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Research Article

How Does the Time-Varying Network Structure Evolve between the EU Carbon Futures Prices and Industrial and Energy-Related Indices? A Study Based on a Time-Varying T-Copula

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Industrial and energy-related industries are major sources of carbon dioxide emissions, and their interdependence, as reflected in the financial field, has attracted the attention of scholars. For the purpose of exploring the evolutionary characteristics of the short-term dynamic correlation coefficient between the EU carbon futures price and the industrial and energy-related indices, this paper selected the settlement price of EU carbon emission quota futures, the MSCI energy I index on three dimensions, and the Dow Jones industrial index and West Texas crude oil futures price, as sample data. Using the time-varying t-copula model to measure the dynamic correlation coefficient between variables, the time-sliding window idea and coarse-grained method were combined to establish the correlation fluctuation mode, and a complex network theory and analysis methods were used to study the evolutionary traits of the time-varying network structure between the EU carbon price and the industrial and energy-related index. The results show that the transmission objects of the key correlation fluctuation modes in the network are stable and maintain their own state with a high probability. Second, the clustering effect exists in the transmission process. Some nodes with high mediating abilities are also the key correlation wave modes in the dynamic correlation evolution network. This study provides ideas for the study of the correlations between multiple variables and is also a useful reference for international investors.

1. Introduction

Due to the swift expansion of the global economy, the emission of greenhouse gases, mainly carbon dioxide, has become a focus in all walks of human life. To cope with the harm caused by climate problems, countries have put forward GHG emission reduction systems, resulting in the swift growth of the carbon emissions trading market. International carbon futures, as important derivative contracts in the financial market, have also attracted much attention [1–3]. This special financial market trades carbon dioxide emission rights as a commodity, and the price of trading is determined by the supply and demand dynamics within the carbon emission market, which in turn is closely linked to the production and demand of the industrial and energy sectors, which account for an increasing share of carbon emissions each year. For investors, the relationship between the carbon futures and the industrial and energy-related stock indices in financial markets is important to observe. Therefore, this paper analyzes the correlation between carbon futures and the industrial and energy-related indices to clarify the link from a financial market perspective and provide a reference for international investors.

Domestic and foreign scholars' studies on the carbon emission trading market mainly focus on the fluctuation of the carbon futures market [4] and the influencing factors of the carbon futures and spot prices [5–7]. Eva and Stefan modeled the changes in the price dynamics and volatility of the potential random price process through a Markov mechanism transformation and the AR-GARCH model and then examined the short-term behavior in the spot prices of carbon dioxide emission allowances and found that its volatility mainly comes from the fluctuation of the demand for carbon quotas [8]. Using an empirical analysis of the three main markets under the EU carbon trading system, scholars such as Daskalakis et al. found that carbon spot prices better approximate a jump-enhanced geometric Brownian motion [9]. Chevallier used empirical research to explore the factors influencing carbon futures yields and found opinions that differ from those of previous market watchers; the link between macroeconomic shocks and carbon markets is weak [10]. Kumar et al. used the vector autoregression approach to investigate the relationship among the stock prices of new energy companies, oil prices, and carbon futures prices, and there was no notable correlation found between the prices of new energy stocks and carbon futures prices [11].

At the same time, certain scholars have also focused their attention on the correlations between the carbon emission market and other markets. The area of research that has received the most focus is the correlation between the carbon emission market and the energy market [12–14]. For example, Kanamura used a supply and demand correlation model to explore the correlation between the EU carbon trading price and energy prices, both theoretically and empirically, showing that energy prices positively impact the EU carbon price [15]. By combining DCC, GARCH, and BEKK models, Zhang and Sun discussed the correlation between the futures price of European carbon and the prices of three fossil energy sources and concluded that the correlation between the coal market and the carbon market was the highest and that the impact of the price decline of three fossil fuels on the fluctuations of carbon prices might be stronger than that of a price rise of the same degree [16].

In the long history of correlation research, scholars have mainly used models such as the Pearson correlation coefficient [17], DCC [18, 19], and copula [20–22]. Among them, the linear correlation between variables can be expressed by the Pearson correlation coefficient; however, it cannot describe complex correlation characteristics such as the nonlinearity between variables [23, 24]. The DCC model focuses on describing the dynamic conditional correlation under the aggregation of financial market volatility [25]. Copula models focus more on the tail correlation characteristics between financial markets [26].

In contrast, Patton constructed the basic form of the time-varying copula function based on the classical theory of copula functions [27]. The time-varying copula [28–32] model comprehensively considers the nonlinear, time-varying, and thick-tailed complex correlation features among financial time series. For example, scholars Zhang and Zhao [33] used the time-varying geometry copula to model the dependence structure between crude oil and natural gas returns and found that there was a strong correlation between the two series. The time-varying t-copula [34] has relatively good goodness of fit and can take into account the upper-tail and lower-tail correlations of financial time series. For example, Gong and Huang [35] used the time-varying conditional t-copula model to study the correlation between the exchange rates of the US dollar, euro, and yen against RMB before and after the reform of China's RMB exchange rate system and revealed that the conditional tail correlation has reference value for the risk management of foreign exchange portfolios. However, the correlation fluctuations between the variables are also indicative of a complex process. The development of complex network theory offers a solid theoretical foundation for exploring the intricacies of complexity science, especially the study of the evolutionary characteristics of the variables, which can be examined through complex network.

In summary, this paper's main contribution lies in examining the time-varying nonlinear correlation coefficient between variables by using the time-varying t-copula model, abstracting the correlation strength between the multivariate variables into a symbolic sequence by using the time-sliding window and coarse-grained method, and combining it with complex network theory. The evolution of the dynamic correlation between the EU carbon quota futures price and the industrial and energy-related indices is mapped in a complex network. The modes of the correlation strengths between the variables are taken as network nodes, and the conversion and conversion frequency between the modes are taken as the edge and weight of the network. By analyzing the network's topology characteristics, the dynamic correlation evolution characteristics of the EU carbon quota futures price and the industrial and energy-related indices are explored.

2. Data and Methods

The main anthropogenic sources of carbon dioxide are the consumption of fossil fuels, industrial production, and urban waste treatment. Crude oil, which dominates traditional energy consumption, is subject to price fluctuations that can impact the economy of a region or country. Based on this analysis, this paper examines the dynamic correlation evolution between the settlement prices of EU carbon futures and indices related to industry and energy.

2.1. Data. As the EU carbon market was established in 2005, at present, it holds the position of being the world's largest and most mature carbon trading market [36]. Therefore, among the many regional carbon trading markets, the EU carbon emission allowance futures settlement price is selected as the object of this study. According to the operational traits of the carbon market in the European Union, which is in an advanced phase, the EU carbon market entered the third operational phase in January 2013, which makes it relatively mature. Therefore, this paper uses daily data for the time frame from January 2013 to December 31, 2022, as the sample size. The MSCI indices are widely used as reference indices in the investment field and are important reference indicators that allow investors to judge investments. In this paper, the MSCI developed market energy I index, the MSCI emerging market energy I Index, and the MSCI global energy I index are selected to investigate the dynamic correlation between the EU carbon futures prices and the energy stock indices in three dimensions. The Dow Jones industrial average, which averages the share prices of 30 blue-chip US companies covering all sectors except transportation and utilities, and the settlement price of West Texas crude oil futures, one of the most significant benchmark oils in international trade, are selected. The six groups of research variables were preprocessed, the date and time of each group of data were aligned, the rest days and missing data were deleted, and 2516 pieces of effective data were obtained by taking the logarithmic rate of return (the above data originates from the WIND database).

