

# **Research Article**

# **Real-Time Frequency Adaptive Tracking Control of the WPT System Based on Apparent Power Detection**

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In wireless power transfer (WPT) systems, inverters are used to achieve high-frequency conversion of DC/AC, and their conversion efficiency and working frequency are key factors affecting the system's power transfer efficiency. In practical applications, many hardware issues, such as power transistor shutdown and loss, are the main reasons that affect the inverter conversion efficiency. On the other hand, the working frequency of WPT systems ranges from hundreds of kHz to a few MHz, and traditional voltage and current phasor estimation requires a very high sampling rate which is difficult to achieve. To overcome these limitations, this paper introduces a phase-shifting full bridge inverter using a zero-voltage switching (ZVS) soft switching technology to optimize the conversion efficiency of the inverter. Meanwhile, apparent power is introduced to detect the operating frequency and phase angle. Combined with an FPGA soft switching control strategy, this approach allows for the quick adjustment of the driving pulse of MOS transistors, as well as the voltage and current at the transmitting end, to a completely symmetrical state in real-time, effectively suppressing frequency offset and achieving efficient frequency tracking control and maximum efficiency tracking (MET) control of the WPT system. Through simulation and experiments, the ZVS soft switching technology has been achieved with the inverter control strategy, leading to improved conversion efficiency. The frequency offset that can be corrected can reach 0.1 Hz using the apparent power detection method, and the maximum transfer efficiency of the WPT system can reach 91%.

# 1. Introduction

With the rapid development of modern industry and technology, requirements for the convenience, intelligence, and safety of power supply are put forward. Wireless power transfer (WPT) technology has attracted widespread attention [1–3], and it has demonstrated application value in charging fields such as electric vehicles, consumer electronics, aerospace, and biomedical [4–7]. In WPT systems, changes in magnetic flux and current of the resonant coil are often caused by factors, such as environmental temperature, coil size, working conditions, and surface effects, resulting in differences between the theoretical resonant frequency and the actual resonant frequency, greatly reducing the system transfer efficiency. Therefore, frequency tracking is one of the key technologies to improve transfer efficiency by keeping the WPT system operating at resonant frequency.

To ensure that the WPT system operates at the resonant frequency point, there are three main control methods: coil topology optimization [8–10], dynamic compensation tuning [11–13], and frequency tracking. Compared to the other two methods, frequency tracking control is widely used in systems due to its easy implementation and fast response [14–21]. In [14], power allocation strategies are achieved targeted by adjusting the resonant frequency of the transmitting end to achieve stable output power that accurately counteracts load imbalance. However, there has been no in-depth research on the speed and accuracy in the

frequency adjustment process. Reference [15] proposes an inverter control strategy based on pulse frequency modulation (PFM) and pulse width modulation (PWM) to achieve automatic tuning of the WPT system, enabling optimal efficiency tracking of the WPT system. In the research of selftuning algorithms and phase detection methods, self-tuning and adaptive frequency tracking control (AFTC) of maximum transfer power for WPT systems through fuzzy PI algorithm is achieved in [16]. In [17], a variable frequency phase shift (VFPS) control strategy is proposed, calculating the instantaneous switching current of the power transistor to achieve zero-voltage switching (ZVS) operation with minimal frequency change. However, in practical applications, accurately collecting the instantaneous switching current can be challenging, which affects the control effect. Reference [18] proposes the frequency tracking algorithms to operate zero phase angle (ZPA) point for WPT systems based on second-order generalized integrator phase-locked loop (SOGI-FLL), reducing switching losses and improving the efficiency of inverters. Considering the manufacturing defects and aging issues of compensating capacitors in WPT systems, Reference [19] suggests applying the Sobol sensitivity method to rank the importance of the deviation of three capacitors on the transmission characteristics of the system. Additionally, a method to track the secondary resonance frequency to improve system performance is designed. A dual objective control strategy (DCS) is proposed in [20], adjusting the operating frequency of the highfrequency power amplifier and the duty cycle of the switching voltage to adapt to changes in distance between coils or output load changes, ensuring that the system can track the maximum power point under the optimal ZVS conditions. Reference [21] proposes an autonomous pulse frequency modulation scheme (APFM) for WPT battery charging; by extracting and processing the current signal on the primary side, the WPT system can adjust its output power by changing its duty cycle, achieving reliable ZVS.

