Research Article

Design Analysis of a Helium Xenon-Printed Circuit Heat Exchanger for a Closed Brayton Cycle Microtransport Reactor

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Microtransport with small size and a wide range of applications is very attractive for the utilization of future reactor modularization. A microreactor uses a closed Brayton cycle (CBC) to achieve high conversion efficiencies at low specific mass. The recuperator is one of the key components of CBC system which recovers heat exhausted from the turbine. A printed circuit heat exchanger (PCHE) is currently the preferred type of recuperator for the closed Brayton cycle due to its high heat transfer efficiency, high compactness, and high pressure and temperature resistance. This work intends to analyze different geometry configurations of a zigzag PCHE to increase its heat transfer efficiency while reducing its weight and size.

The effect of geometric structure parameters such as channel diameter, zigzag pitch length, and zigzag angle on the Nusselt number and Fanning friction factor is investigated using a zigzag PCHE unit model. Based on the numerical simulation, the principle of least squares is employed to carry out a nonlinear fitting of the flow and heat transfer criterion correlation equations. Besides, the maximum value of the Nusselt number and minimum value of the Fanning friction factor are optimized as two conflicting objective functions using the nondominated sorting genetic algorithm-II (NSGA-II), with which a set of optimal solutions is obtained. Meanwhile, the shortest normalized distance is used to determine the compromised solution on the Pareto optimal points, and the independent variable’s sensitivity analysis is performed. Finally, a multiobjective optimization analysis is conducted for the PCHE to achieve lightweight and the high heat transfer efficiency design requirements of SIMONS.

1. Introduction

The Small Innovative Helium-Xenon Cooled Mobile Nuclear Power System (SIMONS) under design has several benefits, including high dependability, high power density, and high thermal conversion efficiency [1, 2]. The schematic diagram of SIMONS is shown in Figure 1. A closed Brayton cycle (CBC) system is applied to SIMONS, which consists of a compressor-generator-turbine integrated system, a recuperator, a precooler, and a fan module. When a CBC system is applied to the SIMONS, priority should be given to the feasibility of its component size. As one of the essential components of SIMONS, a recuperator is adopted to reuse the waste heat exhausted from the turbine and to increase the inlet temperature of the reactor core [3]. According to the work by Lee et al. [4], the mass and volume of the turbomachinery system in the Brayton cycle of a water-cooled small modular reactor are much less than those of the heat exchanger system. As a result, it is necessary to optimize the size and weight of the recuperator and precooler in order to match a lightweight mobile reactor.

A printed circuit heat exchanger (PCHE) with a structure of diffusion-bonded plates and chemically etched channels is a promising plate-type heat exchanger. Compared to traditional heat exchangers [5], the numerous small channels and strong diffusion bonds allow a PCHE to operate at high pressure and high temperature with excellent heat transfer capacity and compact size. Therefore, PCHE is a preferable
The thermal-hydraulic performance for different geometry configurations of PCHE has received plenty of attention in the past few years. By using supercritical carbon dioxides (sCO₂) as working fluid, Lee and Kim [7] analyzed the flow and heat transfer characteristics of various channel cross-sections (semicircular, circular, rectangular, and trapezoidal) of a PCHE. It was found that the heat transfer efficiency of the rectangular cross-section channel was the best, and the heat transfer efficiency of the circular cross-section channel was the worst. Afterwards, Lee and Kim [8] proposed that the zigzag angles of the channels on both sides of the PCHE are the same, which has the highest heat transfer efficiency. Furthermore, Meshram et al. [9] studied the flow and heat transfer characteristics of PCHEs with various channel configurations. They concluded that zigzag channels have better heat transfer capabilities than straight channels but at the expense of higher pressure drop.

In addition, several works have verified experimental results with numerical simulations using helium (He), resulting in corresponding flow and heat transfer criterion correlations. Baek et al. [10] investigated the flow and heat transfer characteristics of PCHE in the low-temperature region. They found that the performance of PCHE was predominantly affected by the axial conduction heat transfer. To better predict the Nusselt number and Fanning friction factor, Kim et al. [11] proposed the Nusselt number correlations of a zigzag PCHE at the Reynolds numbers (Re) from 350 to 1200. Then, Kim and No [12] proposed criterion correlations for various geometric parameters of a zigzag PCHE, i.e., zigzag angle and channel diameter. Yoon et al. [13] proposed the correlations for the laminar and turbulence region using CFD simulation by considering the influence of the geometry configurations, such as zigzag pitch length and zigzag angle. Based on the CFD simulation results, Torre et al. [14] considered the interactions of three geometric parameters of a PCHE including zigzag pitch length, zigzag angle, and bend radius. They proposed the correlations as a function of three geometrical parameters and the Re.