2.2. Model Design. The copula function, first proposed by Sklar [37], is called the dependence function or connection function, which is a nonlinear correlation analysis function that is formed by connecting the marginal distribution functions of the variables. The definition of the N-dimensional copula function is as follows: $C : [0,1]^N \longrightarrow [0,1]$.

C has a base surface and is an n-dimensional increasing function. The edge distribution function of C satisfies $C_n(u) = C(1, \cdots 1, u, 1, \cdots, 1) = u, u\epsilon[0,1]$.

By definition, it can be seen that the copula function is actually a multivariate distribution function with a uniform edge distribution of [0, 1] on the n-dimensional [0, 1] space. Sklar's theorem states that if F is an N-dimensional joint distribution function with an edge distribution of $F_1, \cdots, F_N$, then there must be a copula function $C : [0,1]^N \longrightarrow [0,1]$ that

$$F(x_1, \cdots x_n, \cdots x_N) = C(F_1(x_1), \cdots, F_n(x_n), \cdots, F_N(x_N)). \quad (1)$$

If $F_1, \cdots, F_N$ is continuous, then C can be uniquely determined; conversely, if $F_1, \cdots, F_N$ is a monadic distribution function, then the function F determined by the above equation is an edge distribution $F_1, \cdots, F_N$. The density function corresponding to the joint distribution function F can be further derived in the following:

$$f(x_1, \cdots x_n, \cdots x_N) = c(F_1(x_1), \cdots, F_n(x_n), \cdots, F_N(x_N)) \prod_{n=1}^{N} f_n(x_n)$$
$$c(u_1, \cdots u_n, \cdots u_N) = \frac{\partial c(u_1, \cdots u_n, \cdots u_N)}{\partial(u_1) \cdots \partial(u_n) \cdots \partial(u_N)}, \quad (2)$$

where $c(\bullet)$ is the density function for the copula function, $f_n(x_n)$ is a marginal distribution $F_n(x_n)$. The joint distribution density function can be viewed as consisting of two parts: the product of the copula density function and the edge distribution density function.

In this paper, we study the time-varying correlation coefficients between the EU carbon price and the industrial and energy-related index, which correspond to the two-dimensional copula function.

$$F(x_t, y_t) = C(F_1(x_t), F_2(y_t)), \quad (3)$$

where x_t is the series of EU carbon allowance futures settlement price returns, y_t is the series of stock index returns, $F_1(x_t)$ and $F_2(y_t)$ are divided into the marginal distribution function of x_t and y_t, $F(x_t, y_t)$ is the joint distribution function of $F_1(x_t)$ and $F_2(y_t)$, and C denotes the copula model.

Referring to previous studies [35, 38, 39], the dependency structure among financial variables changes with time, and there is a peak and thick tail characteristic that fluctuates slowly most of the time and violently a few of the time when extreme events occur. According to previous literature studies [33, 40], the time-varying t-copula model can better describe the nonlinear dependence relationship between multiple variables at the same time, especially the tail dependence relationship between each two variables to a certain extent. In other words, the model has a relatively good goodness of fit and can take into account the top-tail and bottom-tail correlations of financial time series. Therefore, the time-varying t-copula model is selected in this paper to capture the time-varying correlation structure between the two variables. In this paper, the time-varying t-copula model is used to investigate the nonlinear correlation, and the expression is as follows:

$$c(F_1(z_{1,t}), F_2(z_{2,t}); \rho_t, n) = \frac{1}{\sqrt{1-\rho_t^2}} \frac{\Gamma(n+2/2)\Gamma(n/2)}{[\Gamma(n+2/2)]^2} \cdot \frac{\left[1 + z_{1,t}^2 + z_{2,t}^2 - 2\rho_t z_{1,t} z_{2,t}/n\left(1-\rho_t^2\right)\right]^{-n+2/n}}{\prod_{i=1}^{2}\left(1 + z_{i,t}^2/n\right)^{-n+2/n}},$$

$$\mathcal{Q}_t = (1 - \beta_1 - \beta_2)\bar{\mathcal{Q}} + \beta_1 z_{t-1} z_{t-1}^{'} + \beta_2 \mathcal{Q}_{t-1},$$

$$r_t = \widetilde{\mathcal{Q}_t}^{-1} \mathcal{Q}_t \widetilde{\mathcal{Q}_t}^{-1} = \begin{bmatrix} 1 & \rho_t \\ \rho_t & 1 \end{bmatrix}, \quad (4)$$

where β_1, β_2, n is an estimated parameter of the time-varying t-copula model, satisfying the constraint $\beta_1 + \beta_2 < 1$, β_1, $\beta_2 \in (0,1)$. $z_{i,t}$ is the standard residual, $F_1(z_{1,t})$ represents the EU carbon price yield series T distribution probability integral transform, and $F_2(z_{2,t})$ represents the stock index yield series T distribution probability integral transform. $\bar{\mathcal{Q}}$ is the covariance matrix of the standard residuals, and $\widetilde{\mathcal{Q}_t}$ is a matrix where the principal diagonal element is $\sqrt{\mathcal{Q}_t}$; the other elements are 0. ρ_t is the dynamic correlation coefficient, and r_t is the dynamic correlation coefficient matrix.

In this paper, the establishment of dynamic correlation wave modes needs to be combined with sliding window and coarse-grained method, and the intensity of dynamic correlation between variables is quantified by time-varying t-copula model through five steps.

Step 1: the return series were first subjected to descriptive statistics and ADF smoothness tests. The results are shown in Table 1. The skewness of each group of return series is not 0, and the kurtosis is greater than 3, which reflects the

Table 1: Descriptive statistics and ADF smoothness test.

Variable	Statistics Skewness	Kurtosis	JB-prob	ADF t-statistic	Prob	Conclusion
EUA	-1.0573	18.7284	0.001***	-52.546862	0.001***	Smooth
MSCI developed/energy I	-1.2329	26.8546	0.001***	-48.089773	0.001***	Smooth
MSCI global/energy I	-1.3529	26.7261	0.001***	-46.716946	0.001***	Smooth
MSCI emerging/energy I	-1.2891	17.1093	0.001***	-43.907300	0.001***	Smooth
DJIA	-0.98025	26.3163	0.001***	-58.088587	0.001***	Smooth
WTI	0.096774	26.2674	0.001***	-50.369561	0.001***	Smooth

Note: $^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$, indicating that it is significant at the significance level of 1%, 5% and 10%.

Table 2: Autocorrelation and ARCH effect tests.

