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Research Article

Study of the Modal Evolution of the Causal Relationship between Crude Oil, Gold, and Dollar Price Series

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The rise and fall of crude oil prices, gold prices, and US dollar exchange rates is of great significance to investors. It is necessary to explore the evolution characteristics of the causal relationship between crude oil, gold, and US dollar, so as to provide reference for investors. We selected the ICE Brent crude oil continuous futures, the ICE WTI crude oil continuous futures, the NYMEX light crude oil continuous futures, the COMEX gold continuous futures, and the US dollar index as the research objects. Based on the theory of pattern causality, the causal relationship between crude oil, gold, and US dollar is divided into positive causal relationship (T), negative causal relationship (F), and disordered causal relationship (D). The causal modal evolution network of crude oil and gold prices against the US dollar index (BWCGG-U) and the causal relationship modal evolution network of the US dollar index on the price of crude oil and gold (U-BWCGG) were constructed. Complex network and sliding window are used to study the evolution characteristics of short-term causality between crude oil and gold and US dollar in 60 days. The results show that (1) the modes of causality between crude oil, gold, and the US dollar index are constantly changing, with a few key modes. The key modes are often transmitted to themselves. (2) The most critical modes of two networks are all FFFF, and the negative causality between gold and the US dollar index is more stable. Investors based on the gold (dollar index) price to judge the dollar index (gold futures) investment program are more reliable. (3) When crude oil and gold show causal modes of FFFF and DDDF, respectively, the emergence and transformation of the modes can be maintained for a period of time to provide a certain guarantee for investors' investment. However, its internal conversion rate is high, which needs to be judged in combination with the actual situation to improve the forecasting ability.

1. Introduction

With the complex economic and financial background in today's world, energy and precious metals have become important points of all countries and investors. As one of the important energy and commodities, crude oil has financial and political properties [1]. The rise and fall of oil prices can affect the world economy to varying degrees, and as oil resources become scarcer and scarcer, investors are increasingly concerned about oil. Gold has both precious metal and monetary properties and is a "hard currency" with a reserve and investment role. It is considered by investors to be the safest investment target and one of the most important commodities [2]. As the main international trade settlement and reserve currency, most international commodity transactions are denominated in US dollars. There is a certain economic logic, and the dollar exchange rate will act on the crude oil or gold market based on speculation and hedging [3], and its price changes will directly change the price of commodities [4]. The price of crude oil or gold affects the dollar exchange rate through channels such as trade and asset investment [5]. This has led to a closer link between the dollar's exchange rate and oil and gold. Therefore, it is necessary to analyze the causal relationship between crude oil and the US dollar, gold prices, and the US dollar exchange rate; judge the mutual influence relationship and change rule; and provide a decision-making reference for relevant investors.

The interaction between crude oil and the US dollar has been well documented. The first is to explore the direction of mutual causality from the perspective of the length of investment period [6], and the other is to explore the dominant parties of mutual causation from the perspective of different periods [7]. The third is to analyze the relationship between crude oil prices and exchange rates from the perspective of importing and exporting countries [8].

Scholars tend to believe that the dollar exchange rate has an impact on changes in oil prices. From a long-term perspective, most studies believe that the US dollar exchange rate has a negative impact on crude oil prices. Akram [9] and Wu et al. [10] established the structural VAR model and Copula-GARCH model, respectively, and found that the decline in the dollar price was the main reason for the rise in crude oil price. Gao et al. (2022) established the SVAR model and proved by the impulse response function method that the US dollar exchange rate has always had a negative impact on crude oil prices, and the fluctuation range is large [11]. In the short term, some scholars believe that the instantaneous fluctuation of the US dollar exchange rate will not cause major changes in the oil market [12]. In addition, the magnitude of the impact over time was analyzed [13].

However, if A has a causal relationship with B, then B will also contain information about A, and the information about A left in B can be used to estimate A. Therefore, it is necessary to explore the causal relationship between them, and some studies have pointed out that oil price fluctuations can change the exchange rate of the United States dollar [14]. Amano and Norden [15] based on cointegration methods found that oil prices had a long-term impact on the dollar after 1972. Using the VECM method, Volkov and Yuhn [16] found that oil price shocks significantly affected the dollar between 1998 and 2012. Wen et al. [17] found that oil prices are a nonlinear Granger cause of the dollar exchange rate. In recent years, the exploration of the mutual causal relationship between crude oil prices and the US dollar exchange rate has gradually developed.

The relationship between gold and US dollar exchange rates mainly focuses on the study of correlation, and it is believed that there is a strong and stable correlation between the two [18] and to be an inverse change relationship [19]. However, there are few studies on the causal relationship between the two. Most studies believe that gold price fluctuations are explained by monetary variables [20, 21]. Xie et al. [22] construct a vector autoregressive (VAR) model to analyze the relationship between the international gold price and the US dollar index using the impulse response function and Granger causal test. The results show that there is not only a negative long-term equilibrium relationship between the US dollar index and the international gold price but also causal correlation in the Granger sense, that is, the rise of the US dollar index will cause the international gold price to fall. Zhang et al. [23] found that the causal relationship between commodity prices, including gold, and exchange rates is stronger than that between exchange rates and commodity prices. However, there are still few studies on the causal relationship between the gold price and the dollar exchange rate.

The above analysis shows that the current exploration of the causal relationship between crude oil and gold prices and the US dollar exchange rate is basically explored from the aspect of one-way influence. This article not only analyzes the impact of the US dollar exchange rate on crude oil and gold but also discusses the impact of crude oil and gold on the US dollar exchange rate, comparing the similarities and differences.

Traditional Granger causal tests are often used to explore causal relationships [24], but this method cannot divide the causal relationship between periods into causal characteristics of different directions; neither can it quantify the causal relationship between periods to obtain the key causal characteristics. Therefore, this paper uses the pattern causality algorithm to explore the causality relationship between complex nonlinear systems from the perspective of dynamics, to measure the causal relationship between crude oil and US dollars and between gold and US dollars [25]. The causality is divided into positive, negative, and disordered causality. At the same time, the causal impact of crude oil and gold prices on the US dollar exchange rate and the causal impact of the US dollar exchange rate on crude oil and gold prices are determined. It should be noted that the causal relationship obtained by the pattern causal algorithm is not factual causation but quantitative causation.

Moreover, most of the above studies are analyzed from a long-term perspective and less explored the changes in the mutual causal relationship of evolution over time in the short-term investment process. From the perspective of short-term investment, we explore the causal relationship between them for investors to diversify investment and improve overall returns to provide a reference. This requires a model that can show the evolution of causality between oil, gold, and the US dollar, respectively. The complex network method [26] and the sliding window method, which play a key role in the analysis of evolutionary characteristics [27], were used to construct the causal modal evolution network of crude oil and gold prices against the US dollar index (BWCG-U) and the causal relationship modal evolution network of the US dollar index on the price of crude oil and gold (U-BWCG).

The goal is to reveal the evolution characteristics of these causal fluctuations and provide a reference for investors to make timely strategy adjustment in the process of short-term investment. In this process, the causal relationship between crude oil prices and the US dollar index needs to be transformed into specific symbols through a coarse grination description [28, 29], to form the causal relationship mode [30]. The causal modes are taken as nodes, and the transition relations between neighbouring modes are taken as edges to construct networks BWCG-U and U-BWCG. Study the intrinsic and complex change law and evolution mechanism of the causal relationship between crude oil, gold price, and US dollar index, help investors understand the short-term causal relationship between crude oil, gold price, and US dollar index and its change law, and provide investors with certain reference for investing in crude oil, gold or US dollar.