Besides sCO₂ and He, air as working fluid in PCHE was also studied. Kar [15] examined the flow and heat transfer characteristics of the PCHE. They developed correlations between flow and heat transfer coefficients with Re and geometric parameters.

SIMONS requires its energy conversion system to achieve the goals of lower system mass and higher conversion efficiency. If He is utilized as the working fluid, due to its low compressibility, more compression processes are needed to achieve the required pressure ratio in an axial compressor, which will increase the volume of the Brayton cycle device. However, the binary gas mixed with heavier noble gases demonstrates better thermal properties than pure He, which can reduce the volume of CBC system [16, 17]. As a result, helium xenon (He-Xe) is a preferred working fluid for microreactor systems to meet the demand of system compactness [18, 19]. He-Xe is chemically inert, which can avoid corrosion problems and other challenges of nonlinear properties of sCO₂ proximity to the critical point [20].

Considering the difference in heat diffusion mechanism of He-Xe from other working fluids [21], it is needed to conduct in-depth research on the heat transfer correlation for He-Xe. In the past years, several experiments have studied the heat transfer characteristics of He-Xe in a circular tube. Taylor et al. [22] studied the heat transfer mechanism of He-Xe in a circular tube and obtained the criterion correlation. Furthermore, Nakoryakov et al. [23, 24] and Vitovsky et al. [25] also experimentally studied the heat transfer mechanism of He-Xe in triangular and cylindrical channels. However, the above He-Xe experimental study results were mostly carried out in a single circular tube.

Different structural parameters and objective functions are included in heat exchanger design, while conventional heat exchanger design approaches can only optimize heat exchangers for a single objective. Genetic algorithm (GA) or nondominated sorting genetic algorithm (NSGA-II) has been performed for the multiobjective optimization of heat

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**Figure 1: Conceptual layout of the SIMONS.**
exchangers. For a plate-fin heat exchanger (PFHE), Peng and Xiang [26] and Xie et al. [27] used GA for multiobjective optimization of its weight and cost. Xie and Wang [28] also investigated a design method for the multiobjective optimization of the weight and efficiency of PFHE using GA. Lee et al. [29] developed an NTU-based performance prediction algorithm and created a heat exchanger database. Wen et al. [30] successfully coupled a multiobjective evolutionary algorithm and a kriging response surface to determine the optimal design parameters for PFHE with serrated fins. Rao and Patel [31] recommended a better teaching-learning-based optimization technique to raise the heat exchanger’s efficiency and lower its total cost. GA was used by Özkol and Komurgoz [32] to optimize calculations as well. They used multiobjective optimization to identify the ideal shape that might satisfy the goals. Saeed and Kim [33] optimized the geometry of PCHE using response surface methodology in combination with the GA. Yang et al. [34] used the NSGA-II coupled with a surrogate model to optimize the PCHE core sizes in order to study the effects of the PCHE core sizes on heat transfer efficiency and pressure drop. Shi et al. [35] optimized the heat transfer performance of the intake portion of a microchannel ceramic heat exchanger using a surrogate model in conjunction with a GA.

In short, the geometrical characteristics of a PCHE substantially impact its thermal-hydraulic performances. However, the working medium for PCHE is typically sCO$_2$, He, or air. There have only been a few reports on the performance study of PCHE using He-Xe as a working medium.

In this work, the thermal-hydraulic performances of PCHE are analyzed using He-Xe as the medium. Meanwhile, more accurate heat transfer correlations will be developed. Besides, this work is aimed at investigating the optimum geometric parameters of PCHE using the multiobjective optimization algorithm.

The geometric modeling, grid independence, numerical method validation, and determination of the molar mass of He-Xe of PCHE are introduced in Section 2. The orthogonal experimental method utilized to select a smaller number of geometric designs for the numerical analysis is described in Section 3. In Section 4, the thermal-hydraulic performance of the PCHE geometry parameters (including zigzag angle, zigzag pitch length, and channel diameter) is analyzed to propose the criterion correlation of the Nusselt number and Fanning friction factor as a function of the geometric structure parameters. Based on the results, the multiobjective optimization of the Nusselt number and Fanning friction factor is obtained using NSGA-II. Besides, fuzzy decision-making is used to obtain the optimal compromised solution in the Pareto optimal points of the Nusselt number and friction factor. Meanwhile, the sensitivity analysis of the PCHE geometric structure parameters is also carried out. In Section 5, the Pareto optimal points of the mass and heat transfer efficiency of the PCHE-type recuperator are obtained in combination with NSGA-II. The conclusions are drawn in Section 6.

