Hindawi
International Journal of Ecology
Volume 2023, Article ID 1457714, 11 pages
https://doi.org/10.1155/2023/1457714

Research Article

Surface Temperature Influences the Population of *Limnothrissa miodon* in Lake Kariba

Farikayi K. Mutasa [1], Brian Jones [2], Itai H. Tendaupenyu [3,4], Tamuka Nhiwatiwa [5], and Mzime R. Ndebele-Murisa [6]

[1]*Department of Applied Mathematics, National University of Science and Technology, P.O. Box AC939, Ascot, Bulawayo, Zimbabwe*
[2]*Department of Statistics and Operations Research, National University of Science and Technology, P.O. Box AC939, Ascot, Bulawayo, Zimbabwe*
[3]*The Zimbabwe Parks and Wildlife Management Authority, P.O. Box CY140, Causeway, Harare, Zimbabwe*
[4]*Botanical Gardens, P.O. Box CY140, Causeway, Harare, Zimbabwe*
[5]*Department of Biological Sciences, University of Zimbabwe, P.O. MP167 Mt. Pleasant, Harare, Zimbabwe*
[6]*School of Wildlife and Ecology, Chinhoyi University of Technology, Chinhoyi, Zimbabwe*

Correspondence should be addressed to Farikayi K. Mutasa; farikayi.mutasa@nust.ac.zw

Received 12 August 2022; Revised 20 March 2023; Accepted 20 April 2023; Published 10 May 2023

Academic Editor: Ram Chander Sihag

Global warming is a serious world problem where earth's temperature has been reported to increase over the years; the aquatic ecosystems are also not the exceptions. But, the effects of this phenomenon on the aquatic ecosystems are not well understood. This study aims to understand the influence of surface temperature on the population density of *Limnothrissa miodon* in Lake Kariba. We constructed a mathematical model on the population dynamics of *Limnothrissa miodon* with nutrients, phytoplankton, zooplankton, and *Hydrocynus vittatus*. Lake surface water temperature was modelled by a cosine function, and the parameters were estimated from data fitting. Numerical simulations were used to determine the stability of the nonautonomous model. Numerical simulation results of the nonautonomous model showed a stable periodic orbit for varying initial conditions, and therefore, instability. Numerical techniques were used to investigate the influence of surface water temperature on *Limnothrissa miodon*. Results from the model with fitted lake surface water temperature data showed that a shift in the optimal temperature for phytoplankton growth from 25°C to 34°C, corresponding to dominance of Cyanophyceae over Chlorophyceae, resulted in a decline in the population density of *Limnothrissa miodon*. Numerical results showed that the population density of *Limnothrissa miodon* declines after an optimum temperature of 30°C for phytoplankton growth. Numerical simulation results suggested that warming of the lake may lead to a decline in *Limnothrissa miodon* population density in Lake Kariba.

1. Introduction

Global warming is one of the major issues confronting the modern civilization [1]. The major concern is the increasing temperature of the biosphere; the aquatic ecosystems are not an exception. Major concern of the aquatic biologists is to understand the adverse effects of this phenomenon on the aquatic communities [2]. With this purpose, this study was conducted to determine the influence of lake surface water temperature on the population density of *Limnothrissa miodon* in Lake Kariba. The Lake Kariba fishery which is shared by both Zimbabwe and Zambia is of paramount importance for the continued survival of the small pelagic clupeid, *Limnothrissa miodon* [3]. *Limnothrissa miodon* (Boulenger, 1906), also referred to as kapenta, is a major source of protein and income for fishing cooperatives, wholesalers, retailers, and the local community [4]. Reports that the kapenta catches are in decline are a cause of concern to many stakeholders in the kapenta fishery and academia. The decline in the catches in kapenta has been attributed to overfishing, predation by *Hydrocynus vittatus*, climate change, and the reduction of nutrient inflow into Lake Kariba. According to Nhambura [5], several cooperatives and companies which make up the Kariba kapenta industry are struggling and facing collapse since the amounts of kapenta in Lake Kariba are dwindling because of the changing climate. The depletion of kapenta may also have a negative effect on the levels of *Hydrocynus vittatus* (tigerfish), which feeds on kapenta and contributes immensely to the economy of Kariba. Therefore, it is important that we formulate and analyse a mathematical model in order to understand the effects of temperature on the population density of kapenta in Lake Kariba.

Ndebele-Murisa [6], recorded the relative cellular concentration of phytoplankton classes in the Sanyati basin of Lake Kariba. The highest concentration was in the less palatable Cyanophyceae (78.05%), followed by Chlorophyceae (9.7%), Dinophyceae (7.2%), Bacillariophyceae (1.7%), Euglenophyceae (1.6%), Chrysophyceae (1.3%), and Xanthophyceae (0.45%). The species with the highest relative concentration of 66.20% was *Cylindrospermopsis raciborskii*, followed by *Microcystis aeruginosa* (2.79%) from the Cyanophyceae class. From the Chlorophyceae class, *Staurastrum johnsonii* (0.67%) had the highest relative concentration, followed by *Coelastrummicroponum* (0.61%) and *Oedogonium* sp. (0.60%). Ndebele-Murisa [6] also recorded the relative concentrations of the Chlorophyceae species *Staurastrum johnsonii* (0.67%), *Coelastrum microponum* (0.61%), and *Oedogonium* sp. (0.60%) in the Sanyati basin. Cynaphytes are associated with the production of toxins [7, 8] and are often unigestible, indigestible, or nutritionally poor [9, 10]. Many species of Chlorophytes are less palatable due to long splines and long processes [11]. In Lake Kariba, *Limnothrissa miodon* mainly feeds on zooplankton, especially the Cladoceran species *Bosmina longirostris* [12, 13]. Machena [14] found out that 99% of the diet composition of *Limnothrissa miodon* is zooplankton. According to Mhlanga [15], *Limnothrissa miodon* is a major prey species of *Hydrocynus vittatus*. Kenmuir [16] found out that 70% of the diet of *Hydrocynus vittatus* consisted of *Limnothrissa miodon*. Environmental factors play an important role in a fishery [17–19], and as a result, it is critical to investigate mathematically the role of temperature in the dynamics of *Limnothrissa miodon* in the Lake Kariba fishery. Chifamba [17] found that maximum temperature was the best predictor of catch per unit effort (CPUE) of kapenta for the period 1970−1996 and suggested that kapenta catch cycles might be related to weather conditions. Magadza [18] estimated a 100−year warming of 4.8°C for the Lake Kariba area, using data from the Kariba meteorological station for the period 1965−2000. The lake warmed by a mean of 1.54°C between 1965 and 1990, corresponding to a warming rate of 0.62°C per decade [18]. Magadza [18] used temperature data from 1965−2000 and determined an air temperature and water temperature maximum turning point of 34.8°C and 28.7°C, respectively, around 1987 − 88 for *Limnothrissa miodon* in Lake Kariba. The kapenta catches increased exponentially prior to the turning point and then decreased linearly afterward. Regression analysis results from a study by Ndebele-Murisa et al. [19] showed that temperature is a major factor in kapenta catches from 1968−2008. Ndebele-Murisa et al. [20] found that maximum temperature contributed to 72% of the variation in kapenta stocks, 68% of kapenta CPUE, and 99% of the change in lake water levels from 1974 to 2010 in Lake Kariba. Sibanda [21] compared the temperature responses of selected phytoplankton classes in laboratory cultures, focusing on Chlorophyceae and Cyanophyceae. According to her findings, the growth rate of Chlorophyceae decreased above 25°C, while that of Cyanophyceae, on the other hand, increased almost exponentially up to 34°C. Magadza [18] observed a transition from Chlorophyceae to Cyanophyceae in Lake Kariba and found out that Cyanophyceae, particularly *Cylindrospermum raciborskii*, now dominate the lake phytoplankton.