Considering that many hardware issues, such as power transistor shutdown and loss in WPT systems, can lead to nonlinear drift of resonant frequency, the application of input voltage and current phasor estimation requires a high sampling rate when the inverter operating frequency reaches MHz, making it difficult to obtain the sampling rate in practical applications. This article proposes to use the input voltage and current of the WPT system to calculate the average values of active and reactive power, thereby obtaining the amplitude and phase of the input impedance. Then, in conjunction with the working mode of the inverter, an adaptive control strategy is introduced to quickly adjust the power MOS transistor driving pulse. This achieves efficient frequency tracking control, balances the current and voltage of the transmission circuit, and ensures that the system operates at maximum transfer efficiency in a resonant state.

The contributions of our work can be described as follows:

 Quantitative analysis was conducted on the source of the input ripple current of the inverter, providing a basis for further filtering out the low-frequency ripple current at the transmitting end of the WPT system and optimizing the frequency tracking algorithm

- (2) A symmetrical PWM with ZVS soft switching technology was designed to achieve precise control of the inverter power transistor, thereby reducing switching losses and improving conversion efficiency
- (3) An innovative method, combining FPGA to detect the operating frequency and phase angle of apparent power, was employed, and the MOS transistor driving pulse is quickly adjusted to achieve efficient frequency tracking and optimal power transmission

This paper is organized as follows. In Section 2, the overall circuit structure of the WPT system is introduced, and the topology structure and working mode of the inverter are analyzed. The real-time frequency adaptive tracking control method is presented in Section 3. Finally, experiments are presented in Section 4 to demonstrate the control performance of the real-time adaptive frequency tracking method. Section 5 provides the conclusions.

#### 2. Frequency Analysis of the WPT System

2.1. Transfer Efficiency of the WPT System. A transmitting and receiving series (SS) resonant WPT system is taken into consideration for the study. In Figure 1, a typical SS topology is illustrated, where  $u_1$  represents a high-frequency AC voltage source,  $i_1$  and  $i_2$  denote the high-frequency resonant currents of the transmitting and receiving loops, respectively.  $L_1$ ,  $C_1$  and  $L_2$ ,  $C_2$  are the inductors and capacitors of the transmitting and receiving loops, with  $R_1$  and  $R_2$ representing the internal resistances of  $L_1$  and  $L_2$ , respectively, and  $R_L$  representing the load resistance of the resonant circuit. The two loops are magnetically linked with coupling coefficient  $k = M/\sqrt{L_1L_2}$ , where M is the mutual inductance between the transmitting and receiving coils.

The following equations, derived from Kirchhoff's voltage law, can be obtained:

$$\begin{cases} \dot{U}_1 = Z_1 \dot{I}_1 - j\omega M \dot{I}_2, \\ 0 = Z_2 \dot{I}_2 - j\omega M \dot{I}_1. \end{cases}$$
(1)

Here,  $Z_1$  and  $Z_2$  are the equivalent impedances of transmitting and receiving ends, which satisfy the following equation:

$$\begin{cases} Z_1 = R_1 + j \left( \omega L_1 - \frac{1}{\omega C_1} \right), \\ Z_2 = R_2 + R_i + j \left( \omega L_2 - \frac{1}{\omega C_2} \right). \end{cases}$$
(2)

The transmitting and receiving coils have the same structure, that is,  $L_1 = L_2 = L$ ,  $R_1 = R_2 = R$ , and  $C_1 = C_2 = C$ .

Therefore, by selecting appropriate parameters for *L* and *C*, the resonant frequency of the circuits on both sides can be determined,  $f = 1/2\pi\sqrt{\text{LC}}$ .



FIGURE 1: The identical circuit model of the WPT system.