2. Model Description

2.1. Geometrical Model. A PCHE contains lots of zigzag channels etched into individual plates, which are stacked together. This work assumes that the inlet mass flow rate,
pressure, and temperature of any channel are constant and use periodic boundary conditions to model a zigzag PCHE unit model for numerical simulation. Figure 2 shows the schematic diagram of the channel design of the zigzag PCHE.

2.2. Mathematical Modeling. In this work, the numerical computation of PCHE is performed using ANSYS Fluent 17.0. The CPU model of the PC used is Intel(R) Xeon(R) Silver 4215R CPU @ 3.20 GHz and 3.19 GHz, and the memory size of the PC is 112 GB. The governing equations are discretized by the finite volume method and fitted by the second-order upwind method. For the turbulence model, choose the shear stress transport (SST) $k-\omega$ model. The SIMPLE is used to solve the coupling pressure and velocity. For residuals, the convergence conditions are all set as $10^{-6}$.

The boundary conditions for the PCHE channel inlet and outlet use mass flow inlet and pressure outlet, respectively. The wall boundaries on the top, bottom, left, and right are periodic, while the front and back walls are designated as adiabatic walls. This work first uses He as working fluid to verify the numerical simulation method. Then, He-Xe is used as working fluid to investigate the PCHE. The He and He-Xe properties are obtained from the National Institute of Standards and Technology Chemistry Web-Book database [36]. The material for the solid section is alloy 617.

The heat transfer parameters across the whole channel are computed and averaged, and the $Re$ is expressed as

$$Re = \frac{q_{m}D_{h}}{\Lambda_{j}\mu},$$

(1)
where $q_m$ is the mass flow, $\mu$ is the viscosity, and $A_c$ is the channel cross-sectional area. Thermal properties such as density $\rho$ and viscosity $\mu$ are determined from volume-weighted average of temperatures in the hot and cold channels. $D_h$ is the hydraulic diameter, which is calculated as

$$D_h = \frac{\pi D}{\pi + 2},$$

where $D$ is the channel diameter. The heat transfer coefficient $h$ is defined as follows:

$$h = \frac{q}{T_w - T_b},$$

where $T_b$ and $T_w$ are the volume-weighted average bulk temperature and area-weighted average wall temperature, respectively.

The average Nusselt number in a channel can be obtained by

$$\text{Nu} = \frac{hD_h}{\lambda},$$

where $\lambda$ is the thermal conductivity.

The Fanning friction factor is calculated as

$$f = \frac{\Delta P D_h}{2 \rho v^2},$$

where $\rho$ is the density, $v$ is the velocity, $\Delta P$ is the pressure drop, and $l$ is the real flow length of the fluid.

2.3. Validation of the Numerical Results. In this work, we use the geometry parameters in the work by Kim et al. [11] to verify the reliability of the numerical model. Table 1 displays the parameters of the PCHE model. The parameters of boundary conditions used in the simulation are listed in Table 2.

A grid independence test is first carried out to obtain the optimum grid number for the zigzag PCHE unit model. Seven different grid sizes are analyzed in the mesh independence test. The mesh dependency test is shown in Figure 3.
Compared to the number of mesh grids of $2.04 \times 10^7$ (mesh 7), the maximum relative deviation of the pressure drop and the Nusselt number of the grid system of $6.86 \times 10^6$ (mesh 5) is less than 1.0%. Therefore, mesh 5 will be used for numerical calculations, as shown in Figure 4. In the fluid zones, dense grids are established, while in the solid regions, coarse grids are used. The boundary grid has five layers, the first of which has a height of 0.01 mm and a growth rate of 1.2 near the wall.

To verify the applicability and accuracy of the turbulence model in terms of flow and heat transfer, we adopt the same simulation conditions as in the work by Kim et al. [11] for comparison. One can see from Figure 5 that in the comparison of the present result for heat flux variation with the work by Kim et al. [11], the calculated results of heat flux in this work are in good agreement with the previous ones [11]. It indicates that the numerical model can accurately simulate the flow and heat transfer characteristics of the PCHE.

### 2.4. Helium Xenon Mixture Gas

The He-Xe’s convective heat transfer coefficient is evaluated as [22]

$$h = \frac{0.023}{D_h^{0.2} L_0} \times \left( \frac{T_f}{T_w} \right)^{\alpha} \times \left[ m^0.8 \lambda^0.35 \rho^0.65 \mu^{-0.15} \right] \times L_0^{0.8} \times \left( \frac{1}{D_h^{0.2} L_0} \right)^{0.8},$$

where $m$ is the He-Xe molar mass and $\dot{N}$ is the molar mass flow rate. The 1st term and the 2nd term on the right of Equation (6) are associated with the operating condition and geometry structure, respectively. The 3rd term is related to He-Xe’s transport and thermodynamic properties. Under the same working conditions and geometrical dimensions, the convective heat transfer coefficient of He-Xe is only affected by its transport and thermodynamic properties. Therefore, Equation (6) can be normalized and expressed as [22]

$$h \propto m^0.8 \lambda^0.35 \rho^0.65 \mu^{-0.15}.$$  \hspace{1cm} (7)

