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Review Article

Design, Optimization, and Application of Nonlinear Energy Sink in Energy Harvesting Device

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Vibration control has been of great interest to scientists and engineers for many years. Although linear vibration absorbers have been shown to be effective in mitigating vibrations at specific frequencies, their vibration reduction effect is usually limited to a narrow frequency bandwidth. Nonlinear energy sinks have attracted attention due to their better vibration reduction effect over a wider frequency bandwidth. In practical applications, the nonlinear energy sink devices can effectively absorb, dissipate, and convert energy from broadband excitation, so as to achieve vibration reduction and energy harvesting. However, research on energy harvesting based on the nonlinear energy sink is less mature than in linear systems. Multiple parameters of device design (e.g., damping size) affect the actual performance of the nonlinear energy sinks, but there is no exact method to simplify the design of the multiparameter nonlinear energy sinks. Since it is more difficult to implement electromagnetic and electrostatic energy harvesters, more research has focused on piezoelectric energy harvesters. This paper summarizes the research on the nonlinear energy sink and energy harvesting technology, including the introduction of the nonlinear energy sink, energy harvesting based on the nonlinear energy sink, and its application in various fields of energy harvesting. The paper also summarizes some important methods for solving the dynamical equations, as well as their advantages and disadvantages. The conclusions provide an outlook on the subsequent research of the nonlinear energy sink technology, such as the introduction of piezoelectric materials with high energy density, the benefits of balanced vibration suppression and harvesting of vibration energy, and the self-tuning of parameters in complex environments. It provides a powerful reference for the popularization and application of energy harvesting technology based on nonlinear energy sinks.

1. Introduction

The early conventional vibration absorption devices were linear and required significant mass to be added to the structure. The linear vibration absorber consists of a damper, a mass block, and a spring. The damping mechanism of the classical tuned mass damper (TMD) was to transfer the vibration energy from the primary structure to the TMD, which dissipates the energy by the damping element [1]. The TMD has a narrow frequency bandwidth and poor robustness. The optimization of a linear system can only broaden the frequency bandwidth to a certain extent, and the absorption of vibration energy is not very efficient. However, the nonlinear system has stronger robustness and can function in wider frequency bandwidth, which is conducive to broadband energy absorption and has the property that the energy transfer is irreversible [2]. The theoretical analyses of the targeted energy transfer (TET) phenomenon are described in detail in Figure 1.

Ding and Chen summarized the design, analysis, and application of nonlinear energy sinks (NESs) [3]. From the perspective of nonlinear dynamics, the vibration reduction mechanism of the device in various engineering fields was analyzed, which provided ideas and inspiration for the

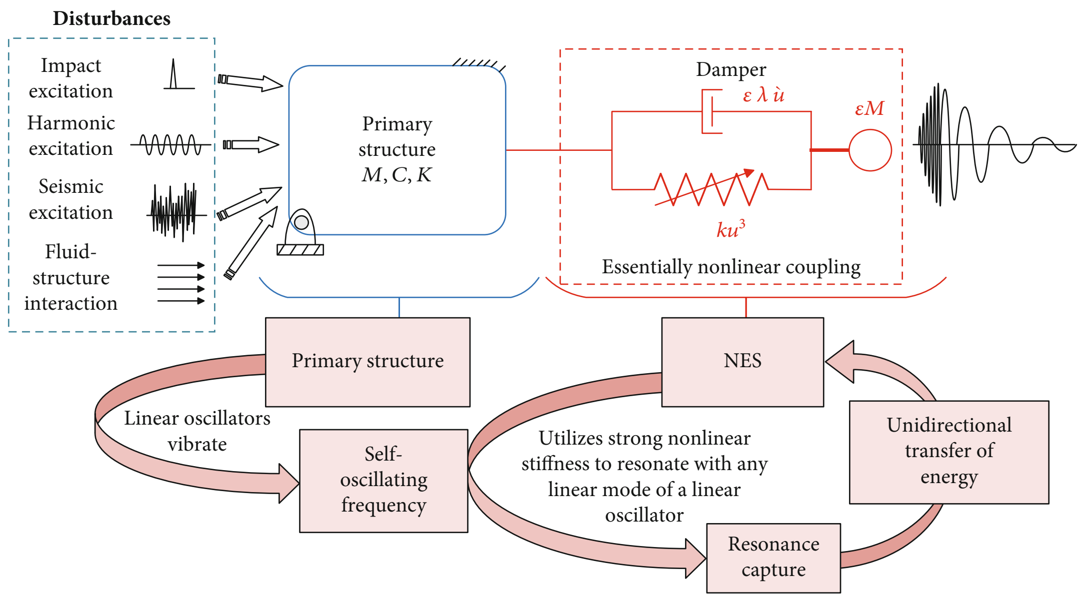


Figure 1: Schematic diagram of irreversibility principle of target energy transfer.

design and application of NES. On this basis, further research shows that the vibration energy can be converted into usable electric energy, which is a promising vibration energy harvesting device.

As shown in Figure 2, NES technology has attracted the attention of many researchers in terms of vibration reduction and energy harvesting. Obviously, the study of vibration energy reuse in the environment has become a hotspot of current research. The problem of how to convert vibration energy into electricity is not difficult to solve. Since kinetic energy is a common energy source, a device to convert kinetic energy into electric energy has been developed by researchers. The conversion device to obtain energy from ambient vibration should have the advantages of relatively uncomplicated design, lightweight, and high stability. Here, we pay special attention to energy harvesting schemes exploiting electromagnetic, piezoelectric, and electrostatic mechanisms and understand that the performance of such techniques in practical applications is considerable. Piezoelectric materials, magnetostrictive materials, and magnetoelectric materials have been applied in energy harvesting, which provides a valuable reference for the improvement of energy harvesting materials and technology [4, 5].

In a wide range of engineering applications, NESs have been introduced as passive nonlinear vibration absorbers and energy harvesters. Because NES can quickly and effectively transfer mechanical energy from the primary structure to nonlinear attachments, it also has the characteristics of simple configuration, broadband vibration attenuation, and high robustness. NES technology has been applied in many fields, such as aerospace, infrastructure [6], and wireless sensor. The energy harvesting of environmental vibration is an important research topic. Wireless sensors are used in remote, harsh, and dangerous environments, and there are problems such as large usage, long node distribution distance, and battery replacement. Therefore, in addition to energy harvesters based on acoustic wave [7, 8], solar energy [9, 10], or thermal energy [11], the use of ambient vibration energy to power wireless sensors in remote areas has been extensively studied [12].

This review paper will emphasize the design, optimization, and application of NES as an energy harvesting device in order to improve energy harvesting performance in various engineering fields from a nonlinear dynamic perspective. Further research work is prospected, which provides a strong reference for the popularization and application of energy harvesting technology based on NES. A summary of highly cited publications on NES research is shown in Figure 3 [13–21].

This paper mainly summarizes three aspects. Section 2 outlines the basic concepts, components, working principles, properties, and advantages of NES, and on this basis expounds the research status of vibration suppression and energy harvesting based on NES. Section 3 introduces energy harvesting and expounds on the performance of energy harvesting based on NES from three energy conversion mechanisms, namely, electromagnetic, piezoelectric, and electrostatic, which provide a valuable reference for the subsequent design of energy harvester. Section 4 summarizes the three fields of aerospace, infrastructure, and wireless sensor and introduces the application of NES in energy harvesting. Section 5 summarizes the status quo of NES technology research and puts forward the future research direction.

2. Overview of the NES

2.1. Basic Concepts of the NES. In the early stage, nonlinear vibration absorbers mainly used nonlinear springs as their stiffness element [22]. Later, researchers named the vibration absorber with strong nonlinear stiffness and were able to realize the unidirectional transfer of vibration energy from the primary structure to the energy-dissipation element as

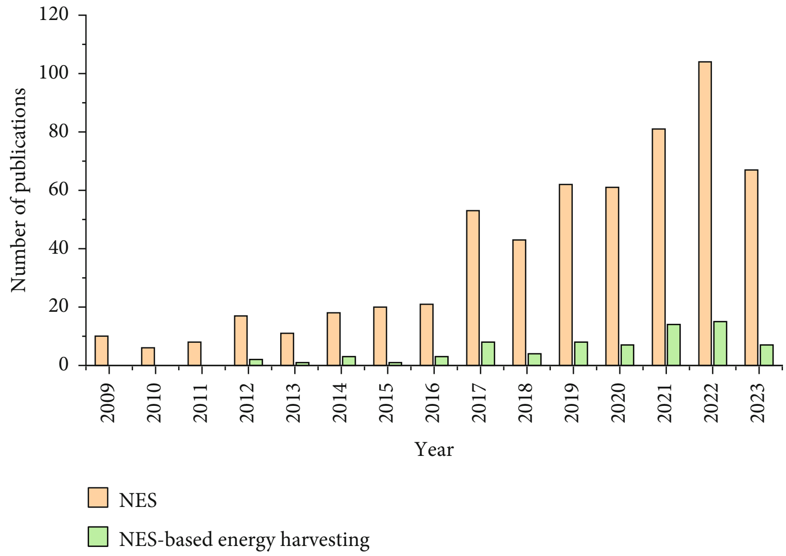


Figure 2: The number of publications on NES and NES-based energy harvesting from the Web of Science database.

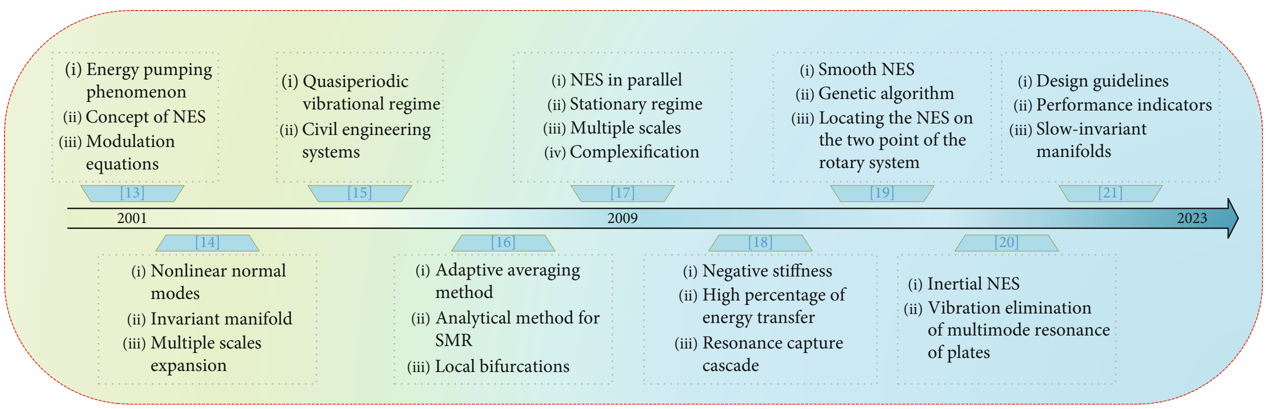


Figure 3: A summary of highly cited publications on NES research.

NES [13, 23, 24]. Therefore, a nonlinear absorber may also be used as a NES and passively absorb energy from a linear system. In addition, when the linear oscillator was weakly coupled to a nonlinear element, it was also called a NES. From the perspective of structure, the NES is mainly composed of three parts, namely, damping element, light additional mass, and strong nonlinear stiffness. It is worth noting that the nonlinear restoring force provided by nonlinear stiffness comes from geometric nonlinear [25, 26], buckling beam [27], magnetic force [28], spring force [29, 30], etc. The NES does not change the natural frequency of the primary system. In addition, the multimode model of the structure with the additional NES is able to capture transient resonances, thus extending its vibration reduction frequency bandwidth. In order to improve the vibration energy absorption performance of the system, a method to estimate the bistable NES stiffness force is proposed [31]. At the same time, the mechanism of capturing and dissipating vibration energy through NES is called target energy transfer, which is characterized by irreversibility and fast transmission speed. When resonance capture occurs, the vibration energy of the primary structure is rapidly transferred to the NES and consumed by the damping elements of the NES. Due to the reduction of vibrational energy, the system will escape from the resonant state and be captured by the new state. When the vibration energy is lower than the critical value, the system no longer satisfies the basic condition of complete transmission of vibration energy and is no longer captured by the new resonance state. During the entire process, most of the vibration energy is consumed within the NES and does not return to the primary structure. Nonlinear normal modes and 1 : 1 resonance were the main topics of NES research [32–34]. The energy pump was caused by the 1 : 1 resonance capture of the system, which enabled the absorber to resonate with any mode of the linear primary system. Different nonlinear normal modes pumped energy in different ways, and effective measures could enhance energy pumping. The energy pumping process can be enhanced by improving the nonlinear normal mode in the undamped system [35]. The curve of the slow invariant manifold explains the phenomenon of resonance capture cascade and quantifies the NES performance [35]. In contrast to the classical resonance capture cascade, the inverted resonance capture cascade induced by the softening stiffness property of the NES dissipates the energy in the low-frequency mode at first [36]. So far, researchers have proposed many NES designs, such as vibroimpact NES (VINES) [37–40], rotational NES (RNES) [24, 41], nonsmooth NES [42], and negative stiffness NES [18, 43, 44]. In terms of how they are configured, they can be categorized as grounded NES and ungrounded NES [25, 45].

2.1.1. VINES. VINES refers to devices that have passive absorption and partial dissipation of shock or seismic energy based on the principle of transferring the energy of the main system to a small attached mass and dissipating the energy through their interaction. Compared to TMD and stiffness-based NESs, impact- and rotary-based NESs perform better in terms of vibration reduction and energy distribution in the structure [46]. VINES provided higher energy density compared to cubic NES energy harvesters [47]. The coupling model of VINES and the linear main structure is shown in Figure 4. The model indicated that the evaluation criterion of the optimal parameter solution was whether 1 : 1 resonance occurred, which provided a reference basis for the optimization of the vibration system with transient TET.

Interestingly, local VINES could influence the global dynamic properties of the primary structures to which it was attached. Depending on the spatial energy distribution within the structure, VINES could interact resonantly with a variety of structural modes over a wide bandwidth. Based on the multiple scales method, analytical and numerical methods for extracting the backbone curve of vibroimpact systems were proposed [49, 50], which deduced the VINES design rules and provided a valuable reference for future design.