Variable	Ljung-Box Q test Statistic	Statistic	Test for the ARCH effect Prob	Prob
EUA	71.4056	35.013734	0.001***	$1.0720e-07^{***}$
MSCI developed/energy I	93.3299	39.900577	0.001***	$1.9288e-11^{***}$
MSCI global/energy I	100.9161	42.104969	0.001***	$8.6320e-13^{***}$
MSCI emerging/energy I	107.3126	54.101719	0.001***	$6.0507e-14^{***}$
DJIA	388.2387	474.939167	0.001***	0^{***}
WTI	88.1167	237.124476	0.001***	$1.5793e-10^{***}$

Note: $^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$, indicating that it is significant at the significance level of 1%, 5% and 10%.

characteristics of financial time series that fluctuate slowly most of the time and violently when extreme events occur very little of the time; that is, the series has the characteristics of a peak and thick tail. The JB statistics show that the p value is less than 0.01, implying that the original hypothesis of a normal distribution is rejected; that is, the normal distribution is not obeyed. It is again verified that the EU carbon quota futures yield and the industrial and energy-related index yield series have peak and thick tail distribution characteristics. Second, the original assumption of the ADF unit root test is that the tested sequence has a nonstationary unit root sequence. Each group of series in the table exhibits a p value of less than 0.01, and the original hypothesis is rejected at a 1% level of significance, which means that each group of return series is a smooth series with good statistical properties.

Step 2: the purpose of the autocorrelation test is to determine the form of the GARCH model mean equation, and if an autocorrelation exists, then the GARCH mean equation uses an autoregressive process. The original assumption of this test is that there is no autocorrelation in the series. After testing, Table 2 displays the results. The p values of the autocorrelation tests for the series of EU carbon quota futures price returns and the industrial and energy-related index returns are all less than 0.01. The original hypothesis was rejected with a significance level of 1% which implies that there is a significant autocorrelation in the groups of series, and therefore, to eliminate the autocorrelation, the AR process must be incorporated into the mean equation of the subsequent GARCH family model.

Step 3: the residuals of the mean equation were tested for ARCH effects, and the original hypothesis was that there was no ARCH effect in the residual series. As shown in Table 2, the test reveals that the p values of the residuals of the mean equation test of the log returns of the EU carbon quota futures settlement price and the other sets of log returns are all less than 0.1, and the original hypothesis was rejected with a significance level of 10% which implies that there is an ARCH effect on the residual error from the mean equation, which satisfies the prerequisite for conducting GARCH family modeling.

Step 4: in the GARCH modeling, the descriptive statistics show that there are spikes and thick tails in each group of series, so the cumulative distribution function of the residual series normalized by the GARCH model under the skewed t-distribution is called a marginal distribution of the copula function. The GARCH model under the skewed distribution is shown below:

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t, \\ \varepsilon_t &= \sigma_t z_t, \\ \sigma_t^2 &= \omega + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2, \end{aligned} \tag{5}$$

where $\omega > 0$, $\alpha_i > 0 (i = 1, \cdots, q)$, $\beta_i > 0 (i = 1, \cdots, p)$, $\sum_{i=1}^{p} \beta_i + \sum_{i=1}^{q} \alpha_i < 1$, and z_t obey a mutually independent skewed t-distribution. The standardized residual of GARCH modeling was extracted, and the ARCH effect was tested. If there was no ARCH effect, the heteroscedasticity was eliminated by the GARCH model, and the GARCH modeling was

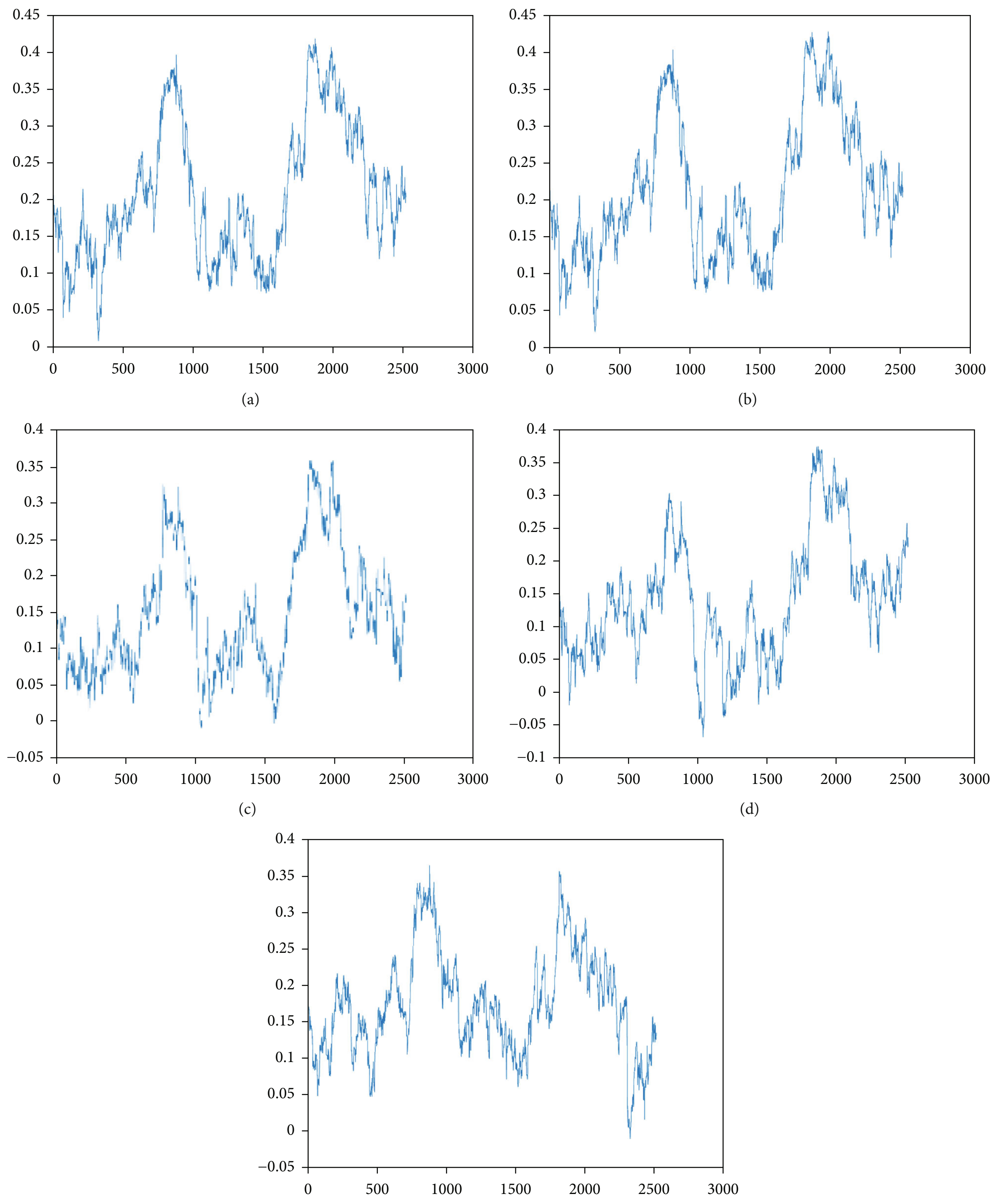


Figure 1: Correlation fluctuation series graph. (a) EUA-MSCI developed/energy I, (b) EUA-MSCI global/energy I, (c) EUA-MSCI emerging/energy I, (d) EUA-DGIA, and (e) EUA-WTI).