The relative convective heat transfer coefficient is determined by [22]

$$\frac{h}{h_{\text{He}}} = \left( \frac{m}{m_{\text{He}}} \right)^{0.8} \left( \frac{\lambda}{\lambda_{\text{He}}} \right)^{0.35} \left( \frac{\rho}{\rho_{\text{He}}} \right)^{0.65} \left( \frac{\mu}{\mu_{\text{He}}} \right)^{-0.15}.$$  \hspace{1cm} (8)

Figure 6 shows the influence of the molar mass of the mixture on its relative convective heat transfer coefficient at different temperatures and pressures. One can see that temperature and pressure have little influence on the relative convective heat transfer coefficient. When $m_{\text{He-Xe}}$ is 15 g/mol, gas mixtures have a heat transmission coefficient that is 6.3% higher than pure He. A higher heat transfer efficiency can be achieved by choosing a medium with a higher heat transfer coefficient. Therefore, we choose He-Xe at 15 g/mol as the medium.
3. Taguchi Method

Compared to the exhaustive method, the orthogonal design method uses only a limited number of cases to calculate the effect of different parameters, which can significantly reduce the computation time [37, 38]. Since this work is aimed at investigating the thermal-hydraulic characteristics of different structural factors such as channel diameter \((D)\), zigzag pitch \((l_z)\), and zigzag pitch angle \((\alpha)\), the Taguchi method is employed to optimize the design parameters for maximum efficiency.

Figure 8: Velocity magnitude with various zigzag pitch lengths \((\alpha = 25^\circ, D = 1.5 \text{ mm})\).
length ($L_z$), and zigzag angle ($\alpha$) of the zigzag PCHE, a three-level and three-factor orthogonal design is conducted according to the parameters and ranges displayed in Table 3. The orthogonal permutation and combination cases are shown in Table 4, where $L_z/\delta$ and $D/\delta$ are the dimensionless parameters and $\delta$ represents the distance between adjacent channels.
4. Results and Discussion

4.1. Thermal-Hydraulic Performance of the PCHE. Since He-Xe’s thermal diffusion mechanism differs from that of other working fluids [21], a thorough investigation of He-Xe’s turbulent heat transfer mechanism for a PCHE is conducted. As mentioned in Section 2.4, when the molar mass of He-Xe is 15 g/mol, it has the highest convective heat transfer coefficient. So the He-Xe with a molar mass of 15 g/mol is used as the medium in the following heat transfer and flow analysis.

4.1.1. Effect of PCHE Geometry Parameter. In this subsection, the performances of He-Xe in a PCHE with different zigzag parameters are analyzed.

The velocity contours at various cross-sections of the 5th pitch of different zigzag angles are presented in Figure 6 for Re = 1036. In internal fluid flows, the central area where the fluid accelerates to make up for the drop in axial velocity brought on by boundary layer expansion is referred to as the accelerating core [39]. It is evident from Figure 7 that the velocity field of the fluid acceleration core changes periodically. The direction of the fluid is changed by the periodic inertial force in the periodic channel, which makes the boundary layer thinner, thereby enhancing convective heat transfer. In addition, with the increase of the zigzag angle, the acceleration core area of the corresponding section will increase. This is because the increase of the zigzag angle leads to a more drastic change of the flow direction, a greater damage degree of boundary layer, and a stronger convective heat transfer in the channel. Therefore, the larger the zigzag angle, the higher the heat transfer efficiency, whereas the larger the resistance in the channel.

Figure 8 shows the velocity distribution of the PCHE at different unit pitches (72 mm to 104 mm from the hot-side inlet) when Re is kept at 1047. It can be concluded that if the total zigzag pitch length is a constant value, as the unit pitch length decreases, the number of pitches will increase, the change frequency of fluid flow direction will increase, and the velocity will also increase. Meanwhile, a fluid...
backflow occurs at the turning point, resulting in increased disturbance, a greater heat transfer capacity, and accordingly a higher pressure drop.

For a PCHE, the distance $\delta$ between the adjacent channels of the single plate and the distance $t_w$ between the cold plate and the hot plate only affect its robustness, while their influences on the thermal-hydraulic performance can be neglected [40]. Therefore, for the analysis of PCHE performance with different channel diameters in this work, $\delta$ and $t_w$ are kept constant. Figure 9 shows the variation of the Nusselt number and pressure drop for different channel diameters and mass flow rates.