How the temperature describes and influences the dynamics of the kapenta population in Lake Kariba has not been studied. A deterministic model involving nutrients, phytoplankton, zooplankton, kapenta, and tigerfish, as well as temperature as an environmental factor, is yet to be developed and tested. The kapenta model will aid in our understanding of the aquatic ecosystem dynamics in the kapenta fishery in Lake Kariba and we will be able to explain the influence of lake surface water temperature on kapenta levels. We hypothesized that (i) a shift from Chlorophyceae to Cyanophyceae will lead to a decline in kapenta population density and (ii) warming of the lake has an adverse effect on the population density of kapenta. The objectives of the study were to (i) formulate and analyse a mathematical model with data fitting, (ii) determine the optimum temperature for phytoplankton growth for the fitted lake surface water temperature data, (iii) compare numerical results on kapenta population density for the mathematical models with the Chlorophyceae and Cyanophyceae phytoplankton classes, and (iv) determine the effect of lake warming on kapenta population density.

2. Materials and Methods

2.1. Study Area. Lake Kariba is located in a tropical area with seasonal rainfall on the Zambezi river between latitudes 16°28′ to 18°04′ S and longitudes 26°42′ to 29°03′E [22], has a volume and surface area of 160 km^3 and 5364 km^2, respectively, and has an average width of 19.4 km, with the widest section measuring 40 km [23]. The lake is 486 m above sea level, and the shoreline is about 2164 km long [3, 23]. The kapenta fishery in Lake Kariba is highly mechanised, licence-controlled, and is shared by Zimbabwe and Zambia [6]. A map of Lake Kariba and its fishing basins is shown in Figure 1.

2.2. Data Collection. The surface water temperature data for Lake Kariba were provided by the University of Zimbabwe Lake Kariba Research Station (UZLKRS), and the data were collected at Station 7, Sanyati mouth, 1st cross, Gachegache

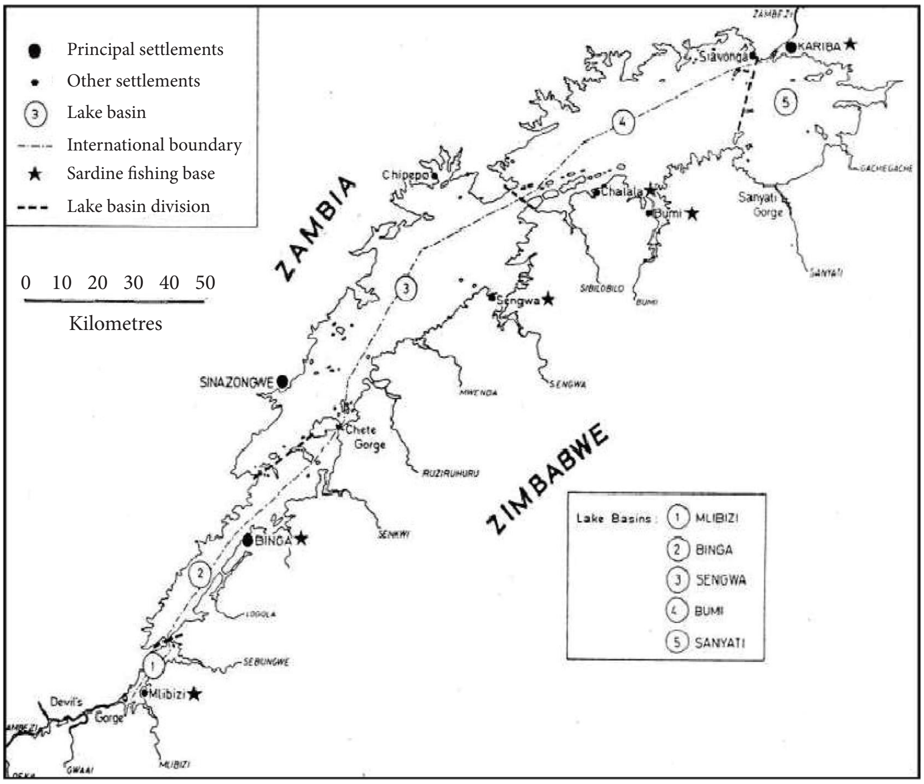

Figure 1: Fishing basins in Lake Kariba [24].

mouth, Charara, Nyaodza, and Nyenje for the period July 2014 to December 2017 [25].

3. Model Formulation

The model has 5 classes whose densities are functions of time: $N(t)$ denoting the concentration of nutrients, $P(t)$ is the population density of phytoplankton, $Z(t)$ is the zooplankton population density, $L(t)$ is the density of the *Limnothrissa miodon* population, and $R(t)$ is the density of the *Hydrocynus vittatus* population. The *Limnothrissa miodon* model [26] is developed to include the effect of temperature on growth of phytoplankton. The coefficient of temperature for the growth of phytoplankton [27] is assumed to be

$$g(T) = e^{-2.3\left|\left(T-T_{opt}\right)/\left(T_{opt}-T_{\min}\right)\right|}, \tag{1}$$

where $T = T(t)$ is lake surface water temperature, T_{opt} is optimal temperature for phytoplankton growth, and $T_{\min}$ is the minimum temperature for phytoplankton growth. The water temperature is modelled by

$$T(t) = b + c\cos(\omega t + \phi), \tag{2}$$

where the parameters b, c, ω, and ϕ are estimated from data fitting. $T(t)$ is the fitted temperature in C, b is the mean water temperature, c is the amplitude, ω is the angular frequency $(2\pi/12)$, t is time in months, and ϕ is a phase angle. The nonlinear dynamical system is

$$\begin{cases} \dfrac{dN}{dt} = \pi_0 - \alpha_0 N - \beta_1 NP, \\ \dfrac{dP}{dt} = \pi_1 g(T(t))NP - \alpha_1 P - \beta_2 PZ, \\ \dfrac{dZ}{dt} = \pi_2 PZ - \alpha_2 Z - \beta_3 ZL, \\ \dfrac{dL}{dt} = \pi_3 ZL - \alpha_3 L - \sigma L^2 - qEL - \dfrac{\beta_4 LR}{d+L}, \\ \dfrac{dR}{dt} = \dfrac{\pi_4 LR}{d+L} - \kappa\eta R - \alpha_4 R, \end{cases} \tag{3}$$

with initial condition

$$\begin{cases} N(0) = \psi_1(0), P(0) = \psi_2(0), \\ Z(0) = \psi_3(0), L(0) = \psi_4(0), \\ R(0) = \psi_5(0), \psi_i(0) > 0, i = 1,2,3,4,5, \end{cases} \tag{4}$$

and defined as

$$\Omega = \left\{(N,P,Z,L,R) \in \mathbb{R}^5 \mid N \geq 0, P \geq 0, Z \geq 0, L \geq 0, R \geq 0\right\}. \tag{5}$$

To be the mathematically feasible region, nutrients enter the water body at the rate π_0 where $\pi_0 > 0$ is a constant; β_1 is an uptake rate; and β_2, β_3, and β_4 are the predation rate coefficients. The α_i's for $i = 0, 1, 2, 3, 4$ are depletion rate coefficients. The π_i's for $i = 1, 2, 3, 4$ are the phytoplankton, zooplankton, *Limnothrissa miodon*, and *Hydrocynus vittatus* growth rate coefficients, respectively. The coefficient σ is a positive constant for the crowding of the *Limnothrissa miodon* population. Kapenta are harvested at a rate qEL, where q is the catchability coefficient and E is the effort measured as boat nights. The growth rate of *Hydrocynus vittatus* is $(\pi_4 ZL/d + L)$ [28] and d is the population density of kapenta at which specific growth rate becomes half of its saturation value. The tigerfish are harvested at a rate $\kappa\eta R$, where κ is the catchability coefficient and η is the effort.