From (1) and (2), the currents generated in the transmitting and receiving loops can be calculated as follows:

$$\begin{cases} \dot{I}_{1} = \frac{Z_{2}\dot{U}_{1}}{Z_{1}Z_{2} + \omega^{2}M^{2}}, \\ \dot{I}_{2} = \frac{j\omega M\dot{U}_{1}}{Z_{1}Z_{2} + \omega^{2}M^{2}}, \end{cases}$$
(3)

where  $\omega$  is the resonant angular frequency, satisfying  $\omega = 2\pi f$ . When resonance occurs, that is,  $Z_1 = R$ ,  $Z_2 = R + R_L$ . The input power  $P_{\text{in}}$  and output power  $P_{\text{out}}$  of  $R_L$  can be expressed as follows:

$$\begin{cases} P_{\rm in} = \frac{U_1^2 (R + R_L)}{R^2 + RR_L + \omega^2 M^2}, \\ P_{\rm out} = \frac{\omega^2 M^2 U_1^2 R_L}{\left(R^2 + RR_L + \omega^2 M^2\right)^2}. \end{cases}$$
(4)

Here,  $U_1$  is the RMS value of the input voltage. The system TE  $\eta$  can be derived from (4) as follows:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{\omega^2 M^2 R_L}{(R + R_L) (R^2 + R R_L + \omega^2 M^2)}.$$
 (5)

According to (5), when the positions of the transmitting and receiving coils in the WPT system are fixed, the system's transfer efficiency is mainly related to the resonant angular frequency  $\omega$ . Once the resonant frequency deviates from its ideal value, the reactive power will increase and its transfer efficiency will be greatly reduced. The system's working frequency is closely related to the inverter's frequency. To mitigate the switching loss of the power transistor in the inverter and reduce the harmonic content of the voltage in the resonant network, this paper proposes a ZVS soft switching control strategy enabling the two bridge arms to work alternately at high and low frequencies while maintaining a balanced working state of the four power transistors in the inverter.

2.2. Analysis of the Inverter Topology. Figure 2 is a topological structure of the full bridge inverter.  $U_{in}$  is the power supply, the power tubes  $Q_a$  and  $Q_d$  form the leading bridge arm, and their driving voltages are  $u_{ga}$  and  $u_{gd}$ ;  $Q_b$  and  $Q_c$  form a hysteresis bridge arm, with driving voltages of  $u_{gb}$  and  $u_{gc}$ ;  $D_a \sim D_d$  are the freewheeling diode,  $C_a \sim C_d$  are the resonant capacitor,  $L_f$  and  $C_f$  form the inverter side filtering circuit, and  $D_1 \sim D_4$  form the full wave rectifier bridge at the receiving end.  $u_1$  is the voltage at the end of the RLC resonant network, which is a sinusoidal pulse width modulation (SPWM) wave controlled by a ZVS soft switching and synchronized with the modulation signal  $u_r$ .

Figure 3 shows the driving voltages  $u_{ga}$ ,  $u_{gb}$ ,  $u_{gc}$ , and  $u_{gd}$  of four MOS transistors, as well as the output signal diagrams of the H-bridge output terminal voltage  $u_{AB}$ . Here,  $u_c$  is the carrier signal, with amplitude  $U_c$  and frequency  $\omega_c$ ;  $u_r$  is a sine modulated signal with an amplitude of  $U_r$  and a working frequency of  $\omega_r$ . The following is a characteristic analysis of the input current  $i_{in}$  of the inverter to determine the relevant parameters of the carrier and modulation waves.

Ignoring the switching dead zone of the full bridge inverter, the double Fourier series can be represented as the following switching function [22]:

$$f(\omega_{c},\omega_{r}) = V\sin(\omega_{r}t) + \sum_{m=1}^{\infty} \frac{4}{m\pi} J_{0}\left(\frac{mV\pi}{2}\right) \sin\left(\frac{m\pi}{2}\right) \cos\left(m\omega_{c}t\right) + \sum_{m=1}^{\infty} \sum_{n=\pm 1,\pm 2\cdots}^{\infty} \frac{4}{m\pi} J_{n}\left(\frac{mV\pi}{2}\right) \sin\left(\frac{m+n}{2}\pi\right) + \cos\left(m\omega_{c}t + n\omega_{r}t - \frac{n\pi}{2}\right).$$

$$(6)$$

Here, the regulation  $V = U_r/U_c$ ; m = 1, 2, 3, ..., is the harmonic frequency of carrier signal  $u_c$ ;  $n = \pm 1, \pm 2, \pm 3 \cdots$ , is the harmonic order of the modulated wave  $u_r$ ; and  $J_n(x)$  is the Bessel function [23].