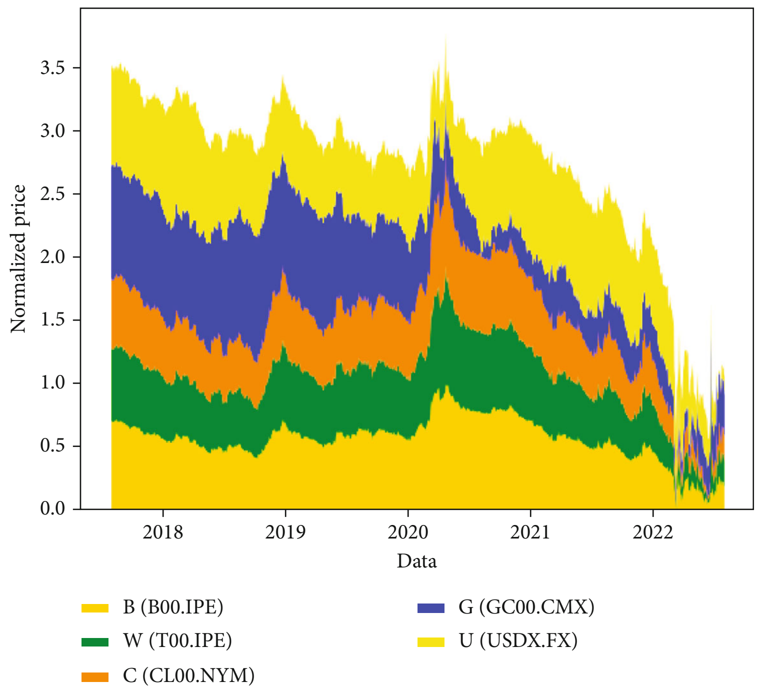

Figure 1: Trend of standardized closing price of crude oil futures and dollar index over time.

Therefore, the contribution of this paper is as follows: first, we not only analyze the impact of the US dollar exchange rate on crude oil and gold but also explore the impact of crude oil and gold on the US dollar exchange rate, which makes up for the lack of research on comparative analysis from the perspective of two-way causation. Secondly, we use the pattern causality algorithm to divide the causal relationship between crude oil, gold prices, and US dollar indices into positive causality, negative causality, and disordered causality to comprehensively reflect the causal characteristics of stock price fluctuations. Finally, from the perspective of short-term investment, we analyze the changes in the causal relationship between crude oil and gold and the US dollar exchange rates over time, which can provide investors with decision-making reference for short-term investment transactions.

The rest of this article is organized below. Section 2 describes the data approach; Section 3 discusses the results of the data analysis, and finally, Section 4 summarizes the findings and provides recommendations for investors to invest in crude oil, gold, and the US dollar.

2. Data and Methodology

2.1. Data. From the perspective of the stock market, the closing prices of the ICE Brent crude oil continuous futures, the ICE WTI crude oil continuous futures, the NYMEX light crude oil continuous futures are selected to represent crude oil, and the COMEX gold continuous futures represent gold. The US dollar index (USDX) is based on the trade settlement volume between major countries in the world and the United States, weighted to calculate the overall strength of the dollar, and uses 100 points as the strength dividing line to measure and evaluate the exchange rate of the dollar against a basket of currencies (Euro, yen, pound, Canadian dollar, Swedish krona, and Swiss franc) [31]. The formula is shown in Equation (1), where 50.14348112 is the result of the index in the base period (March 1973) against base 100. Therefore, this paper chooses US dollar index (USDX.FX) to represent US dollar exchange rate. The closing prices of the four futures and the dollar index were used as research objects.

$$\begin{aligned} \mathrm{USDX} = {} & 50.14348112 \times \mathrm{EURUSD}^{-0.576} \times \mathrm{USDJPY}^{0.136} \\ & \times \mathrm{GBPUSD}^{-0.119} \times \mathrm{USDCAD}^{0.091} \times \mathrm{USDSEK}^{0.042} \\ & \times \mathrm{USDCHF}^{0.036}. \end{aligned} \tag{1}$$

Based on the Wind database, the daily data of all working days of the above research subjects for five consecutive years from July 31, 2017 to July 31, 2022 were selected (http://www.wind.com.cn/en/default.html). A total of 1,216 pieces of data were obtained for crude oil futures and the dollar index, but 57 pieces of data were missing for gold future prices. The Lagrange interpolation method was used to complete the missing data on corresponding working days, and a total of 4,864 pieces of data were finally obtained.

To reduce the influence of the data framework on the causal relationship between crude oil future prices, gold future prices, and US dollar indices, the changes in the three crude oil future prices, gold future prices, and US dollar indices over time are obtained after normalizing the closing price data of the above research objects. As seen from Figure 1, there is a certain same upwards and downwards trend between crude oil futures and gold futures and the US dollar index, and there is a certain correlation. To reveal the complex dynamic characteristics hidden between the crude oil future market, the gold future market, and the US dollar index, it is necessary to explore the causal relationship between the US dollar index and the other two to reveal its internal laws. The correlation between long-term data does not provide investors with much investment advice, so it is necessary to study the change in causality in the short term. In this paper, the causal relationship between crude oil future prices, gold future prices, and the US dollar index is abstracted as a relationship mode by the coarse-grained method. Then, the relationship between causal modes is analyzed by the complex network method, and the inherent complex change laws and evolution mechanisms are studied.

2.2. Pattern Causality between Crude Oil, Gold, and the US Dollar. A system has a tendency to move towards a steady state, which is called an attractor. In complex systems, space-time dynamics are shaped into a unique attractor. The evolution of any variable in the system is determined by the other variables with which it interacts, and the information of any one variable is implicit in the development of other variables. Based on the phase space reconstruction theory, the interaction of complex information is contained in the time series X and Y in the local space-time dynamics between the attractors M_X and M_Y of X and Y. Based on this, pattern causation can be judged based on the accuracy of the pattern "memory" about M_X embedded in M_Y. This causality is not factual causation but quantitative causation. When the patterns in M_Y accurately recall the patterns of M_X and have the same (or opposite) properties, a positive (or negative) causal relationship from X to Y is obtained. M_X's patterns can be recalled from M_Y, but they are neither the same nor the opposite, and this pattern is called disorder causation.

Therefore, the theory of pattern causality can be adopted to explore the causal relationship between the three crude oil future prices X_1, X_2, X_3, gold future prices X_4, and the US dollar index Y. First, we need to use the phase space reconstruction theory to reconstruct the attractors (M_{x1}, M_{x2}, M_{x3}, M_{x4}, and M_y) of the time series variables X_1, X_2, X_3, and X_4 and the US dollar index time series Y using the theory of phase space reconstruction [32]. Then, the average mode of the points on the attractor is obtained based on the theory of symbolic dynamics [33]. Finally, crude oil future prices and the causal relationship between gold future prices and the US dollar index are inferred by exploring the pattern characteristics of crude oil and gold attractors M_{x1}, M_{x2}, M_{x3}, and M_{x4} and US dollar index attractors M_y [10]. The specific steps are as follows:

Step 1. Reconstruct the attractors M_{x1}, M_{x2}, M_{x3}, M_{x4}, and M_y.

To dig for more information on the time series of crude oil futures, gold futures, and the US dollar index, it is necessary to find another new system that is in some sense equivalent to the original system. Drawing on the embedding theorem proposed by Takens [34], we select the appropriate embedding dimension m and delay time τ to reconstruct the attractors M_{x1}, M_{x2}, M_{x3}, M_{x4}, and M_y with the same structure as the original system topology was reconstructed. We use a pseudoscientific neighbour algorithm to determine both $m = 3$ and $\varepsilon = 1$[35]. Then, for each pair of crude oil futures price time series X_1, X_2, and X_3 and gold future price time series X_4 and the US dollar index time series Y, the reconstructed attractors M_{x1}, M_{x2}, M_{x3}, M_{x4}, and M_y with the same topology structure as the original system are obtained. There is a one-to-one mapping between the five groups of attractors obtained by reconstruction and the attractor M of the original system. Therefore, we analyze the relationship between crude oil future prices, gold future prices, and the US dollar index by studying their attractor characteristics. Figure 2 is a schematic diagram of the attractors of the reconstructed time series X_1 and Y. Taking the reconstructed B00. The IPE crude oil future price X_1 attractor is used as an example, and each point on M_{x1} can be expressed as

$$x_1(t) = (X_1(t), X_1(t-\varepsilon), \cdots, X_1(t-(m-1)\varepsilon)), \quad t = 1 + (m-1)\varepsilon \text{ to } t = L. \tag{2}$$

Step 2. Obtain the average mode on the point of attractor M_{x1}.

