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Research Article

Research on the Quantitative Causal Transmission of Stock Price Fluctuations of Listed Companies in the Rare Earth Industry Chain

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Rare earth is an important strategic mineral resource, and the study of the causes of stock price fluctuations of rare earth listed companies and their transmission can enable all parties in the market to have a clearer understanding of current market relationships. In this study, the quantitative causality analysis model and complex network method are used to analyze each link within the rare earth industry chain and the relationship between regions from the quantitative causality of stock price fluctuations among listed companies in the rare earth industry. The main transmission paths of fluctuations in each link between companies are also given. It is found that the quantitative causal relationship between the price fluctuations of rare earth industry stocks has positive causation, negative causation, and unclear causation, and the dominant causal relationship is mostly positive causation. The positive causal relationship between enterprises in the same region is significantly greater than that between enterprises in different regions. The negative causal relationship and unclear causal relationship between enterprises in different regions are significantly greater than the negative causal relationship and unclear causal relationship between enterprises in the same region.

1. Introduction

Rare earth is a strategic resource known as "the vitamins of industry" and is divided into light rare earth and heavy rare earth according to mineral characteristics. Since rare earth is used by the military and in the metallurgical, petrochemical, and other industries, the development of the rare earth industry is vital for modern industrial development. The rare earth industry chain has an enterprise group structure with certain intrinsic connections, and the upper, middle, and downstream links of the industrial chain are responsible for the development of rare earth ores, rare earth smelting and separation, and their deep processing and application, respectively [1]. Products and value flow between each link: upstream to midstream transmission of products and downstream to upstream transmission of value and information. A change in one link will affect other links in their own development direction and investment and consumption decisions.

At present, scholars' research on rare earths and the rare earth industry mainly focus on the impact of global rare earth resource mining, the impact of rare earths on the environment [2–4], China's rare earth industry policy [5–7], the supply and demand of rare earth resources [8, 9], the development of the rare earth industry chain, and investment in the rare earth stock market.

On the one hand, the research on the rare earth industry chain is to evaluate and analyze the industrial chain. Liying and Shenglian [10] conducted a comprehensive performance evaluation of the rare earth industry chain. Binqing [11] conducted a SWOT analysis and study on the Jiangxi rare earth industry chain in 2012. On the other hand, the influencing factors of the development of the rare earth industry chain are studied. Juan et al. [12] analyzed the financial support efficiency and influencing factors of China's rare earth industry. According to Yiding et al. [13], based on the two major driving factors of efficiency and risk, the main factors hindering the extension of the rare earth industry chain were analyzed. According to Yiding et al. [14], the study concluded that the instability of rare earth raw material supply prices is not conducive to the upgrading of China's rare earth industry. In addition to the research on China's rare earth industry chain, there is also the exploration of the development trend of Russia's rare earth industry [15]. These studies provide a reference for the sustainable development of rare earth element markets and rare earth-related industries in China and the world.

The research on rare earth capital market and investment research, on the one hand, is the study of the investment value and structure of the rare earth stock market [16]. On the other hand, in the analysis of the correlation between rare earth stocks and other industries, Reboredo and Ugolini [17] used the Markov zone system conversion vector autoregressive model to analyze the price spillover effect between rare earth stocks and financial markets. Chen et al. [18] examined the volatility spillover effects and dynamic correlations between international crude oil, new energy, and Chinese rare earth markets. Bouri et al. [19] analyzed the dynamic return and volatility correlations between rare earth stock indices and clean energy, consumer electronics, telecommunications, medical devices, and aerospace and defense indices and showed that extreme market scenarios have a strong impact on both earnings and volatility connectivity dynamics. Zheng et al. [20] studied the asymmetric connectivity and dynamic spillover of China's renewable energy and rare earth markets and described the risk transfer between renewable energy and rare earth market enterprises in the form of network connectivity. However, there is still little research on the rare earth stock market based on the perspective of the industrial chain.

The stock market is closely related to other industries and has high-dimensional complexity. There are a variety of factors, some of which have very close and complex relationships. Many scholars use network analysis to explore these relationships [21–23]. The correlation, causation, cointegration, and other relationships between price fluctuations between multiple stocks are important components of complexity science. However, compared with correlation, causality can explore the core problems of the internal laws of complex systems.

In recent years, many scholars have used a variety of methods to study causality. Granger's causality test has been widely used in economics, finance, ecological environment research, and other fields [24–26], and satisfactory results have been achieved in the field of linearity. However, complex systems in real life are often nonlinear, so scholars have carried out optimization research on the nonlinear Granger test and the traditional Granger causal test, both of which have been widely used [27–29]. In addition to the Granger causal test, Sugihara proposed a cross-convergence mapping (CCM) algorithm in 2012 to explore the causal relationship of complex nonlinear systems from the perspective of dynamics. By applying the CCM algorithm, it was determined that there are three causal relationships in the ecosystem: unidirectional, bidirectional, or driven by external variables [27]. However, this method is only suitable for moderately coupled subsystems and will fail when the coupling subsystem is too weak or too strong. These methods gradually shift from simple linear systems to practical complex nonlinear systems and provide many references for the study of causality. However, these analyses are mainly data-driven analysis methods, which can only reflect the successive linkage relationship on the data and cannot be called causation.

Stavroglou et al. prescribe causal theory (PC) to explore causality. Using symbolic dynamics theory, each variable is symbolized to reduce the influence of noise. According to PC theory, positive causality and negative causality exist in ecosystems. Notably, the financial market is characterized by more complex causal relationships such as disordered relationships, where variable X and variable Y neither "compete" nor "promote" [30]. The complex network method is a new method for studying the relationship between entities and has been widely used in various disciplines in recent years. Some scholars have studied the correlation of financial markets, and many use complex networks to model single-layer networks of stock markets [31–33]. These models can capture the price fluctuation relationship between stocks and, when combined with econometric methods, facilitate studying the spread of financial risks through network indicators, revealing the characteristics of complex systems to a large extent.

Therefore, in order to explore the stock price fluctuation relationship of various links of the global rare earth industry chain and companies in various regions, it provides a decision-making reference for participants in the rare earth stock market. Based on PC theory, this paper uses the phase space reconstruction method to restore the chaotic attractor based on PC theory, constructs the modal state while removing noise interference by symbolic dynamics, and performs weighted average calculation of the mode to obtain quantitative causality. After considering the practical factors and adding others such as regional factors, the proportion of main products, and the weighted return on net assets, a quantitative causal relationship model is formed, and a complex network is constructed from its calculation results. The strength of the causal relationship network between stock prices in the rare earth industry, the weighting degree of the strong causal relationship network, the betweenness centrality, and the clustering coefficient were analyzed considering the industrial chain and the region where the enterprise is located. The maximum fluctuation transmission path is also determined according to the industrial chain links. In this way, we will explore the relationship between companies in the global rare earth industry chain and between regions.

The contribution of this paper is as follows: firstly, through the set phase space reconstruction, symbolic dynamics, and sliding window method, the quantitative causality model is constructed, so as to obtain the positive causality, negative causal relationship, and unknown causal relationship between the stocks of listed companies in the rare earth industry and comprehensively reflect the causal characteristics of the fluctuation between stock prices. Finally, combined with the complex network method, the quantitative causal relationship network of the rare earth industry chain is constructed, the relationship between various links and regions of the rare earth industry chain is analyzed, and the main fluctuation transmission path is found. Based on this, it provides decision-making support for various participants in the stock market.

The rest of this article is organized below. Section 2 describes the data methodology. Section 3 discusses the results of causal strength. Section 4 discusses strong causality in rare earth industry chain stock markets. Section 5 identifies major volatility transmission paths, and finally, Section 6 summarizes the findings and provides recommendations for investors to invest in rare earth stocks.

2. Data and Methods

2.1. Definition of the Rare Earth Industry Chain. The industrial chain consists of upstream enterprises, midstream enterprises, and downstream enterprises. In this study, the upstream rare earth industry chain includes mineral mining, rare earth smelting, ore refining, and other enterprises in the rare earth industry. Mining, smelting, and purification of light rare earths and heavy rare earths are also included.

The midstream industrial chain includes rare earth permanent magnet materials, rare earth catalytic materials, rare earth hydrogen storage materials, rare earth luminescent materials, rare earth polishing materials, and other related product generation, manufacturing, and sales enterprises.

The downstream industrial chain is composed of rare earth permanent magnet product application enterprises (mainly motor consumer electronic equipment generation enterprises with NdFeB magnets as raw materials), motor exhaust gas purification with molecular rare earth catalysis as the main raw materials, petroleum catalytic cracking product generation enterprises, portable electronic devices with nickel-metal hydride batteries as the main raw materials, hybrid electric vehicles, and other production enterprises.

This study focuses on listed companies in the global rare earth industry, so the defined rare earth industry chain is limited to listed companies.

The study selected the data of listed companies in the global rare earth industry chain industry map of the Wonder Database Industry Chain Platform as the research object, because some key companies did not disclose data before 2013, and the research period was from March 1, 2013, to December 31, 2020. Excluding companies listed and delisted in the selected year, there were 42 domestic and foreign listed companies, with a total of 1912 observations, as shown in Figures 1–3. There are 7 stocks of upstream companies, 16 stocks of midstream companies, and 19 stocks of downstream companies, for a total of 42 stocks.

2.2. Construction of a Quantitative Causal Network Model for Price Fluctuations between Stocks in the Rare Earth Industry Chain. Due to the high-dimensional complexity characteristics of stock time series, when constructing the model, it is necessary to first use the phase space reconstruction theory method to analyze the high-dimensional complex system [34]. Then, to facilitate the study and reduce noise interference, symbolic dynamics theory is used to modulate the fluctuation relationship to calculate the strength of the data-based linkage relationship of the stock time series [30]. Finally, combined with the stock price fluctuations of listed companies in the global rare earth industry chain and the main influencing factors of the correlation between stock price fluctuations, a comprehensive model group is designed to calculate quantitative causality, as shown in Equations (1)–(3). There are three kinds of causal relationships between stock price fluctuations, namely, positive causation, negative causation, and unclear causation.

Positive causation indicates the extent to which the price fluctuations of the stocks that are the cause of fluctuation codirectionally affect the price fluctuations of other stocks.

$$\mathrm{CV}(p) = f_p\left(X_1, X_2, \cdots\cdots X_i, \cdots\cdots X_j, \cdots\cdots X_{42}\right). \tag{1}$$

Negative causality indicates the extent to which the price fluctuations of the stocks that are the cause of fluctuation inversely affect the price fluctuations of other stocks.

$$CV(n) = f_n\left(X_1, X_2, \cdots\cdots X_i, \cdots\cdots X_j, \cdots\cdots X_{42}\right). \tag{2}$$

Unclear causation indicates the extent to which the price fluctuation of the stock that is the cause of fluctuation does not affect the price fluctuation of other stocks.

$$\mathrm{CV}(d) = f_d\left(X_1, X_2, \cdots\cdots X_i, \cdots\cdots X_j, \cdots\cdots X_{42}\right). \tag{3}$$

The formula $(X_1, X_2, \cdots\cdots X_i, \cdots\cdots X_j, \cdots\cdots X_{42})$ is the closing price time series of 42 stocks input to the model, and X_i represents the price time series of stock I, with a total of 42 stock price time series. The output of the model is $\mathrm{CV}(p), \mathrm{CV}(n), \mathrm{CV}(d)$, representing three 42×42 matrices, a positive causality matrix, a negative causality matrix, and an unclear causality matrix.

The model realizes the calculation of causality between multiple stocks for price fluctuations and forms three causal matrices $\mathrm{CV}(p), \mathrm{CV}(n), CV(d)$. Table 1 describes the related variables used, $\mathbb{R}$ represents the set of real numbers, $\mathbb{N}$ represents the set of nonnegative integers, $\mathbb{R}^E$ represents the set matrix of real numbers, and $\mho^E$ represents the matrix of symbol sets.

To calculate the quantized causality between two stocks, first, the attractors L_{x_i} and L_{x_j} of two time series $X_i(t)$ and $X_j(t)$ are constructed. Then, symbolic dynamics theory is used to obtain the average mode of each point on the attractor, and finally, the linkage relationship between $X_i(t)$ and $X_j(t)$ is inferred by exploring the characteristics of the modes of attractor L_{x_i} and the points on L_{x_j}. By repeating this process between the pairs, the linkage relationship matrix can be obtained, and the key process of the entire calculation process is shown in Figure 2.

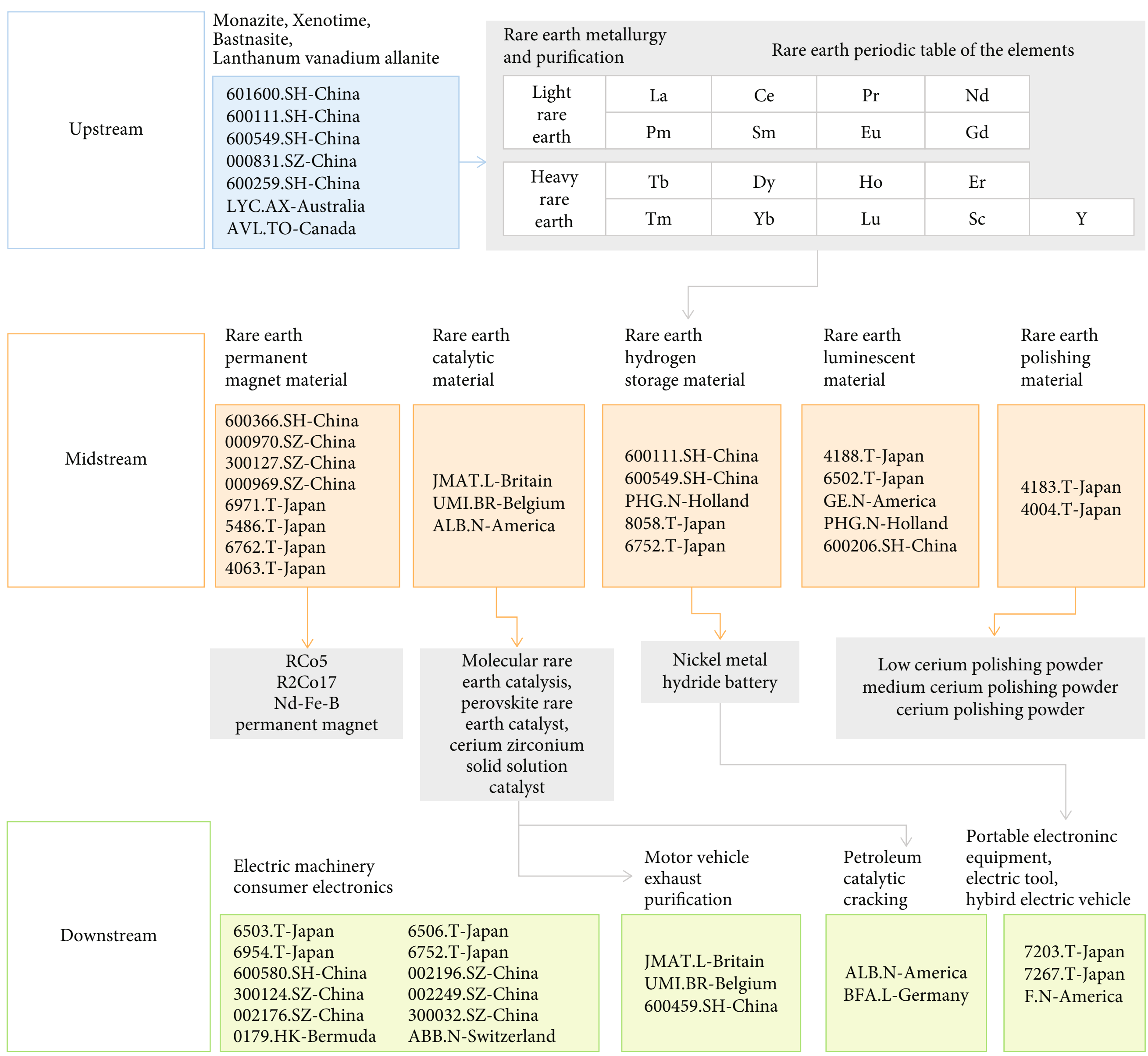


FIGURE 1: Rare earth industry chain industry map.