3.1. Positivity of Solutions

Theorem 1. *Let the initial data be* $N(0) \geq 0, P(0) \geq 0, Z(0) \geq 0, L(0) \geq 0, R(0) \geq 0$. *Then, solutions of* $N(t)$, $P(t)$, $Z(t)$, $L(t)$, *and* $R(t)$ *of system (3) are positive* $\forall t \geq 0$.

Proof 1. It follows from the first equation of system (3) that

$$\dot{N}(t) \geq -\alpha_0 N(t) - \beta_1 N(t)P(t), \forall t \in [0, T], \tag{6}$$

and we obtain

$$N(t) \geq N(0)\exp\int_0^t (-\alpha_0 - \beta_1 P(u))\mathrm{d}u \geq 0, \forall t \in [0, T]. \tag{7}$$

From the second equation of system (3),

$$\dot{P}(t) \geq -\alpha_1 P(t) - \beta_2 P(t)Z(t), \forall t \in [0, T]. \tag{8}$$

By integrating (8), we obtain

$$P(t) \geq P(0)\exp\int_0^t (-\alpha_1 - \beta_2 Z(u))\mathrm{d}u \geq 0, \forall t \in [0, T]. \tag{9}$$

From the third equation of model (3),

$$\dot{Z}(t) \geq -\alpha_2 Z(t) - \beta_3 Z(t)L(t), \forall t \in [0, T], \tag{10}$$

and integrating (10) results in

$$Z(t) \geq Z(0)\exp\int_0^t (-\alpha_2 - \beta_3 L(u))\mathrm{d}u \geq 0, \forall t \in [0, T]. \tag{11}$$

Considering the fourth equation of system (3),

$$\dot{L}(t) \geq -\left(\alpha_3 + qE + \sigma L(t) + \frac{\beta_4 R(t)}{d + L(t)}\right)L(t), \forall t \in [0, T], \tag{12}$$

and integration of (12) results in

$$L(t) \geq L(0)\exp\left[-\int_0^t \left(\alpha_3 + qE + \sigma L(u) + \frac{\beta_4 R(u)}{d + L(u)}\right)du\right] \geq 0, \forall t \in [0, T]. \tag{13}$$

According to the fifth equation of model (3),

$$\dot{R}(t) \geq -(\alpha_4 + \kappa\eta)R(t), \forall t \in [0, T]. \tag{14}$$

Integration of (14) yields

$$R(t) \geq R(0)\exp(-(\alpha_4 + \kappa\eta)t) \geq 0, \forall t \in [0, T]. \tag{15}$$

We therefore conclude that solutions of model (3) with initial conditions (4) remain positive $\forall t \geq 0$. □

3.2. Existence of Solutions

Theorem 2. *A solution of model system (3) is feasible.*

Proof 2. It is essential to demonstrate that all feasible solutions of model (3) are uniformly bounded in $\Omega \subset \mathbb{R}^5$. Let $\{(N(t), P(t), Z(t), L(t), \text{and } R(t)) \in \mathbb{R}^5\}$ be any solution of system (3) with non-negative initial conditions. Let $W(t) = N(t) + P(t) + Z(t) + L(t) + R(t)$, then

$$\begin{aligned}
\frac{dW}{dt} &= \pi_0 - \alpha_0 N - \beta_1 NP + \pi_1 g(T(t))NP - \alpha_1 P - \beta_2 PZ + \pi_2 PZ - \alpha_2 Z - \beta_3 ZL \\
&\quad + \pi_3 ZL - \alpha_3 L - \sigma L^2 - qEL - \frac{\beta_4 LR}{d + L} + \frac{\pi_4 LR}{d + L} - \kappa\eta R - \alpha_4 R, \\
&= \pi_0 - \alpha_0 N - \alpha_1 P - \alpha_2 Z - (\alpha_3 + qE)L - \sigma L^2 - \kappa\eta R - \alpha_4 R \\
&\quad + (\pi_1 g(T(t)) - \beta_1)NP + (\pi_2 - \beta_2)PZ + (\pi_3 - \beta_3)ZL + (\pi_4 - \beta_4)\frac{LR}{d + L}, \\
&\leq \pi_0 - \alpha_0 N - \alpha_1 P - \alpha_2 Z - (\alpha_3 + qE)L - (\kappa\eta + \alpha_4)R, \\
&\leq \pi_0 - \nu W(t),
\end{aligned} \tag{16}$$

where $\nu = \min\{(\alpha_0, \alpha_1, \alpha_2, \alpha_3 + qE, \kappa\eta + \alpha_4)\}$. Thus,

$$\frac{dW(t)}{dt} + \nu W(t) \leq \pi_0. \tag{17}$$

Equation (17) has the solution

$$\begin{aligned} 0 < W(N,P,Z,L,R) &\leq \frac{\pi_0}{\nu}\left(1 - e^{-\nu t}\right) \\ &+ W\left(N_0, P_0, Z_0, L_0, R_0\right)e^{-\nu t}, \end{aligned} \tag{18}$$

as $t \longrightarrow \infty$, (18) becomes

$$0 < W(N,P,Z,L,R) \leq \frac{\pi_0}{\nu}. \tag{19}$$

Therefore, all solutions of the system (3) enter the feasible region,

$$\Omega = \left\{(N(t), P(t), Z(t), L(t), R(t)) \in \mathbb{R}_+^5 \colon W \leq \frac{\pi_0}{\nu} + \varsigma, \forall \varsigma > 0\right\}. \tag{20}$$

This completes the proof of the theorem. □

4. Data Fitting Results

A cosine function in equation (2) was fitted in MATLAB R2016a to the mean monthly surface water temperature data for the period July 2014 to December 2017 in order to estimate and describe the variation in the lake surface water temperature in Lake Kariba. The output obtained from MATLAB was

$$T(t) = 26.89 + 3.027\cos(-0.6267t - 11.49). \tag{21}$$

The estimates of parameters of equation (2) are shown in Table 1. The goodness of fit statistics for the fitted model (21) were sum of squared estimate of errors (SSE)= 1.423, $R^2 = 0.97666$, $R^2_{\text{adj}} = 0.9678$, and root mean square error (RMSE)= 0.4218.

The actual and fitted surface water temperature plots are shown in Figure 2.

The fitted model has a very high R^2_{adj} indicating that the fit quality to the series of the temperature data points is good.

5. Numerical Simulations

For the coefficient of temperature for the growth of phytoplankton, $g(T)$, in (1) we assumed $T_{\min} = 5^\circ$C [27], $T_{\text{opt}} = 25^\circ$C for Chlorophyceae, and $T_{\text{opt}} = 34^\circ$C for Cyanophyceae in Lake Kariba [18]. The plots of the temperature coefficient $g(T)$ versus temperature for Chlorophyceae and Cyanophyceae are shown in Figures 3(a) and 3(b), respectively.