$x_1(t)$ according to the Euclidean distance: $x_1(i)$.

As shown in Figure 3, to test the causal relationship between X_1 and Y, we analyze the average pattern of points based on the nearest neighbours on attractors M_{x1} and M_y. Taking $x_1(t)$ as an example, first select the $m+1$ point closest to $x_1(t)$ according to the Euclidean distance, i.e., $x_1(i)$:

$$x_1(i) = \left(X_1\left(t_{x1(i)}\right), X_1\left(t_{x1(i)} - \tau\right), \cdots, X_1\left(t_{x1(i)} - (m-1)\varepsilon\right)\right), \quad i = 1, \cdots, m+1, x_1(i) \in N_{x_1(t)}, \tag{3}$$

where $t_{x1(i)}$ represents the time to $x_1(i)$ and $N_{x1(t)}$ represents the nearest neighbour set of $x_1(t)$.

Then, we calculate the weight between $x_{1(i)}$ and $x_{1(t)} w_i^{x_1}$, $i = 1, \ldots m+1$, and then transform each $x_{1(i)}$ to the $h_i^{x_1}$.

$$\begin{aligned} w_i^{x_1} &= \frac{e^{-d(x_1(t), x_1(i))}}{\sum_i e^{-d(x_1(t), x_1(i))}}, \\ h_i^{x_1} &= \left(\frac{X_1\left(t_{x_1(i)} - \varepsilon\right) - X\left(t_{x_1(i)}\right)}{X_1\left(t_{x_1(i)}\right)}, \cdots, \frac{X_1\left(t_{x_1(i)} - (m-1)\varepsilon\right) - X_1\left(t_{x_1(i)} - (m-2)\varepsilon\right)}{X_1\left(t_{x_1(i)} - (m-2)\varepsilon\right)}\right), \end{aligned} \tag{4}$$

where d is the Euclidean distance.

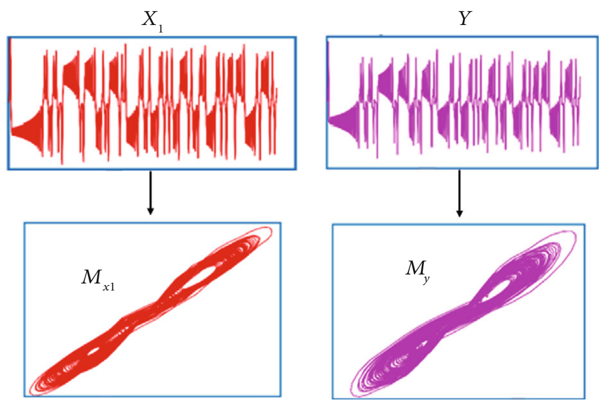

Figure 2: A diagram of the reconstructed attractors M_{x1} and M_y of time series X and Y.

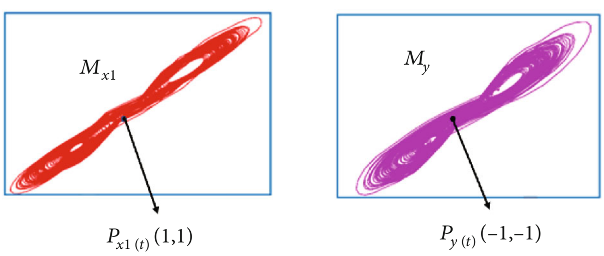

Figure 3: Get the mean mode of the attractor M_{x1} and M_y.

Next, we calculate the weighted sum of $h_{1i}^{x_1} H_{x1(t)}$.

$$H_{x1(t)} = \sum_{i=1}^{m+1} w_i^{x_1} h_{1i}^{x_1} \tag{5}$$

Finally, the average pattern of $x_1(t)$ is obtained by coarse-graining.

$$P_{x_1(t)} = f\left(H_{x_1(t)}\right), \tag{6}$$

where f is a coarse-grained transformation of $f(z_1, \cdots, z_n) = (\text{sgn }(z_1)), \ldots, \text{sgn}(z_n)), (z_1, z_2, \ldots, z_n)$ is an n-dimensional vector, and sgn is the symbolic function n. Then, for points $x_1\,(t) = (0.35, 0.66, 0.33)$,$h_i^{x_1} =$ (0.66-0.35/0.35, 0.33-0.66/0.66) = (0.866,0.5), and finally, its average mode $P_{x_1(t)} = f(h_{x_1(t)}) = (1,-1)$.

Based on the above process, the average patterns of $x_2\,(t)$, $x_3\,(t)$, $x_4\,(t)$, and $y(t)$ can be obtained $P_{x_2(t)}, P_{x_3(t)}, P_{x_4(t)}$, and $P_{y(t)}$.

Step 3. Quantify the strength of causality.

If variable Y is causally related to variable X, then X will also contain information about Y, and Y information left in X can be used to estimate Y. We explore the patterns of oil and gold attractors M_{x1}, M_{x2}, M_{x3}, and M_{x4} and US dollar index attractor M_y to detect the nature of causality between original future prices and gold future prices X_1, X_2, X_3, X_4 and US dollar index Y. As shown in Figure 4, we explore the average pattern (1,1) on the ICE Brent crude oil continuous future crude oil future price x_1 that leads to the average pattern (1,1) on the US dollar index y, and then, positive causality defines the relationship between x_1 and y. When negative causality defines the relationship between x_1 and y, patterns (1,1) on x_1 lead to patterns (-1, -1) on y. In addition, there are disordered causalities that are neither positive nor negative if the pattern in x_1 (1,1) leads to the pattern in y (-1,1)[36].

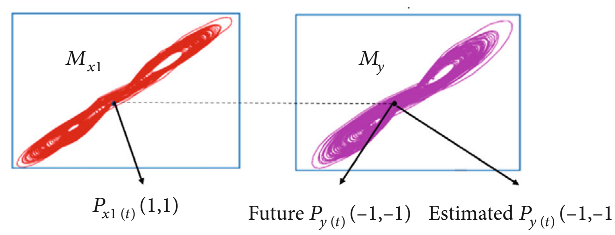

Figure 4: Determine the causal relationship between M_{x1} and M_y.

Additionally, taking the causal relationship between the ICE Brent crude oil continuous future crude oil futures X_1 and the US dollar index Y as an example to quantify the causal strength (CI) of X_1 against Y, we repeat the above process for each point on M_{x1}. For each possible model $P_{x_1(t)}$, we fill the CI matrix with the percentage of cases where the estimated mean model $\widehat{P}_{y(t)}$ is equal to the true mean model $P_{y(t)}$, i.e., causal intensity (CI), as shown in Figure 5. The column vectors in Figure 5 represent the average pattern of US dollar index $y(t)$ within its nearest neighbourhood $E_{y(t)}$. The first row represents the average pattern of the ICE Brent crude oil continuous future crude oil future price $x_1(t)$ coborn with US dollar index $y(t)$ in its nearest neighbourhood $E_{x_1(t)}$. The elements in the matrix represent the value of the causality strength. The value tends to 0, the causality is weaker, and the value tends to 1, and the causality is stronger. Finally, we calculate three causal relationships from X to Y based on the CI matrix. In a CI matrix, the principal diagonal and antidiagonal elements of the matrix correspond to positive causality (red area) and negative causality (green area), respectively, while other elements correspond to disordered causality (blue area). Therefore, the strength of the three causal relationships can be determined as follows:

Strength of positive causality of the ICE Brent crude oil continuous future crude oil futures X_1 against US dollar index Y:

$$\text{PPO} = \frac{1}{9} \sum_{\text{main}} (\text{matrix(CI)}). \tag{7}$$

Strength of negative causality of the ICE Brent crude oil continuous future crude oil future X_1 against US dollar index Y:

$$\text{NNE} = \frac{1}{8} \sum_{\text{back}} (\text{matrix(CI)}). \tag{8}$$

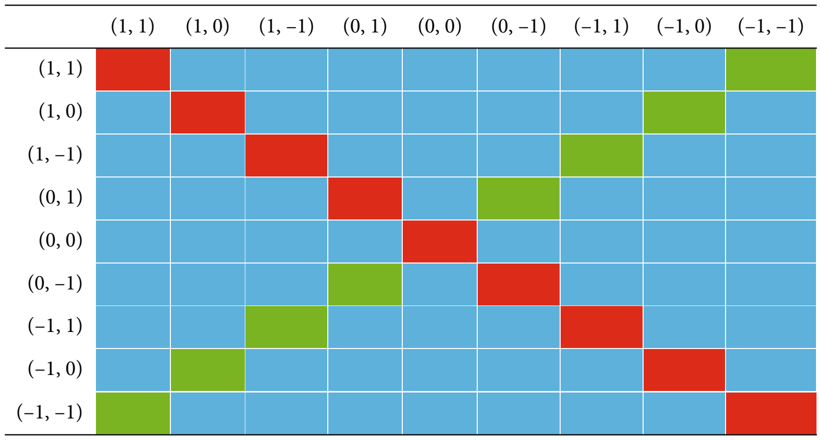

Figure 5: CI matrix ($m = 3$).

Strength of disordered causality of the ICE Brent crude oil continuous future crude oil future X_1 against US dollar index Y:

$$\mathrm{DDI} = \frac{1}{64} \sum_{\text{others}} (\mathrm{matrix}(\mathrm{CI})). \tag{9}$$

Similarly, the causality intensity of the ICE WTI crude oil continuous future X_2, the NYMEX light crude oil continuous future X3, and the COMEX gold continuous future X_4 on US dollar index Y can be calculated. We also calculate the causality intensity of US dollar index Y on the ICE WTI crude oil continuous future X_2, the NYMEX light crude oil continuous future X_3, and the COMEX gold continuous futures X_4.

2.3. Coarsening and Network Construction. To obtain the time-varying causality characteristics of the causal mode between the crude oil price and the US dollar index and between the price of gold and the US dollar index, we use the sliding window method to divide the time series of the research object into several subperiods. The 60-day investment decision line can reflect the long-term trend in the stock market, which provides a guarantee for meeting the characteristics of short-term research cycles, the diversity of short-term causality, and the convertibility of causality. Therefore, we select 60 trading days as the length of the sliding window and one trading day as the sliding step as a subperiod. In each subperiod, the causal modes of crude oil and gold prices on the US dollar index and the causal modes of the US dollar index on crude oil and gold prices are obtained. Taking the causal mode in each subperiod as the node, the causal relationship modality between adjacent subcycles is converted to edges, and the causal modal evolution network of crude oil and gold prices against the US dollar index (BWCG-U) and the causal relationship modal evolution network (U-BWCG) of the US dollar index on the price of crude oil and gold are constructed. Based on this analysis, we analyze the time-varying transformation characteristics of causality between crude oil price, the price of gold and the US dollar index at the quarterly line scale. The specific steps are as follows:

Step 1: determine the causality signs of T00.IPE crude oil futures, CL00.NYM crude oil futures, G00.CMX gold futures, and US dollar index USDX.FX

Using the ICE WTI crude oil continuous futures and the US dollar index as an example, the corresponding positive, negative, and dark causality intensity values are obtained according to the above determination method of causality intensity, and the causality relationship is abstracted into symbol representation according to Formula (10).

$$pe_i = \begin{cases} \mathrm{P}, & \mathrm{MAX}(\mathrm{PPO}, \mathrm{NNE}, \mathrm{DDI}) = \mathrm{PPO} \\ \mathrm{F}, & \mathrm{MAX}(\mathrm{PPO}, \mathrm{NNE}, \mathrm{DDI}) = \mathrm{NNE}\,. \\ \mathrm{D}, & \text{other} \end{cases} \tag{10}$$

If the positive causality between T00.IPE oil futures and the US dollar index is the strongest, it is denoted as P, and the strength of negative causality is the largest, denoted as F, and all other cases are denoted as D.

Step 2: define causal modes

After determining the causal sign between crude oil, gold prices, and the US dollar index, the eight symbol patterns shown in the chart shown in Figure 6 are formed. The row time series is the cause, and the column time series is the result. The figure shows the causal relationship between the rise and fall of the price of crude oil future BOO. IPE against the price of the US dollar index is the negative causal relationship F. Through the process shown in Figure 2, the causal mode is constructed, the causal mode FDTF of the US dollar index change caused by the price change of crude oil and gold is formed, and the causal mode FDFD of the causal relationship of the price change of crude oil and gold is caused by the change of the US dollar index.

Step 3: construct the modal evolution network BWCG-U and U-BWCG of the causal relationship between crude oil, the price of gold, and the US dollar index

A causal mode is obtained in each sliding window, and the causal mode is transformed over time. Since the transition between the two causal modes repeatedly occurs during the conversion process, the conversion between the causal modes forms a network. To reflect the evolution of

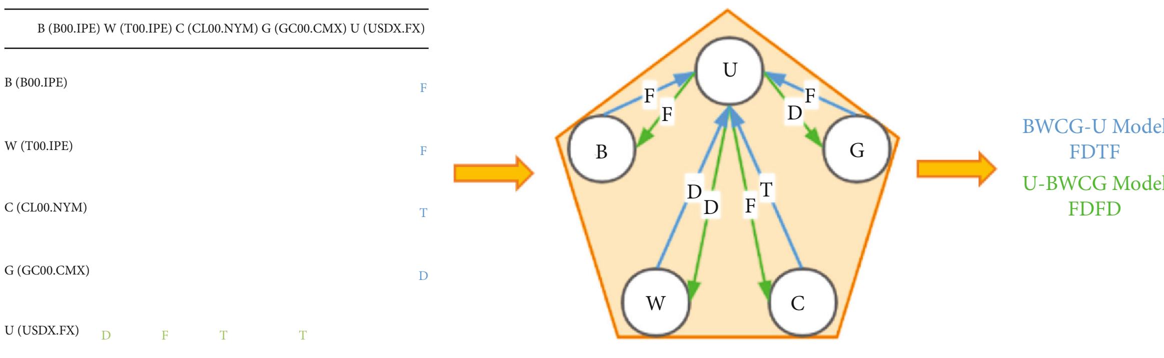

Figure 6: The process of constructing the modal causation between crude oil, gold, and US dollar index.

modalities, this paper takes the causal relationship modality between crude oil, gold prices, and US dollar indices as nodes and the conversion relationship between adjacent causal modes as edges. The corresponding network is constructed with the weight of the conversion frequency between the two causal modes as the edge. The network construction process is shown in Figure 7.

3. Results

Based on the above steps, network BWCG-U and network U-BWCG, as shown in Figures 8 and 9, are obtained. BWCG-U has 80 nodes and 782 edges, and U-BWCG has 81 nodes and 841 edges. The size of the node in the figure represents the weighted degree value of the node. The larger the node, the higher the frequency of the causality mode being converted into other modes and the frequency of other modes being converted into this mode. Nodes of the same color are in the same community. Green is community 1, orange is community 2, blue is community 3, and purple is community 4. The direction and thickness of the curve represent the conversion direction of the causal mode and the conversion frequency between each other, respectively, and the clockwise direction is its conversion direction, and the thicker the curve, the greater the conversion frequency of the two causal modes.