2.1.2. RNES. Figure 5 shows the free eccentric rotor with mass m and radius r_0. The rotor is installed inside a main oscillator of mass M and linear stiffness C. The coefficient of linear viscous damping around the axis of the rotator is γ. The level of external excitation can lead to three states of the eccentric attachment in Figure 5. Among them, a high level of excitation can make the NES dissipate the majority of the energy. The rotary-oscillatory NES was obtained by using an elastic arm instead of a rigid coupling arm on the basis of a RNES. The rotary-oscillatory NES improves the energy absorption and dissipation efficiencies over a wider range of initial input energy compared to the RNES [52]. The rotary-impact NES combines the synergistic effects of current rotary-based and impact-based NESs, and the device improves the efficiency of directional energy transfer [53].

2.1.3. Nonsmooth NES. A NES with nonsmooth stiffness characteristics is called a nonsmooth NES [53]. As shown in Figure 6, the nonlinear stiffness of spring could be composed of linear springs with varying stiffnesses.

The device made the structure of the system compact, simple, and adjustable, especially under moderate excitation.

2.1.4. Negative Stiffness NES. A negative stiffness NES is a NES device with negative stiffness characteristics, and its

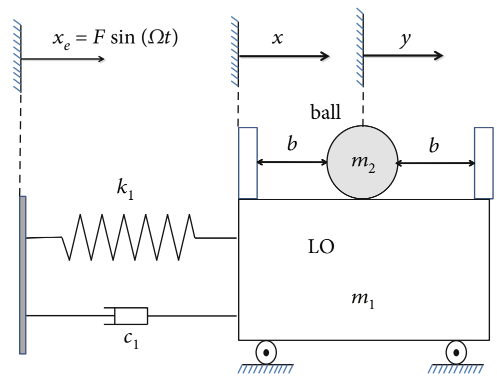


Figure 4: The schematic diagram of the VINES system [48].

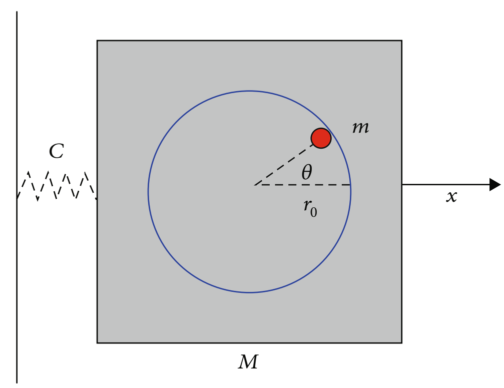


Figure 5: Scheme of primary mass with attached eccentric rotator [51].

model is shown in Figure 7. The motion governing equation of the coupled system was written as equations (1a) and (1b). The control force provided by NES was expressed as equation (2).

$$M\ddot{u} + C\dot{u} + Ku - F_c = -M\ddot{u}_g, \tag{1a}$$

$$m_{\mathrm{NES}}(\ddot{u} + \ddot{z}) + F_c = -m_{\mathrm{NES}}\ddot{u}_g, \tag{1b}$$

$$\begin{aligned} F_C &= F_{\mathrm{NES}} + F_{\mathrm{SF}} \\ &= 2F_s\left[1 - L\left({L_0}^2 + (z - z_0)^2\right)^{-1/2}\right] + F_{\mathrm{SF}}. \end{aligned} \tag{2}$$

F_{NES} stood for the force of negative stiffness. Sliding friction was referred to as F_{SF}. k_{NES} stood for the spring stiffness. L and L_0 represented the physical unloading length and compression length of the spring, respectively. NESs can broaden the frequency bandwidth of damping vibration energy, and their internal nonlinearity plays a significant role in the dynamic properties of the system. This article summarizes and generalizes several common NES devices, as shown in Table 1.

2.1.5. Nongrounded NES. Nongrounded NES is a mass block coupled to a vibrating mechanical system by a highly nonlinear spring, as shown in Figure 8.

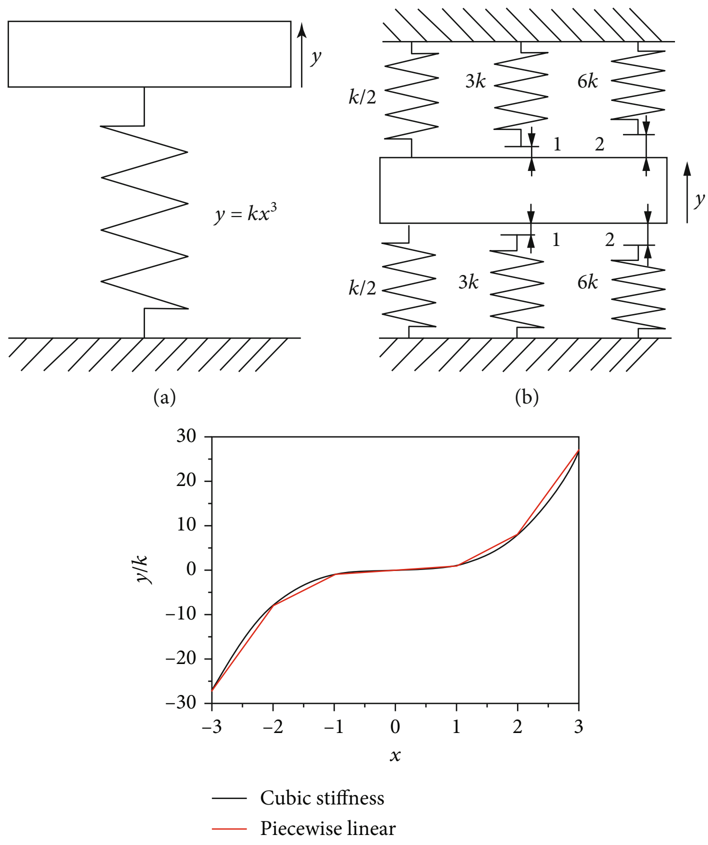


Figure 6: Structure of the NES: (a) nonlinear spring, (b) assemble of linear springs, and (c) comparison of reaction forces [54].

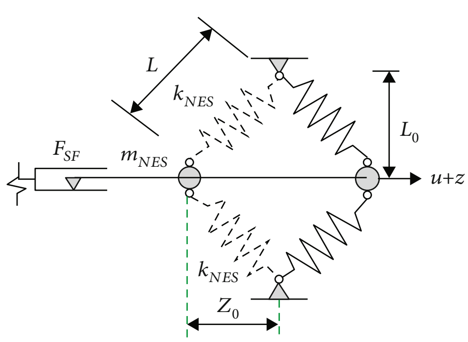


Figure 7: Top view of the NES layout [44].

Compared to the grounded NES configuration, the nongrounded NES configuration is much more efficient. As a result, the most popular NES is the nongrounded NES. Nongrounded NES is generally regarded as a vibration absorber for shock and periodic excitation, and few researchers have examined the mechanisms of vibration propagation that are isolated by nongrounded NES [55].

2.1.6. Grounded NES. Grounded NES is less well known and is connected to the mechanical system by a weakly linear spring while being grounded by a highly nonlinear spring, as shown in Figure 9. In the ungrounded configuration, the TET effectively generates about 89% energy dissipation. In contrast, grounded systems do not have significant energy transfer to the NES due to the lack of a continuous nonlinear normal mode under excitation.

An important advantage of grounded NES is that its design is significantly less constrained by mass than nongrounded NES [45]. Grounded NES can effectively suppress transient and steady-state vibration in rotor system [57].

2.2. Features and Advantages of NES. In recent years, NES has replaced linear devices in some areas, mainly used to suppress the vibration of a variety of structures. Because NES can attenuate the energy of the primary structure in a short time, the vibration absorption efficiency of NES is significantly higher than that of traditional passive vibration absorbers. Moreover, bistable systems [58, 59] and tristable systems [60, 61] were capable of large interwell motion between multiple equilibrium positions. At the same time, the NES added less mass than the traditional vibration absorber, which was especially important for building vibration reduction. In addition, the natural frequencies of buildings are variable (such as the aging of components). Tuning mass dampers, however, requires additional adjustment of the natural frequency (NES does not). By studying the energy pumping phenomenon under different excitations [15], it was concluded that NES had good performance under earthquake excitation. In order to optimize the performance of the vibration energy harvester, it was necessary to

Table 1: Introduction to NES design types.

Num.	Ref.	Type	Advantage	Disadvantage	Improvement
1	[24, 41]	Rotational nonlinear energy sink	Fast and efficient response to attenuation, compact design, and strong robustness	Dynamics are complex and require large initial energy	Reduces the time required for total energy to be reduced to a specified level and lowers energy thresholds
2	[37–40]	Vibroimpact nonlinear energy sink	Small overall amplitude of structural transient response, high vibration impact mitigation capability, and low mass ratio	High-speed impact and large amplitude excitation are not easy to achieve. Finding a solution is difficult and difficult to analyze.	Combination with smooth NES, bringing the energy-free impact mode, and using the Hilbert transform to analysis
3	[42]	Nonsmooth nonlinear energy sink	Remove the undesirable stable higher branch. The effective excitation range level is increased. Strong robustness	The disappearance of strongly modulated responses. The amplitude of the main structure increases under moderate excitation	Improved elastic elements to achieve a nonsmooth NES with ideal falling stiffness
4	[18, 43, 44]	Negative stiffness nonlinear energy sink	Low sensitivity to changes in damping and stiffness, high energy utilization rate, and maintaining high performance at excessive input energy	The structural design is more complex. Negative stiffness realization is not ideal	Maintain sufficient performance and robustness under various external incentives

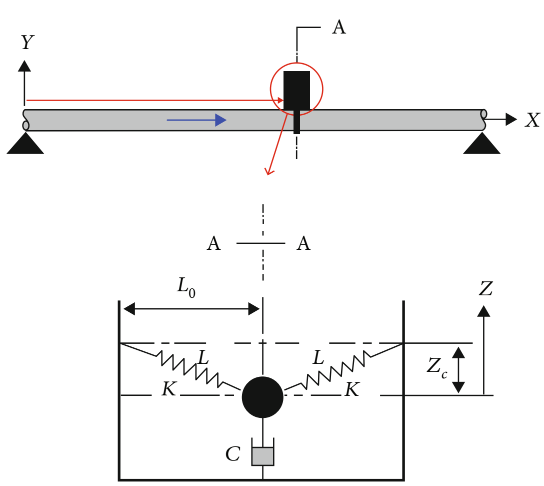


Figure 8: Configuration schematic diagram of nongrounded NES [25].

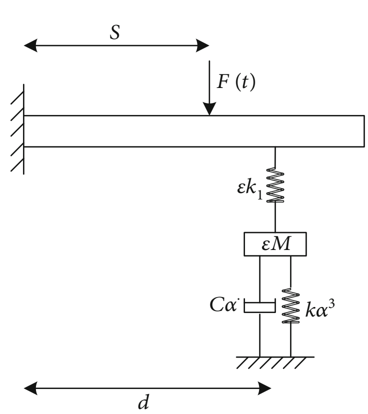


Figure 9: Configuration schematic diagram of grounded NES [56].

analyze the mechanism of TET, such as the optimal energy value of excited nonlinear normal mode [62, 63]. On the one hand, the nonlinear coefficient did not affect the energy exchange between the main system and NES [64]. Nonlinear stiffness elements could enhance energy-pumping performance [32]. On the other hand, TET could be implemented simultaneously in two linear oscillators [65]. Although the current research on TET has limitations, its wide applicability is undeniable. TET improves the stability, mechanical accuracy, and service life of the primary structure. In addition, TET can improve the vibration energy conversion efficiency of the primary structure, making it possible for practical applications of long-term autonomous energy replenishment.

Compared with the linear system, the harvesting power of the nonlinear energy harvesting system under impulse excitation was significantly improved [66]. Based on the model in Figure 10, a NES adjustment method and modeling selection for uncertain and deterministic situations were proposed in a pioneering way [67, 68]. The optimal parameter solution of the NES is obtained at the intersection of the folded singularity and the detached resonance curve (i.e., the red point in Figure 10(b)). In addition, observation of Figure 10(b) reveals that the upper part of region II at $\delta = -0.15$ is included in the region II corresponding to $\delta = 0.15$. This means that the adjustment of the NES should be determined by the uncertainty $\delta = \delta_{min}$ when the natural frequency of the primary system is uncertain.

High-frequency vibration energy could be acquired by utilizing dynamic instability caused by strong nonlinearity under impulse excitation [69]. Compared with traditional linear vibration absorbers with the same mass, stiffness, and damping, the system composed of a ring vibration isolator and NES had better energy absorption effects under harmonic excitation [70]. By comparing the difference in vibration damping effect of NES on a nonlinear plate and linear plate, the performance and dynamic response of the

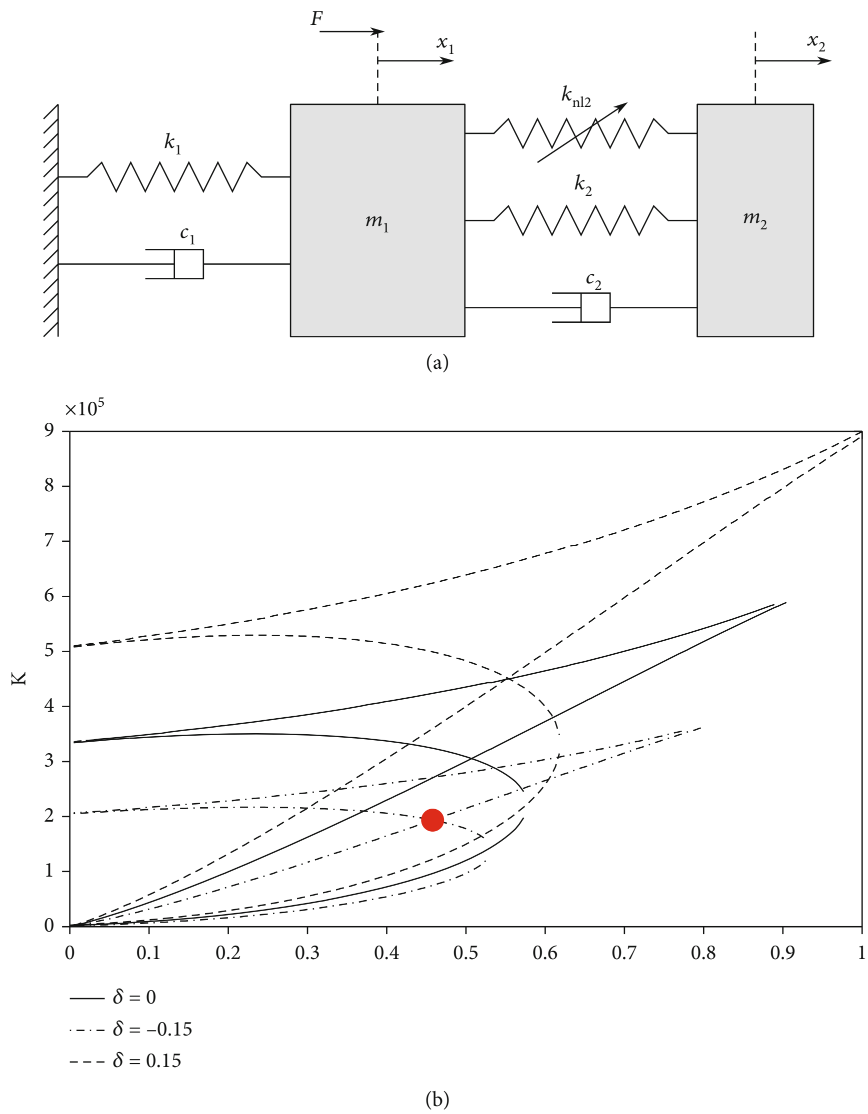


Figure 10: (a) Linear oscillator coupled to a nonlinear vibration absorber. (b) Sizing zones of the NES for various uncertainties.

two combinations under high excitation amplitude were obtained [71]. In addition, by analyzing the vibration behavior and excitation amplitude changes of nonlinear simply supported beams coupled to NES, it was proved that the TET phenomenon could suppress the transient vibration of different mechanical systems [72]. In general, under different excitation conditions, NES all showed satisfactory performance. By analyzing the properties of NES, we explain how to select appropriate system parameters to enhance the TET phenomenon. Energy thresholds, material selection, and stiffness can be considered when designing NES to improve vibration suppression and energy harvesting performance.