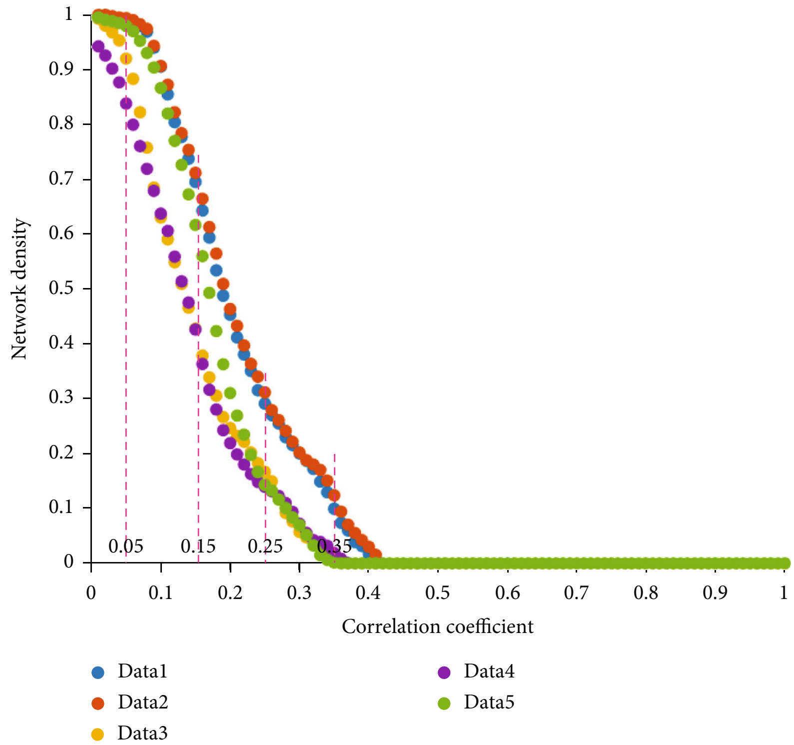


Figure 2: The network density of each group of variables under different thresholds (data1:EUA-developed/energy I; data2:EUA-global/energy I; data3:EUA-emerging/energy I; data4:EUA-emerging/energy I; data5:EUA-WTI)

	EUA-developed/ energy I	EUA-global/ energy I	EUA-emerging/ energy I	EUA-DJIA	EUA-WTI	Coarse-grained mode
1	0.206748464	0.213146086	0.149150628	0.144545861	0.173792837	Sliding window
2	0.206416056	0.211854429	0.142061024	0.146986099	0.174433904	
3	0.203607294	0.209369551	0.14352321	0.145285421	0.174203423	
4	0.192228452	0.19689048	0.135323257	0.131738751	0.171727411	
5	0.193259412	0.198496273	0.139796018	0.138968559	0.171840689	mmrrm
6	0.190808692	0.195613324	0.135814699	0.128503464	0.171561197	mmrrm
7	0.182419603	0.186978372	0.134304653	0.116476784	0.167027277	mmrrm
8	0.179846013	0.185497491	0.139862456	0.11489462	0.168345782	mmrrm
9	0.180283272	0.185751708	0.137385584	0.114853086	0.167720595	mmrrm
……	……	……	……	……	……	
2516	0.203550056	0.210849257	0.169887354	0.221759532	0.136006735	mmmmr

Figure 3: Correlativity fluctuation mode coarse-granulation process.

successful. The ARCH effect hypothesis was accepted by all the sequences; that is, the pairwise variable GARCH modeling was successful.

Finally, this paper chooses a time-varying t-copula model with a relatively good fit and the ability to take into account the upper and lower tail correlations of the financial time series to capture the time-varying correlation structure between two variables and establishes a joint distributed time-varying t-copula model to obtain the fluctuation process of the time-varying nonlinear correlation between the EU carbon emission allowance futures price and the other five groups of variables, as shown in Figure 1, within a certain range of the nonsmooth fluctuations.

2.3. Coarse-Grained Processing of Correlated Fluctuating Modes. The correlation coefficients take values in the range [-1, 1]. In this paper, the strength of the variables' correlation is defined according to the values of the five groups of

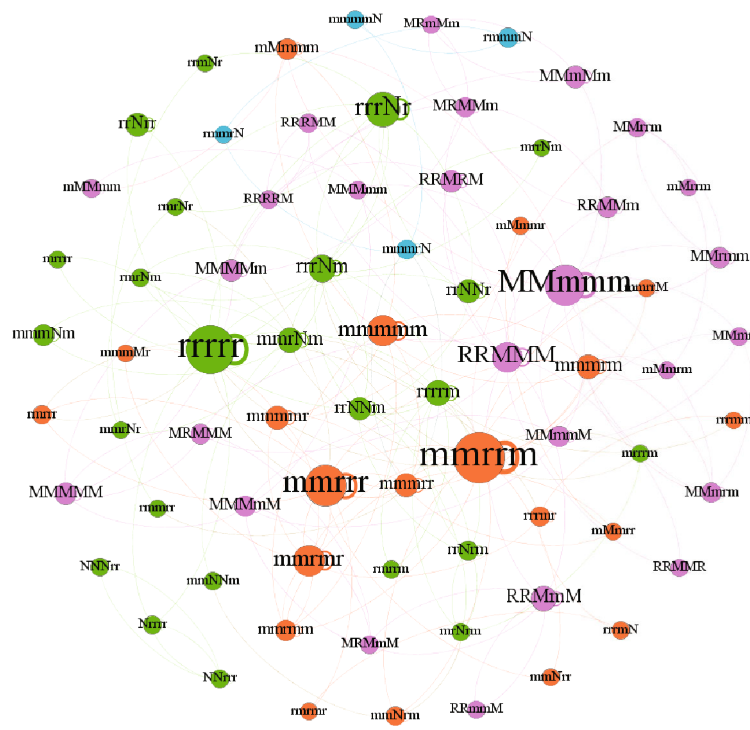


Figure 4: Time-varying network of correlations between EU carbon futures prices and industrial and energy-related indices.

time-varying correlations studied, and the strength of the correlation over a certain fluctuation interval is replaced by an alphabetical abstraction. When dividing the intensity interval, the extreme values and median values of the time-varying correlation coefficients of each group of variables are considered, while the network density of each group of variables under different correlation coefficients is referred to in Figure 2, and the mean value of the threshold inflection point of each group is taken, thus forming the intensity interval of the correlation coefficients shown in formula (6).

$$\begin{cases} R, 0.35 < r_{xy} \leq 1 \text{; strong positive correlation,} \\ M, 0.25 < r_{xy} \leq 0.35 \text{; comparatively strong positive correlation,} \\ m, 0.15 \leq r_{xy} \leq 0.25 \text{; medium positive correlation,} \\ r, 0.05 \leq r_{xy} \leq 0.15 \text{; comparatively weak positive correlation,} \\ N, -0.05 \leq r_{xy} < 0.05 \text{; no correlation,} \\ y, -0.15 \leq r_{xy} < -0.05 \text{; comparatively weak negative correlation,} \\ q, -0.25 \leq r_{xy} < -0.15 \text{; medium negative correlation,} \\ Q, -0.35 \leq r_{xy} < -0.25 \text{; comparatively strong negative correlation,} \\ P, -1 \leq r_{xy} < -0.35 \text{; strong negative correlation.} \end{cases} \quad (6)$$

Coarse-granulation is the time-varying correlation coefficient between the above obtained EU carbon price and the industrial and energy-related index. In order to better classify the intensity of the correlation coefficient and facilitate better analysis, the correlation coefficient between the obtained variables is divided into the intensity interval represented by different letters. In this way, the whole system interval is converted into a letter sequence, and the study of the coarse-grained letter sequence is equivalent to the study of the corresponding time series. Overall data sliding and mode formation are shown in Figure 3. The correlation coefficients between the settlement price of EU carbon emission quota futures and the settlement price of the MSCI developed market energy I index, MSCI emerging market energy I index, MSCI global energy I index, Dow Jones industrial index, and West Texas crude oil futures under each time sliding window are denoted by letters in the corresponding correlation strength interval. Under each sliding window is a coarse-grained mode consisting of five letters representing the strength of the time-varying correlation coefficient between the EU carbon price and the industrial and energy-related indices.