3.1. Key Causal Modes. By identifying the key causal modes in the network, we can understand the main characteristics of the causal relationship between crude oil, gold prices, and the US dollar index in the process of time change. We take the modes that often occur during the transformation of causality modes as the key causal modes, and the weighting value of the node can indicate the number of occurrences of causality modes in the network, which are used to measure this feature. The larger the weighting value, the more times the mode is transformed by other nodes, the more times it is transformed into other modes, and the more stable the modal is in the process of modal evolution of causality between crude oil, gold prices, and the US dollar index, respectively. Equation (11) is the calculation formula of weighted degree value:

$$w_i = w_i^{\text{in}} + w_i^{\text{out}} = \sum_{j\in N_i} w_{ji} + \sum_{j\in N_i} w_{ij}, \tag{11}$$

where N_i is the number of neighbour nodes of node i, w_{ji} is the frequency from node j to node i, that is, the weight of the edge from node i to node j, and w_{ij} is the same.

As shown in Figures 10 and 11, 20% of the nodes of BWCG-U occupy 58.5% of the weighted degree value, and 19.7% of the nodes of U-BWCG occupy 53.3% of the weighted degree value. It shows that both the BWCG-U and U-BWCG networks conform to the power-law distribution characteristics, and most of the modes in the network are associated with fewer mode types, which means that a few modes are in a stable state in the network. Identifying key modes and finding their conversion modes better identifies the conversion characteristics of the modes of causality between the crude oil future price, the gold future price, and the US dollar index to provide relevant investment suggestions for investors with different risk-bearing capacities.

Table 1 shows the three causal modes with the largest weighted values in BWCG-U and U-BWCG. The key causal modes of BWCG-U are FFFF, DDDF, and TTTF. The causal relationship between the three crude oil future prices and the US dollar index often shows the same positive, negative, or unclear characteristics, and there are many cases where they are all negative. The category of causality between gold prices and the US dollar index is more stable than the category of causality between crude oil prices and the US dollar index. The key causal modes in U-BWCG are FFFF, DFFF, and DDDF. The causal relationship of the US dollar index to BOO.IPE shows unclear characteristics with a high probability, while the causal relationship of T00.IPE and CL00.NYM often shows negative characteristics. Similarly, the causal relationship between the US dollar index and the price of gold shows a more stable negative characteristic, indicating that the negative causal relationship between the price of gold and the US dollar index is more stable, and investors can buy two products at the same time to avoid investment risks.

3.2. Mediation of Mode Transmission. During the evolution of crude oil, gold price, and the US dollar index cause and effect mode, if one model is on the shortest path connected to the other two modes, it will have an intermediary role in the transmission process of these two models and has

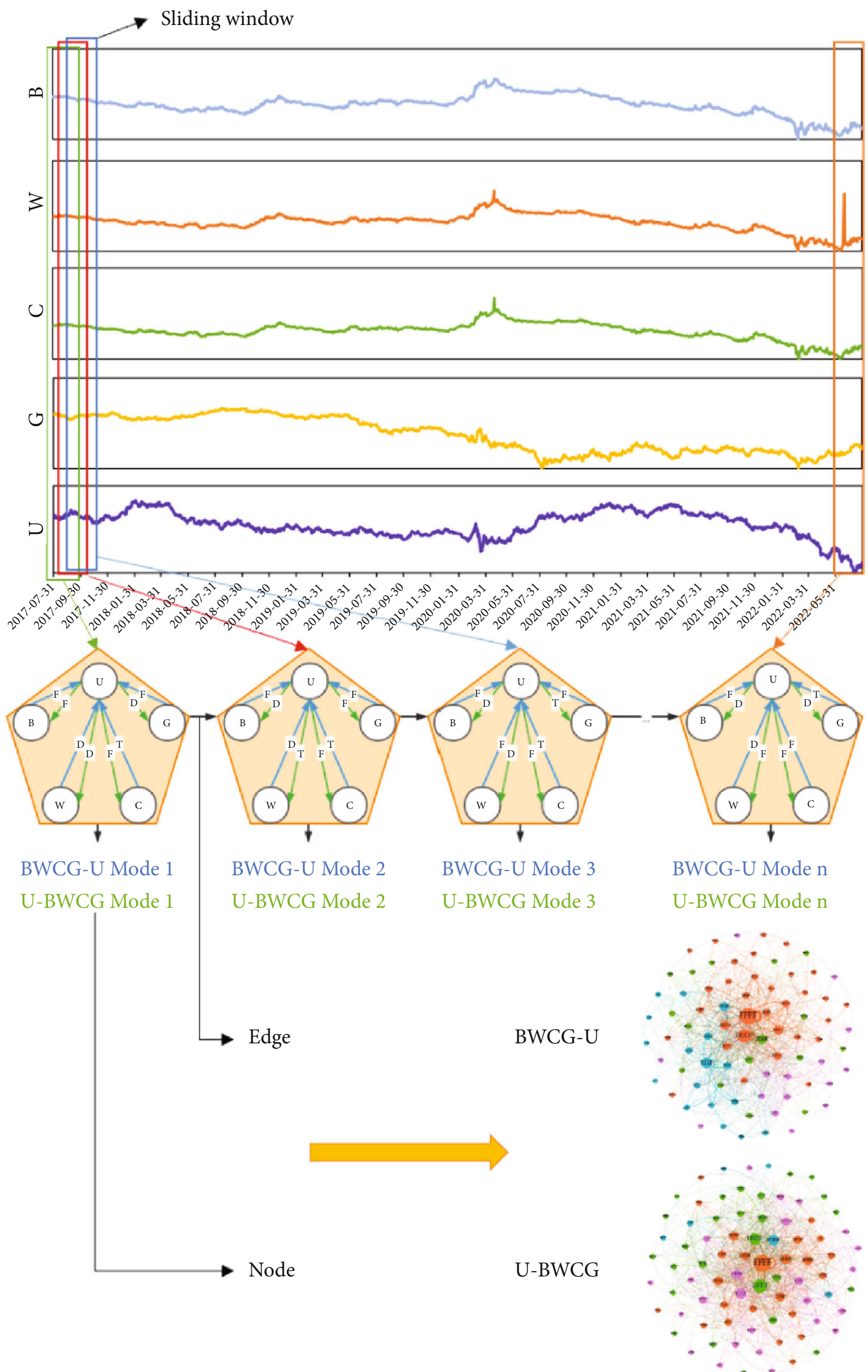

Figure 7: The construction process of the network.

intermediate centrality. The higher the intermediary center of a pattern, the stronger the ability to control information during the transmission process of this mode. In complex networks, some patterns can only be transmitted through mediation patterns. The analysis of the betweenness centrality of the model can provide reference for the price changes of crude oil, gold, and dollar index for investors. The normalized betweenness centrality of common nodes in complex networks represents the mediating ability of nodes. The betweenness centrality B_i of causality model i is defined as

$$B_i = \frac{\sum_j^n \sum_k^n \left(g_{jk}(i)/g_{jk} \right)}{n^2 - 3n + 2}, j \neq k = i, j < k. \tag{12}$$

Among them, $g_{ik}(i)$ is the shortest number of mode j and mode k in mode i as an intermediary. g_{ik} is the shortest number of paths between mode j and mode k. The value of the B_i is between 0 and 1. The larger the value, the stronger the intermediary capacity of the causal mode.

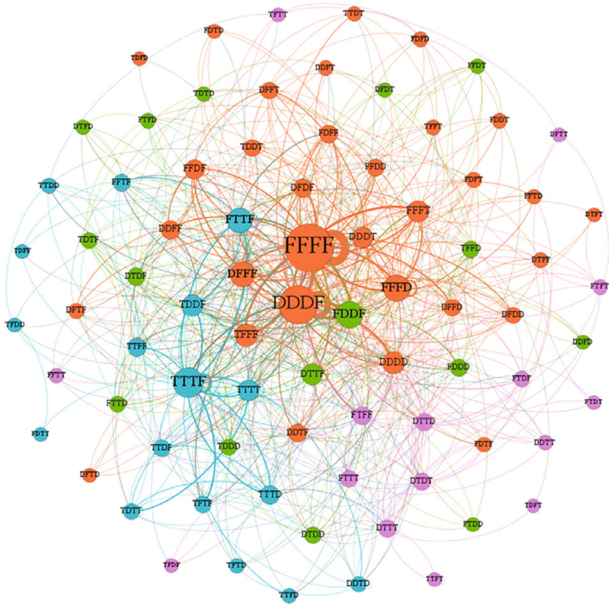

Figure 8: Network BWCG-U.