5.1. Chlorophyceae and Cyanophyceae. In this subsection, we compared numerical results on kapenta population density for the mathematical models with the Chlorophyceae and Cyanophyceae phytoplankton classes. We simulated model (3) using the fitted equation for T in (21) and the default parameter values in (22) to illustrate the effect of T_{opt} on the dynamics of model (3).

$$\begin{aligned} &\pi_0 = 20.3, \alpha_0 = 0.0096, \beta_1 = 0.3, \pi_1 = 0.3, \alpha_1 = 0.032, \beta_2 = 0.18, \pi_2 = 0.0095, \\ &\alpha_2 = 0.08, \beta_3 = 0.3, \sigma = 0.01, \pi_3 = 0.36, \alpha_3 = 0.05, q = 0.00027999, E = 500, \\ &\kappa\eta = 0.16, \beta_4 = 0.4, \pi_4 = 0.4, d = 2, \text{and } \alpha_4 = 0.018. \end{aligned} \tag{22}$$

The parameter values in equation (22) were obtained from published data, while the others were estimates. For the numerical simulations, a fourth-order Runge–Kutta numerical scheme written in Wolfram Mathematica 11 was used. The units of the variables N, P, Z, L, and R in the model system (3) are μgl^{-1}. Numerical simulation results in Figures 4–6 show a decline in the population density of phytoplankton, zooplankton, kapenta, and tigerfish as T_{opt} changes from 25°C to 34°C.

The time series plots and phase portraits in Figures 4–6 show oscillatory behaviour and a periodic orbit of period 1, respectively. Analysis results from the model (3) with the fitted surface water temperature (21) show that if T_{opt} shifts from 25°C to 34°C, which corresponds to the shift in dominance of Cyanophyceae to Chlorophyceae, then there will be a decline in the population density of kapenta. So, if the less palatable Cyanophyceae species become dominant in Lake Kariba, we expect a decrease in kapenta abundance in the lake. Pliński and Jóźwiak [29] found that phytoplankton progressed from Bacillariophycea to Chlorophyceae and then Cyanophyceae as water temperature rose. According to Solomon et al. [30], the decline in zooplankton production was due to a decrease in the more palatable Chlorophyceae as a result of rising water temperatures. Ndebele-Murisa [6] recorded a relative cellular concentration of 78.05% and 9.7% for Cyanophyceae and Chlorophyceae, respectively, in Lake Kariba, showing the dominance of Cyanophyceae over Chlorophyceae. Simulation results in Figures 4–6 are therefore in agreement with findings from Solomon et al. [30] and Magadza [18].

5.2. Temperature Changes in Lake Kariba. In order to determine the optimum temperature for phytoplankton growth for the fitted lake surface water temperature data which result in the maximum kapenta population density, T_{opt} in (2) varied from 25°C to 35°C using the fitted equation for T in (21) for model (3) with $T_{\min} = 5^\circ$C. The plot in magenta in Figure 7 shows the maximum and the curve in blue shows the minimum values of $L(t)$.

Table 1: Parameter estimates from data fitting.

Coefficient	Estimate	95% confidence bounds
b	26.89	(26.54, 27.24)
c	3.027	(2.644, 3.411)
ω	−0.6267	(−0.6868, −0.5666)
ϕ	−11.49	(−11.91, −11.07)

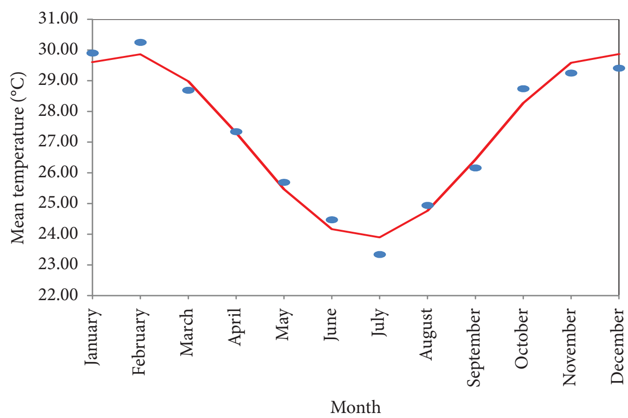

Figure 2: Actual (colour blue) and fitted (colour red) surface water temperature.

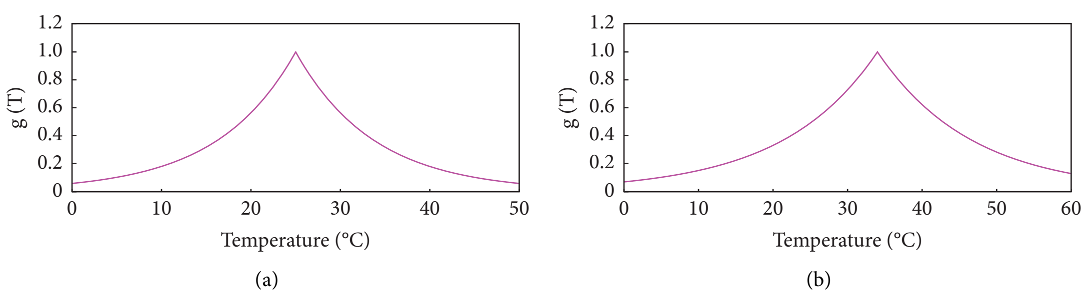

Figure 3: Plot of coefficient of temperature for the growth of phytoplankton $g(T)$ versus temperature for (a) Chlorophyceae and (b) Cyanophyceae.

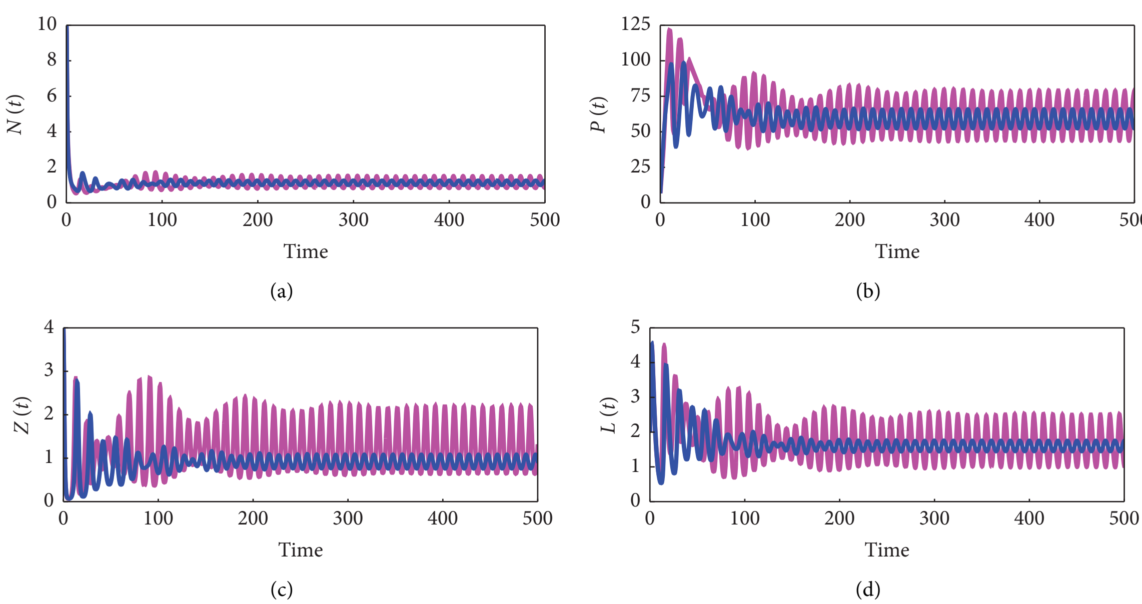

Figure 4: Time series plot of (a) nutrients, (b) phytoplankton, (c) zooplankton, and (d) *Limnothrissa miodon* for model (3) with assumed initial condition: $N(0) = 10, P(0) = 7, Z(0) = 4, L(0) = 2, R(0) = 0.5$ using the default parameter values. $T_{opt} = 25^{\circ}$C (colour magenta) and $T_{opt} = 34^{\circ}$C (colour blue).