As shown in Figure 9, the Nusselt number and pressure drop enlarge with mass flow for the same channel diameter. It demonstrates that an increase in mass flow rate can improve heat exchange effectiveness, but the frequency of fluid-to-channel contact increases, increasing the flow resistance. For a constant mass flow rate, the Nusselt number and pressure drop enlarge as the channel diameter decreases.

To in-depth analyze the influence of channel diameter, the velocity vectors of the various channel diameters are shown in Figure 10. One can find that for a constant inlet mass flow rate, the smaller the channel diameter, the faster the fluid velocity, the easier it is to form turbulent flow, the stronger the heat transfer capacity, and the larger the Nusselt number. However, a higher flow rate would result in a greater channel pressure drop. Therefore, heat exchange ability and pressure loss should be considered when selecting the suitable channel diameter for a PCHE.

4.1.2. Nusselt Number Correlation. Since the fluid cannot form a fully developed flow in a zigzag PCHE, the Nusselt number cannot hold a constant value as in a straight PCHE. Therefore, a new correlation for He-Xe in a zigzag PCHE should be proposed.

Previous studies [13, 14] have developed correlations to account for the effects of zigzag angle and zigzag pitch length in PCHE, but their channel diameters are held at a constant value. However, as mentioned in Section 4.1.1, the channel...
diameter also has an important impact on the flow and heat transfer of a PCHE. Therefore, it needs to fit the Nusselt number as a function of \( D/\sigma, L/\delta, \alpha \), and \( \text{Re} \) based on the requirements of engineering applications.

Figure 11 shows the variation of the Nusselt number with different relative channel diameter ratios, relative pitches, and zigzag angles of PCHE hot-side channels. The line frames in Figures 11(a)–11(c) highlight the geometric parameter values in which the Nusselt number is the largest, which shows that the zigzag angle has a greater effect on the Nusselt number than the zigzag pitch length. In addition, when the Re is smaller, the Nusselt number is less affected by the geometric parameters. This is because the increase of the Re will cause the increase of the fluid velocity in the channel, and it can be seen that the heat transfer capacity of the PCHE will also be enhanced, so the Nusselt number will increase. As shown in Figure 11(d), with different Re, the variation of the Nusselt number caused by the identical variation of channel diameter is almost the same. The structural analysis of the cold-side channel is similar to the hot-side channel as shown in Figure 12.

The least squares curve fitting (lsqcurvefit) function in MATLAB is used in this work. The fitted function \( g(x) \) is shown as follows:

\[
g(x) = \frac{(a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4)}{(1 + a_5x_5 + a_6x_6)},
\]

where \( a_i \) is the undetermined coefficient and \( x_i \) is the independent variable.

\( R^2 \) is the coefficient of determination [41], which is a statistic number used to measure the goodness of fit. The maximum value of \( R^2 \) is 1. The closer the value of \( R^2 \) is to 1, the better the fitting degree. \( R^2 \) can be evaluated by

\[
R^2(y, \hat{y}) = 1 - \frac{\sum_{i=0}^{\text{samples}-1} (y_i - \hat{y}_i)^2}{\sum_{i=0}^{\text{samples}-1} (y_i - \bar{y})^2},
\]

where \( y \) is the measured value, \( \hat{y} \) is the predicted value, and \( \bar{y} \) is the average value of the simulation. Figure 13 shows the Nusselt number nonlinear fitting results, where the \( R^2 \) is 0.97, indicating that the fitted correlation of the criterion correlation for the Nusselt number is excellent.

The correlation of the Nusselt number is proposed as follows:

\[
\begin{align*}
\text{Nu}_h & = \frac{-1.8\alpha + 2.5 \times 10^{-2}(L/\delta) + 3.3 \times 10^{-3}(D/\delta) + 2.5 \times 10^{-3} \text{Re}_h}{1 - 1.13\alpha + 1.1 \times 10^{-2}(L/\delta)} \quad \text{for} \quad 220 < \text{Re} < 5000, \\
\text{Nu}_c & = \frac{-2.1\alpha + 2.5 \times 10^{-2}(L/\delta) + 3.6 \times 10^{-1}(D/\delta) + 2.6 \times 10^{-3} \text{Re}_c}{1 - 1.14\alpha + 1.1 \times 10^{-2}(L/\delta)} \quad \text{for} \quad 20 \leq \frac{L}{\delta} \leq 60, \\
& \quad \pi \leq \alpha \leq \frac{\pi}{6}, \\
& \quad 1.6 \leq \frac{D}{\delta} \leq 5.
\end{align*}
\]

The least squares nonlinear regression method is used to fit the data obtained from the numerical simulations (\( R^2 = 0.97 \)). The Nusselt number acquired from the numerical simulations and the calculation outcomes of the suggested Nusselt number correlation equation is compared in Figure 13. The two sets of findings are basically coincident. Therefore, the proposed Nusselt number correlation satisfies the requirements of engineering applications.
4.1.3. Fanning Friction Factor Correlation. In this subsection, the variation of the Fanning coefficient of friction with the Re for different zigzag pitch length ratios, zigzag angles, and channel diameter ratios of a zigzag PCHE is investigated, as shown in Figure 14. Line frames in Figures 14(a)–14(c) highlight the geometric parameter values with the smallest Fanning friction factor. It can be concluded that the zigzag angle has a greater influence on the Fanning friction factor than the zigzag pitch length. In addition, as the Re increases, the decline tendency of the Fanning friction coefficient becomes smooth. Figure 14(d) shows that the Fanning friction coefficient increases with channel diameter under the same Re. Furthermore, the channel diameter has a more significant effect on the Nusselt number when the Re is small, but this impact decreases as the Re increases. Similar to the hot-side channel, the cold-side channel's structural analysis is illustrated in Figure 15.

The correlation nonlinear fitting model of the Fanning friction factor is shown in Equation (9). Figure 16 shows the Fanning friction factor nonlinear fitting results, where \( R^2 \) is 0.99, indicating that the proposed correlation is excellent.

The relationship between Re, zigzag angle, relative channel diameter ratio, and relative zigzag pitch length ratio is used to construct the flow correlation. This relationship can be determined by
\begin{equation}
\begin{align*}
\left\{ \begin{array}{l}
 f \cdot \text{Re}_h = \frac{-10.8\alpha + 1.0 \times 10^{-1}(lz/\delta) + 8.1 \times 10^{-1}(D/\delta) + 1.0 \times 10^{-2} \text{Re}_h}{1 - 1.9\alpha + 1.3 \times 10^{-1}(lz/\delta)}, \\
 f \cdot \text{Re}_c = \frac{-15.6\alpha + 1.3 \times 10^{-1}(lz/\delta) + 1.2(D/\delta) + 1.3 \times 10^{-2} \text{Re}_c}{1 - 1.8\alpha + 1.1 \times 10^{-2}(lz/\delta)},
\end{array} \right. \\
220 < \text{Re} < 5000,
20 \leq \frac{lz}{\delta} \leq 60,
\frac{\pi}{36} \leq \alpha \leq \frac{\pi}{6},
1.6 \leq \frac{D}{\delta} \leq 5.
\end{align*}
\end{equation}

The results obtained by the numerical simulations are fitted using the least squares nonlinear regression method \((R^2 = 0.9)\). Figure 16 shows that the numerical simulation results of the Fanning friction factor fit excellent with the proposed correlation equation. Therefore, the proposed correlation equation can describe the friction coefficient when both the hot and cold working fluids are He-Xe for heat exchange.

4.2. Multiobjective Optimization

4.2.1. Optimization Procedure. The procedure of multiobjective optimization is shown in Figure 17. The population size is set to 400; the number of evolutionary generations is set
4.2.2. Optimization Results. Since the flow and heat transfer characteristics of a PCHE are closely related to its geometric structure parameters, this work is aimed at optimizing the geometric structure parameters. The ultimate goal of parameter optimization is to achieve one or several optimization objectives of a PCHE, such as the largest Nusselt number, the smallest friction coefficient, or the smallest mass. However, the above-mentioned performance objectives are often mutually restricted, and it is impossible to meet all the requirements simultaneously. In this work, based on the numerical simulation results combined with the multiobjective genetic algorithm, the optimal design is carried out by considering channel diameter, zigzag pitch length, and zigzag angle as design variables.

The multiobjective optimization algorithm can obtain a number of the Pareto optimal points, but in practical applications, only one of the solutions corresponds to the final optimization scheme. The best compromise option can be chosen from the Pareto optimal points if a decision-making procedure based on the shortest normalized distance [42] is utilized.

By comparing the distance between each Pareto optimal point and the ideal virtual solution in Figure 18, one can find that the nondominated solution with the smallest distance is the closest to the ideal virtual solution, which is an optimal compromise point. Figure 19 shows the independent variable solution set for the Pareto optimal points.

The Latin hypercube sampling approach is used to sample 400 sets of sample data in order to examine each variable’s sensitivity to the two target functions in the compromise solution produced by fuzzy decision-making [43]. As shown in Figure 20, the zigzag angle has the most significant effect on the Nusselt number and Fanning friction factor, while the channel diameter has the least effect. The above results are of great significance in selecting geometric parameters of a PCHE to improve its performances.

5. A Case Study

Lightweight is the main goal of CBC system design, in which the mass of the recuperator accounts for a relatively large

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**Figure 16:** Fanning friction factor nonlinear fitting results.

**Figure 17:** A flowchart of the NSGA-II optimization process.

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to a maximum number of 500; the crossover probability and mutation probability are both set to 0.6 and 0.02, respectively.
Therefore, reducing the mass of the recuperator is essential for a lightweight CBC system. In addition, the heat transfer efficiency of the recuperator has a great influence on the CBC system efficiency. This work proposes a method combined with NSGA-II to optimize the heat transfer efficiency and mass of PCHE-type recuperator.

The mass and heat transfer efficiency of the PCHE-type recuperator are taken as the objective function, and its independent variables include zigzag pitch length \( x_1 \), channel diameter \( x_2 \), channel length \( x_3 \), and zigzag angle \( x_4 \). The ranges of values for these independent variables are \( 8 \text{ mm} < x_1 < 32 \text{ mm}, 1 \text{ mm} < x_2 < 3 \text{ mm}, 200 \text{ mm} < x_3 < 800 \text{ mm}, \) and \( 5^\circ < x_4 < 45^\circ \). Equation (13) shows the optimization model of the PCHE for minimizing its mass \((M_{\text{min}})\) and for maximizing its heat transfer efficiency \((\eta_{\text{max}})\). Taking the lightest weight and the highest efficiency as the objective function, the performance design requirements of the PCHE-type recuperator are shown in Table 5.

\[
\begin{align*}
\min M &= F_1(x_1, x_2, x_3, x_4), \\
\max \eta &= F_2(x_1, x_2, x_3, x_4).
\end{align*}
\]

A PCHE-type recuperator mainly consists of PCHE-type recuperator core, pressing plate, cocurrent flow, splitting box, and header pipe, while the PCHE-type recuperator core accounts for most of the weight. Therefore, the mass optimization of PCHE-type recuperator is mainly aimed at the PCHE-type recuperator core part. The calculation formula of the mass of PCHE-type recuperator is as follows:

\[
M_{\text{core}} = n \cdot M_{\text{single, channel}} = n \cdot \rho_{\text{material}} \left( \frac{H \times L \times W}{2} - \frac{(\pi r + D) \times L}{\text{channel volume}} \right).
\]

The calculation formula for heat transfer efficiency of PCHE-type recuperator is as follows:

\[
\eta = \frac{T_{\text{c,out}} - T_{\text{c,in}}}{T_{\text{h,in}} - T_{\text{c,in}}},
\]

Figure 21 shows the Pareto optimal points for all the PCHE configurations constrained by two objective functions (heat transfer efficiency and mass). Each point in the Pareto optimal points represents an optimal solution. Table 6 provides five representative (A–E) solutions of the Pareto optimal points, which can provide a reference for SIMONS’ PCHE-type recuperator design.

6. Conclusions

The current work conducts an optimized analysis of a PCHE in a closed He-Xe Brayton cycle. Flow and heat transfer characteristics of the zigzag PCHE with He-Xe as the medium are studied. Key geometric variables, including the Nusselt number and Fanning friction factor, are analyzed concerning the changes of geometric parameters, such as channel diameter, zigzag angle, and zigzag pitch length. A method using NSGA-II for the multiobjective optimization of PCHE is proposed. The following conclusions are drawn from this work:

(a) The thermal-hydraulic performances of He-Xe in a zigzag PCHE are studied. For a constant Re, it can be concluded that both the Nusselt number and pressure drop increase as the zigzag angle increases, the channel diameter decreases, or the zigzag pitch length decreases. When the Re varies, the zigzag angle has a more significant effect on the Nusselt number than the zigzag pitch length. In addition, the zigzag angle and zigzag pitch length have more
significant effects on the Nusselt number as the channel diameter increases.

(b) The heat transfer correlations are proposed for the zigzag PCHE, which are suitable for $220 < Re < 5000$, $Pr = 0.296$, $5° < \alpha < 35°$, $20 < l_z/\delta < 60$, and $1.66 < D/\delta < 5$.

(c) The flow correlations for the zigzag PCHE have been developed, which are valid for $50 < Re < 5000$, $5° < \alpha < 35°$, $20 < l_z/\delta < 60$, and $1.66 < D/\delta < 5$.

(d) The Pareto optimal points of flow and heat transfer are obtained based on the proposed correlations. Then, the compromise solution is obtained by the shortest normalization method, and the sensitivity analysis of the three independent variables of the multiobjective optimization is carried out. The zigzag angle has the greatest influence on the Nusselt number and Fanning friction factor.

(e) Aiming at the PCHE-type recuperator of CBC system, a method combined with NSGA-II is proposed, and the multiobjective optimization of lightweight and high heat exchange efficiency is carried out.

Table 5: Performance design parameters of the PCHE-type recuperator.

<table>
<thead>
<tr>
<th>Design parameters</th>
<th>Hot side</th>
<th>Cold side</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass flow (kg/s)</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Pressure (MPa)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Inlet temperature (K)</td>
<td>970.45</td>
<td>485.04</td>
</tr>
<tr>
<td>Maximum length (m)</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>Maximum width (m)</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Maximum height (m)</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Maximum pressure drop (kPa)</td>
<td>50</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 6: The structural parameters of the multiobjective optimization results for PCHE-type recuperator.

<table>
<thead>
<tr>
<th>Points</th>
<th>$l_z$ (mm)</th>
<th>$D$ (mm)</th>
<th>$L$ (mm)</th>
<th>$\alpha$ (°)</th>
<th>Heat transfer efficiency ($\eta$)</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>16.3</td>
<td>1.2</td>
<td>182</td>
<td>25</td>
<td>0.875</td>
<td>875</td>
</tr>
<tr>
<td>B</td>
<td>17.6</td>
<td>2</td>
<td>259</td>
<td>18</td>
<td>0.906</td>
<td>1215</td>
</tr>
<tr>
<td>C</td>
<td>21.4</td>
<td>1.9</td>
<td>393</td>
<td>29</td>
<td>0.931</td>
<td>1656</td>
</tr>
<tr>
<td>D</td>
<td>15</td>
<td>1.6</td>
<td>524</td>
<td>21</td>
<td>0.952</td>
<td>2412</td>
</tr>
<tr>
<td>E</td>
<td>13.6</td>
<td>2.5</td>
<td>733</td>
<td>35</td>
<td>0.965</td>
<td>3433</td>
</tr>
</tbody>
</table>
And the corresponding Pareto optimal points are obtained, which have referential significance to the engineering design of the PCHE-type recuperator.

In summary, the correlations are proposed for the PCHE applied in the He-Xe closed Brayton cycle in SIMONS. Further research of thermal-hydraulic performance of this zigzag PCHE will be carried out in an experimental loop, which can provide design criteria for the PCHE using He-Xe as the working medium.

**Nomenclature**

- \(a_i\): Empirical coefficient
- \(A_c\): Channel cross-sectional area (m²)
- \(c_p\): Specific heat (J kg⁻¹ K⁻¹)
- \(D\): Diameter (m)
- \(D_h\): Hydraulic diameter (m)
- \(f\): Frictional factor
- \(h\): Heat transfer coefficient (W m⁻² K⁻¹)
- \(H\): Height of solid body (m)
- \(He\): Helium
- \(L\): Length of channel (m)
- \(l_z\): Zigzag pitch length (m)
- \(m\): Molar mass (g mol⁻¹)
- \(M_{core}\): Core mass (kg)
- \(M_{min}\): Minimum mass (kg)
- \(M_{single, channel}\): Single channel mass (kg)
- \(Nu\): Nusselt number
- \(N\): Molar mass flow rate (mol s⁻¹)
- \(q\): Heat flux (W m⁻²)
- \(q_m\): Mass flow rate (kg s⁻¹)
- \(P\): Pressure (Pa)
- \(ΔP\): Pressure drop (Pa)
- \(R^2\): Coefficient of determination
- \(Re\): Reynolds number
- \(t_{w}\): The distance between the cold plate and the hot plate (m)
- \(T_{c,in}\): Cold-side inlet temperature (K)
- \(T_{c,out}\): Cold-side outlet temperature (K)
- \(T_{h,in}\): Hot-side inlet temperature (K)
- \(T_{h,out}\): Hot-side outlet temperature (K)
- \(T_i\): Inlet temperature (K)
- \(T_w\): Area-weighted average wall temperature (K)
- \(T_b\): Volume-weighted average bulk temperature (K)
- \(ΔT\): Logarithmic mean temperature difference (°C)
- \(v\): Velocity (m s⁻¹)
- \(W\): Width of solid body (m)
- \(x_i\): Independent variable
- \(Xe\): Xenon
- \(y\): Measured value
- \(y\): Predicted value
- \(\bar{y}\): Average value of simulation.

**Greek Symbols**

- \(α\): Zigzag angle (°)
- \(\lambda\): Thermal conductivity (W m⁻¹ K⁻¹)
- \(μ\): Viscosity (Pa s)
- \(ρ\): Density (kg m⁻³)
- \(δ\): Distance between adjacent channels (m)
- \(η\): Heat transfer efficiency.

**Data Availability**

The numerical simulation data used to support the findings of this study were supplied by Hui Yin under license and so cannot be made freely available. Requests for access to these data should be made to Hui Yin at yinhui@sinap.ac.cn.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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**References**