2.4. Construction of Complex Networks. Mapping the time-varying correlation between the EU carbon future prices and industry and the energy-related indices in a complex network facilitates the analysis of the distribution characteristics and the interrelationships between the modes in the system through network indicator characteristics. As shown in Figure 3, this paper implements the mode formation process through a time-sliding window so that the correlation fluctuation mode at time t is the basis for the formation of the correlation fluctuation mode at time $t+1$ so that the transition of the correlation fluctuation mode has direction and transferability, and then a time-varying network of correlation fluctuation modes is constructed, as shown in Figure 4. In this directed weighted network, the correlation fluctuation modes between the EU carbon quota futures price and the other five groups of the logarithmic rate of

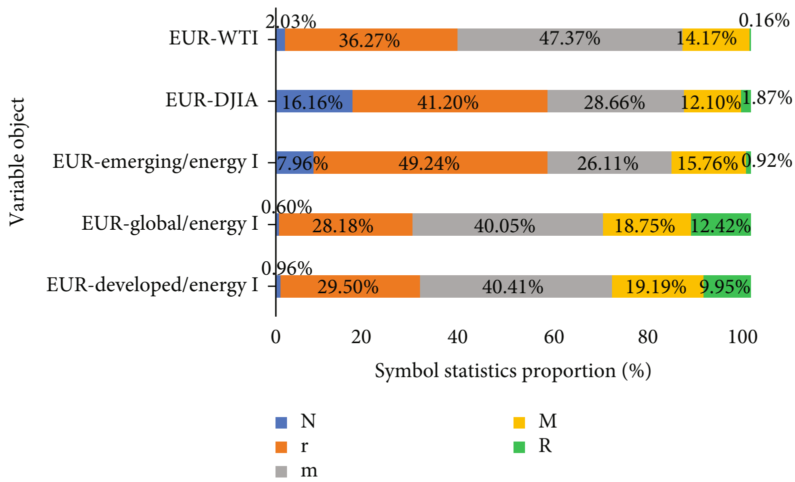


Figure 5: Correlation symbol statistics proportion.

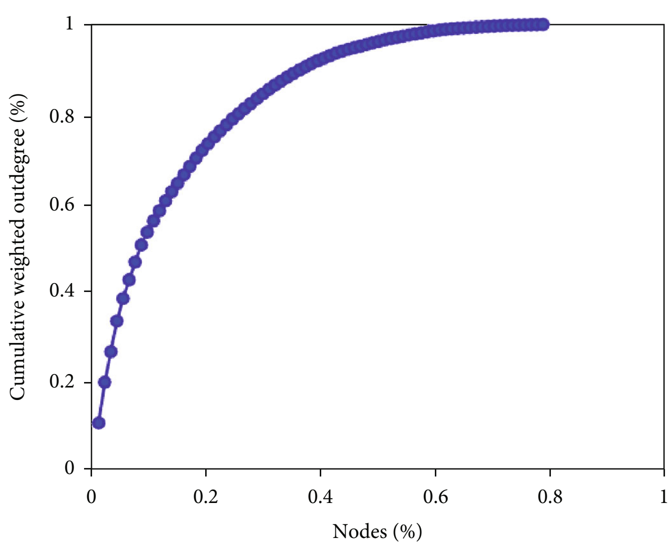


Figure 6: Cumulative distribution of weighted outdegree of time-varying correlation evolutionary networks.

return series are taken as nodes, the conversion is taken as the edge, and the edge weight is determined by the frequency of mode conversion.

One mode consists of five symbols, each of which represents the correlation strength between the settlement price of the EU carbon emission quota futures and the settlement price of the MSCI developed market energy I index, MSCI emerging market energy I index, MSCI global energy I index, Dow Jones industrial average, and West Texas crude oil futures. The time-varying network of the EU carbon quota futures prices and the industrial and energy-related indices is explained by using the weighted outdegree and weighted outdegree outdistribution, clustering coefficient, intermediate centrality, and other complex network characteristics.

3. Results and Analysis

3.1. Statistics of Correlation Fluctuation Modes. The correlation coefficient between each group of data and the EU carbon quota futures price was calculated, and every group showed an overall positive correlation. Five groups of symbol sequences representing the correlation strength between the EU carbon quota futures prices and MSCI developed/energy I, MSCI global/energy I, MSCI Emerging/energy I, Dow Jones Industrial Average, and WTI futures prices were obtained by a coarse-grained symbolic abstraction of the correlation strength. The five groups of symbol sequences were statistically analyzed.

Table 3: Key correlation modes and their weighted outdegree in networks.

Node	Weighted outdegree	The proportion of total weighted outdegree
mmrrm	247	0.09836471843
rrrrr	231	0.09199522103
mmrrr	174	0.06929510155
MMmmm	174	0.06929510155
rrrNr	128	0.05097570688

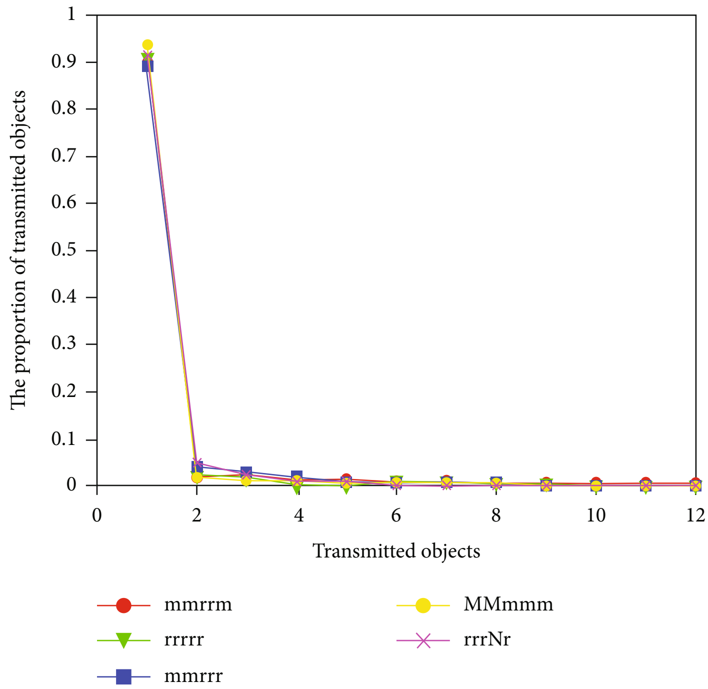


Figure 7: The proportion of transmitted objects in the main transmission submodel (weighted outdegree is not less than 120).