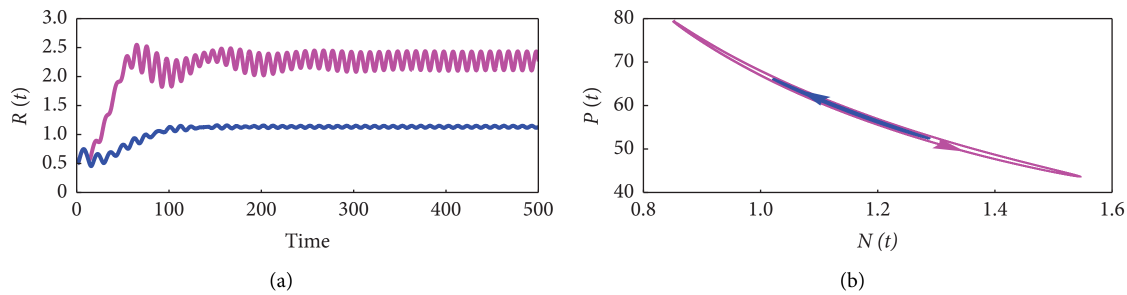

Figure 5: (a) Time series plot of tigerfish; (b) phase portrait of nutrients and phytoplankton with assumed initial condition: $N(0) = 10, P(0) = 7, Z(0) = 4, L(0) = 2, R(0) = 0.5$ using the default parameter values. $T_{opt} = 25^{\circ}$C (colour magenta) and $T_{opt} = 34^{\circ}$C (colour blue).

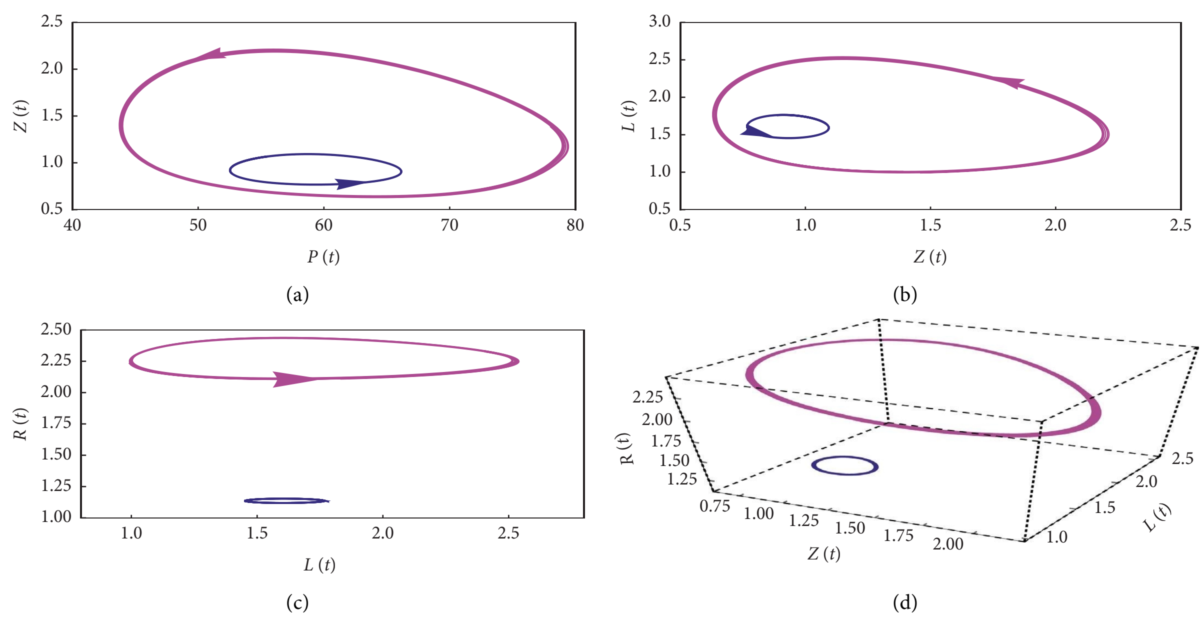

Figure 6: Phase portraits of (a) phytoplankton and zooplankton, (b) zooplankton and *Limnothrissa miodon*, (c) *Limnothrissa miodon* and tigerfish, and (d) zooplankton, *Limnothrissa miodon*, and tigerfish for model (3) with assumed initial condition: $N(0) = 10, P(0) = 7, Z(0) = 4, L(0) = 2, R(0) = 0.5$ using the default parameter values. $T_{opt} = 25^{\circ}$C (colour magenta) and $T_{opt} = 34^{\circ}$C (colour blue).

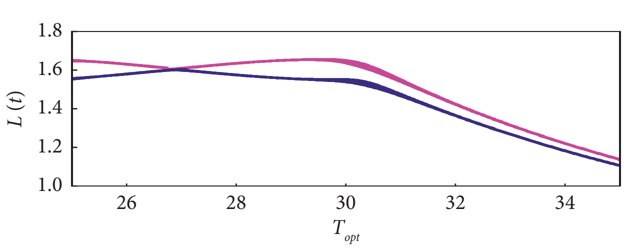

Figure 7: Plot of kapenta versus T_{opt} for model (3) with assumed initial condition: $N(0) = 10, P(0) = 7, Z(0) = 4, L(0) = 2, R(0) = 0.5$ using the default parameter values and $\gamma_2 = 0.005$.

Figure 7 shows that the kapenta population density maximum curve had a maximum at about 30°C and declined thereafter. Therefore, based on that result, it can be concluded that the optimal temperature for kapenta is approximately 30°C. The result is not far off from the breakpoint of 28.7°C for the kapenta catch obtained by Magadza [18].

In order to analyse model (3) for the warming of the lake water, we assumed that the lake warms by a rate of 0.62°C per decade [18]. We added 0.62°C to the mean surface water temperature data for the period 2014 – 2017 to obtain an estimate $T_1(t)$ of the mean surface water temperature after a decade. We projected further for the second decade by adding 0.62°C to $T_1(t)$ to get $T_2(t)$. We modelled the water temperatures $T_1(t)$ and $T_2(t)$ with (2), and the fitted models in (23) and (24) were