2.2.1. Analysis of High-Dimensional Complex Systems. The stock market constitutes a high-dimensional complex system. Observing the high-dimensional system information of the system requires an appropriate methodological system. The phase space reconstruction theory can well analyze the high-dimensional complex system. Chaotic attractors L_{x_i} and L_{x_j}, which are topologically isomorphic to the original chaotic system of stock price fluctuation, are reconstructed by using the coordinate delay reconstruction method of phase space reconstruction [35]. There is a one-to-one mapping between the points on L_{x_i} and L_{x_j} and the points on attractor L of the original chaotic system of stock price fluctuation, so there is a one-to-one mapping between the reconstructed attractors L_{x_i} and L_{x_j}. Therefore, we can study the relationship between stock X_i and stock X_j by exploring the characteristics of attractors L_{x_i} and L_{x_j}.

For stock $X_i(t), t = 1, \cdots, k$, choose the appropriate embedding dimension f, delay τ, and reconstruct the attractor L_{x_i}. Any point on it $x_i(t)$ is shown in the following equation:

$$x_i(t) = (X(t), X(t-\tau), \cdots, X(t-(f-1)\tau)), t = 1 + (f-1)\tau, \cdots, k. \tag{4}$$

In this paper, the C-C algorithm is used to determine $f = 3$ and $\tau = 1$ at the same time [36].

2.2.2. Modulation of Points on Stock Time Series Chaotic Attractors. Symbolic dynamics are used to modulate the points on the reconstructed stock price time series attractors, which can effectively reduce the influence of noise. Let $f : R^n \longrightarrow R^{n-1}$ be the sign conversion function, and map the n-dimensional vector to the $n-1$ dimensional vector.

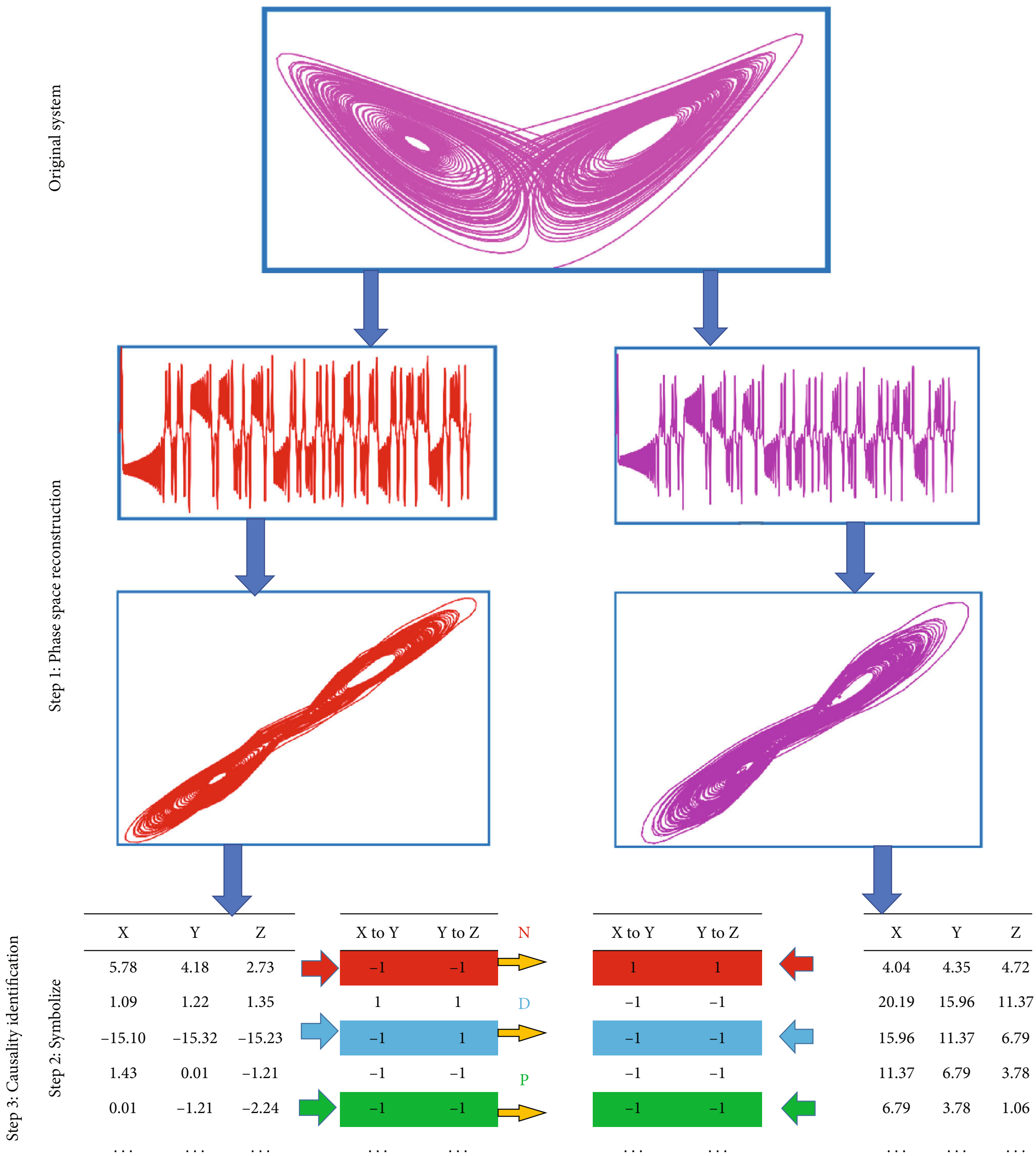


Figure 2: Some of the key processes of the algorithm. Note: as shown by the Lorenz model, in order to discuss the causal relationship between stock X_i and stock X_j from the Lorenz model, the first step is to carry out phase space reconstruction of stock X_i and stock X_j, respectively. Then, the vectors of attractor stock X_i and stock X_j are modalized, respectively. Finally, the linkage relationship between the X_i and stock X_j models at the same time is distinguished. P = positive, N = negative, D = unclear.

When embedding dimension $f = 3$, for L_{x_i} any point,

$$x_i(t) = (X(t), X(t-\tau), X(t-2\tau)). \tag{5}$$

If $X(t) > X(t-\tau) > X(t-2\tau)$, for its modal, $Mx_i(t) = f(x_i(t)) = (-1,-1)$.

If $X(t) = X(t-\tau) = X(t-2\tau)$, for its modal, $Mx_i(t) = f(x_i(t)) = (0, 0)$.

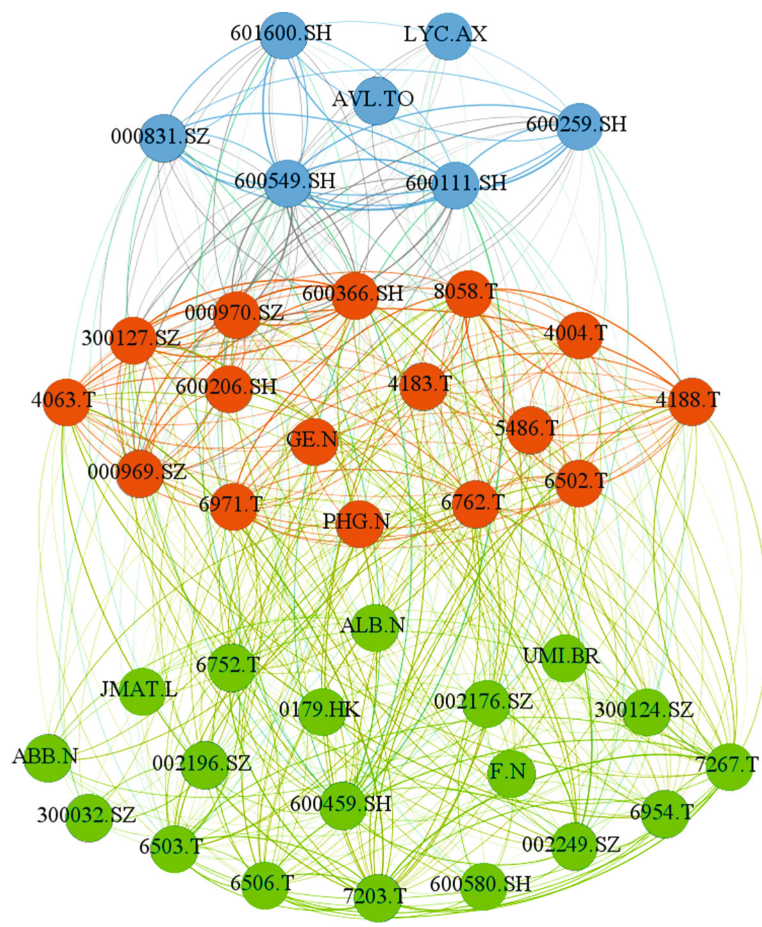


FIGURE 3: Quantitative causality network diagram of rare earth industry chain.

TABLE 1: Describes the relevant variables.

Variable	Description
$X_i, X_j \in \mathbb{R}$	Stock i and j time series (set of real numbers)
$X_i(t) \in \mathbb{R}$	The value corresponding to time t in the stock i time series
$k \in \mathbb{N}$	Stock time series length (1912)
$t \in \mathbb{N}$	Time
$f \in \mathbb{N}$	Embedding dimensions
$\tau \in \mathbb{N}$	Delay time
$L_{x_i} \in \mathbb{R}^E$	An attractor reconstructed by stock i to be topologically isomorphic to the original system
$H_{x_i} \in \mathbb{R}^E$	Neighborhood of the attractor L_{x_i} of stock X_i time series reconstruction
$M_{X_i(t)} \in \mho^E$	Current mode of $X_i(t)$
$\mathrm{CV}_{ji}(\text{positive}) \in \mathbb{R}$	The strength of the positive causal relationship between stock j and stock i
CV	Causality matrix for price fluctuations between stocks

If $X(t) < X(t-\tau) < X(t-2\tau)$, for its modal, $Mx_i(t) = f(x_i(t)) = (1, 1)$.

Then, calculate the mean modes of the points on the attractor. For example, there are two modes:

$M_{x_i(1)} = (1,-1)$, $x_i(1) = (2, 6, 3)$, $k_1 = (6-2/2, 3-6/6)$, with weight $w_1 = 0.3$.

$M_{x_i(2)} = (-1, 1)$, $x_i(2) = (4, 2, 8)$, $k_2 = (2-4/4, 8-2/2)$, with weight $w_2 = 0.7$.

$$k_t = \sum k_i w_i = (2,-0.5) * 0.3 + (-0.5,3) * 0.7 = (0.25,1.95). \tag{6}$$

Then, the average mode of $x_i(1)$, $x_i(2)$ is $M_{(X_i(1),X_i(2))} = (1, 1)$.

2.2.3. Calculation of Linkage Relationship Strength. To calculate the strength of the linkage relationship (CV) from point $X_j(t)$ on stock X_j to point $X_i(t)$ on stock X_i, first, find $f+1$ points on attractor L_{x_i} closest to the f-dimensional variable $x_i(t)$. Then, calculate the average mode $Mx_i(t)$ and estimate the mean mode $\widehat{M}x_j(t)$ of $x_j(t)$, which is in the same segment as $x_i(t)$ in neighborhood H_{x_i} of $x_i(t)$. Finally, the ratio of the estimated average mode of $X_j(t)$ to its true mean mode agreement is calculated, which is CV. The value of

CV ranges from zero to one. A smaller value indicates a weaker linkage, and a larger value indicates a stronger linkage. The same method can also be used to calculate the linkage strength (CV) from $X_i(t)$ to $X_j(t)$.

The specific steps to calculate the causal relationship between point $X_j(t)$ on stock X_j and point $X_i(t)$ on stock X_i consist of four steps [37].

(1) Calculate the average mode of $x_i(t)$. Select the $f+1$ points closest to $x_i(t)$, that is, $x_i(t_m)$, and calculate the weights of $x_i(t_m)$ and $x_i(t)$, respectively w_m^x, $m=1,\cdots,f+1$. The closer the points are, the greater the weight. Adjacent components in each $x_i(t_m)$ are differentiated sequentially to find the mode, which gives h_m^x, $m=1,\cdots,f+1$. The weighted sum of nearest neighbor points is then calculated, i.e., $H_{x_i(t)}$. Finally, $H_{x_i(t)}$ is modularized according to the transfer function, and $Mx_i(t)$ is obtained, which is the average mode of $x_i(t)$

$$
\begin{aligned}
&x_i(t_m)=(X(t_m),X(t_m-\tau),\cdots,X(t_m-(f-1)\tau)),x(t_m)\in N_{x_i(t)},\\
&h_m^x=\left(\frac{X_i(t_m-\tau)-x_i(t_m)}{X_i(t_m)},\cdots,\frac{X_i(t_m-(E-1)\tau-X_i(t_m-(E-2)\tau}{X_i(t_m-(E-2)\tau}\right),\\
&w_m^x=\frac{e^{-d(x(t),x(t_i))}}{\sum_i e^{-d(x(t),x(t_i))}}, d \text{ is Euclidean distance},\\
&H_{x_i(t)}=\sum_{m=1}^{f+1}w_m^x h_m^x,\\
&M_{x_i(t)}=f\left(H_{x_i(t)}\right).
\end{aligned}
\tag{7}
$$

(2) Estimate the average mode of $X_j(t)$ over the same period. $x_j(t)$ is the point at the time corresponding to $x_i(t)$. In this case, the selection of the near point of $x_j(t)$ is the value corresponding to the time of $x_i(t)$ near the point, i.e., $\widehat{x}_j(t_m), t_m=t_{x_i(t_1)},\cdots,t_{x_i(t_{f+1})}$

$$
\begin{aligned}
&\widehat{x}_j(t_m)=\left(\widehat{X}_j\left(t_{x_{im}}\right)\right),\widehat{X}_j\left(t_{x_{im}}-\tau\right)\cdots,\widehat{X}_j\left(t_{x_{im}}-(f-1)\tau\right)\right),\\
&\widehat{N}_{\widehat{x}_j(t)}=\widehat{X}_j\left(t_{x_1}\right),\cdots,\widehat{X}_j\left(t_{x_{f+1}}\right),\\
&h_m^{\widehat{X}_j}=\left(\frac{\widehat{X}_j\left(t_{x_2}\right)-\widehat{X}_j\left(t_{x_1}\right)}{\widehat{X}_j\left(t_{x_1}\right)},\cdots,\frac{\widehat{X}_j\left(x_{t_E}\right)-\widehat{X}_j\left(x_{t_{E-1}}\right)}{X_j\left(x_{t_{E-1}}\right)}\right),\\
&\widehat{H}_{X_j(t)}=\sum_{m=1}^{f+1}w_m^{x_i}h_m^{X_j},\\
&\widehat{M}_{X_j(t)}=f\left(\widehat{H}_{X_j(t)}\right).
\end{aligned}
\tag{8}
$$

(3) Finally, the true mean mode of $x_j(t)$ is calculated

$$
\begin{aligned}
&x_j(t_m)=\left(X_j(t_m),X_j(t_m-\tau),\cdots,X_j(t_m-(f-1)\tau)\right),x_j(t_m)\in N_{x_j(t),}\\
&h_m^{x_j}=\left(\frac{X_j(t_m-\tau)-X_j(t_m)}{X_j(t_m)},\cdots,\frac{X_j(t_m-(f-1)\tau)-X_j(t_m-(f-2)\tau)}{X_j(t_m-(f-2)\tau)}\right),\\
&w_m^{x_j}=\frac{e^{-d\left(x_j(t),x_j(t_m)\right)}}{\sum_i e^{-d\left(x_j(t),x_j(t_m)\right)}}, d \text{ is Euclidean distance},\\
&H_{x_j(t)}=\sum_{m=1}^{f+1}w_m^{yx_j}h_m^{x_j},\\
&M_{x_j(t)}=f\left(H_{x_j(t)}\right).
\end{aligned}
\tag{9}
$$

(4) The process is repeated for any $x_i(t)$ on L_{x_i}, filling the CV matrix with the proportion that the estimated average mode agrees with the true average mode

Figure 4 is a CV matrix of the linkage relationship from $X_j(t)$ to $X_i(t)$ when the embedded dimension $f=3$, and so on for other embedding dimensions. The first column of the matrix is $x_i(t)$, the mean mode on its nearest neighbor $H_{x(t)}$, and the first row is the average mode of $x_j(t)$ at the same time as $x_i(t)$ on its nearest neighbor $H_{x_j(t)}$. The elements in the matrix are the ratio of the estimated average mode of the contemporaneous $x_j(t)$ to the true $x_j(t)$ of the contemporaneous segment, with values ranging from zero to one. The green part of the main diagonal element of the matrix indicates that the $x_j(t)$ mean mode is consistent with the $x_j(t)$ mean mode, that is, the positive linkage relationship. The red part of the secondary diagonal element indicates that the $x_j(t)$ mean mode is the opposite of the $x_j(t)$ mean mode, that is, the negative linkage relationship, while the blue part indicates the unclear linkage relationship. The main diagonal elements are averaged to obtain the strength of the positive linkage relationship from $X_j(t)$ to $X_i(t)$, and the secondary diagonal elements are averaged to obtain the strength of the negative linkage relationship from $X_j(t)$ to $X_i(t)$. The average number of elements other than the primary diagonal element and the secondary diagonal element is the strength of the unclear linkage relationship from $X_j(t)$ to $X_i(t)$.