As shown in Figure 5, symbol m occupies a relatively high proportion in the symbol sequence of the correlation between the EU carbon futures price, MSCI developed/energy I index, MSCI global/energy I index, and WTI futures price, all of which reach more than 40%. As demonstrated, there exists a medium positive correlation between the price of EU carbon quota futures and the three variables. The ratio of symbol r in the symbol sequence of correlation between the EU carbon futures price and MSCI emerging/energy I index and Dow Jones industrial average is 49.24% and 41.2%, respectively. There is a comparatively weak positive correlation between the price of EU carbon quota futures and the two datasets.

In this paper, 95 types of modes should appear theoretically through coarse-grained processing, but only 74 modes appear in practice. Figure 6 displays an analysis of the cumulative distribution of nodes' weighted outdegree. In the figure, approximately 26.81% of the nodes account for 80.96% of the total weighted outdegree. The weighted outdegree of the correlation fluctuation modes has the characteristics of a power law distribution, that is, a few nodes are connected to most nodes in the network, and the network with power-law characteristics belongs to the scale-free network. It indicates that there are fluctuating modes in the correlation of the EU carbon quota futures price and the industrial and energy-related index time-varying network that are statistically significant and nonrandom, and a few fluctuation modes contribute significantly to the transmission process to a considerable extent. It can be seen that if the correlation transformation between the EU carbon quota and the industrial and energy-related index is sought, the key modes and their characteristics can be identified, and the direction of their transformation mode can be found, which will offer pertinent investment recommendations to international investors with varying risk tolerances.

3.2. Identifying Key Correlation Fluctuation Modes. In this paper, the weighted outdegree is applied to identify the key correlation fluctuation modes. This index is a comprehensive index of the local information of the complex network nodes. The greater the value, the higher the transmission capacity of the node. The weighted outdegree of a node is defined as follows:

$$w_i^{\text{out}} = \sum_{j\in N_i} w_{\text{ij}}, \tag{7}$$

where w_i^{out} denotes the weighted outdegree and N_i and w_{ij} denote the number of neighboring nodes of node i and the weight of node i to node j, respectively.

Figure 6 indicates that the node's weighted outdegree conforms to the characteristics of a power law distribution. In this paper, according to the ranking of the weighted outdegree of the correlation fluctuation modes, as shown in Table 3, it is revealed that the weighted outdegree of the first five modes is notably higher than that of the remaining modes, thus identifying these modes as key correlation fluctuation modes. The first mmrrm mode indicates that the EU carbon quota futures price has a moderate positive correlation with the MSCI developed/energy I index and MSCI global/energy I index, a comparatively weak positive correlation with the MSCI emerging/energy I index and Dow Jones industrial index, and a moderate positive correlation with the West Texas crude oil futures price (WTI). In this mode, the EU carbon quota futures are positively correlated with the other four groups of international indices and the futures

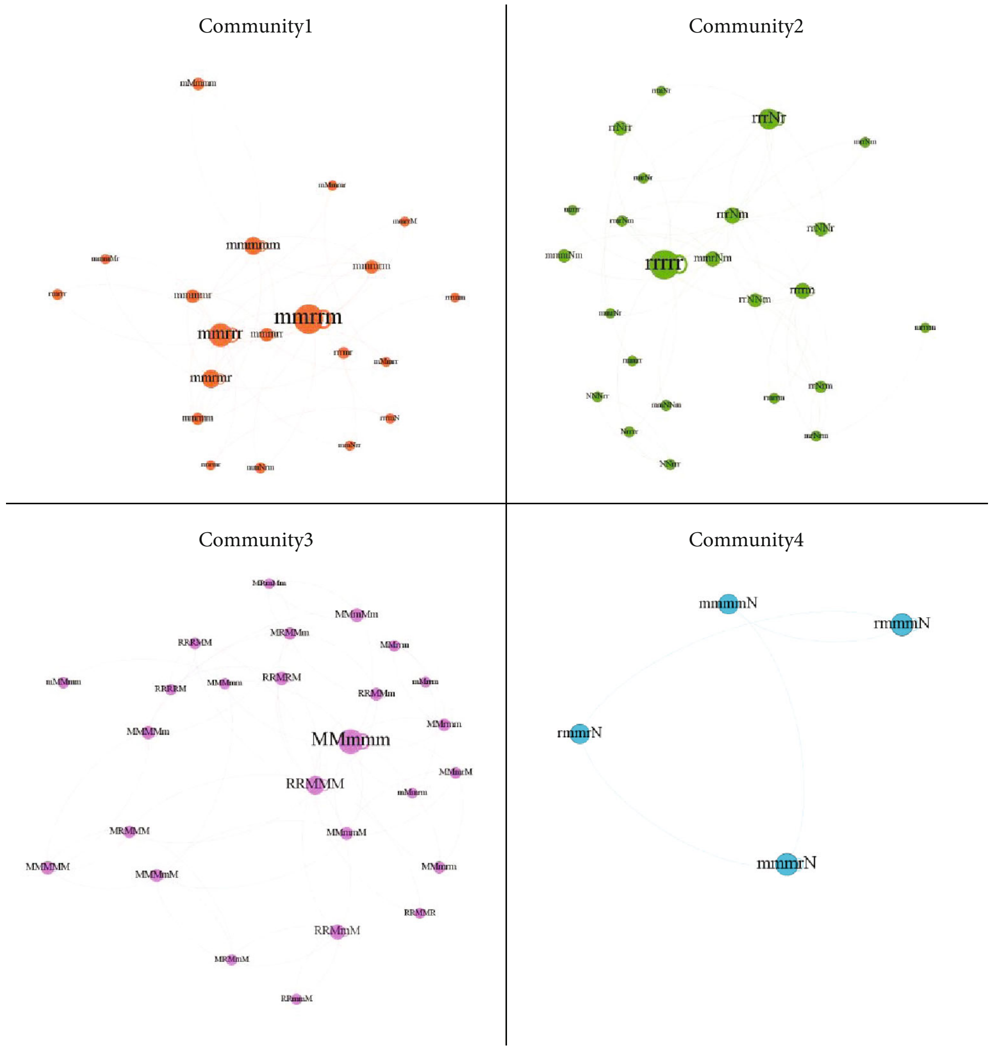


Figure 8: Clustering effect in dynamic correlation evolutionary networks (community 1 : 27.03%; community 2 : 32.43%; community 3 : 35.14%; community 4 : 5.41%).

Table 4: Characteristics of community structure.

	Community1	Community2	Community3	Community4
Number of network nodes	20	24	26	4
Number of network edges	63	73	83	8
Average shortest path length	3.014	3.191	3.901	2
Average clustering coefficient	0.47	0.512	0.494	0.333

price of crude oil; however, this strength is relatively low compared with the strong and comparatively strong positive correlation, indicating that the probability of stocks falling or rising simultaneously is not high. In this scenario, it is suggested that international investors can portfolio their investments in EU carbon quota futures and several other stocks or futures to avoid an excessive concentration of funds.

3.3. Transmission Mode. There are 74 types of modes in the time-varying evolutionary network of correlation fluctuation modes constructed in this paper. In theory, there will be 74 $\times$ 74 connecting edges for the mutual conversion of modes; however, in practice, there are only 254 edges. It shows that the main transmission modes are controlled by a few correlation fluctuation modes, which significantly affect the network information transmission. Figure 7 is the proportion chart of the top 5 key correlation fluctuation mode transmission objects. In the figure, the horizontal coordinate represents the transmission objects, the number 1 on the x-axis denotes transmission to itself, and the vertical coordinate denotes the probability of correlation fluctuation mode transmissions.

Mode mmrrm propagates to itself with a probability of 89.88%. The mode rrrrr propagated to itself with a probability of 90.91%. The mode mmrrr propagated to itself with a

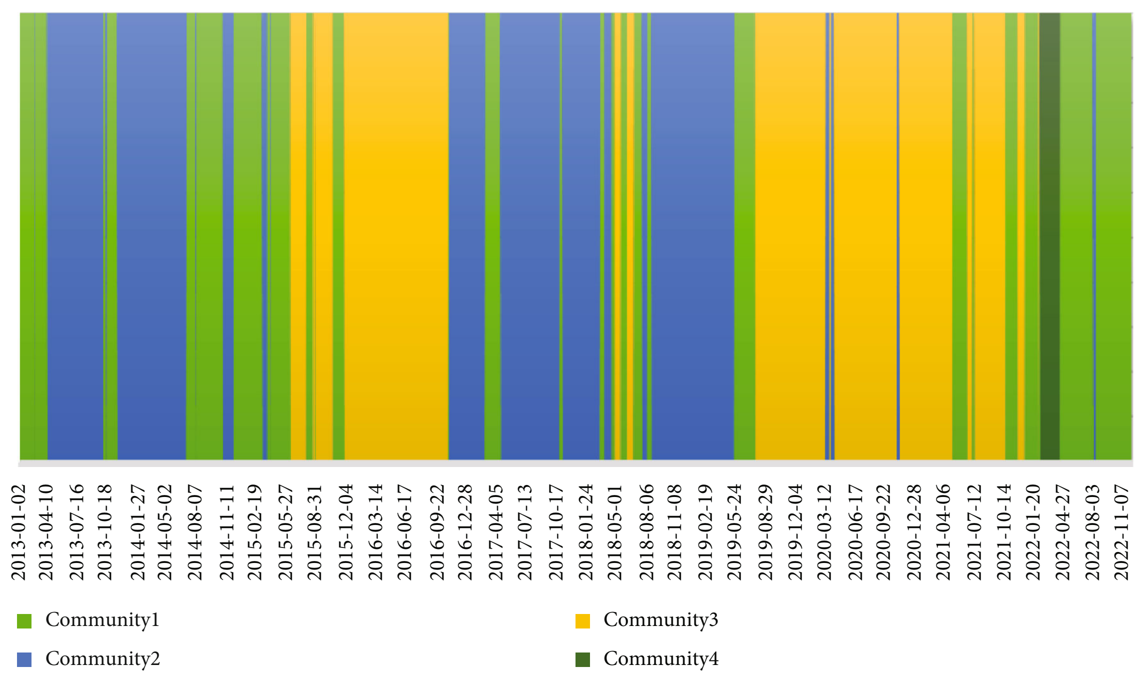
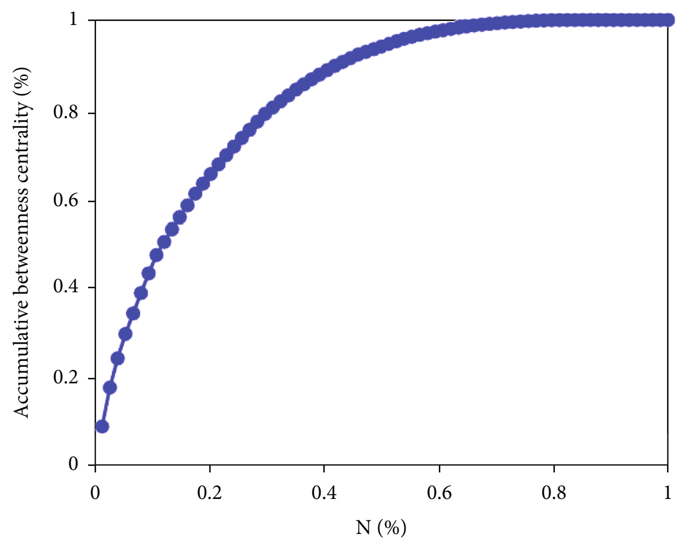


Figure 9: A graph of four communities over time.

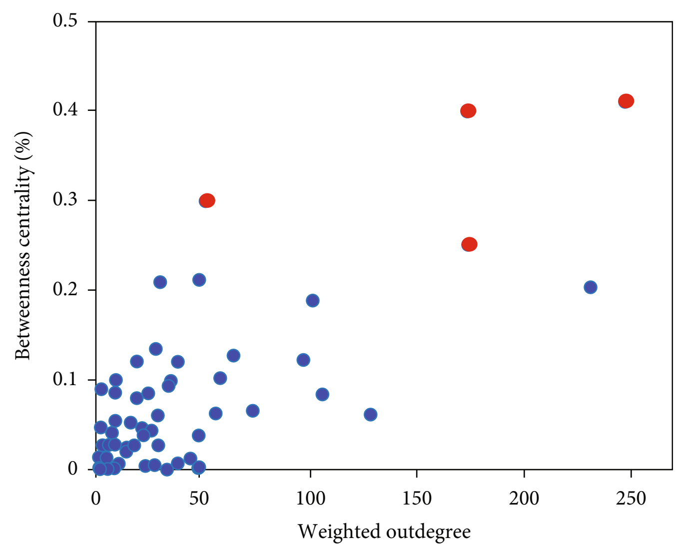
probability of 89.08%. Modal MMmmm and rrrNr transmit to themselves with a 93.68% and 91.41% probability, respectively. The five key modes are transmitted to themselves with a high probability and are relatively stable in the transmission process, indicating that when the key modes appear, the correlation fluctuation mode tends to maintain its own state for a period of time. International investors are advised to refer to these key modes when making investment choices. For example, in the MMmmm correlation fluctuation mode, the EU carbon quota futures price has a strong positive correlation with the MSCI developed/energy I and MSCI global/energy I stocks and has a moderate positive correlation with the MSCI emerging/energy I, Dow Jones Industrial Average, and West Texas crude oil futures. In the next period of time, it will still maintain its own status with a probability of more than 93%. At this time, international investors can make investments according to the fluctuation of the EU carbon quota futures price. Meanwhile, investors with a risk appetite can choose to invest in the EU carbon quota futures portfolio, MSCI developed/energy I, and the MSCI global/energy I stocks.

3.4. Clustering Effect. The convenience of cluster analysis makes it an effective way to comprehend the impacts of fluctuation clustering during transmission. An effect of fluctuation clustering is that some correlated fluctuation modes transform each other more frequently. Under the condition that the sliding window length is 5 days, the transmission network nodes in this study are divided into 4 communities according to the transmission frequency between the nodes. The node transmissions within the same community are closer, and the probability of node transmissions is higher than that of intercommunity node transmissions, indicating that the correlation fluctuation modes of the EU carbon quota futures prices and the industrial and energy-related indices within the community have a relatively high transmission frequency, and the conversion between the modes is relatively stable.