$$T_1(t) = 27.51 + 3.027\cos(0.6267t - 1.076), \tag{23}$$

$$T_2(t) = 28.13 - 3.027\cos(-0.6267t + 4.218). \tag{24}$$

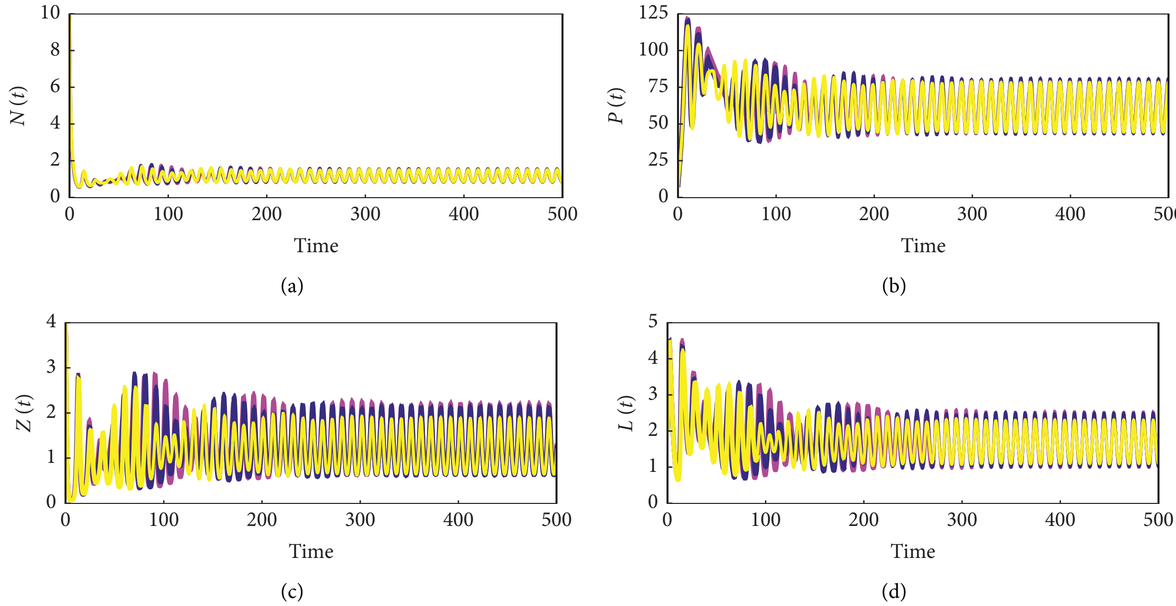

Figure 8: Time series plot of (a) nutrients, (b) phytoplankton, (c) zooplankton, and (d) *Limnothrissa miodon* for model (3) with assumed initial condition: $N(0) = 10, P(0) = 7, Z(0) = 4, L(0) = 2, R(0) = 0.5$ using the default parameter values. Actual mean surface water temperature (colour magenta), $T_1(t)$ (colour blue), and $T_2(t)$ (colour yellow).

The fitted models were then substituted into (1). We simulated model (3) using some initial condition and compared numerical results from model (3) using the 2014 – 17 data, $T_1(t)$, and $T_2(t)$, with $T_{opt} = 25^\circ$C and $T_{\min} = 5^\circ$C in (1). Numerical results which illustrate the influence of lake warming on the concentration of nutrients and population density of phytoplankton, zooplankton, kapenta, and tigerfish are shown in Figures 8–10.

Results from Figures 8–10 show that as the lake warms, there is a corresponding decline in the population density of zooplankton, therefore resulting in a decline in the kapenta and tigerfish population density in the lake. Therefore, warming of the lake may have a negative effect on the population density of kapenta in the Lake Kariba fishery, and the environmental factor temperature may have a significant impact on kapenta abundance.

6. Discussion

In light of global warming, there is a knowledge gap regarding the effects of temperature increase on the density of *Limnothrissa miodon* in Lake Kariba. In this paper, we formulated and analysed a model that has nutrients, phytoplankton, zooplankton and *Limnothrissa miodon*, and temperature as an environmental factor. The phytoplankton growth rate, phytoplankton mortality, grazing on phytoplankton, zooplankton growth rate, zooplankton mortality, grazing on zooplankton, and *Limnothrissa miodon* mortality were assumed to be Holling type I forms. Feeding on *Limnothrissa miodon* and *Hydrocynus vittatus* growth was assumed to be Holling type II forms. Positivity and existence of solutions to model (3) were investigated. Numerical simulations were done for the model with default parameter values. Mathematical modelling provided an overall picture of the dynamics of the model variables. Assuming that existing trends in surface lake water temperature will continue, we used numerical simulations to demonstrate that global warming influences the *Limnothrissa miodon* population density in Lake Kariba. The thermal response of *Limnothrissa miodon* to global warming was considered in the context of thermal optima.

For the model with fitted lake surface water temperature data for the period 2014 – 2017, we used numerical simulations and investigated the effect of a shift in the optimal temperature for phytoplankton growth from 25°C to 34°, corresponding to dominance of Cyanophyceae over Chlorophyceae, and results showed a decline in the population density of *Limnothrissa miodon*.

The optimum temperature for phytoplankton growth for the fitted lake surface water temperature data for the period 2014 – 2017 varied from 25°C to 35°C, and simulation results showed a maximum kapenta population density at approximately 30°C and the result is similar to the breakpoint of 28.7°C for kapenta catch obtained by Magadza [18]. According to Abowei [31], a species' existence is threatened if the water temperature exceeds its upper limit of toleration. Furthermore, according to McDonald et al. [32], lake warming to abovementioned optimum temperatures can result in a reduction in the production of species like lake trout.

In order to explore the influence of global warming on *Limnothrissa miodon* population density, we projected lake surface water temperature for the next two decades and used data fitting and numerical simulations to predict the population density of *Limnothrissa miodon* in Lake Kariba. Our findings show that the population density of *Limnothrissa miodon* declines as the lake warms and this is in agreement with Abdellaoui et al. [33], who used a time series analysis

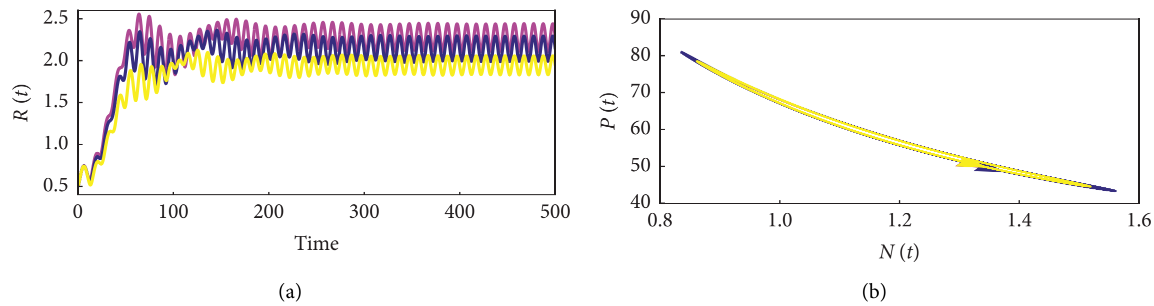

Figure 9: (a) Time series plot for tigerfish and (b) phase portrait of nutrients and phytoplankton for model (3) with assumed initial condition: $N(0) = 10, P(0) = 7, Z(0) = 4, L(0) = 2, R(0) = 0.5$ using the default parameter values. Actual mean surface water temperature (colour magenta), $T_1(t)$ (colour blue), and $T_2(t)$ (colour yellow).

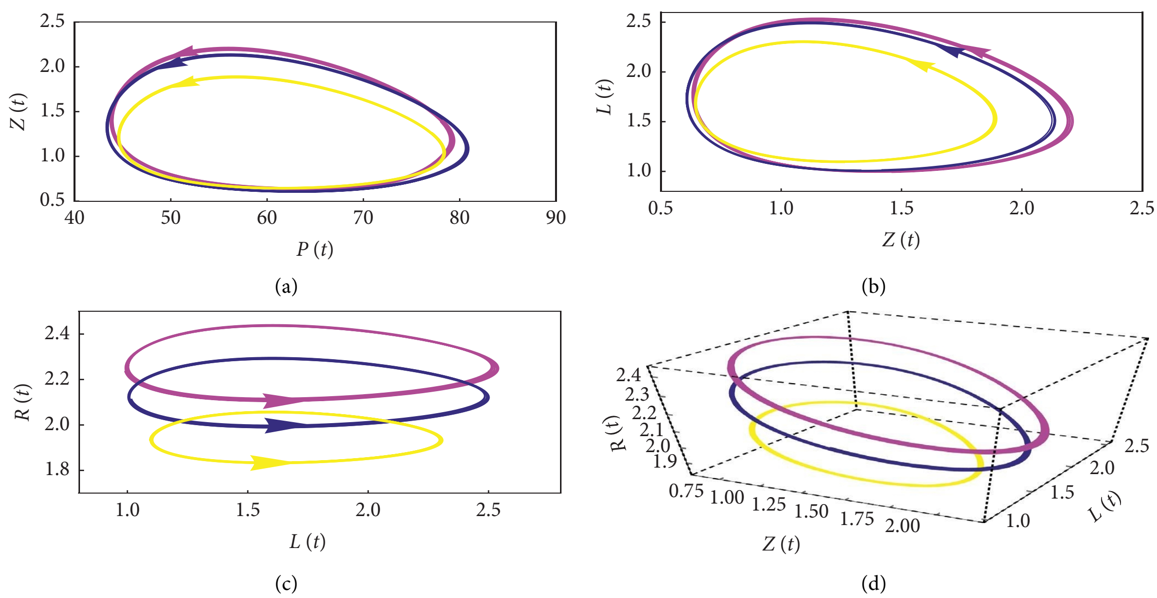

Figure 10: Phase portraits of (a) phytoplankton and zooplankton, (b) zooplankton and *Limnothrissa miodon*, (c) *Limnothrissa miodon* and tigerfish, and (d) zooplankton, *Limnothrissa miodon*, and tigerfish for model (3) with assumed initial condition: $N(0) = 10, P(0) = 7, Z(0) = 4, L(0) = 2, R(0) = 0.5$ using the default parameter values. Actual mean surface water temperature (colour magenta), $T_1(t)$ (colour blue), and $T_2(t)$ (colour yellow).

and modelling approach on assessing the impact of temperature and chlorophyll variations on the fluctuations of sardine abundance in Al-Hoceima. Their results showed an inverse relationship between fluctuations of sardine catch per unit effort and sea surface temperature and that sea surface temperature is the most important parameter affecting the abundance of small pelagic fish in the Moroccan Mediterranean Sea. Similarly, Lam et al. [34] used simulation models in their study and anticipated that climate change will decrease fish catches globally by 7.7% by 2050. Our numerical simulation results are in agreement with findings of the European Environmental Agency's research [35], which showed that lakes are impacted by climate change, mainly by temperature rises [35–37]. Our results are congruent with previous studies that have shown significant adverse effects of warming on freshwater ecosystems [38–41]. Our findings are also supported by Mohammed and Uraguchi [42], who suggested that climate change will have an adverse impact on the already strained fish resources in sub-Saharan African countries. The highlights of the study are as follows:

(i) A cosine function was fitted to mean lake surface water temperature data and the accuracy of the fitted model was analysed

(ii) Default parameter values and different initial conditions were used to run numerical simulations for a nutrient, plankton, kapenta, and tigerfish model with a temperature coefficient for phytoplankton growth

(iii) Numerical simulation results of the non-autonomous model showed a stable periodic orbit for varying initial conditions, and therefore, instability

(iv) Lake warming has a negative impact on the more palatable Chlorophyceae, thus leading to decline in kapenta density in Lake Kariba

(v) Kapenta population density starts to decline after $T_{opt} = 30^{\circ}C$

(vi) A decadal warming rate of 0.62°C results in a decline in zooplankton population density, which leads to a decline in the kapenta population density

7. Conclusions

Numerical results showed that the population density of kapenta declines after the lake surface water temperature surpasses 30°C. Simulation results also showed that lake warming has a negative effect on the more palatable Chlorophyceae population density, resulting in a decline in the density of kapenta in Lake Kariba. Therefore, we conclude that lake surface water temperature influences the dynamics of *Limnothrissa miodon* since rising temperatures have been shown to have a negative impact on the kapenta population density in Lake Kariba. Lake surface water temperature influences the productivity in the water body, altering the mean and amplitude of plankton, kapenta, and tigerfish population density oscillations. Rising temperatures may lead to the disappearance of the more palatable Chlorophyceae and an abundance of the less palatable Cyanophyceae. This may result in a decline in zooplankton populations, and this may adversely affect the kapenta population. Therefore, lake warming may result in a decrease in the kapenta population and reduced kapenta catches in the lake, with the probable consequence of reducing the number of fishing vessels as the fishing will no longer be profitable. This can have adverse effects on the local communities and the local economy. In light of these findings, the governments of Zambia and Zimbabwe and all stakeholders are, therefore, encouraged to take action toward reducing the impacts of climate change in line with the Paris agreement, which was adopted in 2015.

For future studies, we intend to model the dynamics of *Limnothrissa miodon* with the invasive crayfish.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors would like to express their gratitude to the Lake Kariba Fisheries Research Institute and the University of Zimbabwe Lake Kariba Research Station for providing the data used in this study. Dr. Nobuhle Ndlovu, Ms. Adroit T. Chakandinakira, and Ms. P Chirozva deserve special recognition. Research Board Grant number RDB/90/18 from the National University of Science and Technology supported this study.

References

[1] D. Steel, C. T. DesRoches, K. Mintz-Woo, and K. Mintz-Woo, "Climate change and the threat to civilization," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 119, no. 42, Article ID e2210525119, 2022.

[2] S. Hitz, J. Smith, and J. Smith, "Estimating global impacts from climate change," *Global Environmental Change*, vol. 14, no. 3, pp. 201–218, 2004.

[3] G. Paulet, "Kapenta Rig Survey of the Zambian Waters of Lake Kariba," 2013, https://www.fao.org/3/az411e/az411e.pdf.

[4] G. Bell-Cross and B. Bell-Cross, "Introduction of *Limnothrissa miodon* and *Limnocaridina tanganicae* from lake tanganyika into Lake Kariba," *Fish Res Bull Zambia*, vol. 5, pp. 207–214, 1971.

[5] F. Nhambura, *Kariba's Kapenta under Threat"*, The Herald, Glasgow, Scotland, 2014.

[6] M. R. Ndebele-Murisa, *An Analysis of Primary and Secondary Production in Lake Kariba in a Changing Climate*, University of the Western Cape, Bellville, South Africa, 2011.

[7] C. D. Amsler, J. B. McClintock, and B. J. Baker, "Secondary metabolites as mediators of trophic interactions among Antarctic marine organisms," *American Zoologist*, vol. 41, no. 1, pp. 17–26, 2001.

[8] G. A. Codd, "Cyanobacterial toxins: occurrence, properties and biological significance," *Water Science and Technology*, vol. 32, no. 4, pp. 149–156, 1995.

[9] D. E. Arnold, "INGESTION, assimilation, survival, and reproduction by DAPHNIA pulex fed seven species of blue-green algae 1, 2," *Limnology & Oceanography*, vol. 16, no. 6, pp. 906–920, 1971.

[10] R. De Bernardi and G. Giussani, "Are blue-green algae a suitable food for zooplankton? An overview," *Hydrobiologia*, vol. 200-201, no. 1, pp. 29–41, 1990.

[11] J. Severiano, C. B. Amaral, A. S. Diniz, and A. Moura, "Species-specific response of phytoplankton to zooplankton grazing in tropical eutrophic reservoirs," *Acta Limnologica Brasiliensia*, vol. 33, 2021.

[12] H. M. Masundire, "Seasonal variation in size of *Bosmina longirostris* OF Müller in tropical Lake Kariba, Zimbabwe," *SIL Proceedings, 1922-2010*, vol. 24, no. 3, pp. 1455–1459, 1991.

[13] J. J. Mandima, "The food and feeding behaviour of *Limnothrissa miodon* (Boulenger, 1906) in Lake Kariba, Zimbabwe," *Hydrobiologia*, vol. 407, pp. 175–182, 1999.

[14] C. Machena, *A Study of the Primary Productivity of the Sanyati Basin of Lake Kariba and the Application of a Regression Model to Estimate Fish Yields in the lake*, University of Zimbabwe, Harare, Zimbabwe, 1983.

[15] W. Mhlanga, "A study of Hydrocynus vittatus. Castelnau in Lake Kariba, Zimbabwe," *African Journal of Tropical Hydrobiology and Fisheries*, vol. 9, no. 1&2, pp. 25–35, 2000.

[16] D. H. S. Kenmuir, "Some aspects of Hydrocynus vittatus castelnau (tigerfish) research at Lake Kariba," *Newslett. Limnol. Soc. sth. Afr.*vol. 17, pp. 33–38, 1971.

[17] P. C. Chifamba, "The relationship of temperature and hydrological factors to catch per unit effort, condition and size of the freshwater sardine, *Limnothrissa miodon* (Boulenger), in Lake Kariba," *Fisheries Research*, vol. 45, no. 3, pp. 271–281, 2000.

[18] C. H. D. Magadza, “Indications of the effects of climate change on the pelagic fishery of Lake Kariba, Zambia-Zimbabwe,” *Lakes & Reservoirs: Science, Policy and Management for Sustainable Use*, vol. 16, no. 1, pp. 15–22, 2011.

[19] M. R. Ndebele-Murisa, E. Mashonjowa, and T. Hill, “The implications of a changing climate on the Kapenta fish stocks of Lake Kariba, Zimbabwe,” *Transactions of the Royal Society of South Africa*, vol. 66, no. 2, pp. 105–119, 2011.

[20] M. Ndebele-Murisa, T. Hill, L. Ramsay, T. Hill, and L. Ramsay, “Validity of downscaled climate models and the implications of possible future climate change for Lake Kariba’s Kapenta fishery,” *Environmental Development*, vol. 5, pp. 109–130, 2013.

[21] P. Sibanda, *Comparative estimates of growth rates of some chlorophyceae and Cyanophyceae at different temperatures*, University of Zimbabwe, Harare, Zimbabwe, 2005.

[22] L. Madamombe, “The economic development of the kapenta fishery Lake Kariba (Zimbabwe/Zambia),” University of Troms, Troms, Norway, 2002.

[23] P. Chipungu, “Review of Draft Proposal for the Management of the Zambian Inshore Fisheries on Lake Kariba,” *Project Report. Zambia-Zimbabwe SADC Fisheries*, 1993.

[24] K. Nyikahadzoi, “Challenges to collective action in the management of the Kapenta fishery in lake Kariba,” *Lakes & Reservoirs: Science, Policy and Management for Sustainable Use*, vol. 14, no. 4, pp. 337–351, 2009.

[25] T. Nhiwatiwa, *Lake Kariba Surface Water Temperature Data*, University of Zimbabwe Lake Kariba Research Station, Harare, Zimbabwe, 2017.

[26] F. K. Mutasa, B. Jones, I. H. Tendaupenyu, T. Nhiwatiwa, and M. R. Ndebele-Murisa, “Bifurcation analysis of a 5D nutrient, plankton, Limnothrissa miodon model with Hydrocynus vittatus predation,” *Journal of Applied Mathematics*, vol. 2022, Article ID 1095441, 21 pages, 2022.

[27] Y. Li, D. Xie, and J. A. Cui, “The effect of continuous and pulse input nutrient on a lake model,” *Journal of Applied Mathematics*, vol. 2014, Article ID 462946, 13 pages, 2014.

[28] C. S. Holling, “The components of predation as revealed by a study of small mammal predation of the European pine sawfly,” *The Canadian Entomologist*, vol. 91, no. 5, pp. 293–320, 1959.

[29] M. Pliński and T. Jóźwiak, “Temperature and N: P ratio as factors causing blooms of blue-Cyanophyceae algae in the Gulf of Gdansk,” *Oeceanol*, vol. 41, no. 1, pp. 73–80, 1999.

[30] S. Solomon, D. Qin, M. Manning, K. Averyt, and M. Marquis, *Climate Change 2007-the Physical Science Basis: Working Group I Contribution to the Fourth Assessment Report of the IPCC*, Vol. 4, Cambridge University Press, Cambridge, England, 2007.

[31] J. F. N. Abowei, “Salinity, dissolved oxygen, pH and surface water temperature conditions in Nkoro River, Niger Delta, Nigeria,” *Advance Journal of Food Science and Technology*, vol. 2, no. 1, pp. 36–40, 2010.

[32] M. E. McDonald, A. E. Hershey, and M. C. Miller, “Global warming impacts on lake trout in arctic lakes,” *Limnology & Oceanography*, vol. 41, no. 5, pp. 1102–1108, 1996.

[33] B. Abdellaoui, A. Berraho, F. Falcini et al., “Assessing the impact of temperature and chlorophyll variations on the fluctuations of sardine abundance in Al-Hoceima (South Alboran Sea),” *Journal of Marine Science: Research & Development*, vol. 7, no. 5, p. 239, 2017.

[34] V. W. Y. Lam, W. W. L. Cheung, R. Gabriel, and U. Rashid Sumaila, “Projected change in global fisheries revenues under climate change,” *Scientific Reports*, vol. 6, no. 1, pp. 1–8, 2016.

[35] P. Kristensen, C. Whalley, F. Zal et al., “EEA European Waters Assessment,” *European Waters Assessment*, 2018.

[36] D. Hering, L. Carvalho, C. Argillier et al., “Managing aquatic ecosystems and water resources under multiple stress—an introduction to the MARS project,” *Science of the Total Environment*, vol. 503-504, pp. 10–21, 2015.

[37] P. Kristensen and W. Vanneuville, “Climate change, impacts and vulnerability in europe 2012,” 2012, https://www.eea.europa.eu/publications/climate-impacts-and-vulnerability-2012.

[38] B. Moss, D. McKee, D. Atkinson et al., “How important is climate? Effects of warming, nutrient addition and fish on phytoplankton in shallow lake microcosms,” *Journal of Applied Ecology*, vol. 40, no. 5, pp. 782–792, 2003.

[39] E. Jeppesen, B. Kronvang, T. B. Jørgensen et al., “Recent climate-induced changes in freshwaters in Denmark,” *Climatic Change and Global Warming of Inland Waters: Impacts and Mitigation for Ecosystems and Societies*, pp. 155–171, 2013.

[40] D. W. Schindler, “The cumulative effects of climate warming and other human stresses on Canadian freshwaters in the new millennium,” *Canadian Journal of Fisheries and Aquatic Sciences*, vol. 58, no. 1, pp. 18–29, 2001.

[41] G. Woodward, D. M. Perkins, and L. E. Brown, “Climate change and freshwater ecosystems: impacts across multiple levels of organization,” *Philosophical Transactions of the Royal Society B: Biological Sciences*, vol. 365, pp. 2093–2106, 2010.

[42] E. Y. Mohammed and Z. B. Uraguchi, “Impacts of climate change on fisheries: implications for food security in Sub-Saharan Africa,” *Global Food Security: Emerging Issues and Economic Implications*, pp. 114–135, Nova Science Publishers, Inc, Hauppauge, NY, USA, 2013.