2.3. Vibration Control Based on NES. The NES was early used for vibration control of various mechanical devices to achieve finer manipulation and protect the stability of the main structure. Although the introduction of nonlinearity can effectively suppress the vibration of the system, it may also make the dynamic response of the system more complicated. It is important to analyze how the NES functions in a variety of structures in order to optimize the NES design. The ratio between the energy located in the NES and the total energy present in the system gives a good indication of how the energy is distributed in the system. Also, the energy ratio is a good indicator for evaluating the vibration reduction performance of the system, which is given by the following equation [72]:

$$\frac{E_{\mathrm{NES}}}{E_{\mathrm{TOT}}} = \frac{T_{\mathrm{NES}} + U_{\mathrm{NES}}}{T_{\mathrm{LO}} + U_{\mathrm{LO}} + T_{\mathrm{NES}} + U_{\mathrm{NES}}}, \quad (3)$$

where T_{NES} denotes the kinetic energy in the NES, T_{LO} denotes the kinetic energy in the linear oscillator, U_{NES} denotes the potential energy in the NES, and U_{LO} denotes the potential energy in the linear oscillator.

Vortex-induced vibrations (VIV) can lead to fatigue failure of high-rise structures. Specific mass ratio and appropriate NES parameters could effectively inhibit VIV and reduce transverse and longitudinal amplitudes [73–75]. The RNES could passively use the kinetic energy in the cylinder to suppress the VIV and reduce the drag [76]. Compared to other energy harvesting strategies to be used in VIV, the use of RNES-energy harvester as an additional device can further improve the maximum average power [77]. Notably, NES can reduce drag in bluff body flow. Geometric nonlinear damping was helpful to improve the energy dissipation efficiency of NES under seismic excitation [78]. The NES system model with negative stiffness and sliding friction is shown in Figure 11. This model cleverly utilized the negative stiffness between equilibrium points and the friction sliding of NES mass to achieve high absorption and dissipation of seismic energy [79, 80].

By analyzing the dynamic behavior of NES and simply supported beam coupling system, the necessary conditions for Hopf bifurcation, saddle-node bifurcation, and strongly modulated responses were obtained [81]. Interestingly, when the nonlinear periodic response of an axially moving beam was studied, it was found that the performance of the nonlinear attachment was positively correlated with the linear stiffness and motion velocity [82].

At present, rotary machinery has put forward higher requirements for vibration suppression devices. In the actual test, the vibration suppression rate of the NES with piecewise linear stiffness reaches 88% [83, 84]. Although a grounded NES was more practical in practical engineering, the actual vibration suppression rate of a grounded NES in suppressing transient and steady-state vibration of the rotor system only reached 68% [57]. In particular, since the grounded NES did not rotate with the rotor system when operating, its unbalanced state had no effect on the rotor subsystem. Bistable NES with buckling beam could withstand higher input energy and suppress vibration, filling the research gap of bistable NES applied to rotor systems [85]. The tristable NES could suppress vibration below 1 mm, which helped to further improve the stability of rotating machinery [86]. In a rotor system with multiple smooth NES, the damping parameter had the most obvious influence on NES performance [19]. The transmission was an effective indicator to evaluate the performance of NES, and it provided reliable data for the parameter optimization of NES [87]. It should be noted that more parametric optimal solutions could be found by using sensitivity analysis and particle swarm optimization [88]. The two sides of the asymmetric NES equilibrium point had strong nonlinearity and weak linearity, respectively, allowing the designer to achieve a simpler design, higher efficiency, and a more compact device [89]. In summary, the NES improvement scheme with grounded, bistable, and tristable structures can effectively improve the performance and robustness of the NES. In recent years, NES can also be used for energy harvesting due to the properties of wideband energy transmission [90, 91].

2.4. Energy Harvesting Based on NES. It is feasible to harvest vibration energy using NESs and energy harvesters. In simple terms, the external vibration causes the mass to oscillate to produce mechanical energy, which is then converted into electrical energy by an energy converter. The system model with an electromagnetic energy harvester is shown in Figure 12. The relative motion between the magnet and the coil in the middle caused the magnetic flux of the coil to change and generate the induced current. The coil was connected to an external device to collect electrical energy.

An energy harvesting device consisting of a fixed ring magnet and a fixed magnet, the device could obtain linear, hardening nonlinear, or softening nonlinear response by adjusting the outer diameter of the fixed ring magnet [93]. Nonlinearity may lead to additional response, which would affect the energy absorption efficiency [94]. Higher output voltage and wider frequency bandwidth could be obtained when the multistable energy harvester chose appropriate high-order nonlinear coefficients [95]. NES was used for the first time to eliminate the unfavorable periodic response in the two-degree-of-freedom vibration system while retaining the strongly modulated response, as shown in Figure 13 [96].

In the face of complex and variable environmental vibration, a device model capable of adjusting nonlinear forces is shown in Figure 14 [97, 98]. When the mass block vibrated up and down, the bearing could roll on the slide. The device obtained nonlinear force suitable for the system by designing personalized slide.

The potential energy of the system could be expressed as follows:

$$U = \frac{K(x_1 - x)^2}{2} - \frac{Kx_1^2}{2} + \frac{K_h(x_0 + S(x))^2}{2} - \frac{K_h x_0^2}{2} + Mgx. \tag{4}$$

In practical applications, continuous energy capture is crucial. Previous energy harvesters mainly considered excitation in a single direction, but the new three-dimensional energy harvesters could adapt to complex environment [99]. In addition, structural failure could be avoided by guiding the energy harvesters to a periodic orbit with a stable vibration amplitude [100]. It was worth noting that energy capture was not only limited to 1 : 1 resonance but also could improve the output power and bandwidth at 3 : 1 internal resonance [101]. In the face of complex external excitations in the environment, designers need to consider the multidirection excitation and vibration reduction protection of the main structure.

2.5. NES Performance Balance in Vibration Suppression and Energy Harvesting. If the device can realize vibration suppression and energy harvesting at the same time, it will be helpful to enhance the practical application value of the device. Energy harvesting and vibration suppression based on TET were not completely opposed, as in the case of nonlinear cascaded systems [102]. In addition, "variant NES" improved the energy transfer and harvesting of vibration by taking advantage of the low damping of fixed beams [103, 104]. It was beneficial to analyze the TET and energy harvesting efficiency of different systems, which provided a valuable reference for future structural optimization [105]. The vibration reduction and energy harvesting performance

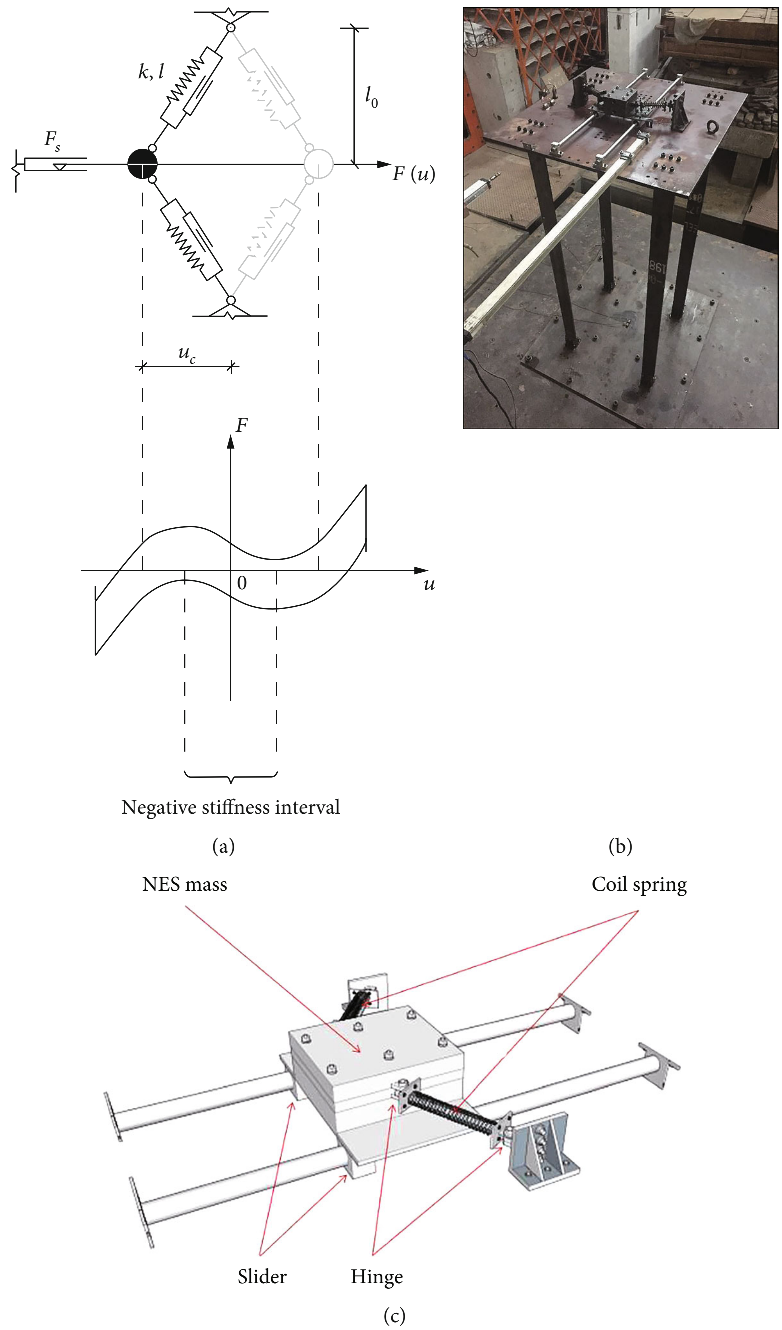


Figure 11: NES with negative stiffness and sliding friction: (a) conceptual layout, (b) the test structure installed on the shake table, and (c) schematic diagram [80].

of the NES-based energy harvester can be evaluated by the following equation:

$$\bar{P}_{\mathrm{el}} = \frac{1}{t_0}\int_0^{t_0} P_{el}(t), \quad \bar{P}_{\mathrm{vis}} = \frac{1}{t_0}\int_0^{t_0} P_{\mathrm{vis}}(t), \tag{5}$$

where $\bar{P}_{\mathrm{el}}$ denotes the average electrical power and $\bar{P}_{\mathrm{vis}}$ denotes the average viscous power. The balance of NES performance in terms of vibration suppression and energy harvesting can be achieved by tuning the values of $\bar{P}_{\mathrm{el}}$ and $\bar{P}_{\mathrm{vis}}$.

By analyzing the multiscale behavior of two different types of two-degree-of-freedom systems, the relation between parameters and performance of the NES system was expounded [106–108]. The multiscale method was used to solve the equations of nonlinear energy harvesting systems [109, 110]. Geometric nonlinearity and material NESs were integrated with the piezoelectric vibration energy harvester to realize vibration control and energy harvesting of single-degree-of-freedom structures [111]. By using the Runge-

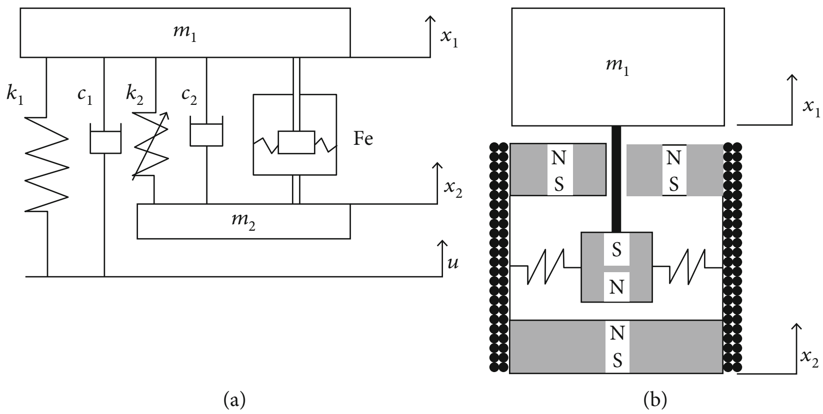


Figure 12: (a) System model of a coupled system of NES and dual-purpose isolator connected to a linear primary system. (b) Schematic diagram of the dual-purpose vibration isolator [92].

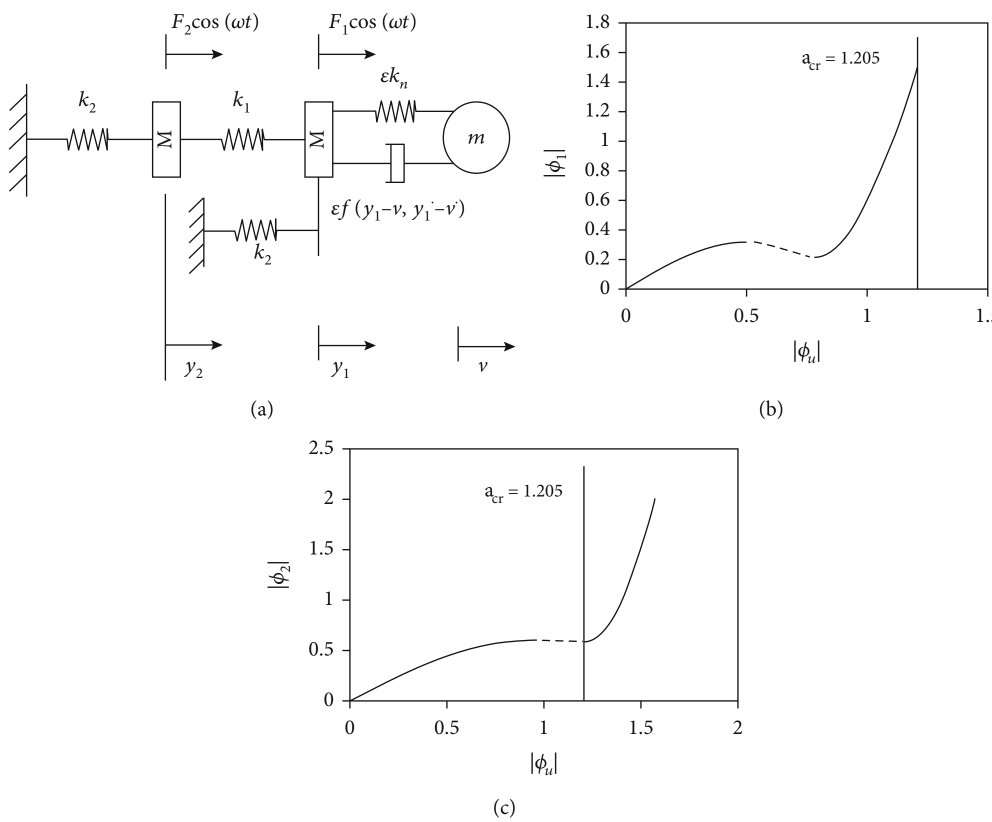


Figure 13: A harmonically excited three-degree-of-freedom system consisting of two linear coupled oscillators and a NES with nonlinear damping characteristics. (a) System mechanical model. (b) Slow invariant manifold projections close to the first excited mode. (c) Slow invariant manifold projections close to the second excited mode.

Kutta method and harmonic balance method, the displacement response of the system could be obtained, and the relationship between the energy harvesting effect of the system and time was analyzed [112, 113]. In Table 2, the methods of analyzing nonlinear equations are summarized.

3. NES-Based Energy Harvesters

Ambient energy usually takes the form of vibration energy, heat energy, and solar energy. Because vibration energy can exist continuously in engineering structures and nature, vibration energy is the main source of energy harvesting. At present, the nonlinear vibration energy harvester mainly includes electromagnetic, piezoelectric, and electrostatic, as shown in Table 3 [126–128].

U is energy density, μ_0 is magnetic permeability, B is the magnetic induction, σ_y is the yield stress, k is the electromechanical coupling coefficient, Y is Young's modulus, ε_0 is vacuum permittivity, E is electric field strength, and α is calculating the ratio between the energy located in the NES E_{NES} and the total energy E_{TOT} present in the system.

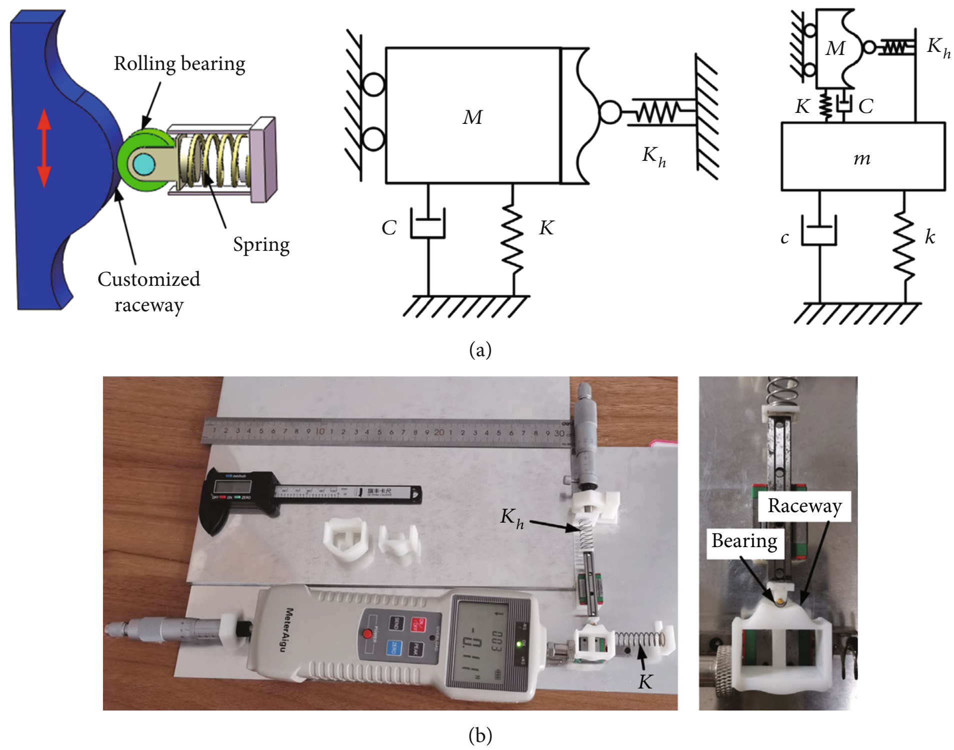


Figure 14: (a) Schematic diagram of the device and its application in vibration energy harvesting, vibration isolation, and NESs. (b) Overall view and partially enlarged view of the experimental device for nonlinear force measurement.

Table 2: The analysis methods for solving NES nonlinear equations.

Number	Ref.	Method name	Advantages or disadvantages
1	[114–116]	Harmonic balance	Clear concept, easy to use, and suitable for a weak nonlinear system and strong nonlinear system. In the case of strong nonlinearity, increasing the order can present the system response more accurately
2	[117–119]	Runge-Kutta	High precision, convergence, stability (under certain conditions), and a large amount of calculation
3	[120, 121]	Complexification-averaging	The solution is fast but can only be solved once. The high-order response of the system cannot be accurately presented in the case of strong nonlinearity
4	[49, 101, 110, 122]	Multiple scale	In theory, it can be solved with any precision, but it is complicated to solve
5	[98, 123, 124]	Perturbation	Solving general physical problems with small parameters is simple and fast with general accuracy. The result depends heavily on the selection of small parameters
6	[78, 124, 125]	Newton-Raphson	The second-order convergence rate is fast. The solution is accurate. The calculation is complicated. Not applicable to high-dimensional data. Errors or an endless loop may occur

3.1. Electromagnetic Energy Harvester. Electromagnetic energy harvester [129] uses the relative motion of magnet and coil to generate electromotive force, so as to obtain electric energy. External magnetic forces could provide a restoring force between the NES and the main system and offset the linear terms of the elastic forces to achieve nonlinear characteristics [126, 130, 131]. When giant magnetostrictive material was applied to NES, the output power of the system would be higher at a specific frequency [132]. Compared to the systems with the NES-only systems, the systems with additional NES combined with a giant magnetostrictive energy harvester are more effective in suppressing vibration and generating electricity [133]. As can be seen from Figure 15, most of the input energy was absorbed by the NES-electromagnetic energy acquisition coupling system.

The electromagnetic damping c_e and the average electric power p of the NES-electromagnetic energy harvester were, respectively, expressed as follows [134, 135]:

$$c_e = \frac{k_t k_e}{R_H},$$
$$p = \frac{2k_t k_e \pi^2 f^2}{R_H}(A_1 - A_2)^2 = 2c_e \pi^2 f^2 (A_1 - A_2)^2. \quad (6)$$

Table 3: Comparison of the three types of nonlinear vibration energy harvester.

	Electromagnetic energy harvester	Piezoelectric energy harvester	Electrostatic energy harvester
Energy density equation	$U = \frac{\mu_0 B^2}{2}$	$U = \frac{\sigma_y{}^2 k^2}{2Y}$	$U = \frac{\varepsilon_0 E^2}{2}$
Energy storage density (mJ/cm^3)	24.8	35.4	4
Output current	High	Low	Low
Output voltage	Typically <1 V	5 V or more	100 V or more
Integration into microsystems	Difficult	Difficult	Easy
Voltage source	Not need	Not need	Need
Output impedance	Relatively low	High	High
Voltage rectification	Difficult	Easy	Easy

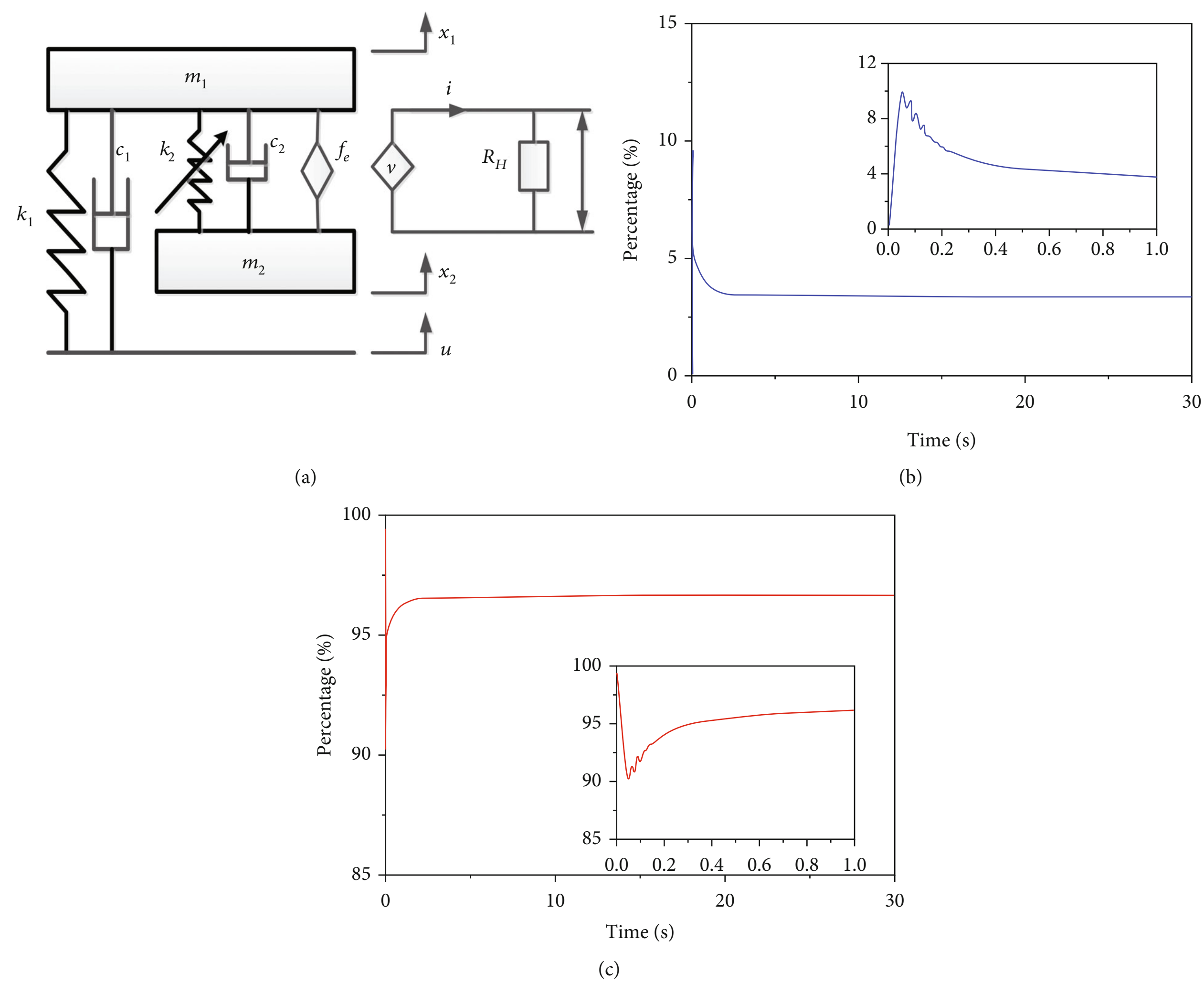


Figure 15: (a) Mechanical model of a coupled system of NES-electromagnetic energy harvesting connected to a linear primary system and electrical schematic diagram of electromagnetic energy harvesting. Energy absorption rate: (b) primary system, (c) NES-electromagnetic energy acquisition coupling system [127].

Under random excitation, compared with the harvester with horizontal and vertical springs, the bistable energy harvester with only horizontal springs had better energy harvesting performance [136]. Lever-type NES was used to collect energy for the vibration of single-degree-of-freedom structures, with the properties of aperiodic and periodic motion [137]. The parameter values of the nonlinear vibration energy harvester had a significant influence on its dynamic behavior [138]. With appropriate parameter design, VINES based on electromagnetic energy conversion could stably collect more than 80% vibration energy, as shown in Figure 16 [47]. Compared to traditional NES energy harvesting devices, VINES-based energy harvesters could be used for instantaneous high-density energy harvesting [139, 140].

In the electromagnetic bistable energy harvesting system, the bistable element could improve the energy harvesting benefit of low-amplitude excitation [141, 142]. A variable-potential NES-coupled electromagnetic harvester reduces the potential barrier height and improves the energy harvesting performance effectively after stiffness optimization [143]. A coupled piezoelectric-electromagnetic structure fully utilizes the positive stiffness and hard nonlinearity of the two elements, which can efficiently convert vibration energy into electrical energy in the range of 5-13 Hz and suppress vibration above 13 Hz [144]. Introducing nonlinear elements into the primary structure could improve the frequency bandwidth and response amplitude, thus improving the energy harvesting efficiency [145, 146]. However, energy harvesting should not be limited to a single excitation direction. The hula-hooping-like nonlinear structure could collect vibration energy from all directions with wideband response [147, 148]. In general, the electromagnetic energy harvester does not require a power supply, and the output current is relatively high. However, the disadvantages are low output power, complex manufacturing process, high design requirements, and difficulty in integrating into the microsystem.