As can be seen in Figure 8, four communities contain different modal types, respectively, and five key modes are distributed in communities 0, 1, and 2. Community 0 contains two key modes, mmrrm and mmrrr. Community 1 contains two key modes, rrrrr and rrrNr, and community 2 contains the key mode MMmmm. Table 4 also shows the characteristics of different sizes, connectivity, and degree of clustering of communities in the complex network, reflecting the diversity of transitions between different modes in the time-varying network of EU carbon quota futures prices and industrial and energy-related indices. Community 0 is a relatively small community with 20 nodes and 63 edges. The average shortest path length is 3.014, indicating that it is easy to convert between nodes in the community, and the conversion period is about 3 days. At the same time, the average clustering coefficient is 0.47, which means that a certain degree of small groups are formed within community 0. Community 1 has 24 nodes and 73 connected edges, the average shortest path length is 3.191, and the clustering coefficient is 0.512, which indicates that the nodes in community 1 have tighter modal transmission, representing a tighter subnetwork. Community 2 is the largest community with 26 nodes and 83 links, with an average shortest path length of 3.901 and a clustering coefficient of 0.494. This means that the information transmission within the community may need to pass through more intermediary nodes; community 3 is the smallest community with only 4 nodes and 8 connected edges, but the average shortest path length is 2.0, and the clustering coefficient is 0.333, which indicates that the modal transmission within the community will be very fast and direct.



(a)



(b)

FIGURE 10: Distribution of transmission mediations in dynamic dependency evolutionary networks.

An analysis of the community networks in complex networks helps to understand the characteristics of the community networks in detail. Figure 9 shows the distribution of associations over time, from which we can see the distribution of communities to which the wave fluctuation modes belong over time. The modes for most of the selected time periods are in community 2 and community 3. As shown in Table 4, communities 2 and 3 contain 72.98% of the node numbers and 61.41% of the edge numbers of the entire network, indicating that various types of correlation fluctuation modes in the time-varying correlation evolutionary network are frequently transformed in communities 2 and 3. According to the calculation, the average path length between nodes is 5.448, indicating that the average conversion cycle of the fluctuation modes of the correlation between the price of EU carbon quota futures, international stock indices, and crude oil futures is 5 days. Focusing on community networks in detail, for example, in community 2, there is the largest average clustering coefficient, and there are relatively short average shortest paths. The fluctuation modes of correlation in community 2 are observable and very closely connected, and the transmission is more stable. This indicates that when the mode type of the correlation fluctuation between the price of EU carbon quota futures and the industrial and energy-related index belongs to the second community, it will be transmitted stably with the mode type in the second community for a period of time. International investors can invest by looking at the modal types within the community.

3.5. Transmission Intermediacy. A node with high intermediation centrality means that it is located on the shortest path between many other modal pairs, which means that it acts as a mediator in the transmission process and is an important condition for changes in the modalities of the correlation fluctuations between communities so that some information about future changes in the correlation between the EU carbon allowance futures prices and the industrial and energy-related indices can be made available to international investors from the mediating modalities. This allows some information on future changes in the correlation between the EU carbon allowance futures prices and the industrial and energy-related indices to become available to international investors from the medium model.

Figure 10(a) indicates that 28.38% of the nodes possess 77.19% of the transmission mediating capacity of the entire network, and the first 6 correlation fluctuation modes bear 38.87% of the mediating capacity of the network. It is also found that some correlation fluctuation modes with high weights have higher mediating abilities.

As shown in Figure 10(b), among the four types of modes with higher mediating abilities than the other modes, three of them are not only key modes in the dynamic correlation evolution network of the EU carbon quota futures price and the industrial and energy correlation index but also have high mediating abilities and have a significant impact on the transmission process between the modes. For example, modal mmrr ($BC = 0.409059$,w_i^{out}=247), modal MMmmm ($BC = 0.39936$,w_i^{out}=174), and modal mmrrr ($BC = 0.250404$,w_i^{out}=174) have both a high mediating power and high weight. From the perspective of the centrality of complex network intermediaries, the emergence of such modes is a precursor to the change in the relationship among EU carbon quota futures prices, global stock market indices, and crude oil futures prices, and international investors ought to focus on the emergence of the media modes.

4. Discussion and Conclusions

In this paper, the daily data from January 2013 to December 31, 2022, during the time period were taken as the sample size interval. The time-varying t-copula model was used in combination with the complex network theory, time-sliding window idea, and coarse-grained method; the correlation fluctuation modes were taken as nodes, and the conversion between the modes and the conversion frequency was taken as the edge and weight. Therefore, a dynamic correlation evolution network with a weighted direction was constructed to study the dynamic correlation evolution characteristics between the EU carbon futures prices and the industrial and energy-related indices. The study found the following:

The study finds that the time-varying evolution network of the EU carbon quota futures price and the industrial and energy-related index is a typical scale-free network, indicating that there are several primary types of correlation modes between the EU carbon quota futures price and the industrial and energy-related index. Only a few of the modes are closely related to other modes, and most of the modes are less related to other modes. From 2013 to the end of 2022, the settlement price yield of the EU carbon quota futures showed a positive correlation with the yield of the industrial and energy-related indices on the whole; however, the correlation intensity was slightly different. The correlation relationship between the price of the EU carbon quota futures and the MSCI developed/energy I index, as well as the MSCI global/energy I index and WTI futures price, is stronger than that between the EU carbon quota futures price and the MSCI emerging/energy I index and Dow Jones Industrial index. It shows that the evolution of the correlation between the EU carbon quota futures price and the industrial and energy-related indices is complicated and dynamic.

Secondly, the dynamic correlation evolution network between the settlement price of the EU carbon quota futures and the industrial and energy-related indices can be established to effectively identify the critical correlation fluctuation modes that have a significant position in the conversion process. International investors can start with the key modal characteristics and the direction of their transformation patterns. When the fluctuation correlation between the EU carbon futures prices and the industrial and energy-related indices within 5 days is relatively weak and positive, the key modes are conveyed to themselves with a likelihood exceeding 90%, indicating that the key modes will be sustained over a duration of time when they appear. International investors with risk aversion can choose an investment portfolio of EU carbon quota futures and other stocks or futures to avoid an excessive concentration of funds.

Third, international investors should pay attention to the clustering effect in modal transmissions. As shown in the long-term distribution analysis, the fluctuation modes of the 5-day correlation are mostly in community 2 and community 3, indicating that once the 5-day correlation between the settlement price of the EU carbon quota futures and the industrial and energy-related index enters the node state of community 2 or community 3, it is likely, with a relatively high probability, to undergo an internal transformation and remain in this state for an extended period. International investors are advised to anticipate the movement of the underlying relationship and pace their investments accordingly at this time.

Finally, through the analysis of the characteristics of node intermediation abilities, it was found that the key modes have high intermediation abilities, simultaneously offering investors with precursor information on a change in the correlation between the EU carbon quota futures and the industrial and energy-related indices and helping international investors to more accurately predict the state of stock prices in the next period.

In summary, this paper not only provides ideas for the study of the correlations between multiple variables but also has reference implications for international investors. Future analyses based on this study should explore the evolutionary characteristics of the dynamic correlations across different time scales and markets.

Data Availability

The data presented in this study are openly available at the wind database (accessed on 1 January 2023).

Conflicts of Interest

The authors declare no conflict of interest.

Authors' Contributions

Ziyang Wang was responsible for the conceptualization, methodology, investigation, software, writing—original draft, writing—review and editing, formal analysis, and data curation. Zhiliang Dong was responsible for the conceptualization, visualization, supervision, project administration, funding acquisition, resources, and validation.

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