The output voltage  $u_{AB}$  of the inverter can be expressed as a function of the DC voltage  $U_{in}$ :

$$u_{\rm AB}(\omega t) = U_{\rm in} \times f(\omega_c, \omega_t). \tag{7}$$

Correspondingly, the inverter outputs the filtered inductance current  $i_{Lf}$ , which can be expressed as follows:

$$i_{\rm Lf}(\omega t) = \frac{u_{\rm AB}(\omega t)}{Z(\omega)},\tag{8}$$

where  $Z(\omega)$  is the output impedance of the inverter at the corresponding angular frequency  $\omega$ .







FIGURE 3: Power transistor drive signal and inverter output signal diagram.

Combining equations (6)–(8), the input current of the inverter can be expressed as follows:

$$i_{\rm in}(\omega_{\rm c},\omega_{\rm r}) = i_{\rm Lf}(\omega t)f(\omega_{\rm c},\omega_{\rm r}) = \frac{u_{\rm AB}(\omega t)f(\omega_{\rm c},\omega_{\rm r})}{Z(\omega)} = U_{\rm in}\left[M\sin(\omega_{\rm r}t) + \sum_{m=1}^{\infty}\sum_{n=\pm 1,\pm 2\cdots}^{\infty}\frac{4}{m\pi}J_{n}\left(\frac{mM\pi}{2}\right)\sin\left(\frac{m+n}{2}\pi\right) \times \cos\left(m\omega_{\rm c}t + n\omega_{\rm r}t - \frac{n\pi}{2}\right)\right]\left[\frac{M}{Z(\omega_{\rm r})}\sin\left(y - \phi(Z(\omega_{\rm r}))\right) + \sum_{m=1}^{\infty}\frac{4}{m\pi|Z(m\omega_{\rm c})|}J_{0}\left(\frac{mM\pi}{2}\right)\sin\left(\frac{m\pi}{2}\right) \times \cos\left(m\omega_{\rm c}t - \phi(Z(m\omega_{\rm c}))\right) + \sum_{m=1}^{\infty}\sum_{n=\pm 1,\pm 2\cdots}^{\infty}\frac{4}{m\pi|Z(m\omega_{\rm c}+n\omega_{\rm r})|}\right]$$

$$\times J_{n}\left(\frac{mM\pi}{2}\right)\sin\left(\frac{m+n}{2}\pi\right) \times \cos\left(m\omega_{\rm c}t + n\omega_{\rm r}t - \frac{n\pi}{2} - \phi(Z(m\omega_{\rm c}+n\omega_{\rm r}))\right)\right].$$
(9)

In (9),  $\emptyset(Z(\omega_x))$  is the impedance angle of the inverter output impedance at angular frequency  $\omega_x$ .

To ignore the high-frequency admittance in the inverter, sufficient carrier wave ratio should be ensured to ensure that the high-frequency output impedances  $Z(m\omega_c)$  and  $Z(m\omega_c + \omega_c)$ 

 $n\omega_r$ ) are much greater than the output impedance  $Z(m\omega_c)$  of the RLC resonant network. Therefore, in this system, the frequency of selected carrier signal  $u_c$  is 1 MHz, and the frequency of modulation signal is 100 kHz. Therefore, the input current in (9) can be further simplified as follows:

$$i_{\rm in}(\omega_{\rm c},\omega_{\rm r}) = \frac{U_{\rm in}V^2}{2|Z(\omega_{\rm r})|}\cos\left(\phi(Z(\omega_{\rm r})) - \frac{U_{\rm in}V^2}{2|Z(\omega_{\rm r})|}\cos\left(2\omega_{\rm r} - \phi(Z(\omega_{\rm r})) + \frac{2U_{\rm in}V}{|Z(\omega_{\rm r})|}\sum_{m=1}^{\infty}\sum_{n=\pm 1,\pm 2\cdots}^{\infty}\frac{1}{m\pi}J_n\left(\frac{mV\pi}{2}\right)\right)$$

$$\times \sin\left(\frac{m+n}{2}\pi\right)\left[\sin\left(m\omega_{\rm c} + (n+1)\omega_{\rm r} - \frac{n\pi}{2} - \phi(Z(\omega_{\rm r})) - \sin\left(m\omega_{\rm c} + (n-1)\omega_{\rm r} - \frac{n\pi}{2} + \phi(Z(\omega_{\rm r}))\right)\right]\right].$$
(10)