The strength of the positive linkage relationship from stock $X_j(t)$ to $X_i(t)$ is shown in the following equation:

$$
\mathrm{CV_{ji}}(\text{positive})=\frac{1}{9}\sum_{\text{main}}(\text{matric}(\text{CV})).
\tag{10}
$$

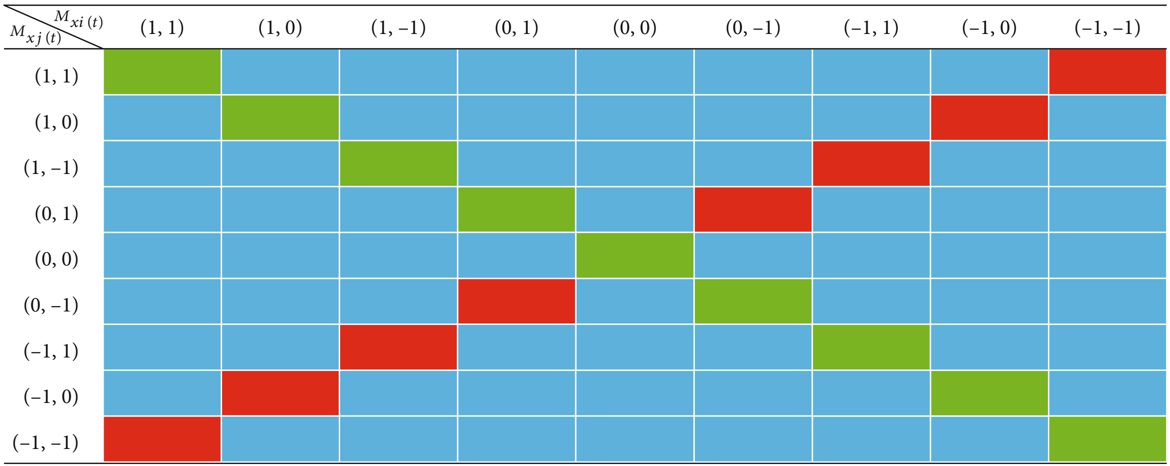


FIGURE 4: CV matrix (dimension $f = 3$).

The strength of the negative linkage relationship from stock $X_j(t)$ to $X_i(t)$ is shown in the following equation:

$$\mathrm{CV}_{\mathrm{ji}}(\text{negative}) = \frac{1}{8}\sum_{\text{counter}}(\text{matric}(\mathrm{CV})). \tag{11}$$

The strength of the unclear linkage relationship from stock $X_j(t)$ to $X_i(t)$ is shown in the following equation:

$$\mathrm{CV}_{\mathrm{ji}}(\text{disordered}) = \frac{1}{64}\sum_{\text{others}}(\text{matric}(\mathrm{CV})). \tag{12}$$

2.2.4. The Quantitative Causality Model of Industrial Chain Characteristics Is Introduced. The strength of causality calculated by the pattern causality model can be considered as a successive linkage relationship in the data or the causal relationship in the physical sense, However, there remains a large gap between the causality in the real sense. Therefore, to use the above calculation methods, this study considers the actual influencing factors of rare earth stock price fluctuations, introduces the "actual factors" σ_{ij} and designs a causal relationship strength calculation model in line with the global rare earth industry chain.

In addition to the technical calculation based on the above fluctuation data and modal probability, many market factors should also be considered when analyzing the quantitative causal relationship between the stocks of listed companies in the rare earth industry. To make the problem solvable, only the proportion of rare earth products in the main business, return on net assets, and regional factors are considered.

The positive quantization causality model is

$$\mathrm{cv}_{\mathrm{ij}}(p) = \mathrm{CV}_{\mathrm{ij}}(\text{positive}) \times \sigma_{\mathrm{ij}}(p). \tag{13}$$

The negative vectorized causality model is

$$\mathrm{cv}_{\mathrm{ij}}(n) = \mathrm{CV}_{\mathrm{ij}}(\text{negative}) \times \sigma_{\mathrm{ij}}(n). \tag{14}$$

The unclear quantitative causality model is

$$\mathrm{cv}_{\mathrm{ij}}(d) = \mathrm{CV}_{\mathrm{ij}}(\text{disordered}) \times \sigma_{\mathrm{ij}}(d). \tag{15}$$

The actual factors in equation σ_{ij} are expressed as shown in the model in Equations (16)–(18).

The positive true factor model is shown in the following equation:

$$\sigma_{ij}(p) = \begin{cases} \dfrac{1}{\alpha_{ij}} & \text{Stocks in the same link of the industrial chain,} \\ \dfrac{\beta_i R_j}{\beta_j R_i\,\alpha_{ij}} & \text{Stocks are in the sequential adjacent links of the industrial chain,} \\ \dfrac{\beta_j R_j}{\beta_i R_i\,\alpha_{ij}} & \text{Stocks are in the reverse order of adjacent links in the industrial chain,} \\ \dfrac{\beta_i R_j}{2\beta_j R_i\,\alpha_{ij}} & \text{Stocks are in the industrial chain that are not adjacent in order,} \\ \dfrac{\beta_j R_j}{2\beta_i R_i\,\alpha_{ij}} & \text{Stocks in the industrial chain in reverse order not adjacent links.} \end{cases} \tag{16}$$

The negative true factor model is shown in the following equation:

$$\sigma_{ij}(n) = \begin{cases} \dfrac{1}{\alpha_{ij}} & \text{Stocks in the same link of the industrial chain,} \\ \dfrac{\beta_j R_i}{\beta_i R_j\,\alpha_{ij}} & \text{Stocks are in the sequential adjacent links of the industrial chain,} \\ \dfrac{\beta_i R_i}{\beta_j R_j\,\alpha_{ij}} & \text{Stocks are in the reverse order of adjacent links in the industrial chain,} \\ \dfrac{\beta_j R_i}{2\beta_i R_j\,\alpha_{ij}} & \text{Stocks are in the industrial chain that are not adjacent in order,} \\ \dfrac{\beta_i R_i}{2\beta_j R_j\,\alpha_{ij}} & \text{Stocks in the industrial chain in reverse order not adjacent links.} \end{cases} \tag{17}$$

The unclear true factor model is shown in the following equation:

Table 2: Characteristics of the influence of the proportion of main products on causality.

Perspective	Influence the process	Regularity
Upstream	β_i is a positive indicator when calculating the upstream-to-midstream causal relationship β_i is a negative indicator when calculating the midstream versus upstream causal relationship β_i halves the impact when calculating causality with downstream	Upward control downward conduction Up is negative, and down is positive Halving the impact between upstream and downstream
Midstream	β_j is a positive indicator when calculating the upstream-to-midstream causal relationship β_j is a negative indicator when calculating the midstream versus upstream causal relationship β_j is a positive indicator when calculating the midstream versus downstream causal relationship β_j is a negative indicator when calculating the downstream versus midstream causal relationship	
Downstream	β_k is a positive indicator when calculating the downstream versus midstream causal relationship β_k is a negative indicator when calculating the causal relationship between midstream and downstream β_k halves the impact when calculating causality with upstream	

Table 3: Characteristics of the impact of ROE on stock causation.

Perspective	Influence the process	Regularity
Upstream	R_i is a negative indicator when calculating the upstream versus midstream causal relationship R_i is a positive indicator when calculating the midstream versus upstream causal relationship R_i halves the impact when calculating causality with downstream	Drive up and control down Down is negative, and up is positive Halving the impact between upstream and downstream

$$\sigma_{ij}(d)=\begin{cases}1 & \text{Stocks in the same link of the industrial chain,}\\ \left(1-\beta_j\right)\times(1-\beta_i)\times\alpha_{ij} & \text{Stocks are in the adjacent links of the industrial chain,}\\ \dfrac{(1-\beta_i)\times\left(1-\beta_j\right)}{2}\times\alpha_{ij} & \text{Stocks in the industrial chain in not adjacent links.}\end{cases} \tag{18}$$

In the above three formulas, α_{ij} is the regional factor. β is the proportion of rare earth products in the main products, and the value range is $0<\beta\le 1$. The proportions of the corresponding main products of upstream stock i, midstream stock j, and downstream stock k are β_i, β_j, β_k. R is the return on equity, upstream stock i, midstream stock j, and downstream stock k corresponding to the return on net assets are R_i, R_j, R_k.

In the same region, supply and demand are less disturbed by various factors, with strong certainty, clear correlation, and a strong positive causal relationship. Product connectivity and capital connectivity are strong, the composition of investor groups is relatively stable, the structural similarity is strong, and the positive causal relationship is strong. In different regions, supply and demand are affected by a variety of factors, capital connectivity is relatively poor, investor group structure is quite different, and investment preferences and other aspects are also relatively different. This can be understood as a heterogeneous market, which has a weakening effect on positive and negative causality and has an enhanced effect on unclear causality. The regional correlation index is recorded as α, and the value range is shown in the following equation:

$$\alpha_{ij}=\begin{cases}1 & \text{When stock i and stock j are in the same region,}\\ 2 & \text{When stock i and stock j are in different regions.}\end{cases} \tag{19}$$

The "industry" factor is expressed in terms of the proportion of rare earth-related products in the main product, i.e., the proportion of the main product is β, and the characteristics of the impact of β on the causal relationship are shown in Table 2. The proportion of nonrare earth products in the main products reflects unclear causality.

The "industrial chain" factor is expressed by "weighted return on net assets" because this indicator can not only reflect the operation of listed companies and the growth and profitability of enterprises but also the prosperity of the industry to a certain extent. The higher the index, the stronger the ability of the enterprise's own capital to obtain income, the better the operational efficiency, and the greater the competitiveness and influence in the industrial chain. The characteristics of the impact on stock causation are shown in Table 3.

The high return on weighted equity in the upstream reflects the rising cost of raw materials and enhanced control over the midstream and downstream enterprises. The influence on causality is mostly negative. The control ability of the downstream is realized through the midstream, but its control power is greatly attenuated. For the upstream, because of its strong influence and high driving force, the impact on causality is mostly positive. For the downstream, the midstream is mainly control-oriented, and the influence on causality is mostly characterized by a negative direction. The downstream has a more obvious driving effect on the midstream, and its influence on causality is mostly positive, while the influence on the upstream drives its development by driving the midstream. However, the degree of influence attenuates considerably but still presents positive influence characteristics overall.

Through the addition of the above three factors, the model calculation results can effectively reflect the degree of causal relationship between the global rare earth industry chain.

2.2.5. Construction of the Quantitative Causal Network of the Rare Earth Industry Chain. Network diagrams provide great visualization. In this study, a causal relationship network was constructed with stocks as nodes, causal relationships as edges, and quantitative causal strength (CV) as edge weights. The schematic diagram is shown in Figure 3. Three directed weighted causal network diagrams (positive causality, negative causality, and unclear causation) were constructed for seven upstream stocks, 16 midstream stocks, 19 downstream stocks, and all 42 stocks, for a total of 12 causal network diagrams.

The causal network diagrams of upstream and downstream stocks are all full network diagrams, and one stock is an isolated point in the causal network diagram of midstream and all stocks, and the others are full network diagrams, but the weights of the network edges are different, indicating that there are various causal relationships of different strengths between the stocks. The weighting degree of 12 causal networks is further used to analyze the strength of causal relationships.

The analysis mainly uses some topological indicators of the constructed network for analysis, including indegree, outdegree, weighted indegree, weighted outdegree, betweenness centrality, and clustering coefficient. These indicators can represent the impact of stock fluctuations on other stock fluctuations in the entire industry, the centrality of stocks in the industry, and the role of the industrial chain, respectively.

3. Causal Strength Analysis

The causal relationship strength model is used to calculate the positive causal relationship strength value, negative causal relationship strength value, and unclear causal relationship strength value between the two pairs of stocks. Next, the strength values of the three causal relationships will be compared and analyzed according to the location of the industrial chain, reflecting the distribution characteristics of the three causal relationships in different links of the industrial chain.

3.1. Positive Causality Strength Analysis. From the distribution of causal relationship strength and its average intensity, the basic situation of a positive causal relationship between stock price fluctuations in each link of the industrial chain is analyzed.

According to the calculation results, the positive causal relationship strength values between the upstream stocks are distributed between 0 and 0.8, where 40% of the values are distributed between 0.3 and 0.8, and the average strength is 0.41, as shown in Figure 5(a).

The positive causality strength values between the two midstream stocks are distributed between 0 and 0.8, of which more than 62% are distributed above 0.3, and their average strength values are 0.35, as shown in Figure 5(b).

The positive causal strength values between the two pairs of downstream stocks are mainly distributed between 0 and 0.7, of which more than 65% are distributed above 0.3, and the average strength value is 0.338, as shown in Figure 5(c).

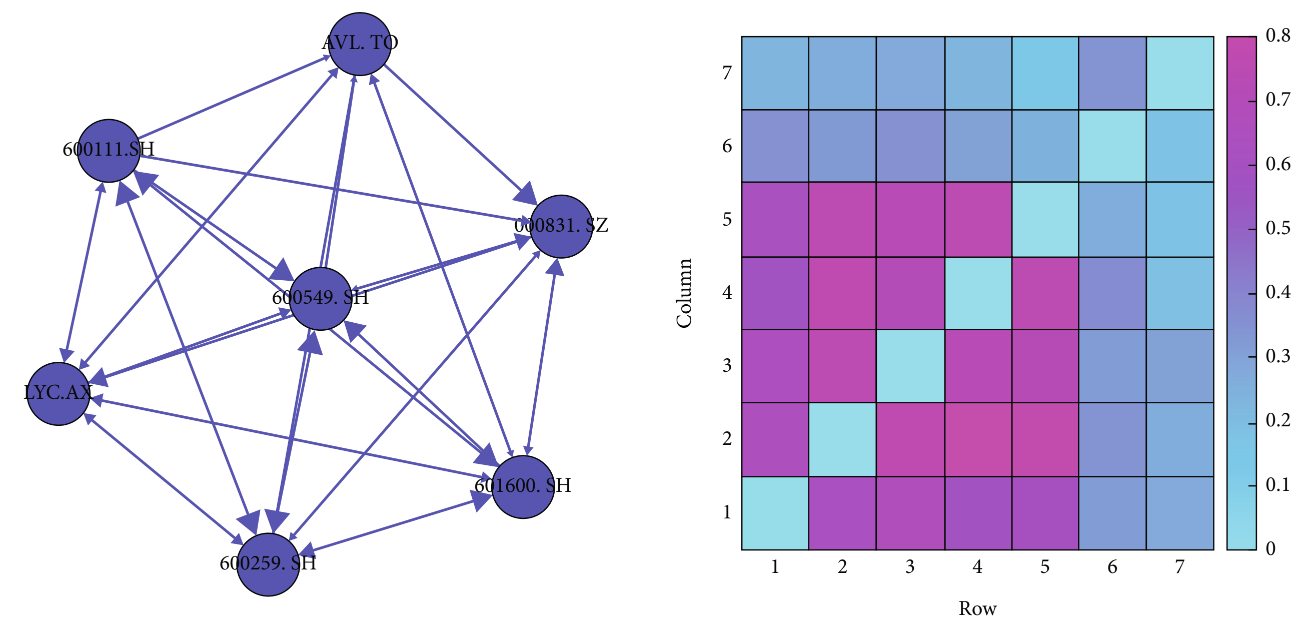
The positive causal relationship strength values between all stocks in the industrial chain are mainly distributed between 0 and 0.8, of which more than 63% are distributed above 0.3, the highest value is 0.8, and the average intensity value is 0.356, as shown in Figure 5(d). This is shown in Table 4.

From the above data and the comparison of Table 4 data, it can be seen that the maximum and average values of the positive causal relationship between the upstream stocks of the rare earth industry chain are relatively large, and the strength of the positive causal relationship between the midstream and downstream stocks weakens in turn, an outcome related to the proportion of main products. Upstream enterprises usually have the largest proportion of rare earth products in the main products, while the main products in the midstream are relatively scattered, and the main products of downstream enterprises are more diversified. The technical content is relatively higher, and the influence of rare earth products in their main products will decline significantly. The decline of main products leads to a decrease in the causal relationship between enterprises in the positive fluctuation of stock prices, which is in line with reality and the characteristics of the data.

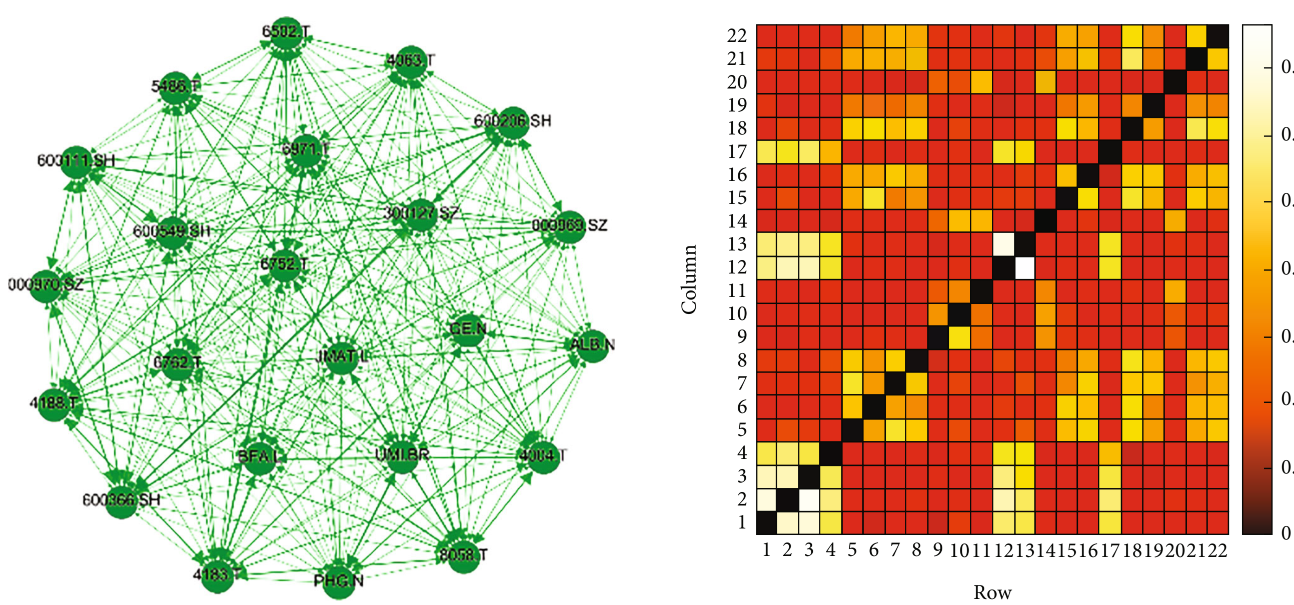
However, the comparison of the positive causal relationship strength data between all stocks shows that its maximum value is significantly higher than the causal relationship between stock price fluctuations within a single link of the industrial chain, mainly due to the performance-driven relationship between enterprises in the upstream and downstream of the industrial chain. The weighted return on net assets of midstream and downstream enterprises will affect the strength of the positive causal relationship between the performance of upstream and midstream enterprises.

3.2. Negative Causality Strength Analysis. From the distribution of causal relationship strength and its average intensity, the basic situation of a negative causal relationship between stock price fluctuations in each link of the industrial chain is analyzed.

The strength values of negative causality between stocks of upstream listed companies are mainly distributed

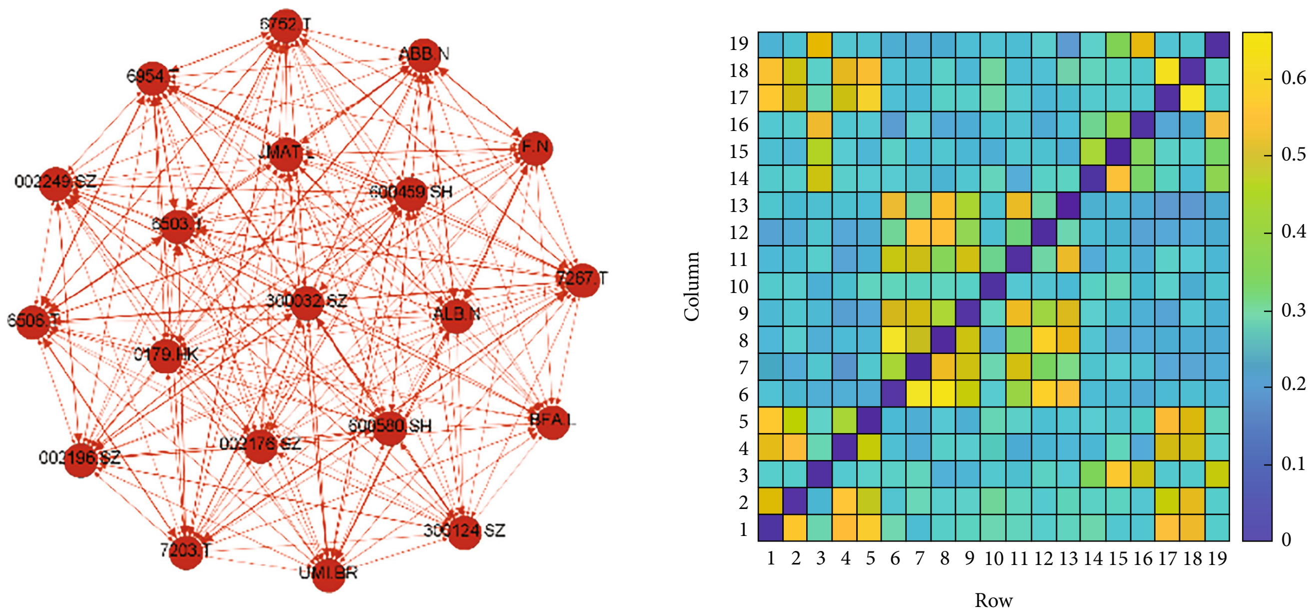


(a)



(b)

Figure 5: Continued.



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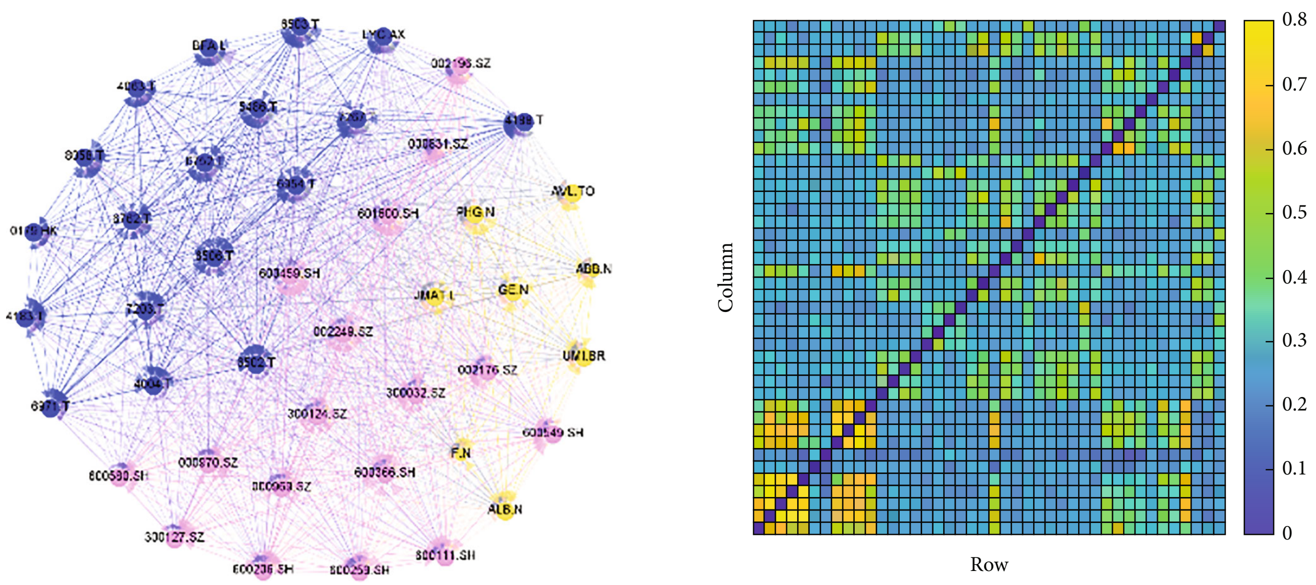


(d)

Figure 5: Positive causality strength network.

Table 4: Comparison of positive causality data.

Industrial chain links	Average intensity	Maximum
Upstream	0.41	0.785
Midstream	0.35	0.766
Downstream	0.338	0.661
Whole	0.356	0.802

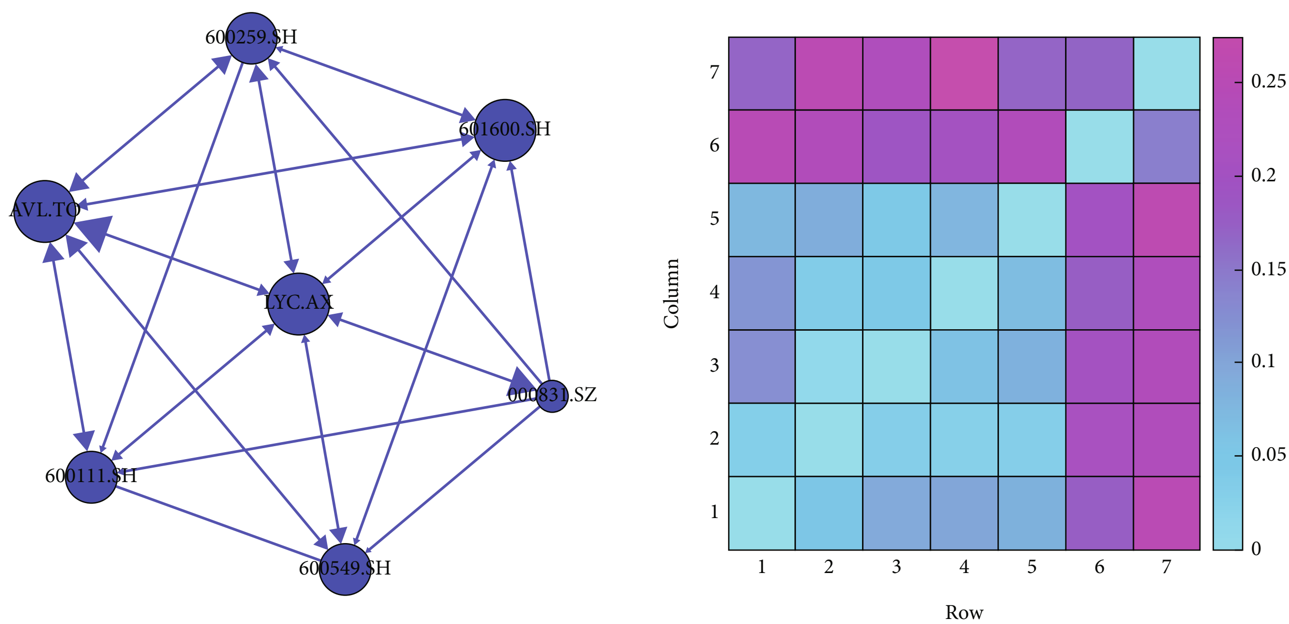
between 0 and 0.1, all values are less than 0.3, and their average strength is 0.123, as shown in Figure 6(a).

The strength values of negative causality between stocks of midstream listed companies are mainly distributed between 0 and 0.3, 92% of the values are less than 0.25, and the average strength is 0.18, as shown in Figure 6(b).

The strength of negative causality between stocks of downstream listed companies is mainly distributed between 0 and 0.3, 91% of the values are less than 0.25, and the average strength is 0.181, as shown in Figure 6(c).

The strength value of the negative causal relationship between all stocks in the industrial chain is mainly distributed between 0 and 0.35, 99% of the values are less than 0.3, and the average strength is 0.182, as shown in Figure 6(d). This is shown in Table 5.

Thus, the strength distribution of negative causality is the opposite of positive causation, with the upstream being the smallest, and the midstream and downstream increasing



(a)

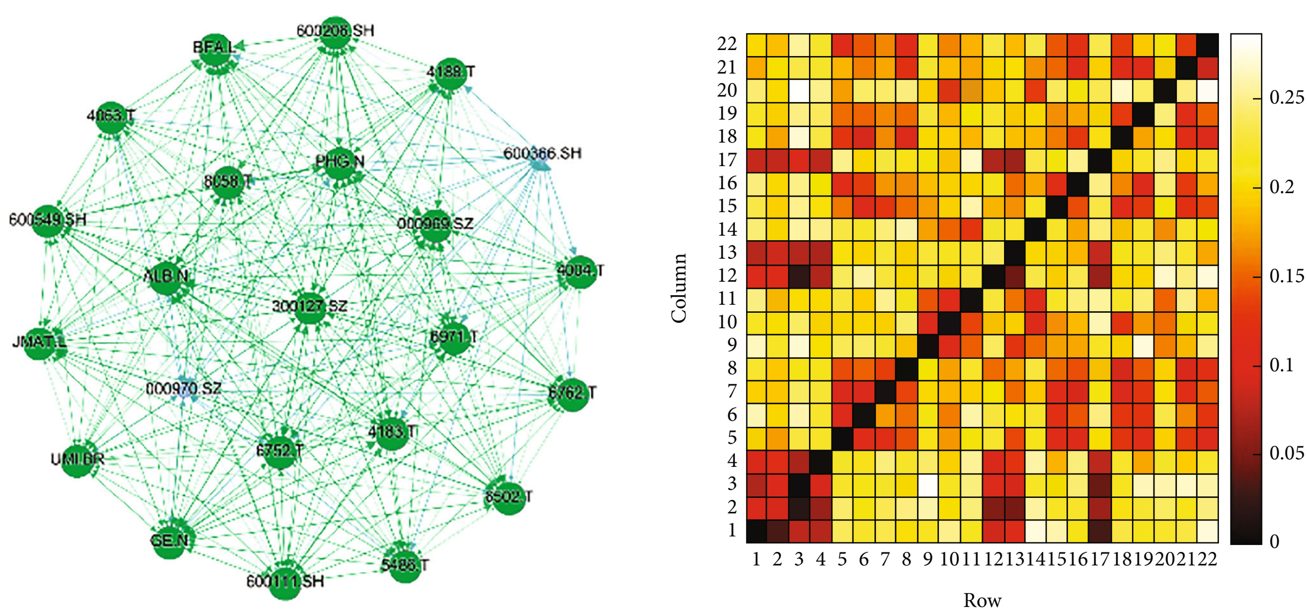


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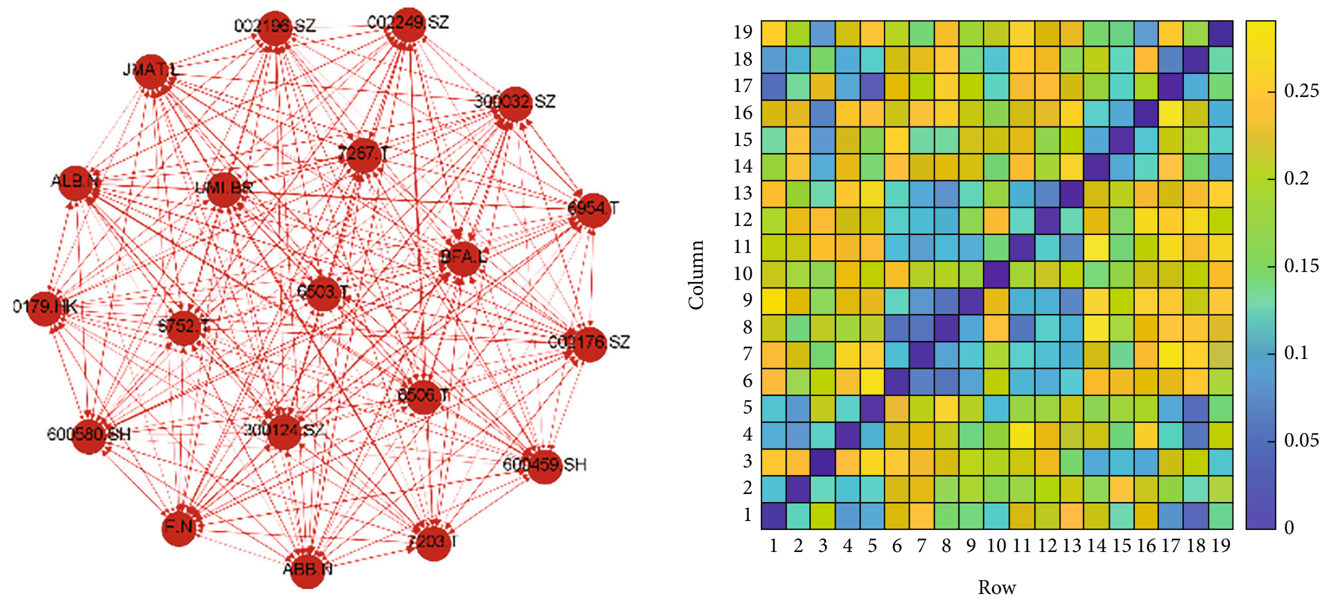
FIGURE 6: Continued.



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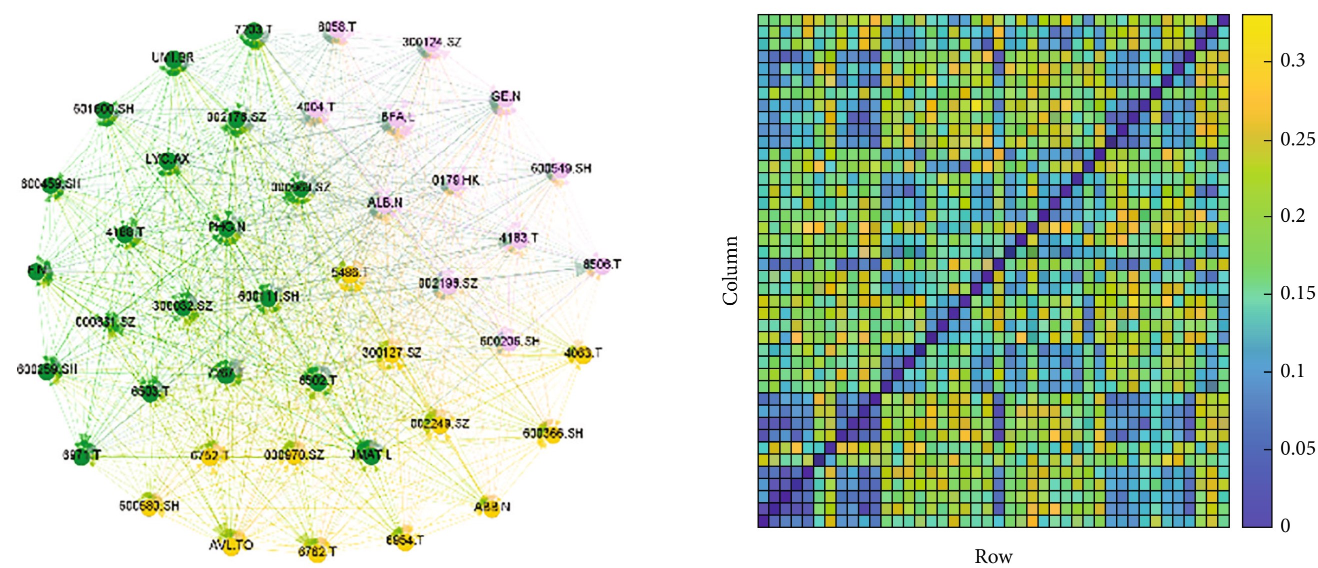


(d)

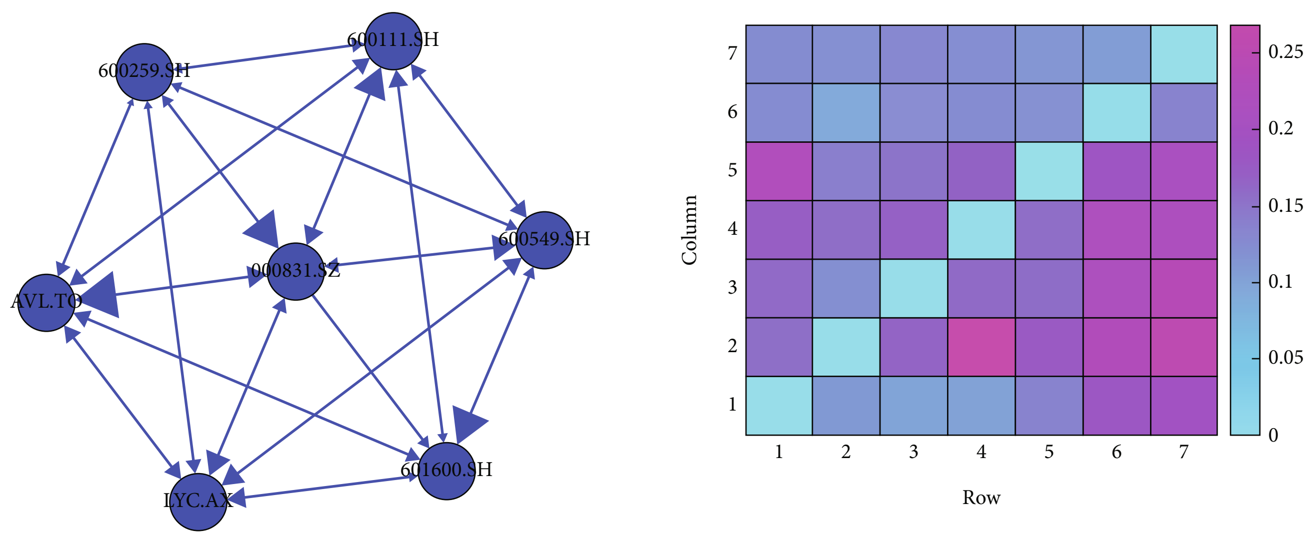
Figure 6: Negative causality strength network.

Table 5: Comparison of negative causality data.

Industrial chain links	Average intensity	Maximum
Upstream	0.123	0.258
Midstream	0.18	0.287
Downstream	0.181	0.291
Whole	0.182	0.331

in turn. However, the overall situation of a negative causal relationship between stocks of listed companies in the industrial chain is slightly different from that of positive causality, and its average strength and maximum value are greater than those between stocks within each link.

On the one hand, the reason for this distribution may be that the weighted return on net assets of upstream and midstream enterprises is high, and the asking price ability of midstream and downstream enterprises is strong, resulting in a negative correlation between causality. On the other hand, it may be because of the high regional concentration of upstream enterprises, and the regional distribution of midstream and downstream enterprises is more dispersed. If upstream firms are more geographically concentrated, the negative impact between them may be more significant. The geographical distribution of middle and downstream enterprises is more dispersed, which may reduce the negative impact between each other.



(a)

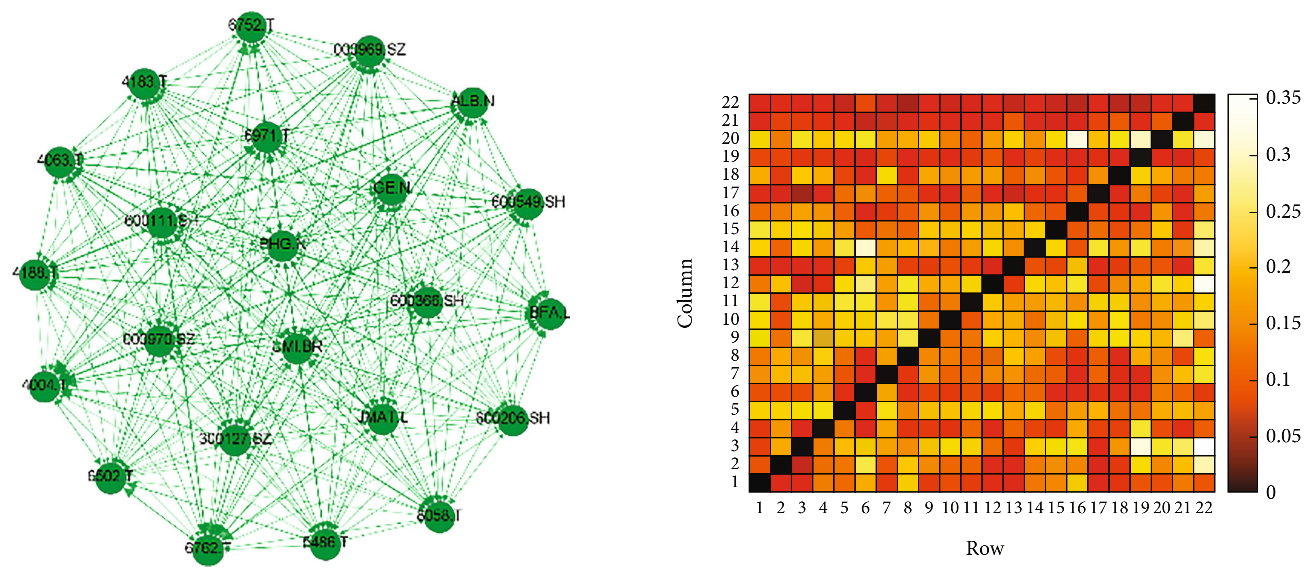


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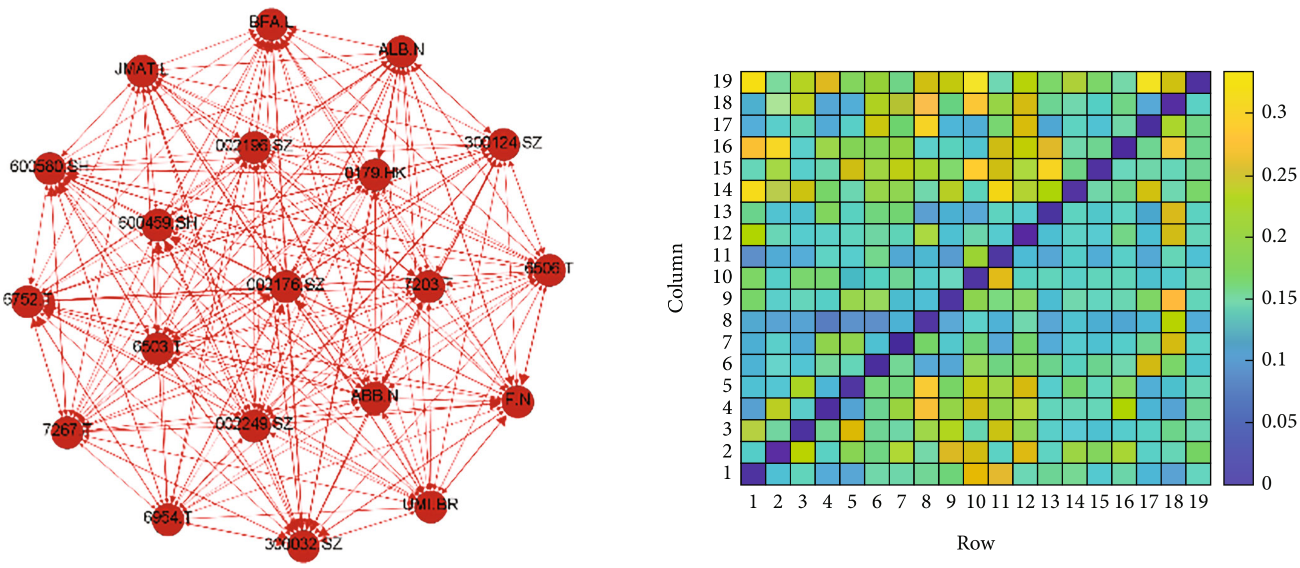
Figure 7: Continued.



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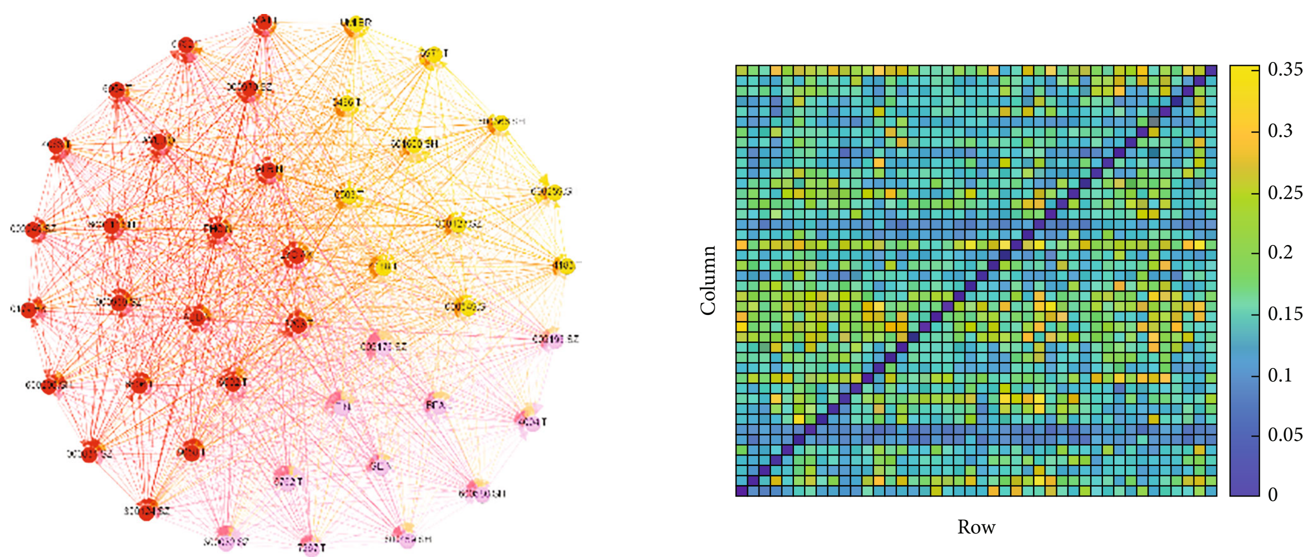

(d)

Figure 7: Unclear causality strength network.

3.3. Unclear Causality Strength Analysis. From the distribution of causal relationship strength and its average intensity, the basic situation of unclear causal relationships between stock price fluctuations in each link of the industrial chain is analyzed.

The strength value of the unclear causal relationship between the stocks of upstream listed companies is distributed between 0 and 0.3, 96% of the values are less than 0.25, and the average strength is 0.136. This is shown in Figure 7(a).

The midstream unexplained causality intensity values are distributed between 0 and 0.35, 98% of the values are less than 0.3, and the average intensity is 0.19, as shown in Figure 7(b).

The downstream unexplained causal intensity values are distributed between 0 and 0.35, more than 98% of the values are less than 0.3, and the average intensity is 0.182, as shown in Figure 7(c).

The intensity value of unclear causality of all listed companies in the industrial chain is distributed between 0 and 0.4, 98% of the values are less than 0.3, and the average strength is 0.184. This is shown in Table 6.

From the distribution of indicators of unclear causal relationship between the stocks of listed companies in each link, one can observe that the average strength and maximum intensity of the unclear causal relationship between the stocks of midstream listed companies are the largest, and the upstream is the smallest.

The unclear causal relationship is mainly reflected in the nonrare earth products in the main products and between listed companies in the same region. This may be because the market for nonrare earth products is more complex

Table 6: Comparison of unclear causal data.

Industrial chain links	Average intensity	Maximum
Upstream	0.136	0.269
Midstream	0.19	0.355
Downstream	0.182	0.335
Whole	0.184	0.355

Table 7: Comparison of causal data by type.

Causal type	Industrial chain links	Average intensity	Maximum
Positive	Upstream	0.41	0.785
	Midstream	0.35	0.766
	Downstream	0.338	0.661
	Whole	0.356	0.802
Negative	Upstream	0.123	0.258
	Midstream	0.18	0.287
	Downstream	0.181	0.291
	Whole	0.182	0.331
Unclear	Upstream	0.136	0.269
	Midstream	0.19	0.355
	Downstream	0.182	0.335
	Whole	0.184	0.355

and uncertain, and price fluctuations are more susceptible to multiple factors. In contrast, the demand and supply of rare earth products in the global market are relatively stable. Upstream companies are more concentrated in the Chinese region, which can lead to relatively clear relationships between them as they operate in similar geographical environments. Midstream companies, on the other hand, are more geographically dispersed, which can lead to stock price movements among them being subject to more unexplained causation. Therefore, because upstream companies are more concentrated in China and the proportion of rare earth main products is high, the unclear causal relationship is weak. The regional distribution of listed companies in the midstream is relatively scattered, the main products are also scattered, and the unclear causal relationship is stronger.

3.4. Overall Situation of Causality Strength. The average strength and maximum values of positive causation, negative causality, and unclear causality are shown in Table 7 below.

The strength of the positive causal relationship between stocks of listed companies in the rare earth industry chain is obviously large, and the strength of negative causality and unclear causality is relatively weak. However, compared with the upstream, midstream, and downstream internals, the maximum value of various causal relationships between all stocks is significantly larger. Since the analysis of all stocks takes into account the industrial chain factors (i.e., the interaction between the upstream and downstream of the industrial chain), the average strength and maximum causality strength of various causal relationships between all stocks are generally high.

4. Strong Causality Analysis of the Rare Earth Industry Chain Stock Market

In this case, to be more representative of reality, in addition to analyzing the distribution of strength and size, it is necessary to target the causal relationship between the three causal relationships (which is relatively the largest and dominant causal relationship) and the causal relationship with the largest absolute intensity.

4.1. Dominating Causality. There are three kinds of causal relationships between different stocks in the rare earth industry chain stock market, each with its own intensity. The causal relationship with the greatest intensity dominates and is called the dominant causal relationship between them.

4.1.1. The Upstream Stock Market Dominates Causality. Figure 8 shows the dominant causal relationship between the seven upstream stocks. Each column represents a dominant causal relationship, with Figure 8(a) representing positive causation, Figure 8(b) representing negative causation, and Figure 8(c) representing unclear causality. It can be clearly seen from the figure that the positive causality dominates between different stocks, and there are also a few negative causal relationships and fewer unclear causal relationships. For example, A.A.M to 000831.SZ and 000831.SZ and LYC.AX to A.A.M are dominated by negative causality. There is no case where unclear causation dominates.

Figure 8 shows that stock prices generally fluctuate in the same direction. While there is a large reverse influence relationship between several pairs of stocks, there is no stock with an unclear relationship as its main relationship. The stocks of listed companies with a negative causal relationship as the dominant relationship often belong to the relationship between different regions, which indicates that there is a competitive relationship between enterprises in the same industrial link between different regions.

4.1.2. The Midstream Stock Market Dominates Causality. Figure 9 specifically shows the dominant causal relationship between 24 (including eight stocks that belong to the upper midstream, or both the midstream and downstream) midstream stocks. It can be clearly seen from the figure that positive causality is also dominant between different stocks, and there are a few negative causal relationships and fewer unclear causal relationships. At the same time, unclear causality is significantly more dominant than negative causation. For example, 000969.SZ to PHG.N, 6752.T to 600206.SH, and GE.N to 4188.T are dominated by a negative causal relationship. Meanwhile, 6971.T and ALB.N; 5486.T and PHG.N; 4063.T and ALB.N; 6752.T and 300127.SZ, GE.N; 600206.SH and PHG.N; 6502.T and 300127.SZ, GE.N; 4183.T and 300127.SZ, JMAT.L; 4004.T and

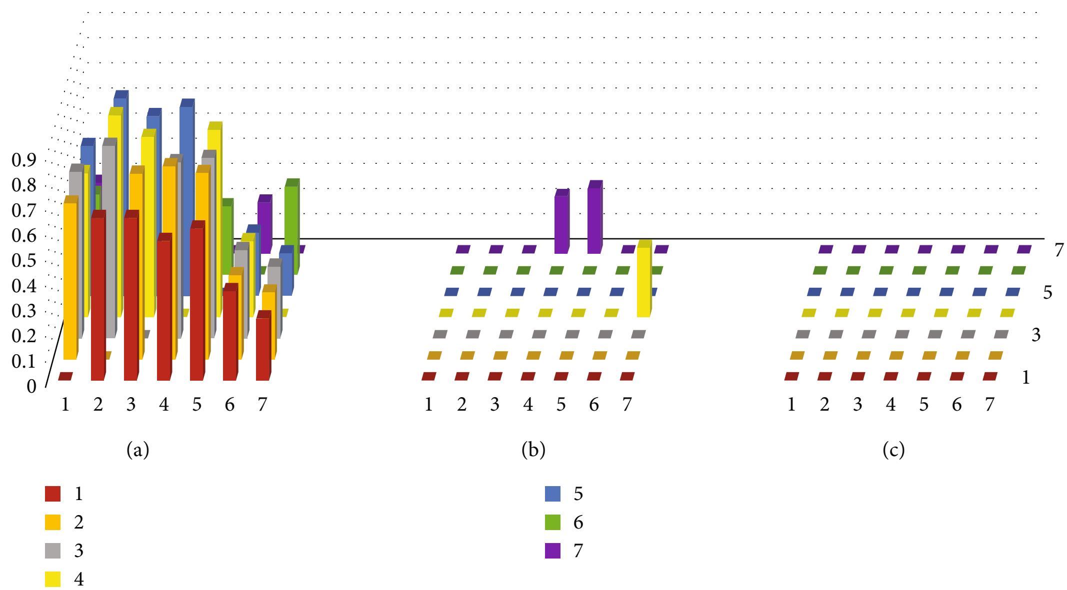

Figure 8: Schematic diagram of upstream dominant causation.

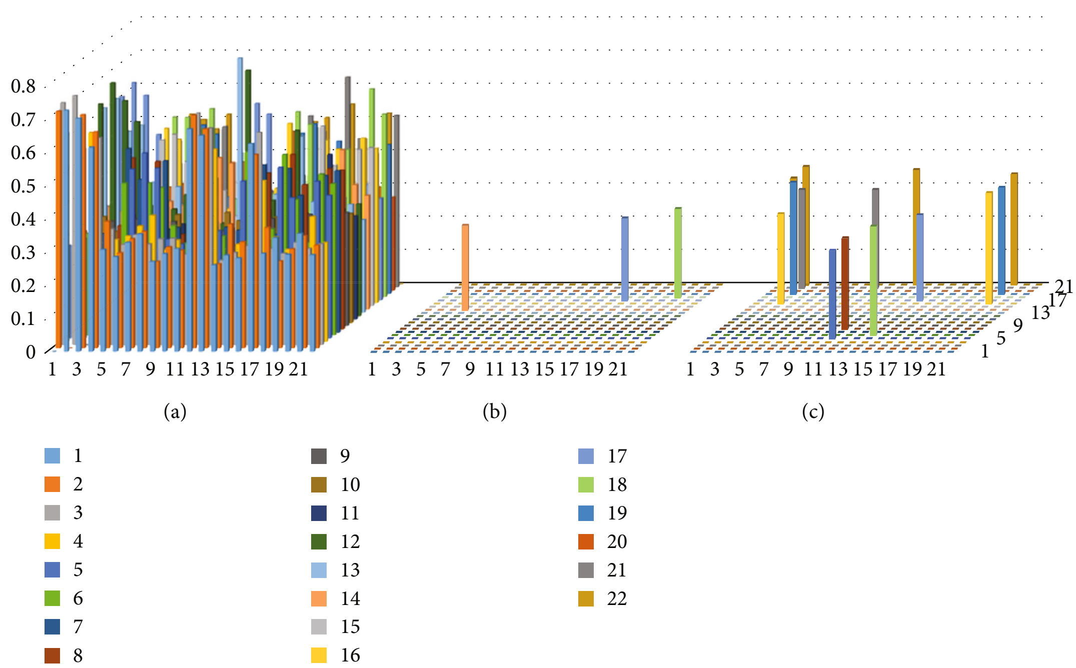

Figure 9: Schematic diagram of dominant causality in the midstream.

000970.SZ, 300127.SZ, 600111.SH, GE.N are dominated by an unclear causal relationship.

In the midstream, there are relatively many situations where the causal relationship of unclear dominates, mainly because the main products are diversified, and nonrare earth products account for a relatively high proportion of the main products, resulting in an unrelated relationship between stock price fluctuations between such companies and other companies.

4.1.3. The Downstream Stock Market Dominates Causality. Figure 10 shows the dominant causal relationship between 20 downstream stocks (including one stock belonging to both midstream and downstream). It can be clearly seen from the figure that the positive causality is also dominant between different stocks, and there are a few negative causal relationships and fewer unclear causal relationships. Unclear causation is slightly less dominant than negative causation.

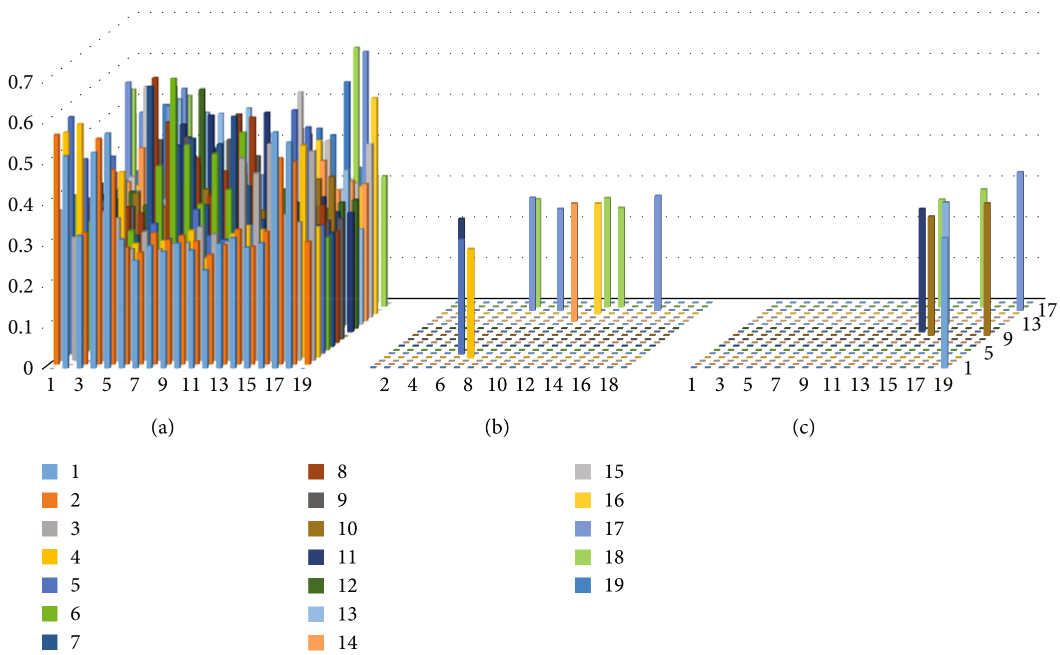

Figure 10: Schematic diagram of downstream dominant causation.

For example, 6954.T and 600580.SH; 6752.T and 002249.SZ; 002176.SZ and 6954.T, JMAT.L; ALB.N and 300032.SZ; 7203.T and 600580.SH, 300032.SZ, 600459.SH; and 7267.T and 600580.SH, 300032.SZ, 600459.SH are dominated by a negative causal relationship. These enterprises are more likely to belong to enterprises in different regions, and there is a certain degree of competition or other constraints between their businesses.

6503.T to F.N; 0179.HK to UMI.BR, F.N; 002176.SZ to JMAT.L; 600459.SH and UMI.BR; 7267.T and 600459.SH, ALB.N; and 7203.T and F.N are dominated by an unclear causal relationship.

4.1.4. The Stock Market of the Whole Industry Chain Dominates the Causal Relationship. Figure 11 specifically shows the dominant causal relationship between the 42 stocks. It can be clearly seen from the figure that the positive causality is also dominant among different stocks, and there are a few negative causal relationships and fewer unclear causal relationships.

From the above analysis and the proportional data shown in Table 8, it can be seen that listed companies in the rare earth industry, no matter which link, are mainly dominated by positive causality. Negative causal relationships and unclear causal relationships dominate in only a very small number of cases. However, the upstream, midstream, downstream, and the overall chain are slightly different due to differences in the relationship between the upstream and downstream, the proportion of main products, and the number of companies between regions.

4.2. Strong Causality Network Construction. To further analyze stocks in the rare earth industry chain, the causal relationship in the three causal network diagrams of the upstream, midstream, downstream, and all stocks were tested for threshold sensitivity, and after screening and filtering, Figures 12(a)–12(d) show the change in various network densities under each threshold.

In Figure 12, it can be seen that for the three causal networks between the upper, midstream, downstream, and all stocks, with the increase in the threshold, the network density is decreasing, and the trend of change is basically the same. After the upstream threshold increases to 0.3, only the positive causal network has an interaction relationship. Similarly, the threshold of the midstream increases to 0.4, the downstream increases to 0.35, all stocks increase to 0.4, and only the positive causal relationship exists. Figure 13 shows the quantitative causal network of upstream, midstream, downstream stocks, and all stocks under a specific threshold.

4.3. Strong Causality Network Analysis. It can be seen in Table 9 that the top five upstream stocks are all listed companies in China, and their network topology indicators are ranked high, indicating that China's domestic listed companies have a strong influence upstream of the rare earth industry chain. Of all the stocks, 600549.SH has the strongest influence.

As seen in Table 10, among the midstream stocks, Japanese stocks occupy nine of the top ten stocks that impact other stocks in the market, indicating that Japanese companies have a great influence in the midstream of the rare earth industry chain.

As seen in Table 11, among the downstream stocks, there are four Japanese companies that have the top five impacts on other stocks in the market, namely, 6503.T, 7203.T, 6954.T, and 6752.T, and only one Chinese company, 600580.SH. Only four of the top ten companies are from the

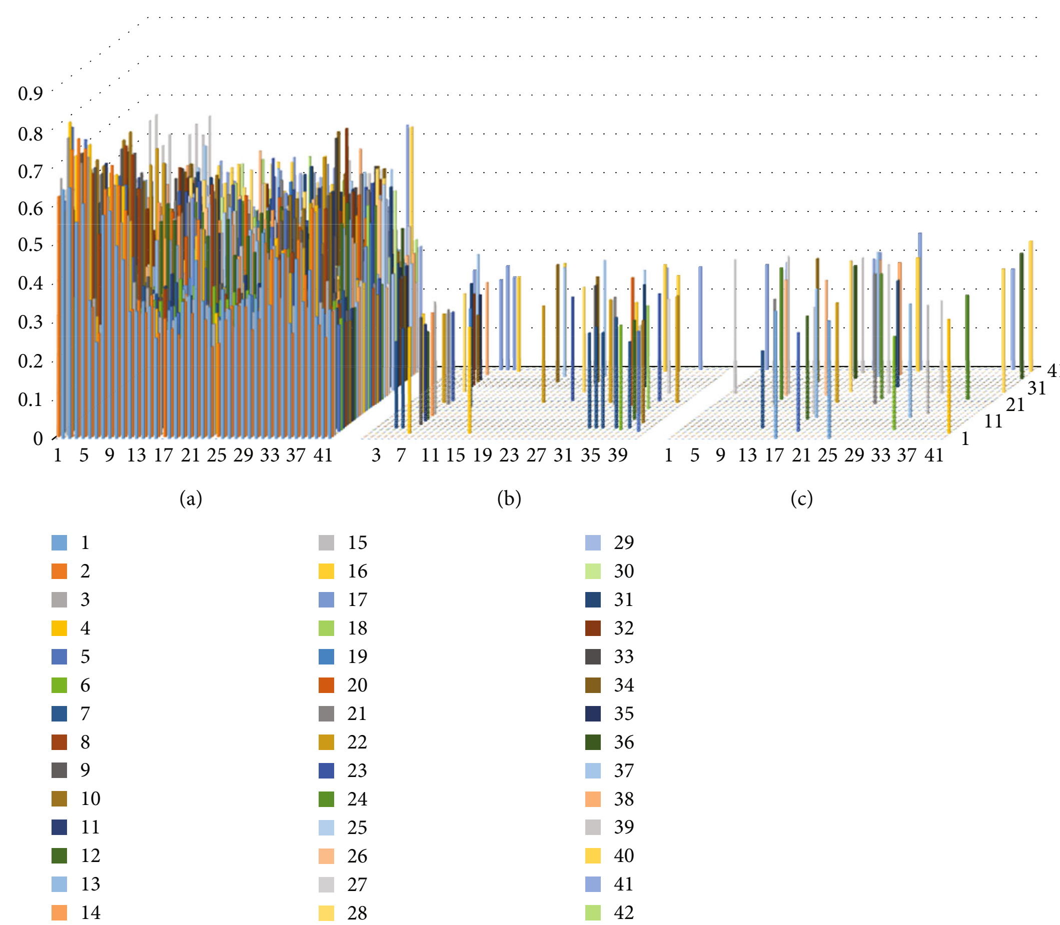

Figure 11: Schematic diagram of the dominant causal relationship of listed companies in the rare earth industry.

Table 8: Various causal relationships in each link of the industrial chain account for the dominant proportion.

Industrial chain links	Positive causal	Negative causal	Unclear causal
Upstream	0.928571	0.071429	0
Midstream	0.963203	0.006494	0.030303
Downstream	0.947368	0.029239	0.023392
Whole	0.939605	0.034843	0.025552

Chinese market, namely, 600580.SH, 300124.SZ, 002176.SZ, and 002249.SZ, while the remaining six overseas companies are all from Japan. It can be seen that while the influence of Japanese companies in the downstream market is also very large, Chinese companies do have some influence.

Ranking by the degree of influence (indegree) of other stocks in the market, China's 002249.SZ and 600580.SH tied for first place, followed by China's 002176.SZ.

As seen in Table 12, among all stocks, the top 10 companies with an impact on other stocks in the market are all Chinese companies, with the largest influence being 600366.SH from the midstream, followed by 002249.SZ from the downstream. Meanwhile, the top fifteen companies have four upstream companies, six midstream companies, and five downstream companies. It can be seen that in the rare earth industry chain stock market, the stock fluctuations of companies from different positions in the industry chain will affect the market fluctuations, while the stock fluctuations of midstream companies have a relatively greater impact on market fluctuations than those from downstream and upstream companies. In the rare earth industry chain, upstream companies are usually involved in the mining and refining of rare earths, downstream companies may be related to the application and processing of rare earths, and midstream companies are in the middle position and may play a key role in the processing, treatment, and distribution of rare earths. For investors, it is important to pay special attention to the stocks of midstream companies, as their volatility can have a significant impact on portfolio performance.

From the perspective of network structure, the interaction between the upstream, midstream, and downstream is also more obvious, and the midstream company 600366.SH and the upstream 002249.SZ and 002196.SZ have the most extensive impact on the fluctuations of other stocks, but the impact intensity is weak. However, 000831.SZ, 000969.SZ, 600111.SH, 000970.SZ, and 300127. SZ are

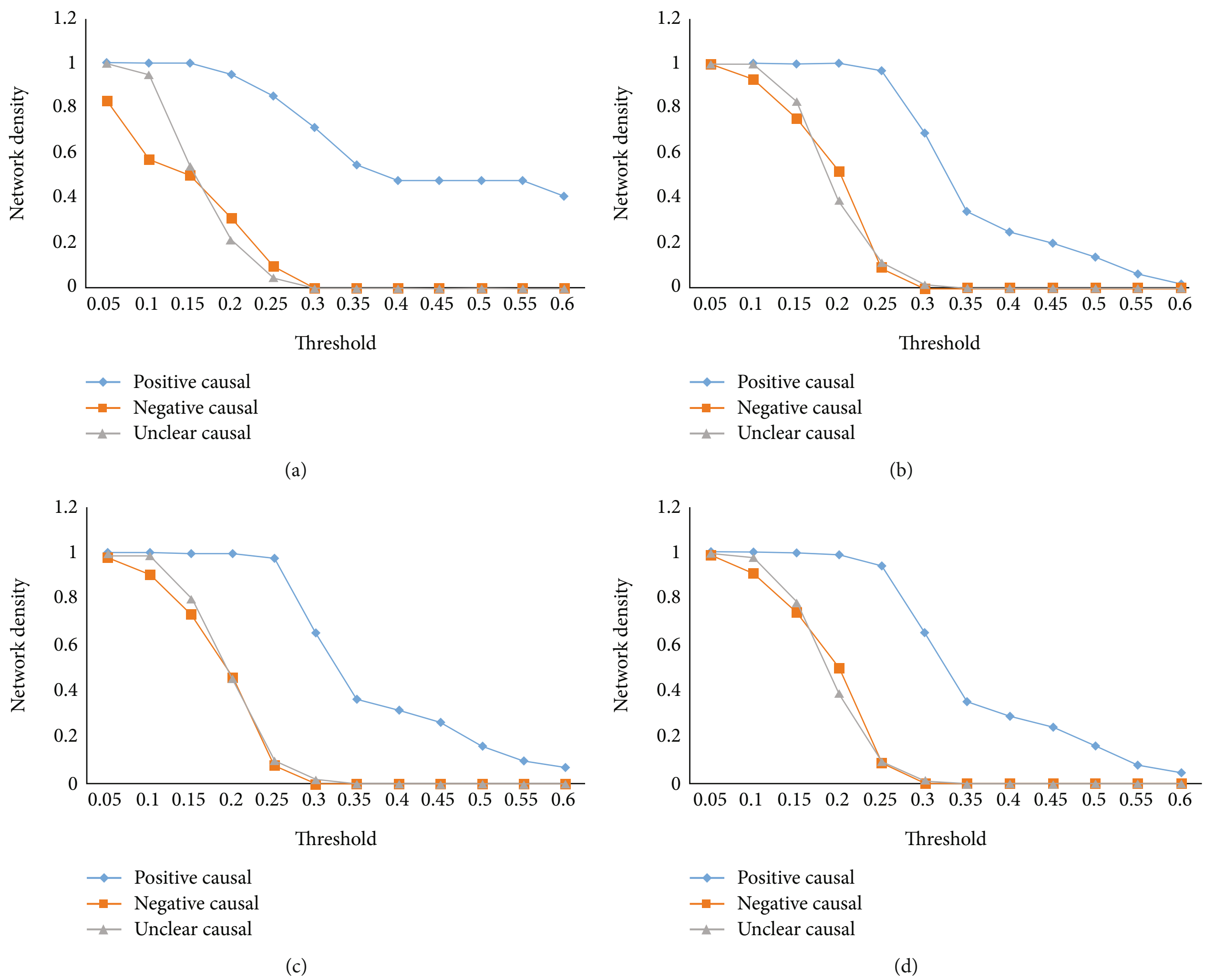

Figure 12: Test diagram of various causal relationship threshold sensitivity tests for stocks in each link of the industry chain.

strongly affected by the fluctuation of other companies' stocks.

The midstream company most affected by the stock fluctuation of the upstream company is 300127.SZ, which is also more influenced by other companies that are also upstream companies, and the intensity is greater. The main reason may be that its stock market value is small, its main business is bigger, and it involves a wide range of fields.

Chinese companies 002249.SZ and 002196.SZ in the downstream impact both upstream and midstream companies. The interaction between upstream 600111.SH, downstream 000970.SZ, and midstream 300127.SZ, and 6971.T is symmetrical.

It can be seen from the division of associations that the interaction between different markets is small, and the fluctuation influence between the same markets is close. There is a close fluctuation correlation between stocks in the Japanese stock market, between stocks in the domestic stock market, and between stocks in the European and American stock markets. However, there is independence between different markets, and each market is affected by specific economic, political, monetary, and social factors that can cause the market to fluctuate differently. The fluctuation correlation between different markets is very weak, and only a few have a certain correlation.

5. The Causal Transmission Path of Stock Price Fluctuations of Listed Companies in the Rare Earth Industry

By quantifying the causal relationship between the price fluctuations of listed companies in the rare earth industry, the strongest fluctuation transmission path from the upstream to the midstream of the industrial chain and then to the downstream can be found, and the fastest path of fluctuation transmission to all stocks can be found.

5.1. Fluctuation Transmission Path along the Industrial Chain. There is a strong relationship between product supply and demand between industrial chains, so stock price fluctuations between upstream and downstream companies in the industrial chain also have a strong correlation, and

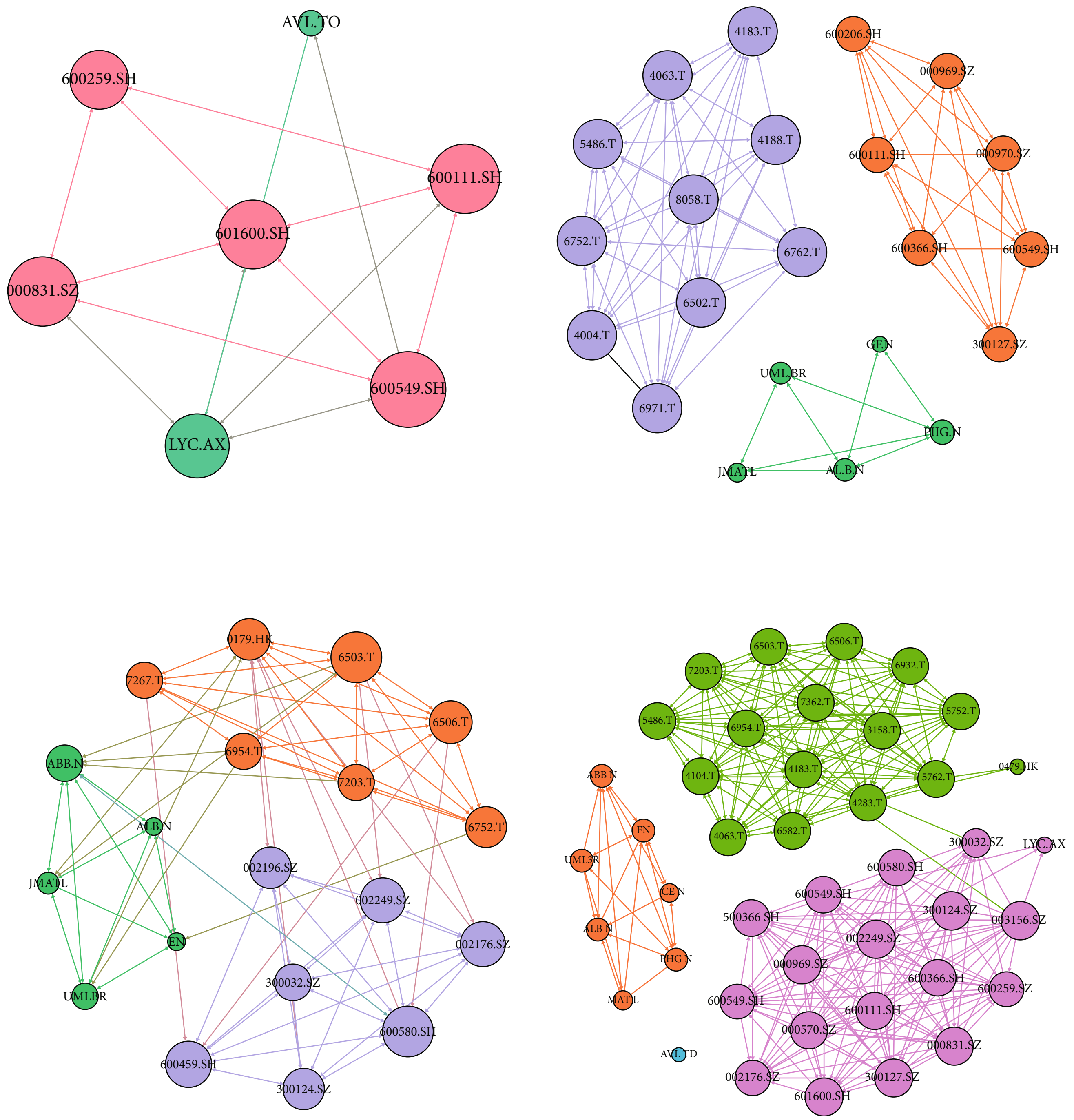

Figure 13: Diagram of various strong causal relationships between stocks in each link of the industrial chain.

the impact of mutual fluctuations is clearer. Table 13 shows the three types of causal relationships, from upstream to midstream to downstream, with the strongest wave transmission paths.

It can be seen in Table 13 that all the positive causal transmission paths are Chinese companies, and the upstream company 000831.SZ has the most fluctuation causality. The midstream distribution is more scattered, the number of 000969.SZ occurrences is relatively large, and the downstream is basically concentrated around company 600459.SH.

Among the companies involved in the following transmission paths are upstream companies, such as 000831.SZ and 601600.SH, that mainly provide rare earth concentrates. Companies 300127.SZ and 600366.SH and other midstream enterprises mainly produce NdFeB permanent magnet materials. Downstream company 600459.SH is mainly involved in precious metal information functional material with very high technical content and precious metal recycling and comprehensive utilization and is the most closely connected enterprise in the industrial chain.

Table 9: Upstream stock network topology indicators.

Stock symbol	Outdegree	Indegree	Weighted degree	Betweenness centrality	Page rank	Clustering coefficients
601600.SH	5	5	10	0.75	0.16	0.9
600111.SH	5	5	10	0.75	0.16	0.9
600549.SH	6	5	11	5.75	0.17	0.633
000831.SZ	5	5	10	0.75	0.16	0.9
600259.SH	4	4	8	0	0.13	1
LYC.AX	4	5	9	5	0.17	0.65
A.A.M	1	1	2	0	0.5	1

Table 10: Various topological indicators of the top 10 stocks in the midstream stock network outreach.

Stock symbol	Outdegree	Indegree	Weighted degree	Betweenness centrality	Clustering coefficients
6971.T (Japan)	9	9	18	0	1
5486.T (Japan)	9	9	18	0	1
6762.T (America)	9	9	18	0	1
4063.T (Japan)	9	9	18	0	1
8058.T (Japan)	9	9	18	0	1
6752.T (Japan)	9	9	18	0	1
4188.T (Japan)	9	9	18	0	1
6502.T (Japan)	9	9	18	0	1
4183.T (Japan)	9	9	18	0	1
4004.T (Japan)	9	9	18	0	1

Table 11: Various topological indicators of the top 10 stocks in the downstream stock network outdegree.

Stock symbol	Outdegree	Indegree	Weighted degree	Betweenness centrality	Clustering coefficients
6503.T (Japan)	10	5	15	13.95	0.38
6506.T (Japan)	8	5	13	6.71	0.52
6954.T (Japan)	7	5	12	3.40	0.57
6752.T (Japan)	7	6	13	72.74	0.57
600580.SH (China)	7	8	15	48.1	0.5
300124.SZ (China)	7	6	13	16	0.81
002176.SZ (China)	7	7	14	18.79	0.64
7203.T (Japan)	7	5	12	1.49329	0.64
7267.T (Japan)	7	5	12	3.00	0.62
002249.SZ (China)	6	8	14	7.16	0.64

The situation reflected that the positive correlation between domestic rare earth listed companies is closer than that between companies in other regions. Different links in the domestic rare earth industry chain may be more closely interconnected, including upstream mining, midstream processing, and downstream applications. The interaction between these companies could lead to more consistent movements in their stock prices.

Among the companies involved in the negative causality transmission path, the upstream is dominated by the Canadian listed company A.A.M.; the midstream companies affected are mainly European, American, Japanese, and Korean companies; and the downstream is mostly Chinese companies, where 002196.SZ is the most negatively affected. Generally, upstream, midstream, and downstream enterprises do not belong to the same region. Thus, there is a clear competitive relationship between companies in different regions.

Among the companies involved in the unclear causal relationship, the upstream is dominated by Chinese companies, the midstream is dominated by Japanese companies, and the downstream is dominated by both Chinese and Japanese companies. The correlation between Chinese and Japanese enterprises in stock price fluctuations is the

Table 12: All stocks in the industry chain are networked to rank various topological indicators of the top 15 stocks.

Stock symbol	Outdegree	Indegree	Weighted degree	Betweenness centrality	Clustering coefficients
600366.SH (China)	17	14	31	7.31	0.81
002249.SZ (China)	17	14	31	6.83	0.81
002196.SZ (China)	17	14	31	151.64	0.73
600111.SH (China)	16	16	32	3.80	0.9
000970.SZ (China)	16	16	32	3.80	0.9
300127.SZ (China)	16	16	32	3.80	0.9
600206.SH (China)	16	15	31	2.75	0.90
300124.SZ (China)	16	15	31	3.73	0.90
601600.SH (China)	15	14	29	1.40	0.95
600549.SH (China)	15	14	29	1.49	0.95
000831.SZ (China)	15	16	31	3.73	0.90
6762.T (America)	15	14	29	15	0.87
4188.T (Japan)	15	14	29	15	0.87
000969.SZ (China))	14	16	30	1.76	0.91
6971.T (Japan)	14	14	28	0	1

Table 13: Fluctuation transmission path in the direction of the industrial chain.

Causal category	Transmission path	Causal strength	
		Upstream⟶midstream	Midstream⟶downstream
Positive causal	000831.SZ⟶300127.SZ⟶600459.SH	0.6781	0.6131
	600259.SH⟶000969.SZ⟶600459.SH	0.6659	0.6633
	600259.SH⟶300127.SZ⟶600459.SH	0.6629	0.6131
	000831.SZ⟶000970.SZ⟶600459.SH	0.6629	0.6424
	601600.SH⟶000969.SZ⟶600459.SH	0.6559	0.6633
	000831.SZ⟶600366.SH⟶600459.SH	0.6519	0.6156
	600259.SH⟶000970.SZ⟶600459.SH	0.6471	0.6424
	000831.SZ⟶600206.SH⟶002176.SZ	0.5821	0.5359
	601600.SH⟶600366.SH⟶600459.SH	0.5671	0.6156
	000831.SZ⟶000969.SZ⟶600459.SH	0.5636	0.6633
Negative causal	A.A.M⟶4063.T⟶002196.SZ	0.2920	0.2523
	A.A.M⟶000970.SZ⟶7267.T	0.2765	0.2659
	A.A.M⟶6762.T⟶300032.SZ	0.2676	0.2409
	601600.SH⟶GE.N⟶002196.SZ	0.2676	0.2712
	A.A.M⟶600366.SH⟶6954.T	0.2656	0.2804
	600259.SH⟶6762.T⟶300032.SZ	0.2566	0.2409
	600259.SH⟶5486.T⟶002249.SZ	0.2560	0.2369
	000831.SZ⟶GE.N⟶002196.SZ	0.2536	0.2712
	600259.SH⟶4183.T⟶600580.SH	0.2529	0.2796
	LYC.AX⟶GE.N⟶002196.SZ	0.2511	0.2712
Unclear causal	601600.SH⟶4188.T⟶6506.T	0.2899	0.2515
	601600.SH⟶GE.N⟶7267.T	0.2801	0.3497
	600259.SH⟶5486.T⟶300032.SZ	0.2376	0.2206
	000831.SZ⟶6762.T⟶600580.SH	0.2327	0.2286
	000831.SZ⟶000969.SZ⟶6503.T	0.1856	0.1945

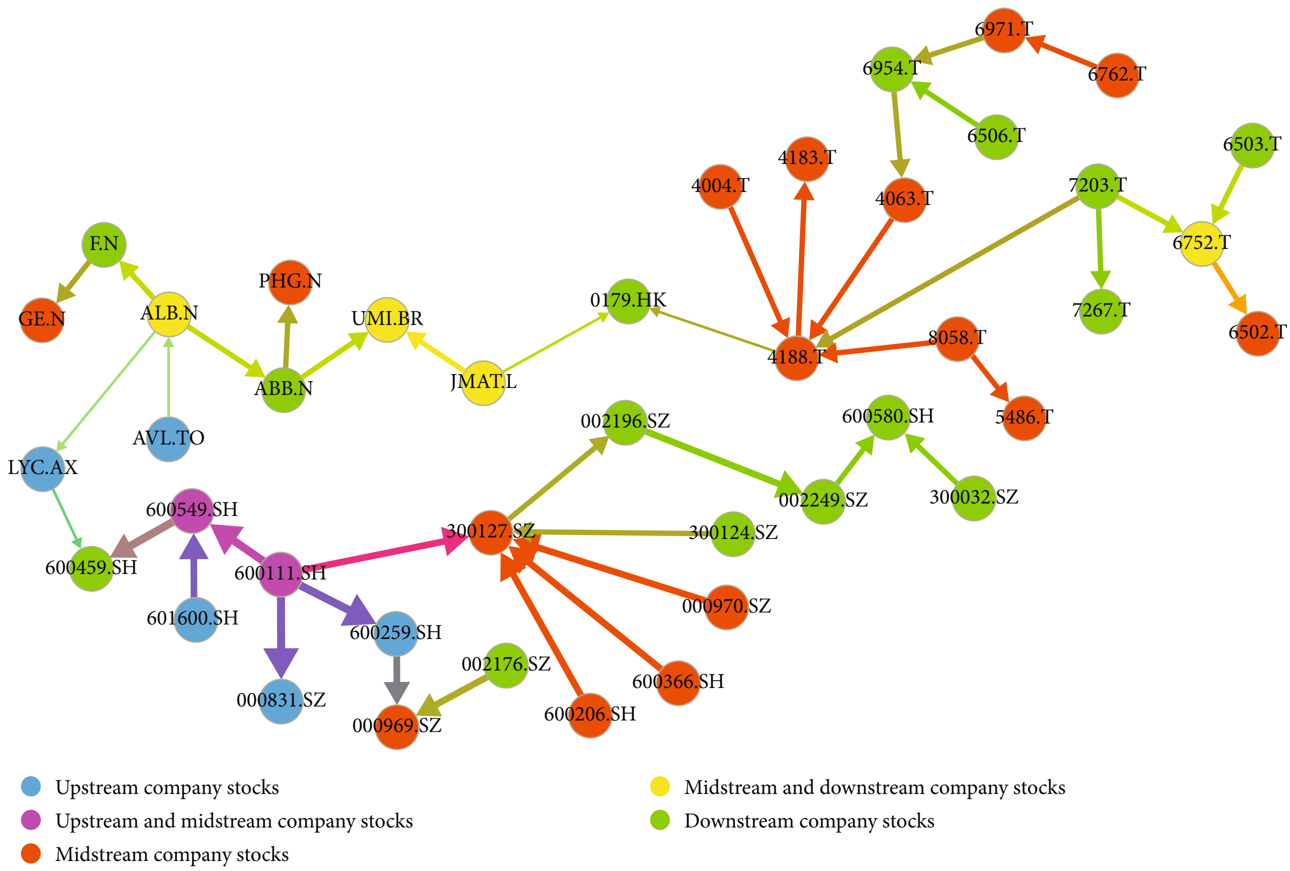

FIGURE 14: Tree diagram with the largest positive causality.

most obvious. China and Japan are two important countries for rare earth supply and demand, but their market positioning, supply chains, and customer bases may differ. This has led to an intercorrelation between their stock price movements.

5.2. Rare Earth Industry Stock Market Share Price Volatility Strongest Conduction Fully Connected Path. According to the three types of causal network diagrams, combined with the idea of a minimum spanning tree, the largest tree diagram of the directed network diagram, which represents the strongest conduction fully connected diagram of the stock market fluctuation of the rare earth industry and the strongest conduction of all 42 stocks, is constructed.

The maximum tree diagram is the most concise and highly transmitted path for fluctuation across all stocks. From the largest tree diagram of positive causality in Figure 14, it can be seen that 300127.SZ, 600111.SH, 600549.SH, ABB.N, 4188.T, 6954.T, and 7203.T play a key role in the overall system. At the same time, it can also intuitively reflect the entire industrial chain, China, Japan, South Korea, Europe, and the United States, three relatively independent and closely interconnected regions. Companies 300127.SZ, 600111.SH, and 600549.SH in China; 4188.T, 6954.T, and 7203. T in Japan; and ABB.N in Europe and the United States are the causal transit centers of the three regional enterprises.

The root node has an upstream-midstream 600111.SH, ALB.N in the midstream and downstream, and 7203.T in the downstream. These stocks have the most critical outward transmission ability, i.e., their fluctuations positively affect the fluctuation of other stocks. At the same time, the root node reflects the regional root nodes of China, Japan, South Korea, Europe, and the United States.

The strongest transmission path of positive causality is within the region, the conduction relationship between regions is not close, and the intensity is small. Companies within regions may be more dependent on similar economic, policy, and market factors, which makes the positive causal relationship between them stronger.

As shown in the maximum tree diagram of negative causality in Figure 15, 6752.T, ABB.N, GE.N, 600366.SH, 002196.SZ, and other stocks are at the key node of negative causal transmission. It can also be seen in Figure 15 that negative causation is often transmitted to nonlocal regional stocks. When one region's economy falters, this can have a knock-on effect on others, especially those with trade and financial ties to the region. This transmission effect manifests itself in the financial market as a decline in stock prices.

As shown in the maximum tree diagram of unclear causality in Figure 16, 300127.SZ, PHG.N., 7267.T., and 4063.T and other stocks are at the key node of unclear causality transmission. It also shows that the unclear causal relationship is similar to the negative causal relationship, and it is often more obvious to nonlocal regional stocks.

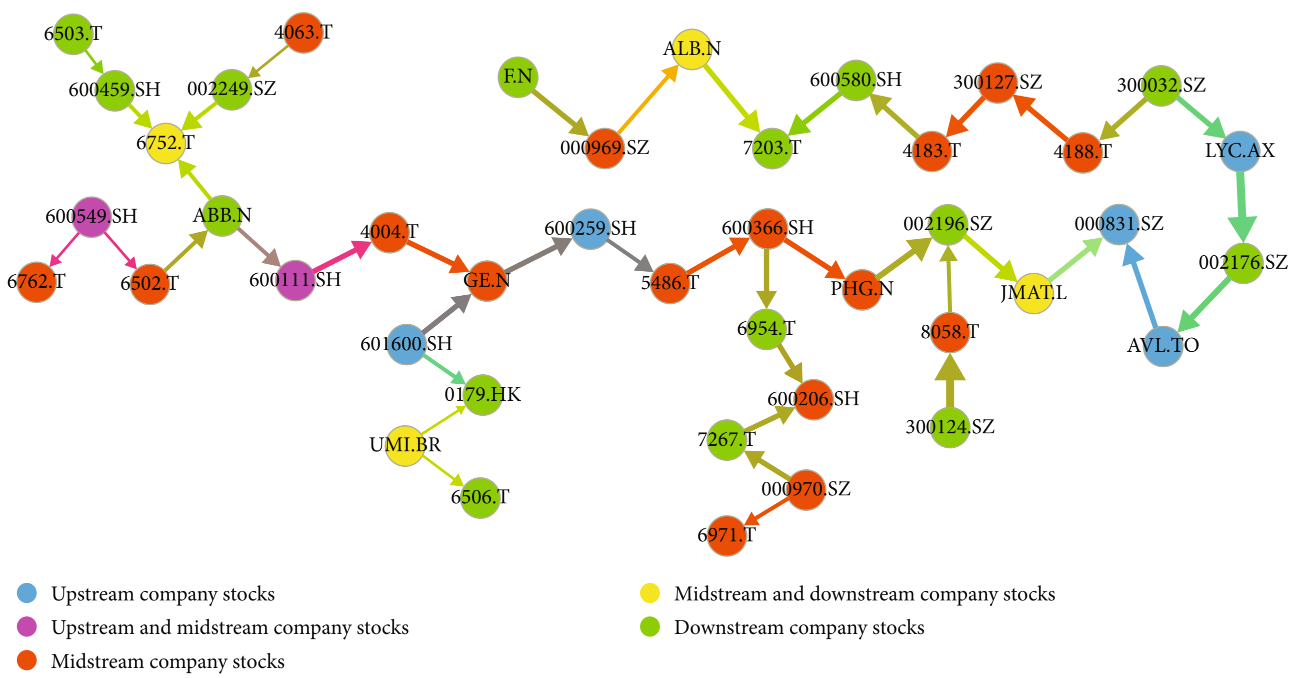

Figure 15: Tree diagram with the largest negative causality.

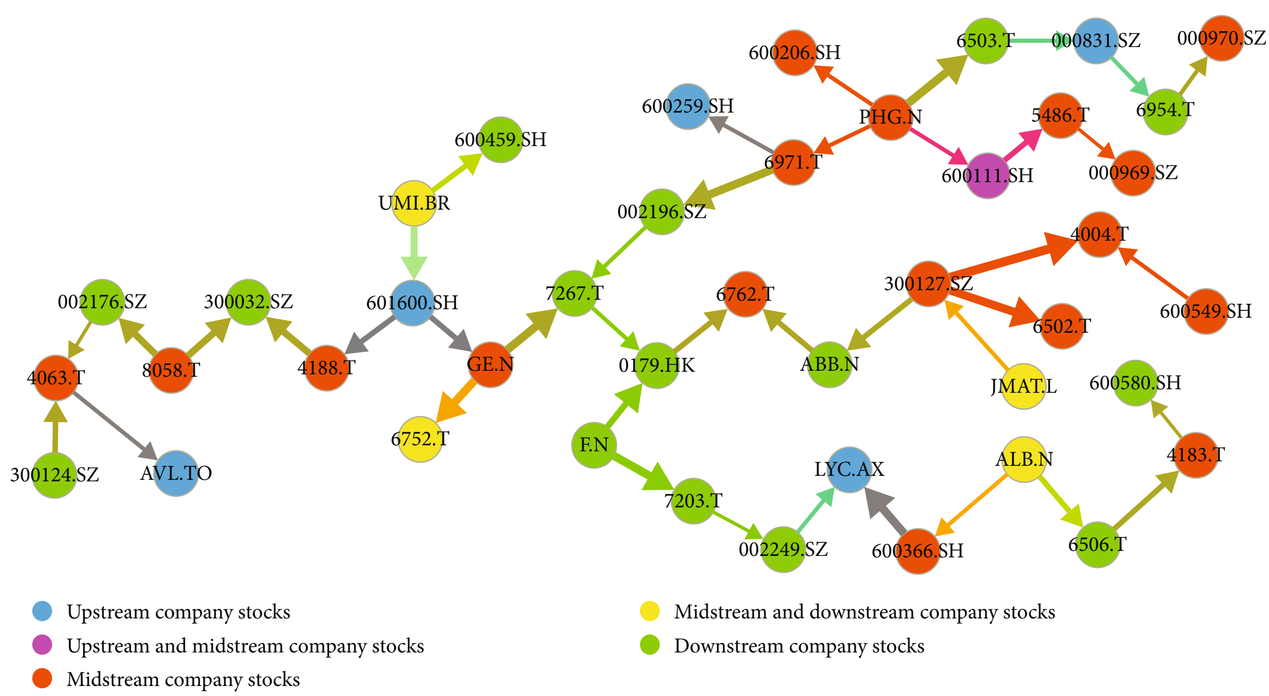

Figure 16: Tree diagram with the largest unclear causality.

6. Conclusion and Discussion

This paper constructs a quantitative causality model based on phase space reconstruction and symbolic dynamics and adds some realistic factors such as regional factor, the proportion of main products, and weighted return on equity. By quantifying the results of causality, a complex network is constructed to analyze the stock price volatility of listed rare earth companies and the causality of the volatility transmission, so as to explore the relationship between various links of the global rare earth industry chain and companies in various regions. Based on this, it provides decision support for all kinds of participants in the stock market. Specific conclusions are as follows:

(1) From the quantitative causality result data, it can be seen that the positive causal relationship between upstream and downstream enterprises in the industrial chain is close, the positive causal relationship between enterprises in the same region is close, and the negative causal relationship or no causal relationship between enterprises in different regions is more obvious. From the overall perspective, the strength of various causal relationships between rare earth industry chain stocks shows that the positive impact between each stock is significantly greater than the negative impact and unclear impact, and investors and managers should pay more attention to their positive impact when paying attention to these stocks

(2) From the perspective of strong causation, it can be seen that China's listed companies have a strong influence on the upstream of the rare earth industry chain, while Japanese companies have the strongest influence in the midstream and downstream. The overall situation of the market shows that the top ten companies in terms of impact on the stocks of other companies are all from China, and the interaction between the upstream, midstream, and downstream is also more obvious. From a regional point of view, stocks in different countries have little influence on each other, and the fluctuation causal relationship between companies within the same country is close, indicating that the same country has close competition and cooperation due to the homogeneity of resources, technology, and talents, resulting in a strong correlation of stock fluctuations within the same market

(3) From the division of associations, it can be seen that the mutual influence between different markets is small, and the fluctuation influence between the same market is relatively close. There is a close fluctuation correlation between stocks in the Japanese stock market, between stocks in the domestic stock market, and between stocks in the European and American stock markets, but the fluctuation correlation between different markets is very weak, and only a few have a certain correlation. The positive causal relationship between stock price fluctuations in China, Japan, Europe, and the United States is relatively low, and the negative causal relationship and unclear causal relationship are relatively high. International investors and regulators can capitalize upon these observed relationships

Data Availability

The data presented in this study are openly available at the wind database.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors' Contributions

Z.D. was responsible for the conceptualization, methodology, investigation, and writing—original draft preparation. Y.J. was responsible for the visualization, project administration, methodology, and software. Z.W. was responsible for the supervision and writing—review and editing. Q.Y. was responsible for the visualization, writing, and reviewing. All authors have read and agreed to the published version of the manuscript.

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