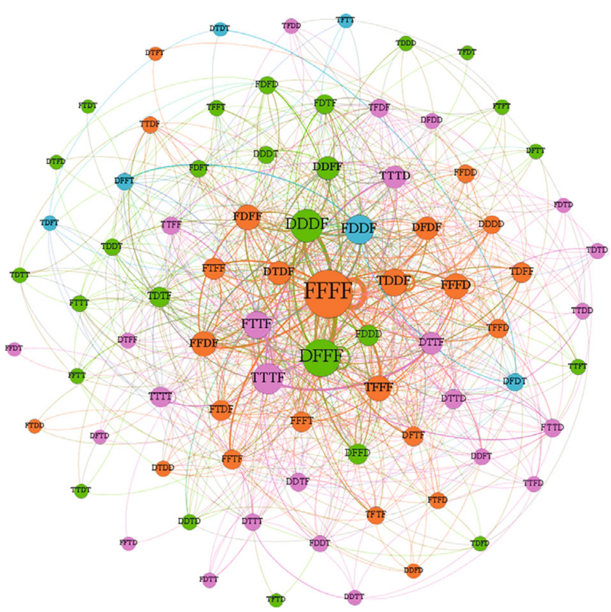

Figure 9: Network U- BWCG.

As shown in Figures 12 and 13, 20% of the nodes in BWCG-U occupy 74.8% of the mediating capacity, and the top three causal relationship modes (FFFF, DDDF, and TTTF) bear 29.7% of the mediating capacity in BWCG-U network. It shows that in the process of causal transmission between crude oil and gold prices and dollar index prices, the key causal mode not only has a strong direct transmission effect on other modes but also has a strong intermediate transmission effect. Among U-BWCG nodes, 19.8% of the nodes bear 63.6% of the mediating capacity, and the top three causal relationship modes (FFFF, DFFF, and FTTF) bear 23.7% of the mediating capacity in the BWCG-U network. In addition to the key causal mode, there are other

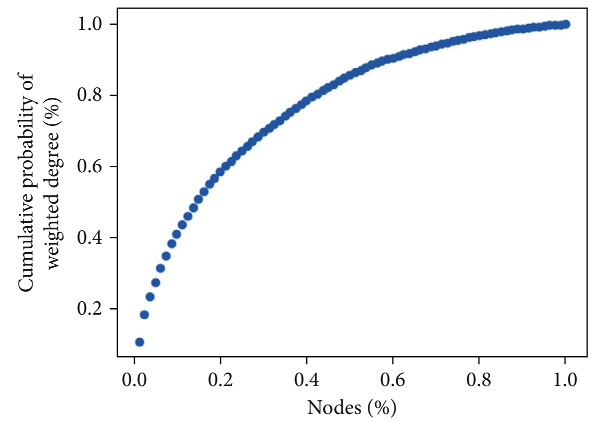

Figure 10: Cumulative distribution of BWCG-U-weighted degree values.

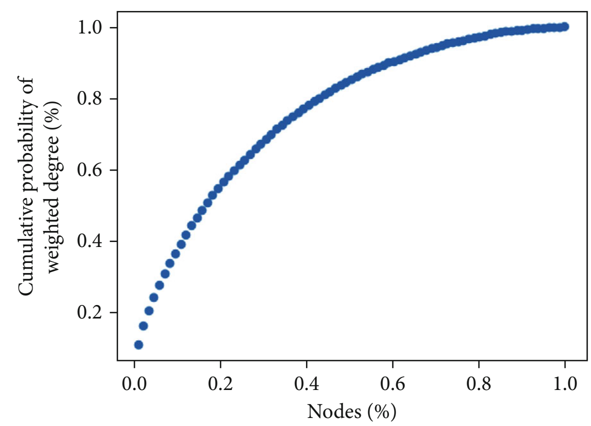

Figure 11: Cumulative distribution of U-BWCG-weighted degree values.

modes similar to FTTF mode in the transmission process of the causal relationship between the dollar index and the price of crude oil and gold. Although they are not key causal modes, they play an important role in the transmission of other modes. DDDF in BWCG-U network ranks the fourth in mediating effect. In general, DDDF in BWCG-U network not only plays the role of critical direct transmission but also has strong mediating effect.

3.3. Critical Transmission Modes. The causal relationship between crude oil, gold prices, and the US dollar index is constantly changing in the short term but with a certain regularity. To explore the regular characteristics, it is necessary to measure the network transmission performance and efficiency, and the average path length and network diameter are often used to measure such characteristics. The average distance between all pairs of nodes in a network is the average path length, which can be used to represent the average minimum transmission time between causal modes. The maximum distance between all pairs of nodes is the network diameter. The average path length is calculated as

$$L = \frac{1}{N(N-1)} \sum_{i \neq j \in V} d_{ij}, \tag{13}$$

Table 1: Key causality modal weight values and proportions of BWCG-U and U-BWCG.

BWCG-U			U-BWCG		
Node	Weighted degree	Proportion of total weighted degree	Node	Weighted degree	Proportion of total weighted degree
FFFF	244	0.106	FFFF	184	0.080
DDDF	178	0.077	DFFF	128	0.056
TTTF	116	0.050	DDDF	102	0.044

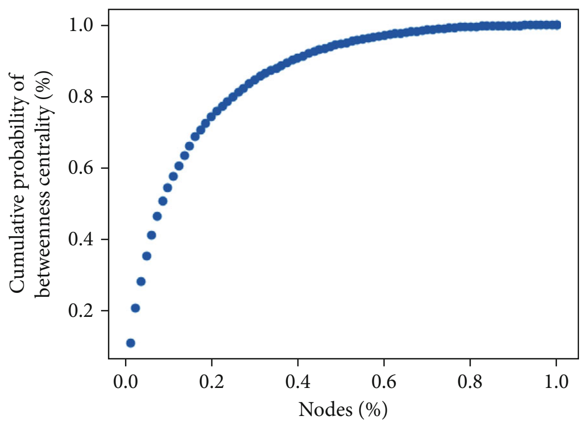

Figure 12: Cumulative distribution of BWCG-U betweenness centrality.

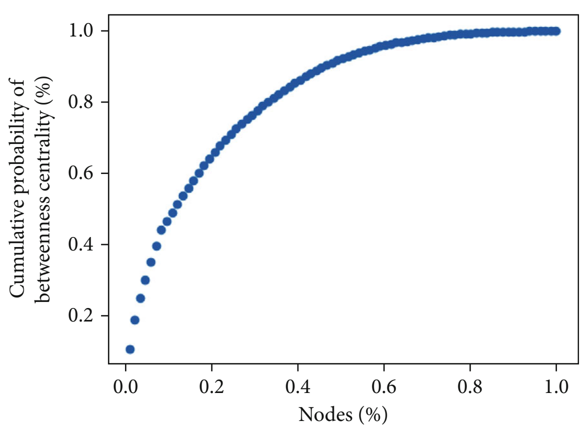

Figure 13: Cumulative distribution of U-BWCG betweenness centrality.

where d_{ij} represents the shortest distance from causal modal i to causal modal j. As seen from Table 2, the average path length of BWCG-U and U-BWCG is greater than 2 and less than 3, indicating that the causal relationship characteristics between crude oil, gold prices, and the US dollar index may change within 2–3 days. It takes a certain amount of time for the causal relationship between crude oil, gold prices, and the US dollar index to change, providing investors with a certain amount of time to adjust their investment strategies.

The network constructed in this paper should theoretically have 81 nodes, namely, 81 causal relationship modes, and 812, that is, 6,561 causal modal transmission relationships. However, there are only 782 transports between causal modes in BWCG-U and 841 causal modes in U-BWCG.

Table 2: Network indicator.

Network	Node	Edge	Average path length	Network diameter
BWCG-U	80	782	2.205	4
U-BWCG	81	841	2.219	5

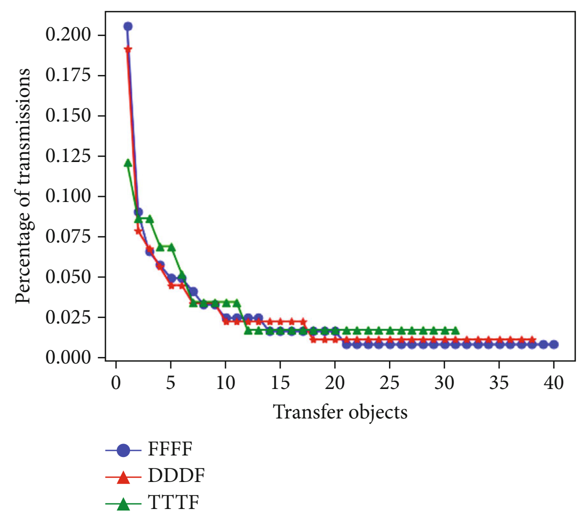

Figure 14: The proportion of objects transmitted by key causality modalities of BWCG-U.

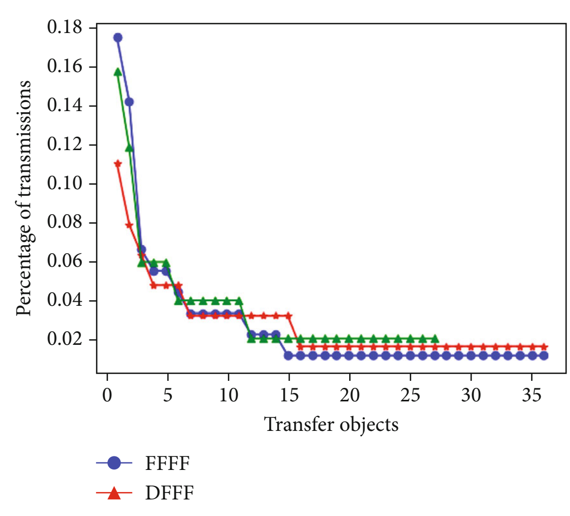

Figure 15: The proportion of objects transmitted by key causality modalities of U-BWCG.

Table 3: The main transport objects for the key mode of BWCG-U and U-BWCG.

BWCG-U				U-BWCG			
Source	Target	Weight	Percentage of transmissions	Source	Target	Weight	Percentage of transmissions
FFFF	FFFF	25	0.205	FFFF	FFFF	16	0.174
DDDF	DDDF	17	0.191	DFFF	DFFF	7	0.109
TTTF	TTTF	7	0.121	DDDF	DFFF	8	0.157

This shows that a few transmission types occupy a key position in the causal mode evolution of crude oil, gold prices, and the US dollar index, respectively. The transmission of the key causal modes identified above is used as the transmission starting mode.

Figures 14 and 15 show the transmission of the BWCG-U and U-BWCG key causal modes, respectively, and it can be seen from the figure that each key causal mode has a variety of transmission objects, but a few transmission objects occupy a high proportion in the transmission process. As shown in Table 3, the main transmission objects of each key causal relationship mode of BWCG-U are themselves, and the proportions of each transmission to themselves are 20.5%, 19.1%, and 12.1%, respectively. These submodes are relatively stable in transmission, which is helpful to judge price movements and investment choices within a certain period. The proportions of the FFFF mode and DFFF to itself in the key causal mode of U-BWCG were 17.4% and 10.9%, respectively, which were smaller than those in BWCG-U. The object with the highest proportion of DDDF modal transmission is not itself but the DFDF modality. This further reflects the important role of the DFFF mode in the evolution of the US dollar index on the causal relationship between crude oil and gold prices.

Figures 16 and 17 show the transmission flow diagram of the most critical causal mode FFFF in BWCG-U and U-BWCG. The size of the traffic in the figure represents the frequency of transmission and the higher the transmission frequency. There are 40 transmission objects in this mode in BWCG-U, which are transmitted to itself at a rate of 20.5%. In addition, it is transmitted to DDDF and TFFF at ratios of 9% and 6.6%, respectively. These three modes account for 36.1% of the FFF modes in BWCG-U, and investors can use these three modes as references in the investment process. There are 36 transmission objects in this mode in U-BWCG, which transmit to itself at a rate of 17.4%. In addition, it is transmitted to DFFF and FFDF at ratios of 14.1% and 6.5%, respectively. These three modes account for 38% of the FFFF modes in the U-BWCG. When investing in the US dollar index based on the price of crude oil and gold futures, if the current price causal mode is FFFF, the causal mode in the latter period is likely to be FFFF or DDDF, at which time investors can mainly judge based on the price trend of gold. When investing in crude oil and gold futures based on the US dollar index, if the current causal mode is FFFF, the causal relationship reflected in the latter investment trading day may be FFFF itself or DFFF. In this case, investors can mainly carry out T00.IPE, CL00.NYM, and GC00.CMX gold future trading.

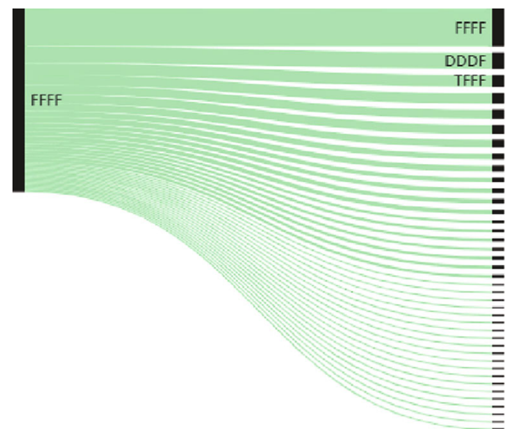

Figure 16: FFFF causality modal transmission flow diagram in BWCG-U.

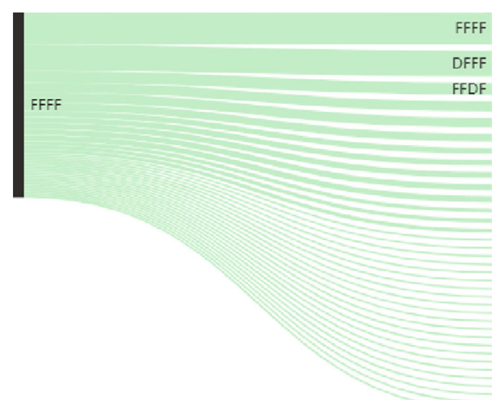

Figure 17: FFFF causality modal transmission flow diagram in U-BWCG.

3.4. The Influence of Clustering Effect on Modal Conversion. The clustering effect of a complex network can aggregate the closely related causal modes in the causal mode evolution network of crude oil price, the price of gold, and the US dollar index into the same community, while the sparsely related causal modes can be divided into different communities. The clustering coefficient C of causal mode i can be calculated by the following formula:

$$C_i = \frac{2E_i}{K_i(K_i - 1)}, \quad (14)$$

where K_i represents the number of causality modes adjacent to causality mode i, and E_i is the actual number of edges

Table 4: Community division and transmission.

Network	Community	The key modal	Number and proportion of modes	Number and proportion of edges	Average clustering coefficient	Average shortest path length
BWCG-U	Community1	FDDF	16 (20%)	46 (5.88%)	0.309	2.293
	Community2	FFFF, DDDF	32 (40%)	225 (28.77%)	0.475	1.959
	Community3	TTTF	17 (21.25%)	67 (8.57%)	0.474	2.113
	Community4	DTTD	15 (18.75%)	32 (4.09%)	0.223	2.65
U-BWCG	Community1	TTTF	25 (30.86%)	112 (13.32%)	0.325	2.221
	Community2	FFFF	24 (29.63%)	186 (22.12%)	0.458	1.828
	Community3	FDDF	6 (7.41%)	10 (1.19%)	0.244	2.3
	Community4	DFFF, DDDF	26 (32.1%)	89 (10.58%)	0.28	2.297

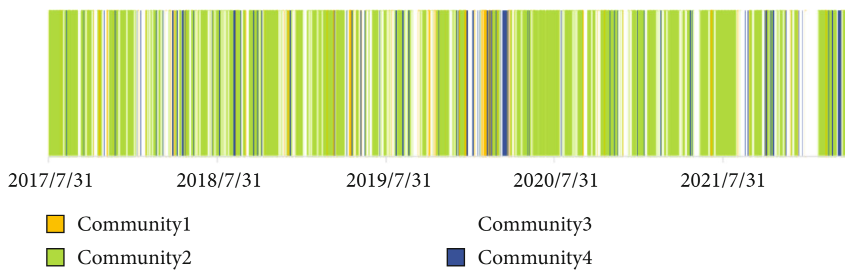

Figure 18: Distribution map of BWCG-U community over time.

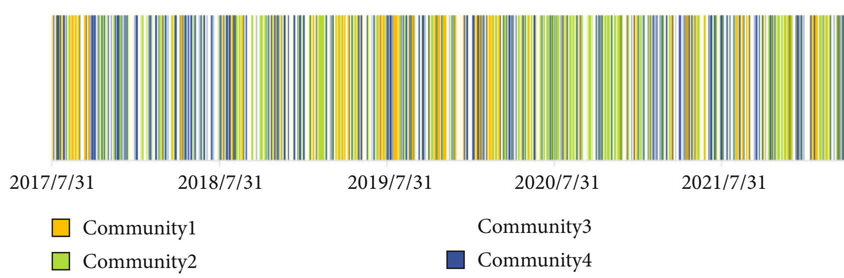

Figure 19: Distribution map of U-BWCG community over time.

between causality modes i and all causality modes in K_i. Causality mode i is connected to other causality modes through K_i, up to a maximum of $K_i(K_i - 1)/2$ edges. The higher the degree of correlation between neighbors of the causality pattern, the higher the clustering coefficient of the causality pattern. The higher the clustering coefficient of the pattern, the more favorable its position in the network.

According to the Girvan-Newman (GN) algorithm, BWCG-U and U-BWCG are divided into four groups, as shown in Figures 4 and 6 . The causal mode categories and mode transmission included in each community are shown in Table 4. The BWCG-U species contains community 1 with FDDF as the key transmission mode, community 2 with FFFF and DDDF as the key transmission mode, community 3 with TTTF as the key transmission mode, and community 4 with DTTD as the key transmission mode. Among them, communities 2 and 3 have more nodes and edges, and their core nodes are the key nodes identified above. Community 2 has a larger average clustering coefficient and a smaller average shortest path, which indicates that the network structure of community 2 is closer. The community of U-BWCG species is community 1 with TTTF as the key transmission mode, community 2 with FFFF as the key transmission mode, community 3 with FDDF as the key transmission mode, and community 4 with DFFF and DDDF as the key transmission mode. Among them, community 2 and community 4 are more critical, and community 2 has a closer network structure.

Figures 18 and 19 show the distribution of node communities over time in BWCG-U networks and U-BWCG networks, respectively. Comparing the two figures, it can be seen that the community evolution caused by the causal modal aggregation of crude oil and gold against the US dollar is more stable, while the community evolution generated by the causal modal aggregation of the US dollar against crude oil and gold is more dispersed. This is because the impact of the dollar on the price of its goods leads to the limited impact of goods on the dollar.

As seen from Figure 18, the causal mode of community 2 occurs most frequently, and more causal modes in community 2 can be transformed internally. Its core modalities, FFFF and DDDF, are the most frequent modalities. In contrast, communities 1, 3, and 4 occur less frequently. Therefore, according to the division of key modal societies, it can be judged that the US dollar index changes with the price of crude oil and gold. When the causality of crude oil price and the price of gold on the US dollar index is negative, or the causality of crude oil price and the US dollar index is not clear, and the causality of the price of gold and the US dollar index is negative, the causality mode has changed to the causality mode of community 2. Because the causal modes in community 2 are more concentrated and uniform over time, the emergence and transformation of modes can often be maintained for a period of time, providing a certain guarantee for investors' investment. However, its internal conversion rate is high, which should be judged in combination with the actual situation.

Figure 19 shows that the probability of community 2 in U-BWCG is greater than the probability of other communities, but the time period of occurrence is relatively scattered and does not form a stable trend over time. There are certain risks in investing in crude oil futures and gold futures based on the US dollar index and the community characteristics.

4. Conclusions

The time series of closing prices from July 31, 2017 to July 31, 2022 of the ICE Brent crude oil continuous futures (B00.IPE), the ICE WTI crude oil continuous futures (T00.IPE), the NYMEX light crude oil continuous futures (CL00.NYM), the COMEX gold continuous futures (GC00.CMX), and the US dollar index (USDX.FX) were selected as the research objects. The sliding window is used to test the short-term causality relationship and transformation between each crude oil price, price of gold, and US dollar index time series and define the describable causality pattern among multivariate time series. The BWCG-U network and U-BWCG network are constructed, and the causal relationship and internal law between crude oil price, the price of gold, and the US dollar index are discussed by analyzing the point intensity distribution, betweenness centrality, main transmission path, clustering effect, and community structure of the network. The following conclusions are drawn:

(1) The causality of crude oil and gold to the US dollar and the causality of the US dollar to crude oil and gold are not single negative causality or positive causality, respectively, but rather change at any time with the transaction. However, a few causal relationship types occupy the dominant position, which not only bear the key direct transmission role but also have strong mediating effect. The FFFF negative causal mode is the most critical causal relationship type. This is because the decline in the US dollar exchange rate is often related to inflation, stock market downturn, etc. at which time the function of crude oil as a strategic material and the value preservation function of gold will be reflected. International crude oil, gold price rise crude oil, and gold exporting countries have more dollars, when buying goods or services of other countries need to be exchanged for euros, yen, and other national currencies, and will increase the value of other currencies and promote the depreciation of the dollar

The main transmission modes of the key causal modes are themselves. Critical causality patterns emerge and remain for a while with high probability. However, the key causal mode of the dollar for oil and gold is transmitted to itself at a lower rate. Paying close attention to the emergence and transformation of key patterns, especially the price movements of crude oil and gold to invest in the US dollar index is a key investment basis for investors.

(2) Based on the key causal modes of crude oil, gold, and the US dollar and their conversion characteristics, it can provide investment references of crude oil, gold, and the US dollar in 60 days for different risk appetite investors. Taking the most critical causal mode FFFF as an example, risk-averse investors can buy crude oil futures and the US dollar index at the same time or gold futures and the US dollar index at the same time to avoid risk. The causal relationship between the price of gold and the US dollar index is more stable and shows negative characteristics. When the price of gold (US dollar index) falls, risk-oriented investors can buy the US dollar index (gold) in the next investment period to obtain income

(3) According to the analysis of the characteristics of community division, in the evolution process of the causality modes of crude oil and gold to the dollar, the 60-day causality mode was mostly concentrated in the FFFF and DDDF as the core community. Combined with the transformation of key causal patterns, it can be seen that when causal patterns become members of the above community, the transformation relationship of causal patterns will have a stable period, which can provide a buffer period for investors. However, its internal conversion rate is high, which needs to be judged in combination with the actual situation to improve the forecasting ability. In the evolution process of the causality mode between the US dollar and crude oil and gold, the causality mode is aggregated and does not form a relatively stable trend with time. It is risky to invest in crude oil futures and gold futures based on the US dollar index on the basis of community characteristics

Data Availability

The data presented in this study are openly available at wind database (accessed on 1 August 2022).

Conflicts of Interest

The authors declare no conflict of interest.

Authors' Contributions

Y.J conducted the conceptualization, methodology, investigation, writing the original draft preparation, writing the review, and editing; Z.D was responsible of the methodology, software, supervision, and funding acquisition; H.A conducted the project administration and visualization. All authors have read and agreed to the published version of the manuscript.

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