According to (10), the input current of the full bridge inverter mainly comprises four components: DC component, carrier subharmonic, second harmonic, and carrier side frequency harmonic. It is evident that the inverter is equivalent to a nonlinear link in the WPT system, and there are also a large number of second harmonics in the input current, in addition to carrier harmonics and edge frequency harmonics. While high-frequency harmonics can be relatively easily filtered out by high-frequency capacitors, the second harmonic, with its lower frequency and higher content, poses challenges for hardware-based filtering. In the ZVS soft-switching process, special attention should be given to second harmonic filtering.

To fully achieve high-frequency inverter, each power transistor is connected in parallel with a capacitor, significantly reducing the stresses of du/dt and di/dt in the circuit, so that the power transistor can easily achieve zero voltage shutdown.

2.3. Analysis of the Working Mode of Inverters. Due to the symmetry of the positive and negative half cycles, the positive half cycle is taken for an example to analyze the six operating modes of the full bridge inverter in the WPT system under ZVS soft switching control. During the positive half cycle, power tubes  $Q_a$  and  $Q_b$  operate at high frequencies, with  $Q_d$  always on and  $Q_c$  always off. The equivalent circuit diagrams of the six working modes are shown in Figures 4(a)-4(f).  $u_{ca}$  and  $C_b$ , respectively, and  $u_{Qa}$  and  $u_{Qb}$  represent the terminal voltages of capacitors  $C_a$  and  $C_b$ , respectively, and  $u_{Qa}$  and  $Q_b$ , respectively The theoretical working waveforms under each working mode are shown in Figure 5.

Mode 1  $(t_0 \sim t_1)$ : the inverter is in a steady state with  $Q_a$ and  $Q_d$  power transistors conducting,  $U_{Ca} = 0$ ,  $U_{Cb} = U_{in}$ , and the current  $i_{Lf}$  on the inductor  $L_f$  increases under the action of DC voltage  $U_{in}$ . When  $i_{Lf}$  reaches the peak current  $i_p$ , i.e., at time  $t_1$ ,  $Q_a$  is turned off, and the current in the RLC resonant network is  $i_{1-p}$ .

Mode 2  $(t_1 \sim t_2)$ : the power tube  $Q_a$  is turned off, and  $i_{Lf}$  is used to charge the bypass capacitor  $C_a$ , causing the voltage at both ends to rise rapidly and discharge  $C_b$ . Due to the very short time  $t_1 \sim t_2$ ,  $i_{Lf}$  has not changed

much and is approximately a constant current source. Until time  $t_2$ , the voltage at both ends of  $C_b$  drops to 0, and the antiparallel diode  $D_b$  conducts.

Mode 3  $(t_2 \sim t_3)$ : at time  $t_2$ ,  $D_b$  naturally conducts, causing  $Q_b$  to open at zero voltage. The inductance current  $i_{Lf}$  is in a continuous flow state until time  $t_3$ ,  $i_{Lf} = 0$ . In this modal process, to ensure that  $Q_b$  achieves ZVS, it is necessary to maintain a dead band time before  $Q_a$  is closed and  $Q_b$  is opened.

Mode 4  $(t_3 \sim t_4)$ : after the current  $i_{Lf}$  drops to 0 at time  $t_3$ , the inverter module enters a zero initial state energy storage state, and the current in  $Q_b$  increases from 0 in a positive direction, when  $Q_b$  is turned off at zero voltage,  $i_{Lf}$  increases in reverse to  $i_{Lf} - i_p$  at time  $t_4$ , and this mode ends.

Mode 5  $(t_4 \sim t_5)$ : after  $Q_b$  is turned off, the bypass capacitor  $C_a$  is discharged and  $C_b$  is charged. The time  $t_4 \sim t_5$  is also very short, and the negative current  $i_{\text{Lf}}$  remains basically unchanged. By time  $t_5$ ,  $U_{\text{Ca}}$  drops to 0.

