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Research Article

Hydromagnetic Stability of Metallic Nanofluids (Cu-Water and Ag-Water) Using Darcy-Brinkman Model

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Thermal convection of a nanofluid layer in the presence of imposed vertical magnetic field saturated by a porous medium is investigated for both-free, rigid-free, and both-rigid boundaries using Darcy-Brinkman model. The effects of Brownian motion and thermophoretic forces due to the presence of nanoparticles and Lorentz's force term due to the presence of magnetic field have been considered in the momentum equations along with Maxwell's equations. Keeping in mind applications of flow through porous medium in geophysics, especially in the study of Earth's core, and the presence of nanoparticles therein, the hydromagnetic stability of a nanofluid layer in porous medium is considered in the present formulation. An analytical investigation is made by applying normal mode technique and Galerkin type weighted residuals method and the stability of Cu-water and Ag-water nanofluids is compared. Mode of heat transfer is through stationary convection without the occurrence of oscillatory motions. Stability of the system gets improved appreciably by raising the Chandrasekhar number as well as Darcy number whereas increase in porosity hastens the onset of instability. Further, stability of the system gets enhanced as we proceed from both-free boundaries to rigid-free and to both-rigid boundaries.

1. Introduction

The concept of nanofluids has improved the heat transfer mechanism by replacing the suspension of micrometer sized particles with nanometer sized particles in conventional fluids. These nanometer sized particles are called nanoparticles which may be metals, metal oxides, carbides, nitrides, or semiconductors. The host liquids may be water, ethylene glycol, propylene glycol, and so forth. The magnificent idea of introducing nanofluids first came into the mind of Choi [1] who claimed the enhanced heat transfer with the addition of nanoparticles. Due to the ultra fine size of nanoparticles, nanofluids have overcome the limitations of micrometer and millimeter sized particles such as settling down in fluid, erosion, and clogging in channel or low thermal conductivity of fluids. Eastman et al. [2] found that dispersion of ultrafine particles in regular fluids improves the physical properties of that fluid. The enhanced physical properties of nanofluids can be utilized in a vast variety of applications [3, 4]. A well comprehensive model for the enhanced thermal conductivity of nanofluids has been given by Wang et al. [5].

The problem of thermal convection for regular fluids has been discussed in length in a treatise by Chandrasekhar [6]. The problem of thermal convection for nanofluids has been initiated by Kim et al. [7]. Tzou [8] studied the problem analytically and used eigenfunction expansion method to solve the conservation equations given by Buongiorno and he found that critical Rayleigh number is reduced with the addition of nanoparticles. Nield and Kuznetsov [9, 10] investigated the onset of thermal convection in a nanofluid layer for porous/nonporous medium. Kuznetsov and Nield [11] further extended the problem using Darcy-Brinkman model. Effect of rotation on thermal convection has been accounted for by Bhadauria and Agarwal [12] and Chand and Rana [13] in porous/nonporous medium and it was

established that addition of Coriolis force term in momentum equation increases the stability of the system. Gupta et al. [14] and Yadav et al. [15] were the first authors for studying convection problem in the presence of imposed magnetic field for bottom heavy and top heavy distribution of nanoparticles, respectively. It was found that magnetic field postpones the onset of thermal convection and the mode of heat transfer is through oscillatory motions for bottom heavy distribution whereas it is through stationary convection for top heavy arrangement of nanoparticles.

The onset of thermal instability of a fluid layer in porous medium has its major application in geophysics particularly in underground reservoirs and in enhanced oil recovery in addition to the usual industrial applications. Nanofluids in porous medium emerge out to have usage in porous foam and microchannel heat sinks which are used for electronic cooling. Applying magnetic field on horizontal layer of nanofluid exhibits some notable features which make it essential to investigate the effects of magnetic field in porous medium. The present paper formulates to present this effect using Darcy-Brinkman model for three types of boundaries: both-free, rigid-free, and both-rigid. The thermal instability problem is analyzed within the framework of normal mode technique and one term weighted residuals method. Lorentz force term is added in the momentum equation in addition to the body and buoyancy forces and an extra viscous term is added in the Darcy equation for consideration of Darcy-Brinkman model and it gives rise to two additional parameters: Darcy number and Chandrasekhar number. Stability of the system gets improved appreciably by raising the applied magnetic field/hence Chandrasekhar number as well as Darcy number whereas increase in porosity hastens the onset of instability. Oscillatory motions are not possible and stability of the system increases appreciably as we proceed from both-free boundaries to rigid-free and to both-rigid boundaries.

2. Formulation of Problem and Conservation Equations in Porous Medium

A horizontal layer of nanofluid of infinite length and thickness d in a homogenous porous medium is considered which is assumed to be at rest initially. Disturbance is caused by heating from beneath the layer so that T_0, T_1 are the temperatures and ϕ_0, ϕ_1 ($\phi_1 > \phi_0$) are the nanoparticle's volume fractions at lower and upper boundaries, respectively (as shown in Figure 1). Porous medium considered here has porosity ε and medium permeability k_1. Gravity force $\mathbf{g} = (0, 0, -g)$ is acting vertically downwards and magnetic field $\mathbf{H} = (0, 0, H)$ is acting in vertically upwards direction. The Darcy velocity is denoted by "$\mathbf{q}_D$" which is related to "$\mathbf{q}$" the nanofluid velocity as $\mathbf{q}_D = \varepsilon\mathbf{q}$.