3.2. Piezoelectric Energy Harvester. Compared with electromagnetic and electrostatic transducers, piezoelectric transducers are widely used in the field of energy harvesting. However, the energy harvesting efficiency of traditional piezoelectric energy harvesters is not ideal in practical application, and the NES-based piezoelectric energy harvester can make up for this shortcoming. In general, compared with traditional piezoelectric energy harvesters, NES-based energy harvesters can effectively improve the efficiency of energy harvesting.

Under simple harmonic excitation and random excitation, the piezoelectric energy harvester could control the vibration of the primary structure and convert the vibration energy into electric energy [149, 150]. Under harmonic excitation, the optimal resistance of the NES-piezoelectric vibration energy harvester could be determined by the harmonic balance method [151, 152]. The aerodynamic loads were modeled using the quasisteady approximation approach, through which the relation between the load resistance and the output power, jump frequency, and bandwidth could be obtained [153]. When considering weak electromechanical coupling, different load resistances could affect the performance of the energy harvester-NES system. Under shock and harmonic excitation, a NES-piezoelectric energy harvester coupled to a pure resistive load circuit obtained higher power [154, 155]. In the mechanical-piezoelectric system, the quasiperiodic responses near the resonant frequency helped to improve the bandwidth of energy harvesting and TET [156]. Furthermore, the electromechanical coupling system based on synchronous charge extraction eliminated the impedance problem of the system's interface circuit so that the energy captured during the system's transient response was not impacted by the load resistance [157, 158]. A novel nonlinear piezoelectric energy harvesting system improved the efficiency of energy harvesting, as shown in Figure 17.

The new pendulum energy harvester under impulse excitation overcame the defects of multidegree of freedom design difficulties [18], complexity, and confusion [160, 161]. Under impulse excitation, hybrid vibroimpact NES (HVI-NES) could display two behavior modes, VINES and TMD, for input energy of different sizes. Its device is shown in Figure 18 [162].

The energy H_R of the primary structure and the energy H_w of the HVI-NES were as follows:

$$\begin{aligned} H_R &= \frac{1}{2}\mu_R\left(\dot{R}^2 + R^2\right), \\ H_w &= \frac{1}{2}\varepsilon\mu_w\left(\dot{\omega}^2 + \beta^2\omega^2\right) + \varepsilon w R. \end{aligned} \tag{7}$$

The ratio of the HVI-NES energy to the total energy was as follows:

$$\rho = \frac{H_\omega}{E_0} = \frac{\varepsilon\mu_w\left(\dot{\omega}^2 + \beta^2\omega^2\right) + 2\varepsilon w R}{2E_0}. \tag{8}$$

Although the HVI-NES lacked nonlinear springs and complex dynamics, the device was capable of operating over a wide frequency bandwidth. The system model with VINES is shown in Figure 19. In the device, chaotic strongly modulated response showed new characteristics, even a small external force could cause a large displacement in the main system [163–166]. Compared with traditional NES, the vibroimpact cubic NES could obtain input energy with a longer bandwidth, significantly improving energy conversion efficiency [167]. In addition, the dynamic equation of the VINES system was obtained based on a semianalytical map-based method, and bifurcation analysis was developed [168]. Obviously, the energy harvester with the combination of multiple structures could effectively improve the efficiency of energy harvesting, and it was a research direction worth exploring.

In both discrete and continuous systems, lever-type NES demonstrated excellent vibration control capability [169, 170]. Compared with a NES-giant magnetostrictive-piezoelectric, a lever-type NES coupled to a giant magnetostrictive-piezoelectric with less added mass showed better performance in both vibration control and energy harvesting [171]. Piezoelectric energy harvesters fabricated from bidirectional functional graded materials can cope with harsh external environments and enhance energy harvesting performance [172]. In a two-degree-of-freedom airfoil containing NES and piezoelectric elements, aeroelastic instability could be well inhibited by increasing the NES mass [173]. In a three-degree-of-freedom airfoil system, the type of response of the system depends on the design parameters of the NES. Also, the NES mass needs to be controlled at 10-15% of the airfoil mass [174]. The NES-giant magnetostrictive-piezoelectric also performs

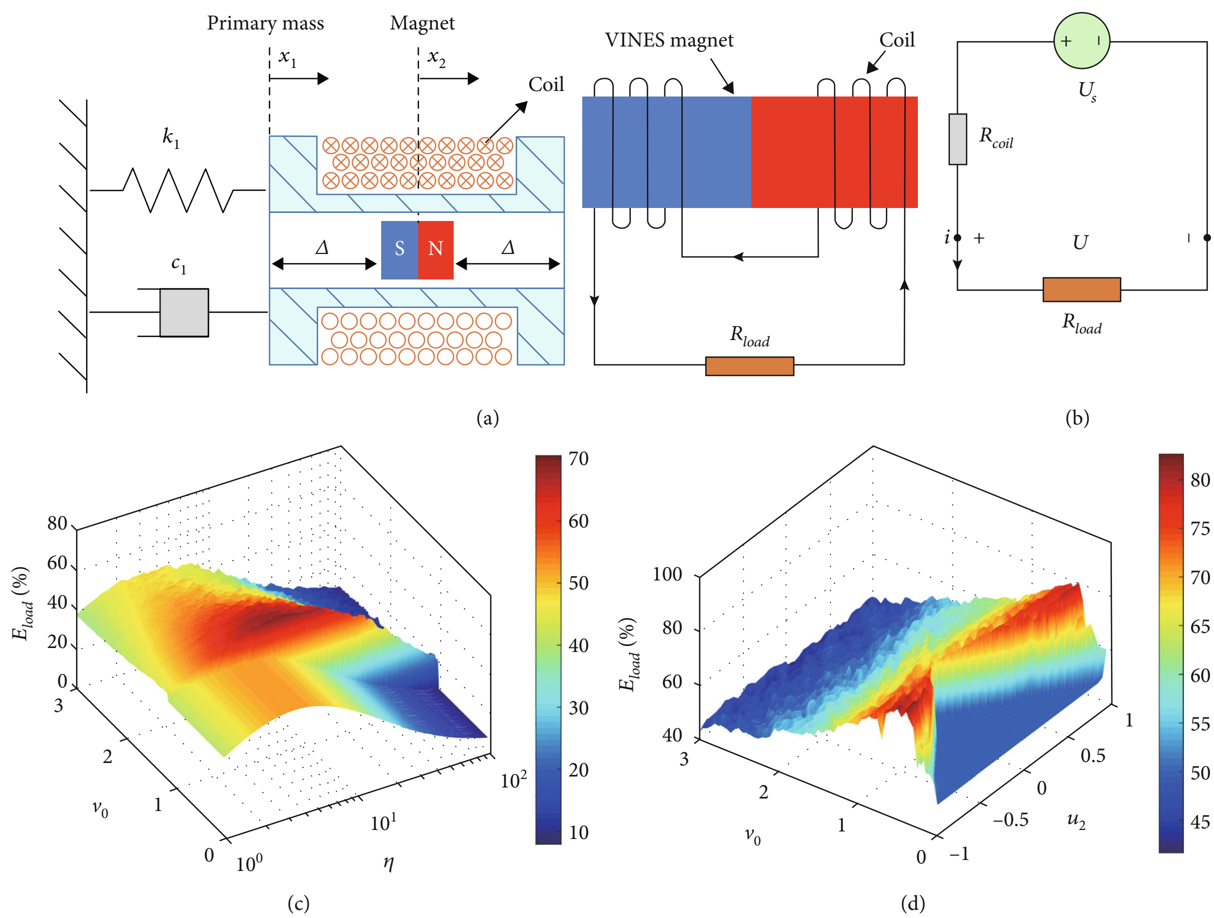


Figure 16: (a) Schematic diagram of a VINES energy harvester. (b) Schematic diagram of the harvesting circuit. Percentage of the harvested energy, E_{load}, from the load resistance as a function of (c) the initial velocity v_0 and resistance ratio η and (d) the initial velocity v_0 and the initial position v_2 of the VINES.

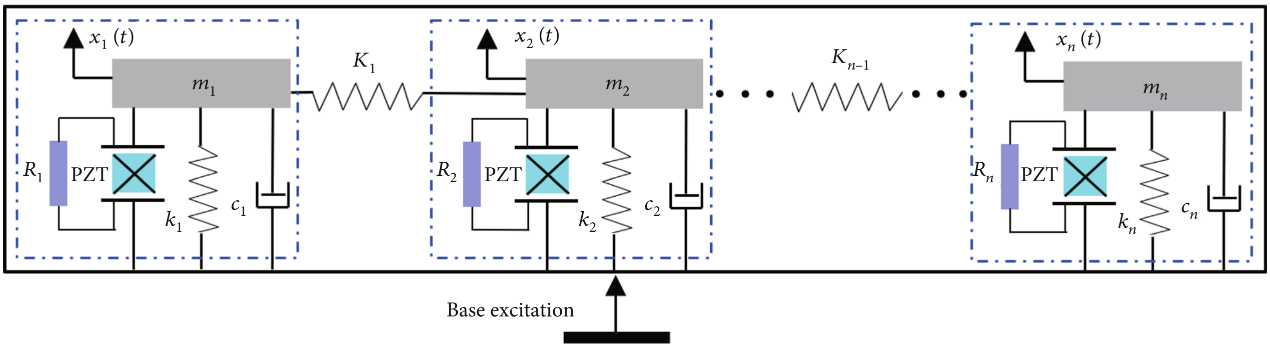
well in suppressing nonlinear aeroelastic response and harvesting energy [175]. NES-giant magnetostrictive-piezoelectric has also been applied to the subsonic aeroelastic system, where the energy harvesting performance of the system is affected by periodic motion and instability [176]. The proposed piezoelectric energy harvester consists of a partially covered piezoelectric cantilever beam, as shown in Figure 20; the electromechanical coupling distribution parameter model of the device was as follows [177]:

$$\begin{aligned} EI(x)\frac{\partial^4 u_{\mathrm{rel}}(x,t)}{\partial x^4} &+ c_s I(x)\frac{\partial^5 u_{\mathrm{rel}}(x,t)}{\partial x^4 \partial t} \\ &+ c_a \frac{\partial u_{\mathrm{rel}}(x,t)}{\partial t} + m(x)\frac{\partial^2 u_{\mathrm{rel}}(x,t)}{\partial t^2} \\ &+ \left(\frac{d\delta(x)}{dx} - \frac{d\delta(x-l_P)}{dx}\right)\vartheta_P\left(L\dot{I}(t) + RI(t)\right) \\ = \Big[\mu_{u_{\mathrm{rel}}}(l,t) &+ \lambda u_{\mathrm{rel}}(l,t)^3\Big]\delta(x-l) \\ &+ [m(x) + M_t\delta(x-l)]a_b \cos\left(w_{b^t}\right). \end{aligned} \tag{9}$$

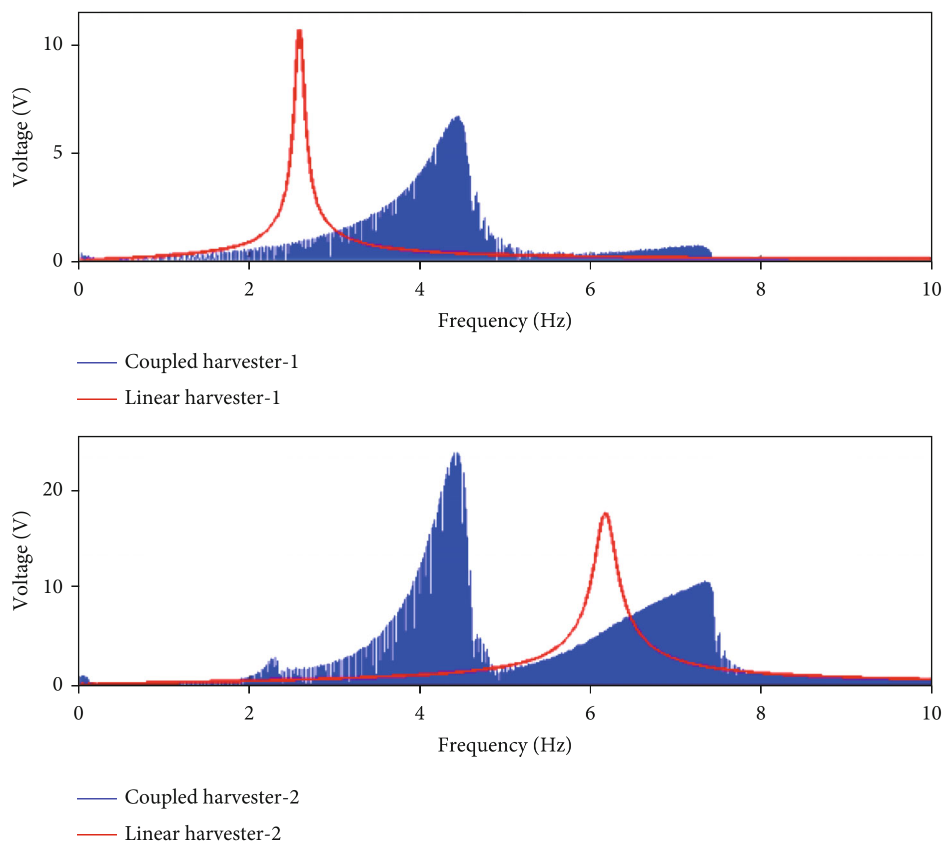
In general, when the cubic magnetic coefficient and inductance parameters of the energy harvester were appropriate, the bandwidth of the energy harvester could reach 40 Hz in the low-frequency range.

The piezoelectric energy harvester with the snap-through buckling structure could collect energy within the range of 0.5-5 Hz, which was helpful in realizing the autonomous energy supplement of wireless sensors [178]. Piezoelectric autoparametric vibration energy harvester achieved chaos control and significantly increased the maximum average power [100, 179]. Introducing a time-delay mechanism into a piezoelectric network and a primary structure could improve the efficiency of energy harvesting [120, 180, 181]. Moreover, the bistable piezoelectric absorber can obtain high-power density at low acceleration, and the structure is simple and stable [182]. With limited amplitude and NES nonlinear stiffness, the piezoelectric nonlinear energy harvester could dissipate and harvest more energy [117, 183]. The piezoelectric energy harvester also does not require a power supply and has the advantages of high output voltage, high efficiency, simple structure, small size, and precise mechanical control. Piezoelectric energy harvesters have low output currents and are not easy to integrate into the microsystem.

3.3. Electrostatic Energy Harvester. Piezoelectric and electromagnetic energy harvesters had defects, which could be



(a)



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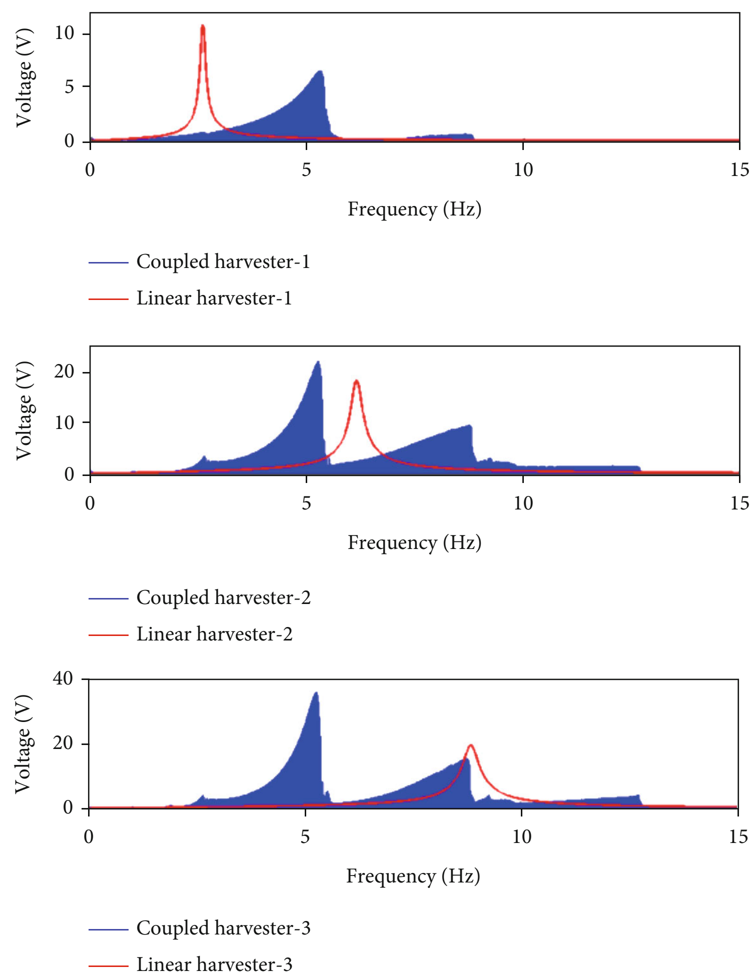
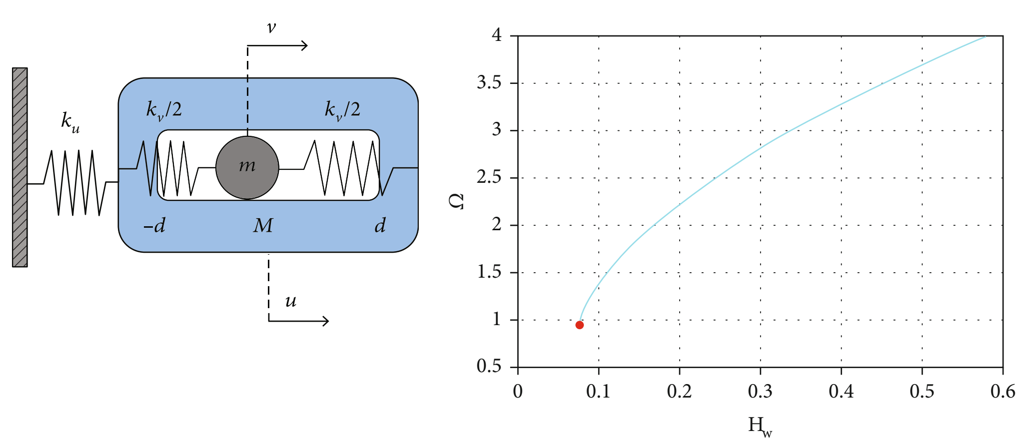


(c)

FIGURE 17: Nonlinear piezoelectric energy harvesting system. (a) System mechanical model. (b) Output voltage with two energy harvesters. (c) Output voltage with three energy harvesters [159].



(a) (b)

FIGURE 18: (a) Sketch of the dynamical system: primary linear oscillator with the HVI-NES. (b) The frequency-energy curve of HVI-NES for 1 : 1 resonance regime.

overcome by electrostatic energy harvesters [128, 184, 185]. An electrostatic vibration energy harvester generates electricity by changing the capacitance. An initial voltage or capacitance is applied to the capacitor before the vibration energy harvesting structure begins to output electricity. A common electrostatic energy harvester is a capacitive

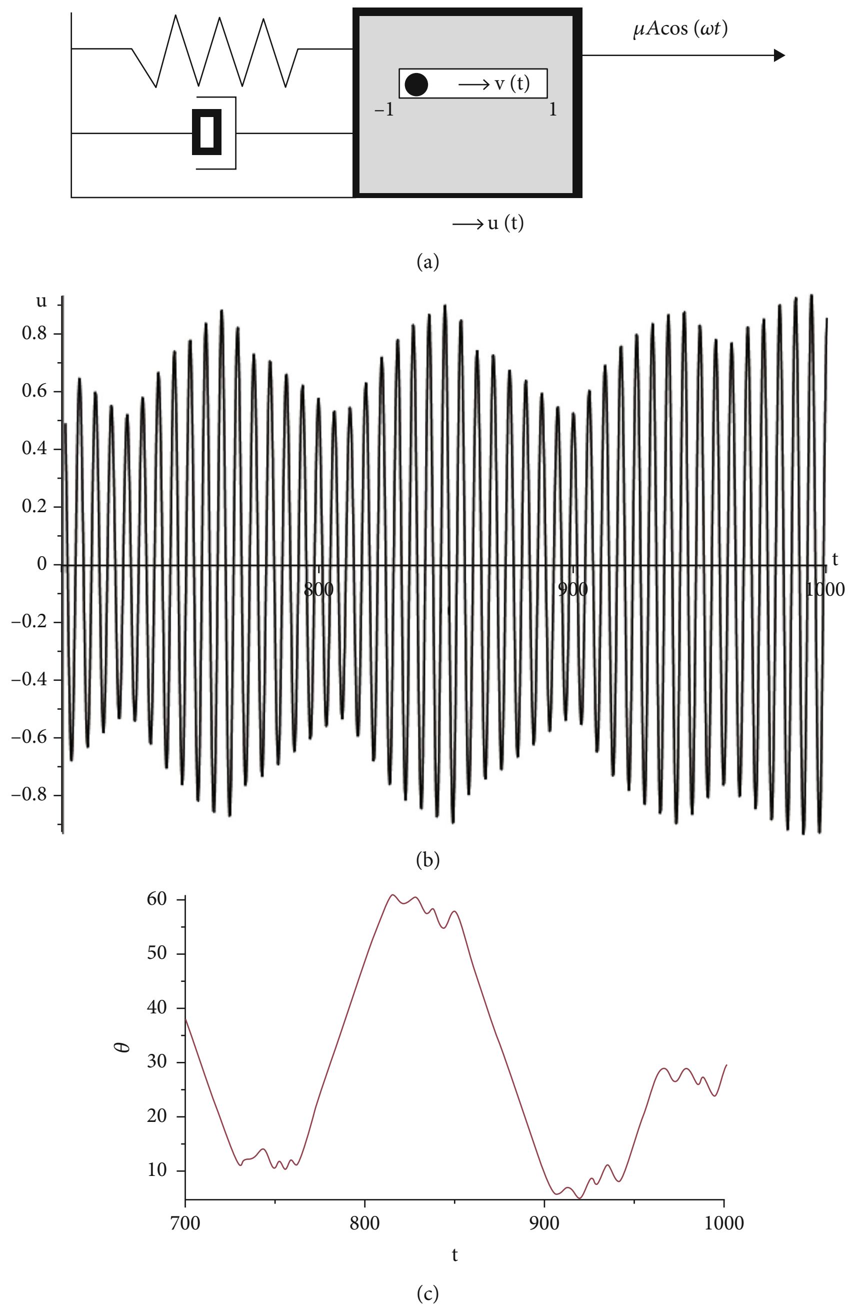
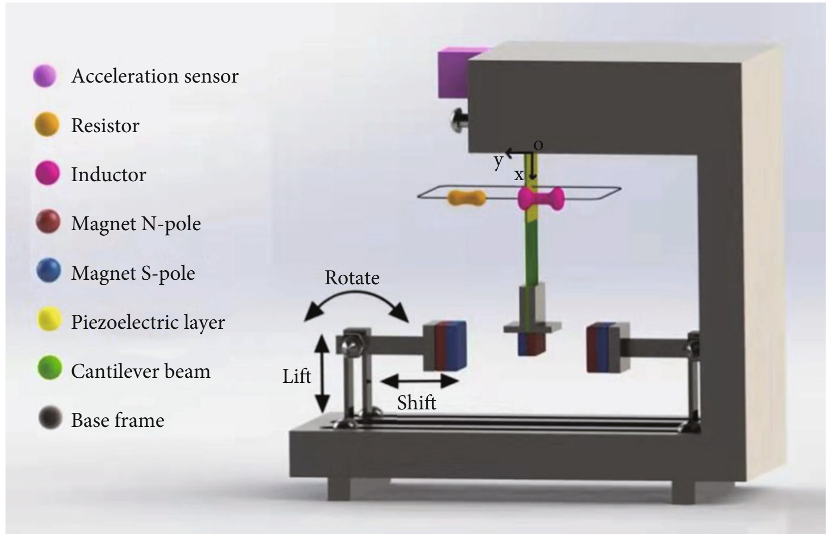
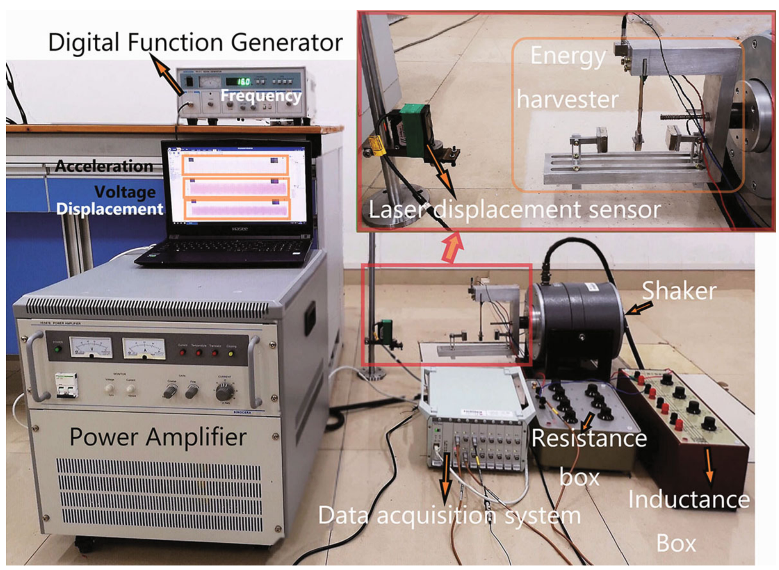


Figure 19: The model comprises a primary forced-damped linear oscillator with the VINES. (a) Sketch of the dynamical system. (b) Displacement of the main oscillator. (c) Angle of the NES rotation [166].

structure consisting of two conductive plates separated by an electric medium. When two plates move relative to each other, the resulting capacitance change produces electrical energy. In practice, when the amount of charge stored in the capacitor changes due to the vibration of the external environment, charge flow is formed in the loop to provide electric energy for the storage device. Compared with other vibration energy harvesters, electrostatic vibration energy harvesters had strong compatibility with integrated circuits, which made it easy to supply power to wireless sensors [186, 187]. The soft spring effect caused by electrostatic force could improve the output power and broaden the working bandwidth [188]. A novel vibration energy harvesting device used dielectric elastomers to convert vibration energy into electrostatic energy and could reduce the volume of the generator while harvesting a large amount of energy [189]. Obviously, each of the three energy conversion mechanisms has its own advantages. The electrostatic energy harvester can also accurately control the mechanical resonance, and more importantly, it can make up for the defects of electromagnetic and piezoelectric energy harvesters which are not easy to be integrated into the microsystem and low voltage. However, electrostatic energy harvesters require additional voltage and higher vibration frequencies.

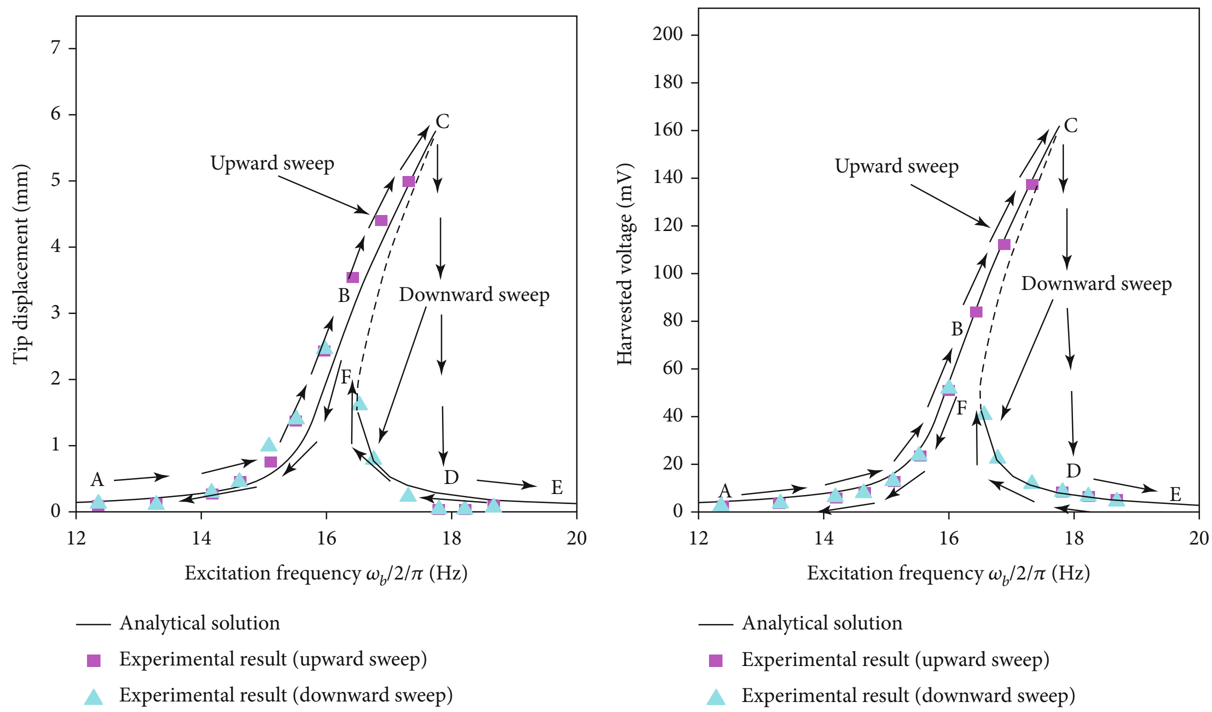


(a)



(b)

Figure 20: Continued.



(c) (d)

Figure 20: Piezoelectric energy harvester consists of a partially covered piezoelectric cantilever beam. (a) System schematic diagram. (b) The experimental setup. (c) The amplitude of the tip displacement varies with excitation frequency. (d) The amplitude of the harvested voltage varies with excitation frequency [177].

Table 4: Research results on the application of energy harvesting in several fields.

Ref.	Field	Year	Research results
[189]	Aerospace	2023	The energy transfer mechanism of plates with nonlinear energy sink and giant magnetostrictive-piezoelectric energy harvester is revealed. The nonlinear energy sink should be mounted in the annular region at the edge of the plate.
[190]	Aerospace	2023	Reduced weight and simplified structure of the energy harvester. High-power density and normalized power density achieved at low amplitude excitation
[191]	Aerospace	2023	Suppresses the vibration of the airfoil by absorption and vibration energy harvesting and supplies power to the monitoring equipment
[192]	Infrastructure	2023	Mechanism of vibration suppression and energy harvesting in pipelines for transporting fluids with a nonlinear energy sink-piezoelectric energy harvester attached revealed
[193]	Infrastructure	2021	The utilization of piezoelectric energy harvesting devices in road traffic can generate energy densities of up to 15.37 J
[194]	wireless sensor	2023	At a base acceleration of 0.8 g, the energy harvester produced a peak power of around 1.38 mW
[195]	wireless sensor	2021	High energy harvesting efficiency under pulsed excitation. Potential to enhance the self-powered capability of wireless sensor

4. NES Energy Harvesting in Various Fields

Partial research results of energy harvesting technology applied in various fields in recent years are shown in Table 4.

4.1. Aerospace. In the aerospace field, rockets and combat aircraft require higher vibration control performance. The rocket is subjected to the pulse force of the engine when launching and the oil pipeline pulsating pressure when the fighter jet is flying. This kind of excitation includes the dynamic load excitation of low-, medium-, and high-frequency bandwidth. Therefore, it is necessary and urgent to carry out relevant research on vibration reduction technology and application. Vibration control and energy harvesting of aerospace structures have become an important step in their design.

The speed range without shimmy has been extended by applying NES to the landing gear, and it is necessary to focus on the speed at which the jumping phenomenon occurs when performing the NES design [196]. Flap-NES can delay

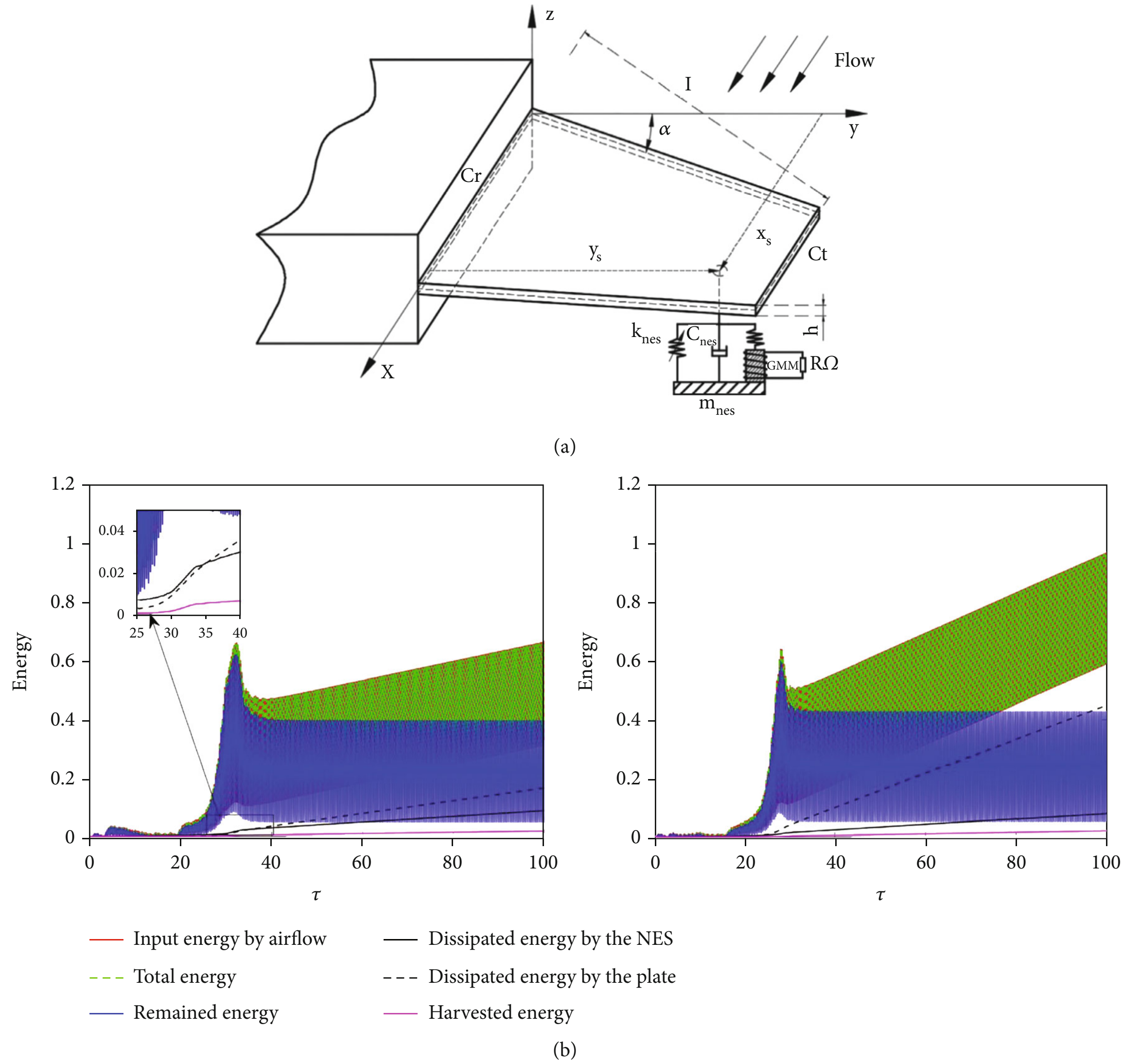


Figure 21: The cantilevered trapezoidal plate structure coupled to a NES-giant magnetostrictive material system. (a) System schematic diagram. (b) Time histories of all the kinds of energy for the system at $\lambda = 324$: the left picture $\zeta = 0.005$ and the right picture $\zeta = 0.025$ [198].

the onset of instability flutter and reduce the amplitude of limit cycle oscillations, thus improving the safety of the aeroelastic system [197]. The design of flap-NES parameters can decrease the computational effort by using a data-driven optimization algorithm. The nonlinear energy harvester with giant magnetostrictive material is shown in Figure 21. This device solved the problem of the aeroelastic response of a cantilever trapezoidal plate in supersonic airflow [114, 198–200]. Lever-type NES-giant magnetostrictive-piezoelectric was used in the whole-spacecraft vibration reduction system and expanded the energy harvesting bandwidth [201, 202]. Within a two-degree-of-freedom airfoil, the installed position of a NES-piezoelectric energy harvester affects the flutter speed and pitch amplitude [191]. It was important to note that the position of the lever-type NES-giant magnetostrictive-piezoelectric fulcrum could be adjusted as needed. The introduction of NES-giant magnetostrictive material has little effect on the natural frequency of the whole spacecraft system [203]. Levitation magnetoelectric energy harvester and NES coupling control system could effectively reduce spacecraft vibration and harvest a large amount of power over a wide frequency range [204]. The bistable piezoelectric composite laminate has the advantages of simple structure and light mass, which can meet the lightweight demand of aerospace equipment [190, 205]. When the bistable piezoelectric composite laminate is utilized in a piezoelectric energy harvester, the device can achieve a higher power density even at lower accelerations [182, 206]. In addition, the large snap-through generated by unsymmetric cross-ply square composite laminated plate when heated could improve the energy harvesting efficiency, and it could also be regarded as a bistable composite shell structure [207]. NES had a good damping effect on the micro-vibration of the flywheel system [121]. In general, energy harvesters can use new materials to improve energy harvesting performance. The acquisition efficiency of the energy harvester is related to excitation frequency and resistance load.

By introducing the NES device into the power system, the system could be used as both an energy harvesting and

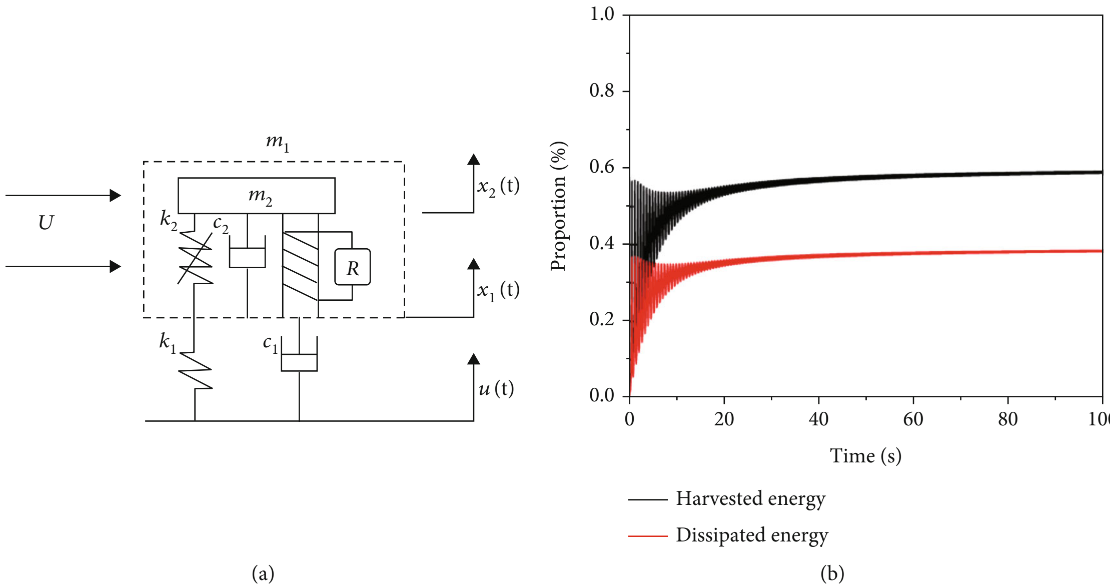


Figure 22: A single-degree-of-freedom structural system consisting of NES and giant magnetostrictive material embedded inside a square blunt body: (a) mechanical model under wind load; (b) percentage of dissipated and harvested energy.

vibration control device [125, 208]. Vibration had an obvious influence on rotating machinery and even led to mechanical failure. In addition, compared with traditional vibration control devices, the rotor-blade-NES system had better vibration suppression performance [83]. The quad-stable piezoelectric energy harvester had the characteristics of a time-varying potential well and low potential barrier so it could effectively capture energy in the low-frequency rotary device [209]. For low-frequency vibration of 5-15 Hz, the energy harvester with bioinspired skeleton structure can effectively convert the mixed vibration into electrical energy [210]. Generally speaking, NES devices can effectively solve the problem of adverse vibration caused by mechanical rotation and extend the service life of the machine.

4.2. Infrastructure. Sound, the form of vibration, also plays an important role in the source of vibration energy. An electroacoustic absorber consisting of a nonlinear shunt and a loudspeaker terminal had TET characteristics in noise reduction propagation and reception paths [211–213]. The nonlinear membrane absorber could be applied in a variety of acoustic systems, which solved the problem that there was no effective low-frequency energy dissipation mechanism in acoustics [214–216]. The lever-type bistable energy harvester could improve the energy harvesting efficiency under high actuation velocity [217].

When small or large structures were subjected to impact loads, wind gusts, and explosions, rotary-impact NES could reduce the vibration amplitude of the structure in a short time, thus protecting the structure from long-term fatigue or damage [53]. It is a good way to improve performance by choosing the right material. Giant magnetostrictive material has features such as high density and fast response time. The integrated system of giant magnetostrictive energy harvesting and nonlinear vibration control under wind load had good vibration reduction and energy harvesting effects, and its device is shown in Figure 22 [118].

The dynamic equation of the integrated system of giant magnetostrictive energy harvesting and nonlinear vibration control was as follows:

$$\begin{aligned} &m_1\ddot{x}_1 + c_1(\dot{x}_1 - \dot{u}) - c_2(\dot{x}_2 - \dot{x}_1) + k_1(x_1 - u) - k_2(x_2 - x_1)^3 \\ &\quad + (\sigma - \sigma_0)\frac{\pi d_{\mathrm{GMM}}^2}{4} + \frac{1}{2}\rho_a D U^2 \sum_{n=1,2,\cdots} a_n\left(\frac{\dot{x}_1 - \dot{u}}{U}\right)^n = 0, \\ &m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1)^3 - (\sigma - \sigma_0)\frac{\pi d_{\mathrm{GMM}}^2}{4} = 0, \\ &\frac{\sigma}{E_m} + \frac{3\lambda_s {H_0}^2}{2(3a - \tilde{\alpha} M_s)^2}\left(1 - e^{\left(\sigma^2(\tilde{\alpha} M_s - 3a)\right)/\left(2E_m\varsigma(3a - \tilde{c} M_s \tilde{\alpha})\right)}\right)^2 \\ &\quad = \frac{x_1 - x_2 + \Delta_0}{l_{\mathrm{GMM}}}. \end{aligned} \tag{10}$$

In the application of building structure engineering, NES could reduce the resonance amplitude of elastic structure by 98% after parameter optimization. In addition, setting a NES at the boundary of the elastic beam could solve the design problems of engineering structure vibration reduction devices [20, 218]. A nonlinear spring-inerter-damper vibration absorber, also known as variant NES, could significantly reduce the nonlinear frequency response peak of the beam [219]. A variant NES device with high nonlinear stiffness and low linear stiffness shares TET and initial energy dependence properties with traditional NES devices [103]. The variant NES device was capable of both vibration suppression and energy harvesting [220].

The system coupled by NES and linear beam had strong robustness to the vibration amplitude change of excitation force after parameter optimization [56, 221]. By using NESs to passively absorb the energy of linear primary structure, a

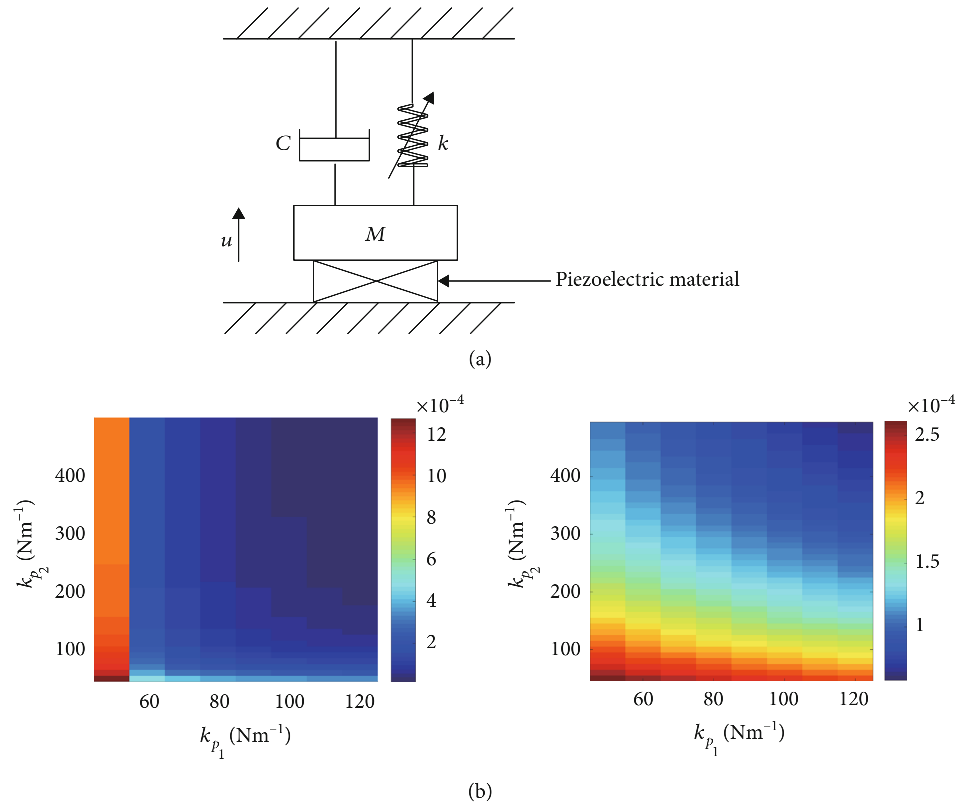


Figure 23: (a) Schematic modeling of a mass-piezoelectric-spring-damper system with a single degree of freedom. (b) Contour projection of harvested power by piezoelectric elements as a function of $k_{p1}k_{p1}$ and k_{p2}: The left figure shows grounding, and the right figure represents ungrounded.

relatively simple and modular vibration isolation foundation could be formed in mechanical systems [222]. Linear systems with nonlinear attachments had the optimal regime of TET and stochasticity robustness for single-mode nonlinear action [223]. The nonlinear energy harvester of a clearance-type NES could capture the impact energy and prolong the energy conversion time [224].

Under the excitation of harmonic load, the smooth NES could suppress the vibration of the fluid-conveying pipeline. The middle position of the pipeline was the best position to connect the NES with the fixed fluid-conveying pipeline [225]. A new integrated approach of piezoelectric energy harvester and NES can effectively suppress pipeline vibration and enable energy harvesting [192]. NES with a piecewise spring device could limit the vibration amplitude and reduce the manufacturing cost of the device. However, the defect of the device was that NES weakens the vibration reduction effect of the linear system [226]. In a two-degree-of-freedom linear structure with coupled piecewise NES, the nonlinear effects of piecewise NES have significant effects on energy transfer and dissipation [227]. Under seismic excitation, magnetic bistable NES could effectively improve the seismic performance of the main structure, and the robustness was much higher than linear TMD and cubic NES [28]. The novel track bistable NES had a good control effect on the vibration of various structures under seismic excitation [228, 229]. The VINES system could effectively improve the seismic capability of the structure by utilizing the TET properties. Compared with installing a single NES on the structure [230], installing multiple NES [231] on the structure could better reduce the structural vibration response [232]. Therefore, the parallel control system of multiple NES was applied to the automotive drivetrain to reduce the vibration amplitude of the transmission [233]. A system composed of multiple NESs and piezoelectric vibration energy harvesters could collect vibration energy under impact excitation, as shown in Figure 23 [234].

The system composed of NES and piezoelectric vibration energy harvester could obtain more electric energy in both grounded and ungrounded configurations, and the results are shown in Figure 24 [235].

A system with multiple NES can suppress vibration amplitudes more effectively than a system with a single NES. At the same time, in terms of energy dissipation and energy harvesting, systems with NES are significantly different in both grounded and ungrounded configurations. Considering the energy harvesting technology on the basis of the vibration reduction of the transmission system, it is feasible to use the engine vibration to power low-energy devices.

As an important part of infrastructure, the highway is regarded as having great potential for vibration energy harvesting in the future. Considering the affordability and durability of piezoelectric materials, capturing the dissipated kinetic energy from roadway traffic through piezoelectric energy harvesting technology is a reliable method [236, 237]. In the piezoelectric energy harvesting device, it was

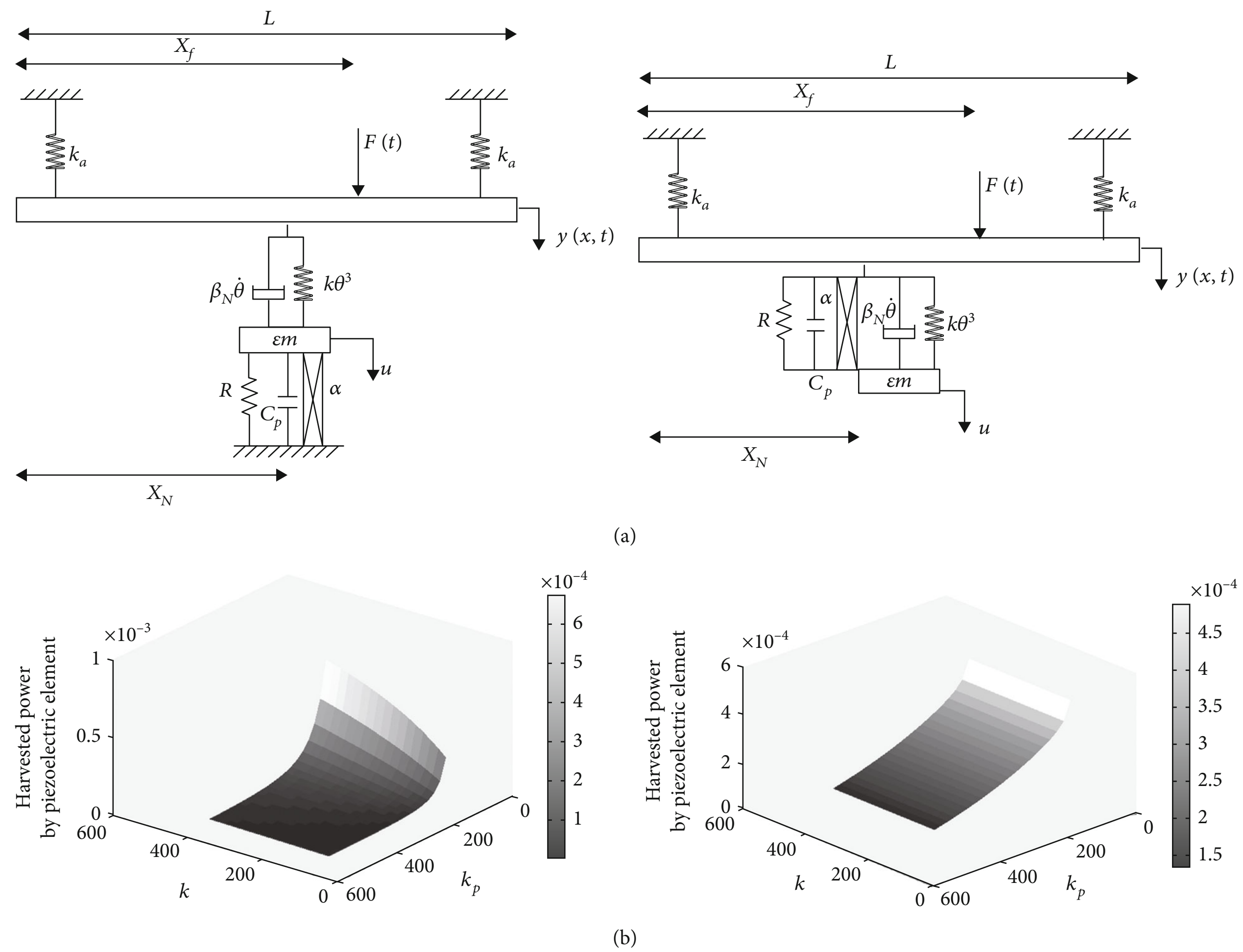


Figure 24: System made up of a NES and a piezoelectric-based energy harvester attached to a free-free beam. (a) NES configurations: grounded and ungrounded. (b) Contour plot of piezoelectric element harvested power versus piezoelectric element stiffness and NES spring stiffness: the left figure shows grounding, and the right figure represents ungrounded.

very important to consider the physical configuration, material selection, and combination with the road surface of the device [193, 236, 238, 239]. A piezoelectric energy harvester with a displacement-amplifying mechanism and high-density materials solved the common problem of low power in the energy harvester, and the vehicle speed was conducive to improving the efficiency of energy harvesting [240]. In general, the practical application of solar panels on the road has some limitations; the root cause is the material's inability to adapt to the complex changes in road conditions. Compared with other materials suitable for energy harvesting, piezoelectric materials have high mechanical strength, high stiffness, and high Curie point and can be used in complex environments. Clearly, piezoelectric energy harvesters are a more cost-effective and reliable solution.

4.3. Wireless Sensor. Energy harvesting in wireless sensor is a new technology, which has wide application value. In many cases, making multiple battery changes to a system is a chore. Therefore, energy harvesting from ambient vibration has become a hot research topic, which aims to power sensors and portable electronic products without batteries, so as to collect energy for self-powered sensor or extend battery life [241]. When the harmonic excitation frequency differs from the natural frequency of the system, the performance of the linear energy harvester was affected [242]. Compared with linear energy harvesting devices, energy harvesters based on NES could simultaneously perform vibration suppression and energy harvesting [66, 243, 244]. Faced with complex environmental excitation, the NES energy harvester overcame the problem of poor robustness of linear energy harvester.

A tunable bistable energy harvester had advantages over pure cubic structures in terms of wideband vibration absorption and energy transfer speed, and it was capable of supplying power to miniature wireless sensors [195]. At present, the linear energy harvesting scheme is the main research of power supply devices in wireless sensor systems. Although the traditional linear energy harvesting model was feasible for energy harvesting circuits in some cases, the corresponding system required more power consumption than the nonlinear energy harvesting model [245]. NES could improve the energy harvesting frequency bandwidth of an energy harvesting system based on a synchronous charge extraction circuit, which was independent of the load resistance in the synchronous charge extraction circuit [115]. NES can provide a new way to power wireless sensors.

5. Conclusion and Prospect

NES has attracted much attention because of its ability to achieve directional energy transfer. Traditional linear vibration absorber is composed of mass, linear spring, and damper. Once the excitation frequency is not equal to the natural frequency of the device, the performance will decline significantly. However, NES only needs the excitation energy to reach the desired threshold to be excited by any vibration frequency. Therefore, the use of NES technology to expand the frequency bandwidth is a viable option. The nonlinear energy harvester can convert the mechanical energy of ambient vibration into electric energy and realize vibration control and energy harvesting at the same time. This review summarizes the development of the NES from 1952 to the present, including the properties and performance benefits of nonlinear dynamics, the design and optimization of energy harvesting systems, and a wide range of engineering applications. It mainly focuses on the realization of nonlinear stiffness and the design of vibration energy harvesting devices. The review shows that there is growing interest in the NES energy harvester research. In future NES research work, the following concerns are worth studying.

(1) There are many kinds of NESs, which can be applied to the energy harvesting problem under different conditions, thus providing a variety of possibilities for the actual design and application. However, because many design parameters will affect the efficiency of the NES, it will increase the difficulty of the design to some extent. Therefore, the simplified design method of NES considering multiple parameters still needs to be developed. In addition, since the vibration energy harvesting performance of the NES is sensitive to the uncertainty of the initial energy, it is necessary to establish a random optimization method that can consider the uncertainty of the initial energy in order to determine the optimal design parameters of NES (e.g., mass ratio and damping ratio)

(2) In some cases, energy harvesting and vibration suppression are contradictory. That is, the energy harvesting strategy wants the main system to keep the large amplitude vibration as long as possible. In contrast, the vibration suppression strategy aims to suppress the vibration of the main system as quickly as possible. NES can be used in a vibration system for both structural vibration suppression and energy harvesting, and the relationship between the two can be balanced to maximize the performance of vibration suppression and energy harvesting

(3) In the face of complex and changeable environmental vibration, NES can improve the frequency bandwidth of the energy harvesting system and can collect vibration energy stably and efficiently from multiple directions rather than a single direction. In addition, NES introduced the autonomous tuning technology to realize the system parameter change with the environmental vibration, so as to achieve better vibration energy harvesting performance

(4) Because the linear energy harvester can only collect energy at a specific frequency, the available frequency bandwidth of such an energy harvester is narrow. However, the NES energy receiver can be excited by any vibration frequency as long as the excitation energy reaches the desired threshold. However, the energy harvesting performance at low frequencies is not as good as that of the optimized linear energy harvester. It is a challenge to realize the improvement of the energy harvesting performance of the nonlinear energy harvester at low frequencies

(5) Since NES is only effective for certain incentives, otherwise, it may worsen dynamic behavior. Therefore, broadening the range of acceptable incentive variations for NES is a subject worth studying

(6) In the biomedical field, the realization of energy harvesting based on NES has potential research value. For example, it is of research value to provide a continuous supply of electrical energy for different small devices such as health monitoring, pacemakers, and localizers by harvesting the tiny vibrational energy of living things

Data Availability

Data are available on request.

Conflicts of Interest

The authors declare no conflict of interest.

Authors' Contributions

The authors confirm the final authorship for this manuscript. All the authors have equally contributed to this manuscript.

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