Mode 6  $(t_5 \sim t_6)$ : at time  $t_5$ ,  $D_a$  naturally conducts, causing  $Q_a$  turn on at zero voltage. At this time, the negative current  $i_{Lf}$  gradually decreases to 0. The input voltage  $U_{in}$  returns to positive direction to provide energy storage of  $L_f$ . The system returns to mode 1, and the next mode cycle begins.

As an important part of the WPT system, the inverter seeks to identify an effective frequency tracking control algorithm by analyzing its workflow for one cycle, thereby reducing the losses of each section of the inverter and improving the transfer efficiency of the WPT system. The introduction of apparent power is employed to detect the operating frequency and phase angle, facilitating precise frequency tracking in the subsequent section.

# 3. Real-Time Adaptive Frequency Tracking Method

The block diagram of the real-time detection phase angle structure is illustrated in Figure 6. The input voltage  $u_1$  and current  $i_1$  are extracted, through the extraction module,



FIGURE 4: Main circuit diagram of the WPT system with constant output voltage control and MET control. (a) Mode 1. (b) Mode 2. (c) Mode 3. (d) Mode 4. (e) Mode 5. (f) Mode 6.



FIGURE 5: Theoretical working waveforms under various working modes.

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which comprises active power  $P_1$ , reactive power  $Q_1$ , and the amplitude of input voltage  $|U_1|$  of the WPT system. Combined with the FPGA frequency control word and phase accumulator, the MOS transistor driving pulse is rapidly adjusted through the ZVS soft switch control strategy to achieve efficient frequency tracking control and optimize the system's transfer efficiency.

3.1. Real Time Phase Angle Detection Method for Apparent Power. The input voltage  $u_1$  and current  $i_1$  are sinusoidal signals. The instantaneous active power can be calculated as follows:

$$p_1(t) = u_1(t) * i_1(t). \tag{11}$$

Considering  $u_1(t)$ ,  $i_1(t)$  contains harmonic components, its expression can be written as follows:

$$u_{1}(t) = \sum_{k=1}^{n} U_{1k} \cos(\omega_{k}t + \beta_{k}), i_{1}(t) = \sum_{k=1}^{n} I_{1k} \cos(\omega_{k}t + \varphi_{k}).$$
(12)

In equation (12),  $U_{1k}$ ,  $I_{1k}$  are the amplitudes of the first harmonic of  $u_1(t)$ ,  $i_1(t)$ ,  $\omega_k$  is the angular frequency, and  $\beta_k$ ,  $\varphi_k$  are the initial phase. Then,  $p_1(t)$  is represented as follows:

$$p_{1}(t) = \frac{1}{2} \sum_{k=1}^{n} U_{1k} \sum_{j=1, j \neq k}^{n} I_{1j} \cos\left[\left(\omega_{k} + \omega_{j}\right)t + \left(\beta_{k} + \varphi_{j}\right)\right] + \frac{1}{2} \sum_{k=1}^{n} U_{1k} \sum_{j=1, j \neq k}^{n} \left\{I_{1j} \cos\left[\left(\omega_{k} - \omega_{j}\right)t + \left(\beta_{k} - \varphi_{j}\right)\right]\right\} + \frac{1}{2} \sum_{k=1}^{n} U_{1k} I_{1k} \cos\left[2\omega_{k}t + \left(\beta_{k} + \varphi_{k}\right)\right] + \frac{1}{2} \sum_{k=1}^{n} U_{1k} I_{1k} \cos\left(\beta_{k} - \varphi_{k}\right).$$
(13)

The DC component of instantaneous power can be calculated as follows:

$$\overline{P} = \frac{1}{2} \sum_{k=1}^{n} U_{1k} I_{1k} \cos(\beta_k - \varphi_k).$$
(14)

The AC component of instantaneous power is represented as follows:

$$V = \frac{1}{2} \sum_{k=1}^{n} U_{1k} \sum_{j=1, j \neq k}^{n} I_{1j} \cos\left[\left(\omega_{k} + \omega_{j}\right)t + \left(\beta_{k} + \varphi_{j}\right)\right] + \frac{1}{2} \sum_{k=1}^{n} U_{1k} \sum_{j=1, j \neq k}^{n} \left\{I_{1j} \cos\left[\left(\omega_{k} - \omega_{j}\right)t + \left(\beta_{k} - \varphi_{j}\right)\right]\right\} + \frac{1}{2} \sum_{k=1}^{n} U_{1k} I_{1k} \cos\left[2\omega_{k}t + \left(\beta_{k} + \varphi_{k}\right)\right].$$
(15)

Therefore, the active power has the following expression:

$$P_{1}(t) = \frac{1}{T} \int_{0}^{T} (\overline{P} + V) d(t) = \frac{1}{T} \left( \int_{0}^{T} \overline{P}(t) dt + \int_{0}^{T} V(t) dt = \overline{P} \right).$$
(16)

From equation (16), it can be concluded that the DC component of instantaneous power is the active power. If an ideal low-pass filter is used to filter instantaneous power, active power can be obtained as follows:

$$P_{1}(t) \triangleq \frac{1}{T} \int_{t-T}^{T} p_{1}(t) d(t) \cong \text{LPF}\{p_{1}(t)\}.$$
(17)

In practical applications, an analog multiplier and LPF with a cut-off frequency of 2 \* f/8 are used, and the instantaneous reactive power  $q_1(t)$  can be expressed as follows:

$$q_1(t) = u_1\left(t - \frac{T}{4}\right) * i_1(t).$$
(18)

At this point, a phase shifter is required. Therefore,  $Q_1(t)$  can be calculated as follows:

$$Q_1(t) \triangleq \frac{1}{T} \int_{t-T}^{T} q_1(t) d(t) \cong \text{LPF}\{q_1(t)\}.$$
 (19)



FIGURE 6: Real time detection phase angle structure diagram.

Finally, the amplitude of the input voltage  $U_1$  of the WPT system can be obtained using a peak detector. The peak detector includes diodes, smooth RC, and LPF, which can be represented as follows:

$$|U_1| = \max\{|u_1(t')|\}, \quad t - T \le t' \le t.$$
 (20)

To calculate the impedance  $Z_1$  of the input terminal, the following equation is used:

$$S_1 = U_1 I_1^* = U_1 \left(\frac{U_1}{Z_1}\right)^* = \frac{|U_1|^2}{Z_1^*},$$
(21)

$$Z_{1} = \frac{\left|U_{1}\right|^{2}}{S_{1}^{*}} = \frac{\left|U_{1}\right|^{2}}{\left(P_{1} - jQ_{1}\right)},$$
(22)

where  $I_1^*, Z_1^*$  is the corresponding conjugate complex number of  $I_1, Z_1$ .

According to (22), it can be seen that only with active power  $P_1$ , reactive power  $Q_1$ , and the amplitude of input voltage  $|U_1|$ , the amplitude and phase angle of the input impedance  $Z_1$  of the WPT system can be calculated as follows:

$$\left|Z_{1}\right| = \frac{\left|U_{1}\right|^{2}}{\sqrt{P_{1}^{2} + Q_{1}^{2}}},$$
(23)

$$\angle Z_1 = \Delta \theta = \arctan\left(\sqrt{\frac{Q_1}{P_1}}\right). \tag{24}$$

3.2. Adaptive Frequency Tracking Control Strategy. The frequency tracking processing program is determined as the highest priority, and FPGA is selected as the digital

controller in this control strategy. The process of generating driving signals  $u_{ga} \sim u_{gd}$  is shown in Table 1, and the specific analysis is outlined as follows.

- (1) Initialization of related parameters: according to the fixed frequency of the WPT system resonance circuit, set the number of timing clock pulses to  $N_0$ .
- (2) Date collection and clock pulse number increment: the amplitude of active power P<sub>1</sub>, reactive power Q<sub>1</sub>, and input voltage |U<sub>1</sub>| are obtained by FPGA through sampling, and the phase difference ∠Z<sub>1</sub> of the input impedance Z<sub>1</sub> and its corresponding system clock pulse number ΔN are calculated.
- (3) Comparison of the clock pulse number increment with the preset threshold  $\delta$ : if it is greater than  $\delta$ , proceed to step (4) for PWM adjustment. If not, consider that the system can automatically eliminate the difference without correction, and return to step (2).  $\delta$  should be reasonably selected based on the primary side circuit of the phase-shifting full bridge transformer and the actual impedance phase difference.
- (4) Adjust the square wave signal. Performing calculations based on  $N = N_0 + \Delta N$ , the square wave signal MB is adjusted with the same frequency and phase of  $\angle Z_1 = 0$ .
- (5) FPGA generates an inverted signal MD based on the signal MB. According to signal MB, FPGA generates signal MA. First, determine the time  $T_{MA} = NDT_{clk}$  for MA to maintain high level; second, determine the dead time  $T_d = N_d T_{clk}$ , which is generally set to 80ns~100ns; when the falling edge of MB arrives, MA first maintains low

TABLE 1: Generating drive signal  $u_{ga} \sim u_{gd}$  using FPGA.

(1)	Initialize parameters: N <sub>0</sub>
	Monitors: $P_1, Q_1$ and $ U_1 $
(2)	Calculate: $Z_1$ and $\angle Z_1$ using (22) and (24)
	Obtain the corresponding count value $\Delta N$
	If $\Delta N > \delta$ , go to step (4);
(3)	Else: go to step (2);
	End if
(4)	Calculate: $N = N_0 + \Delta N$
(4)	Adjust signal: MB
	Generate inversion signal MD based on MB
(5)	Calculate: $T_{MA}$ , $T_d$
(3)	Generate signal MA based on MB
	Generate signal MC based on MD
(6)	Generate driving signals: $u_{\rm ga} \sim u_{\rm gd}$

level for a time period  $T_d$  and then sets it to high level; MA maintains a high level for a time period  $T_{\rm MA}$  and then sets it to a low level. In this process, the duty cycle during phase shift voltage regulation set to D,  $T_{\rm clk}$  is the reference system clock, and  $N_d$  is the number of  $T_{\rm clk}$  corresponding to the set dead time  $T_{\rm MA}$ . The generation process of MC is the same as that of MA, and its reference signal is MD.

(6) The signals MA, MB, MC, and MD are, respectively, converted into u<sub>ga</sub>~ u<sub>gd</sub> through the driving circuit and supplied to the inverter power transistor Q<sub>a</sub>~Q<sub>d</sub>.

# 4. Simulation and Analysis of the Frequency Tracking System

4.1. Simulation Results and Analysis. A model of the WPT system was established based on Figure 1 for the proposed ZVS soft switching technology phase-shifting full bridge inverter and adaptive frequency real-time tracking control algorithm in the MATLAB/Simulink simulation environment. The relevant parameters of the simulation model are consistent with those of the experimental device, as shown in Table 2. Here,  $\omega_c$  represents the carrier angular frequency, and *D* is the distance between the transmitting coil and the receiving coil.

4.1.1. The Simulation of ZVS Realization with Lagging Arm  $Q_C$ . The open critical waveform of ZVS realization with lagging arm  $Q_C$  can be observed in Figure 7, where  $u_{gc}$  is the driving waveform of  $Q_C$ , and  $U_{Qc}$  is the voltage waveform borne by  $Q_C$ . The dashed line 1 in Figure 7 indicates that before  $Q_C$  is opened, the pipe pressure drop  $U_{Qc}$  is 48 V. When  $Q_C$  is turned on, it is evident that  $U_{Qc}$  first drops to zero, and then the pulse voltage  $u_{gc}$  of the transistor rises, representing the process of zero voltage opening. It can be seen that before  $Q_C$  is turned off, the voltage dropping during  $Q_C$  conduction is approximately zero as indicated by the dashed line 2. When  $Q_C$  is turned off,  $Q_C$  receives the turning off pulse  $u_{gc}$  and returns to 48V, achieving zero voltage turning off.

4.1.2. Adaptive Frequency Tracking Simulation. In order to verify the adaptive ability of the frequency tracking algorithm when system parameters change, the capacitor  $C_1$  in the resonant circuit jumps from 12.41 nF to 15.32 nF, and the resonance frequency of the system changes from 100 kHz to 90 kHz. Here are the waveforms of  $u_1$  and  $i_1$  in transmitting side with or without adaptive tracking control, which are shown in Figures 8 and 9.

Figure 8 illustrates the waveform without frequency tracking control. It can be seen that when frequency deviation occurs, in comparison to Figure 9, the amplitudes of  $u_1$  and  $i_1$  have a significant decrease, and a phase difference between  $u_1$  and  $i_1$  appears, which reduces the power factor of the transmitter. