6	14	15	27	28
2	4	7	13	16	26	29	42
3	8	12	17	25	30	41	43
9	11	18	24	31	40	44	53
10	19	23	32	39	45	52	54
20	22	33	38	46	51	55	60
21	34	37	47	50	56	59	61
35	36	48	49	57	58	62	63

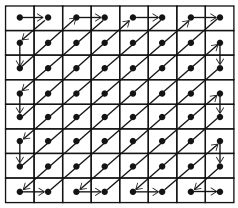


Figure 2: Zigzag mapping of the matrix.

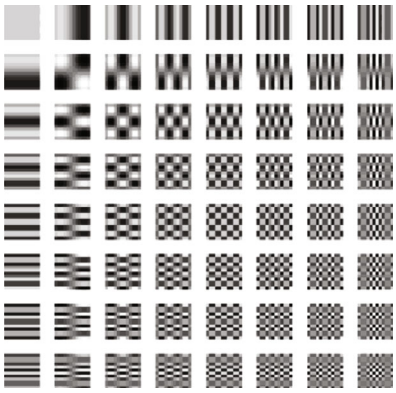


Figure 3: Basic waveforms of 8×8 DCT transformation.

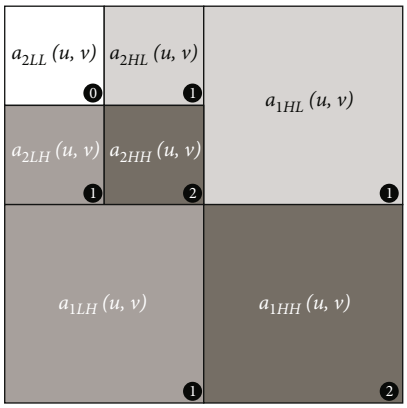


Figure 4: The image is divided into blocks.

The original image is disassembled into rectangular tiles, Figure 4, the image must be divided into blocks because in some cases the image may be too large for the amount of free coding memory, so the blocks are compressed independently as if they were separate images. All blocks have the same dimensions, except for those on the right and bottom border of the image [4–6].

JPEG 2000 has two major disadvantages, it does not coherently modulate the total image (outliers) and its quality degrades when images with a low color depth, such as graphic images, are compressed. Analytics quality or scalability, this issue is a major limitation in terms of using lossless compression in binary images; but at very high compression levels, the perimeter of blocks in the reconstructed image becomes visible. Another problem with JPEG 2000 is that it is too intense in terms of arithmetic, which is not good for digital cameras with limited processing power [4, 7, 10].

5.2.1. Discrete Wavelet Transformation (DWT). The frequency band is divided recursively into two equal parts using octave digital filters. The low-pass filter at the output gives a rough approximation of the signal, and the high-pass filter gives details of the signal. This method is used in the JPEG 2000 standard. If we add the spectrum of the scaling function and the spectrum of the wavelet on the $j+1$ scale [10, 11] this can be expressed by an expression:

$$\varphi\left(2^{j}t\right) = \sum_{k} h_{j+1}(k)\varphi\left(2^{j+1}t - k\right). \tag{7}$$

Since the first scaling function replaces a series of wavelet functions, the expression holds:

$$\Psi\left(2^{j}t\right) = \sum_{k} g_{j+1}(k)\varphi\left(2^{j+1}t - k\right). \tag{8}$$

The signal can be reconstructed from orthogonal wavelet and wavelet transformation coefficients according to the expression:

$$f(t) = \sum_{k} \lambda g_{j-1}(k)\varphi\left(2^{j-1}t - k\right) + \sum_{k} \gamma h_{j-1}(k)\Psi\left(2^{j-1}t - k\right). \tag{9}$$

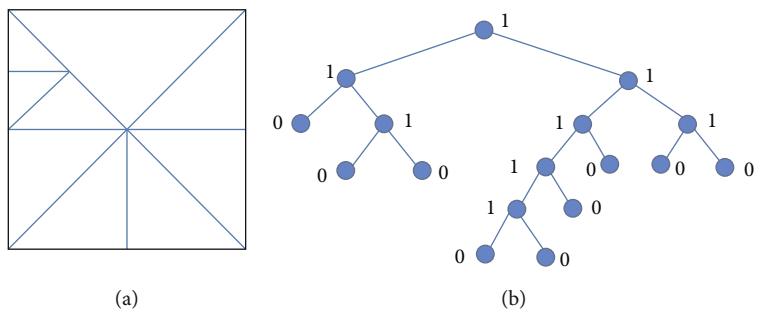
If the scaling function $\varphi j, k(t)$ and the wavelet function $\psi j, k(t)$ are orthonormal, then the discrete wavelet coefficients can be calculated as the convolution of the discrete signal with the wavelet and scaling functions. Low-frequency DWT coefficients are formed by the convolution of signals and scaling functions according to the expression:

$$\lambda_{j-1}(k) = \{f(t), \varphi j, k(t)\} = \sum_{m} h(m - 2k)\lambda_{j(m)}, \tag{10}$$

and DWT coefficients describing details (high frequencies) on the j-th scale, which were obtained by convolution of signals and wavelet functions according to the expression:

$$\gamma_{j-1}(k) = \{f(t), \psi j, k(t)\} = \sum_{m} g(m - 2k)\gamma_{j}(m). \tag{11}$$

It can be concluded that the coefficients on a scale can be calculated based on the coefficients of the previous scale. $h(k)$ is the weighting factor corresponding to the characteristic of a low-pass digital scaling filter, and '$g(k)$' is a digital high-pass wavelet filter. In expressions (10) and (11), the factor k is followed by the number 2, which denotes the subsampling factor. It is possible to design a decomposition and reconstruction function of finite duration, which is very suitable for a concise description of signals of variable properties. [12–14]



(a) (b)

Figure 5: (a) Separation of the image into triangular areas, (b) corresponding binary tree.

6. Compression by Partial Differential Equations (PDE)

Current image compression methods are dominated by concepts that involve discrete cosine transformations (such as the widely-used JPEG standard or discrete wavelike transformation (in JPEG 2000)) and will confirm the idea that there are other possibilities where differential equations may be useful [15, 16].

Decoding is achieved using this scattered data and its interpolation using an appropriate PDE. The partial differential equation chosen is EED. It uses a diffusion tensor that allows smoothing of antagonists while preventing smoothing over them. Although the EED is original and presented as a method to remove noise from the figure, it is particularly useful for interpolation of scattered data [4, 17].

Images can be successfully encoded using binary trees. These binary trees can be used as the backbone of the EED-based codec. However, let us end up with a compression tire, which is based on partial differential equations with perfect quality.

PDE modeling: we use the Laplace equation of the form:

$$\partial^2\mathbf{u}/\partial\mathbf{x}^2 + \partial^2\mathbf{u}/\partial\mathbf{y}^2 = \mathbf{0}, \tag{12}$$

where $\mathbf{x}$ and $\mathbf{y}$ are spatial independent variables in coordinates. Note that with no derivatives in t, Laplace's equation required preconditions, so as not to adopt the possibility of time. The solution is in a rectangular field $(m+1)$ by $(n+1)$, according to the terms of the model boundaries:

$$\begin{aligned} &\mathbf{u}(\mathbf{x},0) = F(\mathbf{x}), \quad \mathbf{u}(\mathbf{x},b) = G(\mathbf{x}), \quad 0 < \mathbf{x} < a, \\ &\mathbf{u}(0,\mathbf{y}) = P(\mathbf{y}), \quad \mathbf{u}(a,\mathbf{y}) = Q(\mathbf{y}), \quad 0 < \mathbf{y} < b. \end{aligned} \tag{13}$$

The first row and the last row of the field are the experimental data; this will lead to inserting the ends of the peaks between the two levels to restore the properties of the original network. The solution is as follows:

$$\mathbf{u}(\mathbf{x},\mathbf{y}) = \sum_{n=1}^{\infty} f_n a_n(a,y) + g_n a_n(x,b-y) + p_n b_n(x,y) + g_n b_n(a-x,y), \tag{14}$$

and the constants f_n, g_n, p_n, and q_n are coefficients of the function, this implies that [18, 19]

$$\begin{aligned} F(\mathbf{x}) &= \sum_{n=1}^{\infty} f_n \sin\left\lceil\frac{n\pi x}{a}\right\rceil, \\ G(\mathbf{x}) &= \sum_{n=1}^{\infty} g_n \sin\left\lceil\frac{n\pi x}{a}\right\rceil, \\ P(\mathbf{y}) &= \sum_{n=1}^{\infty} p_n \sin\left\lceil\frac{n\pi y}{b}\right\rceil, \\ Q(\mathbf{y}) &= \sum_{n=1}^{\infty} q_n \sin\left\lceil\frac{n\pi y}{b}\right\rceil. \end{aligned} \tag{15}$$

In the results inferred from the experiments, the Dirichlet boundary conditions are used by determining the value of the vertices at the boundaries of the rectangle. [19]

6.1. The EED-Based Method. The image compression and decompression scheme that relies on adaptive dilution of image data by triangulation with B-Tree Triple Coding (BTTC) is relatively simple and allows for efficient coding.

To store the triangulation efficiently, we note that the hierarchical division of the triangle leads to the structure of the binary tree encoding. Each triangle that melts us down during the division process is represented by a knot, while the leaves correspond to those triangles that are no longer divided. To store the structure of the tree, a tour of the tree is performed and one bit per node is stored: 1 for a node that has a branch and 0 for a list Figure 5.

EED performs well as an interpolator for sparse data, as it does not rely on triangulation but only on its headers as implicit points, so it optimizes the step of creating sparse interpolation points and stored brightness levels. Since the size of the triangle decreases exponentially with the level index of the binary tree, exponential scaling is also chosen for the threshold. Starting from the small threshold (ε_0) at the roughest level, which is adjusted by a constant factor $\alpha > 1$ per level (steadily in the range between 1.35 and 1.5), which results in

$$\varepsilon_k = \alpha^k \varepsilon_0, \tag{16}$$

at the k level of the binary tree.

(a) (b) (c)

Figure 6: Test image, left to right: (a) JPEG, (b) JPEG 2000, and (c) EED (0.8 bpp).

(a) (b) (c)

Figure 7: Test image left to right: (a) JPEG, (b) JPEG 2000, and (c) EED (0.2 bpp). It is a large mosque that was built in the ninth century by Al-Mutawakkil, the ruler of the Abbasid Empire in the city of Samarra/Iraq.

The EED-based method makes it possible to make all decisions to divide an equal-sized triangle at the same time, with recalculations being limited to one fulfillment for each level of triangulation. This results in faster compression due to less image quality loss, as it is more important to pay attention to the optimal quality than to fast algorithms.

7. Experimental Results Comparison with JPEG, JPEG2000 and EED Standards

A comparison is made between JPEG and JPEG 2000 compression procedures with EED, as they are compression methods that apply different mathematical and equation methods, the maximum values of interpolation and postprocessing by regressing from 256 to 32 degrees brightness level, the adaptive error limit parameter was chosen to achieve compression methods at (0.8 and 0.2) bits per pixel (bpp). A test image (the historic minaret of Samarra—it is a mosque that was built in the ninth century during the Abbasid Empire in the city of Samarra/Iraq) was compressed, measuring 226×223 pixels. Images decoded with a 0.8 bpp image compression ratio using JPEG, JPEG 2000, and EED are shown in Figure 6.

Images decoded with an image compression ratio of 0.2 bpp using JPEG, JPEG 2000, and EED are shown in Figure 7.

8. Results and Discussion

Images decoded with a 0.8 bpp image compression ratio using EEDC, JPEG, and JPEG 2000 are shown in Figure 6. It can be seen from the images that there is no significant degradation due to compression by any of the methods tested; the pictures are of exceptional quality, as every detail in the image can be recognized in Table 1, where we can observe the value of the average absolute errors (AAE) of the image with the different compression degrees.

There are significant quality differences between the decoded images in Figure 7; the images shown above after the encoder is opened and compressed at 0.2 bits per pixel using, JPEG, JPEG 2000, and EED as in Figure 7. We note that there are substantial differences in the image format, as the image in Figure 7(a) was a JPEG, that is, used mathematical equations DCT among the worst images because it does not recognize details except for returning to the original shape of the image, where the block distortions dominate the image. The images in Figures 7(b) and 7(c) that use the

Table 1: The average absolute error value (AAE) for the image with different degrees of compression.

Compression degrees (bpp)	AAE (JPEG)	AAE (EDD)	AAE (JPEG 2000)
0.2	11.25	4.99	4.84
0.4	8.65	3.57	3.66
0.8	2.24	2.52	1.97

format (JPEG 2000, EED) contain details close to the original image and are acceptable for comparison with the previous pattern because they contain the most details, which mean that EED and JPEG 2000 are much better than JPEG.

9. Conclusions

The process of compressing the image is essential to reduce the size of digital information and to reduce the number of bits, thus ensuring the process of transmission and reception through the ascending line and downline in various digital communication fields (cell phones, Internet, television broadcasting, image, etc.), but all methods have the advantage of losing part of the information during the compression and decompression process. Three different methods are presented. Each method uses different mathematical equations (DCT, PDE, and DWT). The partial differential equations are characterized using interpolation characteristics related to the improvement of diffusion edges (edge enhancement diffusion (EED)) when it is necessary to satisfy a very small number of scattered data. The experiments conducted demonstrate that for a high degree of compression, the one-shot partial differentiation approach gives much better results than the widely-used JPEG standard and can also surpass the quality of the highly enhanced JPEG 2000 codec. We hope in the upcoming works to expand future research to include improving compression for video signals, which requires great effort because the video signal represents about 50-60 images per second.

Data Availability

All data, results, and tables are available in the search in a clear detailed manner. It is possible to refer to the results of the equations as well as the test images and the table in the research.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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