The conservation equations for mass, nanoparticles, momentum, and thermal energy in porous medium using Brinkman's model in the presence of magnetic field are [6, 9]

$$\nabla \cdot \mathbf{q}_D = 0, \tag{1}$$

$$\frac{\partial \phi}{\partial t} + \frac{\mathbf{q}_D}{\varepsilon} \cdot \nabla\phi = \nabla \cdot \left[D_B \nabla\phi + D_T \frac{\nabla T}{T} \right], \tag{2}$$

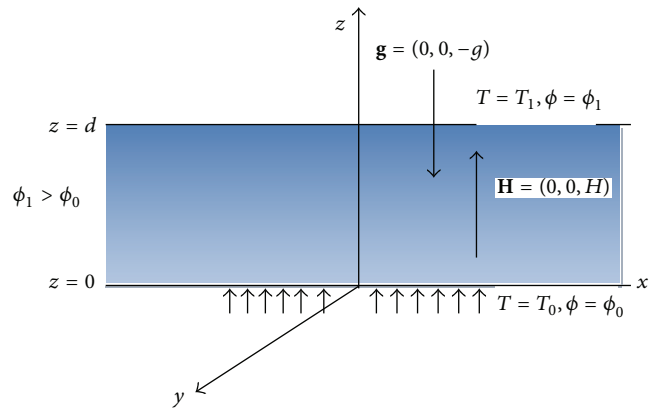

Figure 1: Physical system and geometry.

$$\frac{\rho_f}{\varepsilon} \frac{\partial \mathbf{q}_D}{\partial t} = -\nabla p + \tilde{\mu}\nabla^2 \mathbf{q}_D - \frac{\mu}{k_1}\mathbf{q}_D + \rho\mathbf{g} + \frac{\mu_e}{4\pi}(\nabla \times \mathbf{h}) \times \mathbf{H}, \tag{3}$$

$$(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f \mathbf{q}_D \cdot \nabla T = \left(k_m \nabla^2 T\right) + \varepsilon (\rho c)_p \left[D_B \nabla\phi \cdot \nabla T + D_T \frac{\nabla T \cdot \nabla T}{T} \right], \tag{4}$$

$$\frac{d\mathbf{h}}{dt} = (\mathbf{H} \cdot \nabla) \frac{\mathbf{q}_D}{\varepsilon} + \eta \nabla^2 \mathbf{h}, \tag{5}$$

$$\nabla \cdot \mathbf{h} = 0, \tag{6}$$

where η is the magnetic diffusivity and ρ is the nanofluid's density which is given by

$$\rho = \phi\rho_p + (1 - \phi)\rho_f \cong \phi\rho_p + (1 - \phi)\{\rho(1 - \beta(T - T_0))\}, \tag{7}$$

where $\mathbf{q}_D$, D_B, D_T, ρ_f, ρ_p, $(\rho c)_f$, $(\rho c)_m$, $(\rho c)_p$, ϕ, μ, $\tilde{\mu}$, μ_e, (h_x, h_y, h_z), p, and k_m denote, respectively, the Darcy velocity, the Brownian diffusion coefficient, the thermophoretic diffusion coefficient, the density of the fluid, the density of nanoparticles, the heat capacity of the fluid, the heat capacity of the medium, the heat capacity of the nanoparticles, the nanoparticles volume fraction, the viscosity of the fluid, the effective viscosity, the magnetic permeability, the components of magnetic field, the pressure, and the effective thermal conductivity of the porous medium.

Now the above equations are nondimensionalized using

$$(\tilde{x}, \tilde{y}, \tilde{z}) = \frac{(x, y, z)}{d},$$

$$(\tilde{u}, \tilde{v}, \tilde{w}) = \frac{(u, v, w)\, d}{\alpha_m},$$

$$\tilde{p} = \frac{p k_1}{\mu \alpha_m},$$

$$\tilde{t} = \frac{t\alpha_m}{\sigma d^2},$$
$$\tilde{\phi} = \frac{\phi - \phi_0}{\phi_1 - \phi_0},$$
$$\tilde{T} = \frac{T - T_1}{T_0 - T_1},$$
$$\tilde{h} = \frac{\eta}{H\alpha_m} h,$$
$$\text{where } \alpha_m = \frac{k_m}{(\rho c)_f}, \ \sigma = \frac{(\rho c)_m}{(\rho c)_f}. \tag{8}$$

Then the nondimensional forms of (1)–(6) after omitting the symbol ~ are

$$\nabla \cdot \mathbf{q} = 0, \tag{9}$$

$$\frac{\mathrm{Dr}}{\mathrm{Pr}_1}\frac{\partial \mathbf{q}}{\partial t} = -\nabla p + \mathrm{Dr}\nabla^2 \mathbf{q} - \mathbf{q} - \mathrm{Rm}\,\hat{e}_z - \mathrm{Rn}\,\phi\hat{e}_z + \mathrm{Ra}\,T\hat{e}_z + Q\left[\left(\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x}\right)\hat{e}_x - \left(\frac{\partial h_z}{\partial y} - \frac{\partial h_y}{\partial z}\right)\hat{e}_y\right], \tag{10}$$

$$\frac{\partial T}{\partial t} + \mathbf{q}\cdot\nabla T = \nabla^2 T + \frac{N_b}{\mathrm{Le}}\nabla\phi\cdot\nabla T + \frac{N_a N_b}{\mathrm{Le}}\nabla T\cdot\nabla T, \tag{11}$$

$$\frac{1}{\sigma}\frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon}\mathbf{q}\cdot\nabla\phi = \frac{1}{\mathrm{Le}}\nabla^2\phi + \frac{N_a}{\mathrm{Le}}\nabla^2 T, \tag{12}$$

$$\nabla \cdot \mathbf{h} = 0, \tag{13}$$

$$\frac{\varepsilon}{\sigma}\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}\frac{d\mathbf{h}}{dt} = \left(\frac{\partial u}{\partial z}\hat{e}_x + \frac{\partial v}{\partial z}\hat{e}_y + \frac{\partial w}{\partial z}\hat{e}_z\right) + \varepsilon\nabla^2\mathbf{h}, \tag{14}$$

where

$$\mathrm{Pr}_1 = \frac{\mu}{\rho\alpha_m};$$
$$\mathrm{Pr}_2 = \frac{\mu}{\rho\eta};$$
$$\mathrm{Le} = \frac{\alpha_m}{D_B};$$
$$\mathrm{Ra} = \frac{(\rho g\beta d k_1 (T_h - T_c))}{\mu\alpha_m};$$
$$\mathrm{Dr} = \frac{\tilde{\mu} k_1}{\mu d^2};$$
$$\mathrm{Rm} = \left[\phi_0\rho_p + \frac{\rho(1-\phi_0)}{\mu\alpha_m}\right] g d k_1;$$
$$\mathrm{Rn} = \left(\frac{(\rho_p - \rho)(\phi_1 - \phi_0)}{\mu\alpha_m}\right) g d k_1;$$
$$N_a = \frac{(D_T (T_h - T_c))}{(D_B T_c (\phi_1 - \phi_0))};$$
$$N_b = \left[\frac{\varepsilon(\rho c)_p}{(\rho c)_f}\right](\phi_1 - \phi_0),$$
$$Q = \left(\frac{\mu_e H^2 k_1}{4\pi\eta\mu}\right), \tag{15}$$

the Prandtl number, the magnetic Prandtl number, the Lewis number, the thermal Rayleigh number, the Darcy number, the basic density Rayleigh number, the concentration Rayleigh number, the modified diffusivity ratio, the modified particle-density increment, and the Chandrasekhar number are the various nondimensional parameters, respectively. It is worthwhile to mention that Boussinesq approximation has been used to linearize the system of equations.

3. Primary Flow and Disturbance Equations

The primary flow is described by the state of the system which is at rest whereas temperature, pressure, and volume fraction of nanoparticles are varying in the vertical direction; that is,

$$\mathbf{q} = 0,$$
$$T = T_p(z),$$
$$p = p_p(z),$$
$$\phi = \phi_p(z),$$
$$\phi_p = -N_a T_p + (1 + N_a)(1 - z), \tag{16}$$

and by putting above equations in (11) we get

$$\frac{d^2 T_p}{dz^2} - \frac{(1 + N_a) N_b}{\mathrm{Le}}\frac{dT_p}{dz} = 0. \tag{17}$$

Equation (17) along with the boundary conditions gives the solution as

$$T_p = \frac{1 - e^{-(1+N_a)N_b(1-z)/\mathrm{Le}}}{1 - e^{-(1+N_a)N_b/\mathrm{Le}}}. \tag{18}$$

Using the parametric values of nanofluid parameters (Le ranges from 10^2 to 10^3, N_a is less than 10) and neglecting the terms of second and higher order in the expansion of exponential function, we get the best approximate solution as

$$T_p = 1 - z,$$
$$\phi_p = z. \tag{19}$$

Disturbance on the primary flow is caused by heating from beneath the nanofluid layer and these disturbances are assumed to be small so that

$$\begin{aligned} \mathbf{q}(u,v,w) &= 0 + \mathbf{q}'\left(u',v',w'\right), \\ T &= T_p + T', \\ p &= p_p + p', \\ \mathbf{h}\left(h_x,h_y,h_z\right) &= \mathbf{H} + \mathbf{h}'\left(h'_x,h'_y,h'_z\right), \\ \phi &= \phi_p + \phi', \end{aligned} \tag{20}$$

with $T_p = 1 - z$, $\phi_p = z$. By applying these perturbations to (9)–(14), the resulting equations are linearized by using the concept of linear theory. The obtained system of perturbation equations will be analyzed within the framework of normal modes and single term Galerkin method.

4. Normal Mode Analysis

Now the disturbances are examined by using the technique of superposition of basic modes which are described by the pattern

$$\begin{aligned} &(w,T,\phi,h_z,\xi,\zeta) \\ &\quad = (W(z),\Theta(z),\Phi(z),K(z),X(z),Z(z)) \\ &\quad \cdot \exp\left(ik_x x + ik_y y + st\right), \end{aligned} \tag{21}$$

where k_x and k_y represent the wave numbers in horizontal and vertical directions, respectively, and s represents the growth rate parameter. The system of perturbation equations reduces to

$$\begin{aligned} &\frac{\mathrm{Dr}}{\mathrm{Pr}_1} sZ = \mathrm{Dr}\left(D^2-\alpha^2\right) - Z + QDX, \\ &\frac{\mathrm{Dr}}{\mathrm{Pr}_1} s\left(D^2-\alpha^2\right)W = \mathrm{Dr}\left(D^2-\alpha^2\right)^2 W \\ &\quad - \left(D^2-\alpha^2\right)W + \mathrm{Rn}\,\alpha^2\Phi - \mathrm{Ra}\,\alpha^2\Theta \\ &\quad + QD\left(D^2-\alpha^2\right)K, \\ &\left\{\frac{\varepsilon}{\sigma}\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}s - \varepsilon\left(D^2-\alpha^2\right)\right\}K = DW, \\ &\left\{\frac{\varepsilon}{\sigma}\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}s - \varepsilon\left(D^2-\alpha^2\right)\right\}X = DZ, \\ &s\Theta - W - \left(D^2 - \frac{N_b}{\mathrm{Le}}D - 2\frac{N_aN_b}{\mathrm{Le}}D - \alpha^2\right)\Theta \\ &\quad + \frac{N_b}{\mathrm{Le}}D\Phi = 0, \\ &\frac{1}{\sigma}s\Phi - \frac{W}{\varepsilon} - \frac{N_a}{\mathrm{Le}}\left(D^2-\alpha^2\right)\Theta - \frac{1}{\mathrm{Le}}\left(D^2-\alpha^2\right)\Phi = 0. \end{aligned} \tag{22}$$

After the process of elimination, the above set of equations reduce to

$$\begin{aligned} &\left[\frac{\mathrm{Dr}}{\mathrm{Pr}_1}s\left(D^2-\alpha^2\right) - \mathrm{Dr}\left(D^2-\alpha^2\right)^2 + \left(D^2-\alpha^2\right)\right] \\ &\quad \cdot \left[\varepsilon\left(D^2-\alpha^2\right) - \frac{\varepsilon}{\sigma}\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}s\right]W \\ &\quad + \left[\varepsilon\left(D^2-\alpha^2\right) - \frac{\varepsilon}{\sigma}\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}s\right]\left[\mathrm{Ra}\,\alpha^2\Theta - \mathrm{Rn}\,\alpha^2\Phi\right] \\ &\quad + Q\left(D^2-\alpha^2\right)D^2W = 0, \\ &s\Theta - W = \left(D^2-\alpha^2\right)\Theta + \frac{N_b}{\mathrm{Le}}(D\Theta - D\Phi) - 2\frac{N_aN_b}{\mathrm{Le}} \\ &\quad \cdot D\Theta, \\ &\frac{1}{\sigma}s\Phi + \frac{W}{\varepsilon} = \frac{N_a}{\mathrm{Le}}\left(D^2-\alpha^2\right)\Theta + \frac{1}{\mathrm{Le}}\left(D^2-\alpha^2\right)\Phi \\ &\quad = 0, \end{aligned} \tag{23}$$

where $D \equiv d/dz$ and $\alpha = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number in nondimensional form. Now, single term approximation of Galerkin weighted residuals approach is adopted to solve (23). In this approach, choice of trial functions W_p, Θ_p, and Φ_p depends on the relevant boundary conditions and we write

$$\begin{aligned} W &= \sum_{p=1}^{N} A_p W_p, \\ \Theta &= \sum_{p=1}^{N} B_p \Theta_p, \\ \Phi &= \sum_{p=1}^{N} C_p \Phi_p. \end{aligned} \tag{24}$$

For one term approximation we put $p = 1$. Let us substitute (24) into (23) and make use of orthogonality to the trial functions; we obtain a system of three equations in three unknowns A_1, B_1, C_1. Elimination of these unknowns from the obtained set of equations gives the eigenvalue equation.

5. Results and Discussions

5.1. Free-Free Boundaries and Special Cases. The boundary conditions on both-free boundary surfaces are

$$\begin{aligned} W &= 0, \\ D^2W &= 0, \\ \Phi &= 0, \\ \Theta &= 0, \\ &\text{at the lower and upper boundaries;} \\ K &= 0, \end{aligned} \tag{25}$$

on the boundaries [6]. Trial functions satisfying the above boundary conditions can be chosen as $W = A_1 \sin \pi z$, $\Theta = B_1 \sin \pi z$, and $\Phi = C_1 \sin \pi z$. Let us substitute this solution in (23) and follow the process of integration and elimination of A_1, B_1, C_1 from the obtained set of equations; the eigenvalue equation becomes

$$\begin{aligned}&\left[-\frac{\mathrm{Dr}}{\mathrm{Pr}_1}sJ - \mathrm{Dr}J^2 - J\right]\left[\varepsilon J + \frac{\varepsilon}{\sigma}\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}s\right](s+J)\left(\frac{s}{\sigma}+\frac{J}{\mathrm{Le}}\right)\\&+\left[\varepsilon J + \frac{\varepsilon}{\sigma}\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}s\right]\\&\cdot\left[\mathrm{Ra}\,\alpha^2\left(\frac{s}{\sigma}+\frac{J}{\mathrm{Le}}\right)+\mathrm{Rn}\,\alpha^2\left(\frac{s+J}{\varepsilon}+\frac{JN_a}{\mathrm{Le}}\right)\Phi\right]\\&+Q\pi^2 J(s+J)\left(\frac{s}{\sigma}+\frac{J}{\mathrm{Le}}\right)=0.\end{aligned} \tag{26}$$

At the state of marginal stability, when the amplitudes of small disturbances grow or damp aperiodically then the transition from stability to instability takes place via a stationary pattern of motions which is described by $s = i\omega = 0$. Then the eigenvalue equation (26) reduces to

$$\begin{aligned}\mathrm{Ra}^{\mathrm{stat}} &= \frac{1}{\alpha^2}\left[\mathrm{Dr}\left(\pi^2+\alpha^2\right)^3+\left(\pi^2+\alpha^2\right)^2\right.\\&\left.+\frac{Q\pi^2}{\varepsilon}\left(\pi^2+\alpha^2\right)\right]-\mathrm{Rn}\left[\frac{\mathrm{Le}}{\varepsilon}+N_a\right],\end{aligned} \tag{27}$$

$$\begin{aligned}\mathrm{Ra}^{\mathrm{stat}}+\mathrm{Rn}\left(\frac{\mathrm{Le}}{\varepsilon}+N_a\right) &= \pi^2\frac{(1+x)}{x}\left[\mathrm{Dr}\,\pi^2(1+x)^2\right.\\&\left.+\pi^2(1+x)+\frac{Q}{\varepsilon}\right],\end{aligned} \tag{28}$$

$$\begin{aligned}\mathrm{Ro} &= \pi^2\frac{(1+x)}{x}\left[\mathrm{Dr}\,\pi^2(1+x)^2+\pi^2(1+x)+\frac{Q}{\varepsilon}\right],\\&\text{where } \mathrm{Ro} = \mathrm{Ra}^{\mathrm{stat}}+\mathrm{Rn}\left(\frac{\mathrm{Le}}{\varepsilon}+N_a\right),\ \alpha^2=\pi^2 x.\end{aligned} \tag{29}$$

Case 1. When $(\mathrm{Rn} = 0, Q = 0)$, (28) turns out to be

$$\mathrm{Ra}^{\mathrm{stat}} = \pi^2\frac{(1+x)}{x}\left[\mathrm{Dr}\,\pi^2(1+x)^2+\pi^2(1+x)\right], \tag{30}$$

which confirms that our result coincides with the Brinkman model of Kuznetsov and Nield [11]. The values of thermal Rayleigh number and wave number at which instability sets in (critical values) are found by taking $(d\mathrm{Ra}/dx)_{x=x_c} = 0$.

Case 2. When $\mathrm{Dr} = 0, Q \neq 0$, (29) reduces to

$$\mathrm{Ro} = \pi^2\frac{(1+x)}{x}\left[\pi^2(1+x)+\frac{Q}{\varepsilon}\right]. \tag{31}$$

As a function of x, (31) gets its lowest value when

$$x^2 = \frac{Q}{\varepsilon}+1. \tag{32}$$

$$\begin{aligned}&\text{For } (Q=0),\\&x_c = 1\,(\alpha_c=\pi),\\&\mathrm{Ra}_c = 4\pi^2-\mathrm{Rn}\left(\frac{\mathrm{Le}}{\varepsilon}+N_a\right),\end{aligned} \tag{33}$$

$$\mathrm{Ro}_c = \mathrm{Ra}+\mathrm{Rn}\left(\frac{\mathrm{Le}}{\varepsilon}+N_a\right) = 4\pi^2, \tag{34}$$

which is in confirmation with the outcome of Horton-Rogers Lapwood model of Nield and Kuznetsov [9]. By omitting the nanoparticle's term and magnetic field term we get the critical Rayleigh number as $\mathrm{Ra}_c = 4\pi^2$.

Case 3. When $\mathrm{Dr} \to \infty, Q \neq 0$, (28) reduces to

$$\begin{aligned}&\frac{1}{\mathrm{Dr}}\left(\mathrm{Ra}^{\mathrm{stat}}+\mathrm{Rn}\left(\frac{\mathrm{Le}}{\varepsilon}+N_a\right)\right)\\&=\pi^2\frac{(1+x)}{x}\left[\pi^2(1+x)^2+\frac{1}{\mathrm{Dr}}\pi^2(1+x)+\frac{1}{\mathrm{Dr}}\frac{Q}{\varepsilon}\right].\end{aligned} \tag{35}$$

The right-hand side attains its minimum value at $\alpha = \pi/\sqrt{2}$ and its minimum value is given by

$$\frac{1}{\mathrm{Dr}}\left(\mathrm{Ra}^{\mathrm{stat}}+\mathrm{Rn}\left(\frac{\mathrm{Le}}{\varepsilon}+N_a\right)\right) = \frac{27}{4}\pi^4, \tag{36}$$

$$\mathrm{Ra}^{\mathrm{stat}} = \frac{27}{4}\pi^4\mathrm{Dr}-\mathrm{Rn}\left(\frac{Le}{\varepsilon}+N_a\right). \tag{37}$$

From (34) it is clear that in the absence of nanoparticles and magnetic field critical value of thermal Rayleigh number is $\mathrm{Ra}_c = 4\pi^2$ for the case when Darcy number vanishes ($\mathrm{Dr} = 0$). The value of critical Rayleigh number for the case when Dr tends to infinity is $\mathrm{Ra}_c = 657.5$ (from (37)) which coincides with its value for the regular fluid. Thus when Dr is large, the nanofluid behaves like a regular fluid. It is clear from (27) that the suspension of nanoparticles in conventional fluids lowers the critical value of Rayleigh number as all the parameters $\mathrm{Rn}, \mathrm{Le}, N_a$ are positive for the present configuration of nanoparticles and the expression $\mathrm{Rn}(\mathrm{Le}/\varepsilon + N_a)$ appears with negative sign. Thus the system with the distribution of nanoparticles at the top of the fluid layer is less stable as compared to regular fluid and bottom heavy distribution of nanoparticles.

5.2. Rigid-Free Boundaries. Let us now consider the lower boundary of the fluid to be a rigid surface while the upper

boundary surface is free. Then the relevant conditions on the boundary surfaces for W are

$$\begin{aligned} W &= 0, \\ DW &= 0, \\ &\text{at lower boundary,} \\ W &= 0, \\ D^2W &= 0, \\ &\text{at upper boundary.} \end{aligned} \tag{38}$$

The trial function appropriate to these boundaries is

$$\begin{aligned} W_1 &= z^2(1-z)(3-2z), \\ \Theta_1 &= z(1-z), \\ \Phi_1 &= z(1-z). \end{aligned} \tag{39}$$

By making use of single term Galerkin method and orthogonality, the eigenvalue equation in the present case becomes

$$\begin{aligned} &\frac{420}{390}\varepsilon\left[\left(\frac{\mathrm{Dr}}{\mathrm{Pr}_1}s+1\right)\left(\frac{36}{5}+\frac{24}{35}\alpha^2+\frac{19}{630}\alpha^4\right)\right. \\ &\quad\left.+\mathrm{Dr}\left(\frac{108}{5}\alpha^2+\frac{36}{35}\alpha^4\frac{19}{630}\alpha^6\right)\right]\left[\frac{s}{\sigma}+\frac{10+\alpha^2}{\mathrm{Le}}\right] \\ &\quad\cdot\left(10+\alpha^2+s\right)+\frac{420}{390}\frac{\varepsilon}{\sigma}\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1} \\ &\quad\cdot s\left[\left(\frac{\mathrm{Dr}}{\mathrm{Pr}_1}s+1\right)\left(\frac{12}{35}+\frac{19}{630}\alpha^2\right)\right. \\ &\quad\left.+\mathrm{Dr}\left(\frac{36}{5}+\frac{24}{35}\alpha^2+\frac{19}{630}\alpha^4\right)\right]\left[\frac{s}{\sigma}+\frac{10+\alpha^2}{\mathrm{Le}}\right]\left(10\right. \\ &\quad\left.+\alpha^2+s\right)+Q\frac{420}{390}\left[\frac{36}{5}+\frac{12}{35}\alpha^2\right]\left[\frac{s}{\sigma}+\frac{10+\alpha^2}{\mathrm{Le}}\right] \\ &\quad\cdot\left(10+\alpha^2+s\right)-\varepsilon\left[\left(\frac{3}{10}+\frac{13}{420}\alpha^2\right)\right. \\ &\quad\left.+\frac{\varepsilon}{\sigma}\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}s\frac{13}{420}\right]\mathrm{Ra}\,\alpha^2\left[\frac{s}{\sigma}+\frac{10+\alpha^2}{\mathrm{Le}}\right] \\ &\quad-\varepsilon\left[\left(\frac{3}{10}+\frac{13}{420}\alpha^2\right)+\frac{\varepsilon}{\sigma}\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}s\frac{13}{420}\right]\mathrm{Rn}\,\alpha^2\left[10\right. \\ &\quad\left.+\alpha^2\left(\frac{1}{\varepsilon}+\frac{N_a}{\mathrm{Le}}\right)+\frac{s}{\varepsilon}\right]=0. \end{aligned} \tag{40}$$

For stationary convection, the eigenvalue equation (40) takes the form

$$\begin{aligned} &\mathrm{Ra}^{\mathrm{stat}}+\mathrm{Rn}\left[\frac{\mathrm{Le}}{\varepsilon}+N_a\right] \\ &\quad=\frac{28}{13\alpha^2}\left[\frac{1}{3}\left\{\left(4536+432\alpha^2+19\alpha^4\right)\right.\right. \\ &\quad\left.+\mathrm{Dr}\left(13608\alpha^2+648\alpha^4+19\alpha^6\right)\right\}+\frac{72Q}{\varepsilon}\left(21\right. \\ &\quad\left.\left.+\alpha^2\right)\right]\left(\frac{10+\alpha^2}{126+13\alpha^2}\right). \end{aligned} \tag{41}$$

When $\mathrm{Dr}=0$ and $Q=0$, (41) attains its minimum value as $\mathrm{Ro}_c=56.97$ at $\alpha_c=3.96$. In the presence of magnetic field, that is, for $\mathrm{Dr}=0$, $Q=50$, and $\varepsilon=0.4$, the critical value increases appreciably as is given by $\mathrm{Ro}_c=1882.11$ at $\alpha_c=13.22$. Thus magnetic field has a strong stabilizing effect for the case of rigid-free boundaries. When Dr is large, comparable to unity, that is, $(\mathrm{Dr}=1, Q=50, \varepsilon=0.4)$, critical Rayleigh number is given by $\mathrm{Ro}_c=998.823$ at $\alpha_c=1.581$, while, in the presence of magnetic field $(\mathrm{Dr}=1, Q=50, \varepsilon=0.4)$, it increases significantly; that is, $\mathrm{Ro}_c=5052.82$ at $\alpha_c=4.493$. This shows that magnetic field and Darcy number both inhibit the onset of convection and contribute largely towards the stability of the system.

5.3. Rigid-Rigid Boundaries. Let us now consider that both boundaries of the fluid layer are rigid. Therefore, vanishing of normal and horizontal components of velocity to the rigid surface (no slip condition) leads to the following conditions on boundaries:

$$\begin{aligned} W &= 0, \\ \mathrm{D}W &= 0, \\ \Theta &= 0 \\ \Phi &= 0, \\ &\text{at lower and upper boundary;} \\ K &= 0, \\ DK &= 0, \end{aligned} \tag{42}$$

for perfectly conducting boundaries. The suitable trial functions satisfying these boundary conditions are

$$\begin{aligned} W_1 &= z^2(1-z)^2, \\ \Phi_1 &= z(1-z), \\ \Theta_1 &= z(1-z). \end{aligned} \tag{43}$$

Using the approximation of one term Galerkin method and orthogonality, the eigenvalue equation becomes

$$\begin{aligned}
&\frac{14}{3}\varepsilon\left[\left(\frac{\mathrm{Dr}}{\mathrm{Pr}_1}s+1\right)\left(\frac{24}{30}+\frac{4}{105}\alpha^2+\frac{1}{630}\alpha^4\right)\right.\\
&\quad\left.+\mathrm{Dr}\left(\frac{72}{30}\alpha^2+\frac{6}{105}\alpha^4+\frac{1}{630}\alpha^6\right)\right]\left[\frac{s}{\sigma}+\frac{10+\alpha^2}{\mathrm{Le}}\right]\\
&\quad\cdot\left(10+\alpha^2+s\right)+\frac{14}{3}\frac{\varepsilon}{\sigma}\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}\\
&\quad\cdot s\left[\left(\frac{\mathrm{Dr}}{\mathrm{Pr}_1}s+1\right)\left(\frac{2}{105}+\frac{1}{630}\alpha^2\right)\right.\\
&\quad\left.+\mathrm{Dr}\left(\frac{24}{30}+\frac{4}{105}\alpha^2+\frac{1}{630}\alpha^4\right)\right]\left[\frac{s}{\sigma}+\frac{10+\alpha^2}{\mathrm{Le}}\right]\\
&\quad\cdot\left(10+\alpha^2+s\right)+Q\frac{14}{3}\left[\frac{24}{30}+\frac{2}{105}\alpha^2\right]\left[\frac{s}{\sigma}\right.\\
&\quad\left.+\frac{10+\alpha^2}{\mathrm{Le}}\right]\left(10+\alpha^2+s\right)-\varepsilon\left[\left(\frac{2}{30}+\frac{1}{140}\alpha^2\right)\right.\\
&\quad\left.+\frac{\varepsilon}{\sigma}\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}\frac{s}{140}\right]\mathrm{Ra}\,\alpha^2\left[\frac{s}{\sigma}+\frac{10+\alpha^2}{\mathrm{Le}}\right]\\
&\quad-\varepsilon\left[\left(\frac{2}{30}+\frac{1}{140}\alpha^2\right)+\frac{\varepsilon}{\sigma}\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}\frac{s}{140}\right]\mathrm{Rn}\,\alpha^2\left[10\right.\\
&\quad\left.+\alpha^2\left(\frac{1}{\varepsilon}+\frac{N_a}{\mathrm{Le}}\right)+\frac{s}{\varepsilon}\right]=0.
\end{aligned}\tag{44}$$

Let us discuss the case of stationary convection by putting $s = 0$ in (44). The expression for Rayleigh number for both-rigid boundaries becomes

$$\begin{aligned}
&\mathrm{Ra}^{\mathrm{stat}}+\mathrm{Rn}\left[\frac{\mathrm{Le}}{\varepsilon}+N_a\right]\\
&\quad=\frac{28}{3\alpha^2}\left[\frac{1}{3}\left\{\left(504+24\alpha^2+\alpha^4\right)\right.\right.\\
&\quad\left.\left.+\mathrm{Dr}\left(1512+36\alpha^2+\alpha^4\right)\alpha^2\right\}+\frac{4Q}{\varepsilon}\left(42+\alpha^2\right)\right]\\
&\quad\cdot\left(\frac{10+\alpha^2}{28+3\alpha^2}\right).
\end{aligned}\tag{45}$$

When $\mathrm{Dr} = 0$ and $Q = 0$, (45) attains its minimum value as $\mathrm{Ro}_c = 72.94$ at $\alpha_c = 4.791$. In the presence of magnetic field, that is, for $\mathrm{Dr} = 0, Q = 50, \varepsilon = 0.4$, the critical value increases appreciably as is given by $\mathrm{Ro}_c = 2108.45$ at $\alpha_c = 15.95$. Thus magnetic field predominantly stabilizes the nanofluid layer system for the case of rigid-rigid boundaries. When Dr is large, comparable to unity, ($\mathrm{Dr} = 1, Q = 50, \varepsilon = 0.4$), critical Rayleigh number is given by $\mathrm{Ro}_c = 1986.99$ at $\alpha_c = 1.92$ while, in the presence of magnetic field ($\mathrm{Dr} = 1, Q = 50, \varepsilon = 0.4$), it increases significantly, that is, $\mathrm{Ro}_c = 7514.95$ at $\alpha_c = 5.19$. Thus, both Chandrasekhar number and Darcy number contribute significantly towards the stability of the system and this stability increases as we move from both-free boundaries to rigid-free boundaries and then to both-rigid boundaries.

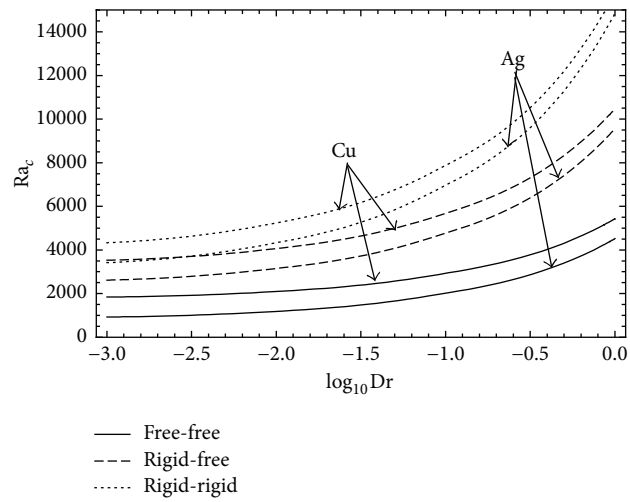

Figure 2: Plot of Ra_c with Darcy number for fixed $Q = 250$ and $\varepsilon = 0.4$.

6. Numerical Results and Discussion

Let us now consider the numerical/graphical investigation of the problem by considering numerical values of various parameters under consideration ($Q = 250, \varepsilon = 0.4, \mathrm{Dr} = 0.01$) for rigid-rigid, rigid-free, and free-free boundaries. To carry out computations, (27), (41), and (45) are used for both-free, rigid-free, and both-rigid boundaries, respectively. The values of nanofluid parameters for $\Delta\phi = 0.001$ are $\mathrm{Rn} = 0.392$, $N_a = 0.5$, and $\mathrm{Le} = 5000$ for Cu-water nanofluid and $\mathrm{Rn} = 0.465$, $N_a = 0.5$, and $\mathrm{Le} = 5000$ for Ag-water nanofluid. Figures 2–7 show the graphical results for stationary convection for Cu-water nanofluid and Ag-water nanofluid in which concentration of nanoparticles at the upper boundary is more than that at the lower boundary.

Figures 2-3 are plot of Ra_c and α_c with the variation in Darcy number for Cu-water and Ag-water nanofluids for three different boundaries. Clearly, Ra_c increases and α_c decreases with the increase in Dr for all the three boundaries. Further, the curves for rigid-free boundaries lie between the curves for both-rigid and both-free boundaries which confirm the earlier result that rigid-rigid boundaries exhibit higher stability than rigid-free and both-free boundaries. Also, the curves showing the effect of Darcy number for Ag-water nanofluid lie below the curves for Cu-water nanofluid which means that Cu-water nanofluid exhibits higher stability as compared to Ag-water nanofluid in the present configuration.

Figures 4-5 correspond to the variation of Ra_c and α_c with the variation in Chandrasekhar number for fixed values of $\mathrm{Dr} = 0.01, \varepsilon = 0.4$ and Figures 6-7 show the variation of porosity for fixed $\mathrm{Dr} = 0.01$, $Q = 250$ for both types of nanofluids. The values of Ra_c as well as α_c increase appreciably with the increase in Chandrasekhar number and decrease significantly with the rise in porosity. Thus the applied magnetic field and porosity have opposing effects of stabilizing/destabilizing the fluid layer system. Further, (27), (41), and (45) show that thermal Rayleigh numbers

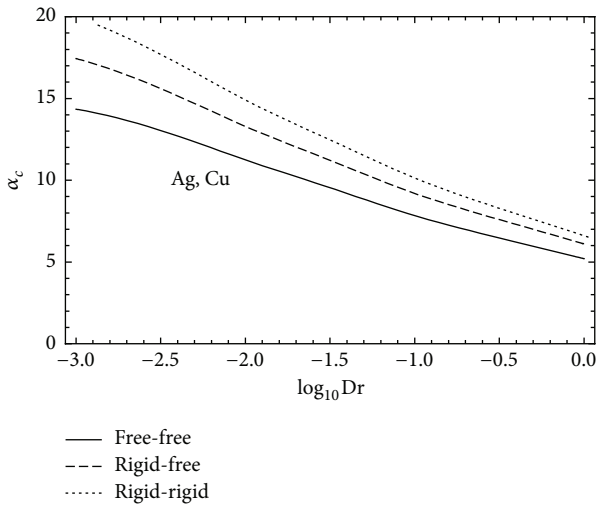

Figure 3: Plot of α_c with Darcy number for fixed $Q = 250$ and $\varepsilon = 0.4$.

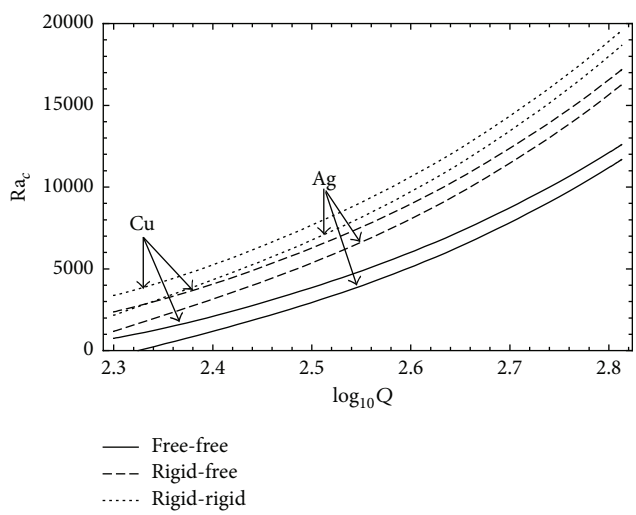

Figure 4: Plot of Ra_c with Chandrasekhar number for fixed Dr = 0.01 and $\varepsilon = 0.4$.

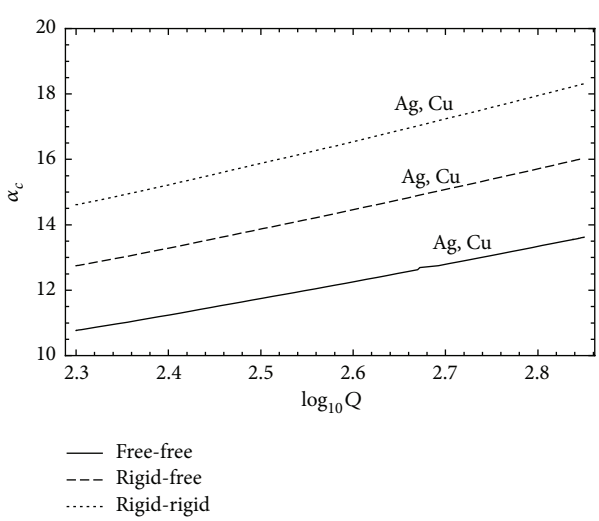

Figure 5: Plot of α_c with Chandrasekhar number for fixed Dr = 0.01 and $\varepsilon = 0.4$.

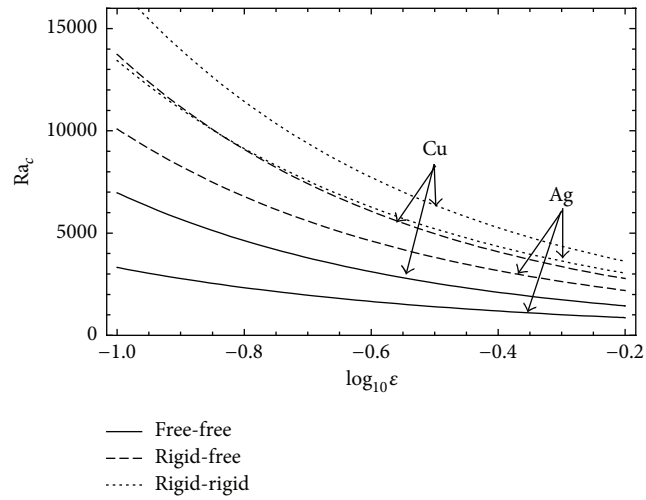

Figure 6: Plot of Ra_c with porosity for fixed Dr = 0.01 and $Q = 250$.

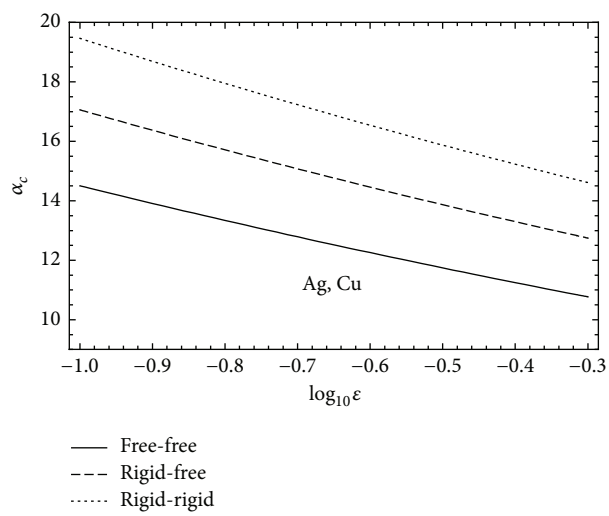

Figure 7: Plot of α_c with porosity for fixed Dr = 0.01 and $Q = 250$.

for stationary convection are independent of both Prandtl number and magnetic Prandtl number. Further increase in concentration of nanoparticles makes the system unstable by decreasing the critical value of thermal Rayleigh number. It is due to the fact that the parameters Rn, Le, N_a are all positive and the expression $\mathrm{Rn}(\mathrm{Le}/\varepsilon + N_a)$ appears with negative sign. However this can well be compensated by increasing the magnetic field and Darcy number. Further the stability of Cu-water nanofluid is higher than that of Ag-water nanofluid in the presence of magnetic field in porous medium.

7. Conclusions

Keeping in mind the application of flow through porous medium in the presence of magnetic field in geophysics particularly in underground reservoirs, enhanced oil recovery, soil sciences and in hydrology; we have investigated the impact of vertical magnetic field on a nanofluid layer using Darcy-Brinkman model for three different boundaries.

The presence of nanoparticles is an essential feature of these processes which introduces two additional effects: Brownian motion and thermophoretic forces. The analysis is carried out within the framework of linear stability theory, normal mode analysis, and single term Galerkin approximation. It is found that the instability sets in through the mode of stationary convection instead of oscillatory motions. The condition for the occurrence of oscillatory motions is that the two buoyancy forces (density gradient of nanoparticles and density variation due to heating from the bottom) must act in opposite directions. The value of critical Rayleigh number is decreased to an appreciable extent due to the presence of nanoparticles on the top whereas it increases for the bottom heavy configuration of nanoparticles. This is due to the fact that the top heavy arrangement works in tandem together with the known fact of thermal conductivity enhancement. While for bottom heavy arrangement of nanoparticles, buoyancy forces play antagonistic roles which results in generating oscillatory motions. It is this aspect that triggers the reversing trend in the two modes of configuration. From (28), it is clear that the suspension of nanoparticles in conventional fluids lowers the critical value of Rayleigh number as all the parameters Rn, Le, N_a are positive for the present configuration of nanoparticles and the expression $\mathrm{Rn}(\mathrm{Le}/\varepsilon + N_a)$ appears with negative sign. Thus the system with the distribution of nanoparticles at the top of the fluid layer is less stable as compared to regular fluid and bottom heavy distribution of nanoparticles. It is figured out that the mode of stationary convection is independent of both; the Prandtl number and magnetic Prandtl number. It has been found that the critical values of wave number and Rayleigh number $(\alpha_c, \mathrm{Ra}_c)$ exhibit a significant rise/fall with the rise in magnetic field parameter Q/porosity ε. Thus magnetic field is found to delay the onset of convection while porosity advances the same. With the increase in Darcy number, Ro_c increases while α_c decreases. It seems that the heat transfer characteristic of the nanofluid will get enhanced with the increase in Darcy number and magnetic field. Further, as one shifts from both-free boundaries to both-rigid boundaries, α_c shows an appreciable increase along with moderate increase in Ro_c. Thus, the system with both-rigid boundaries is found to have more stability as compared to rigid-free boundaries which in turn are more stable than free-free boundaries. Also, Cu-water nanofluid exhibits higher stability than Ag-water nanofluid in the present configuration of the system.

Competing Interests

The authors declare that they have no competing interests.

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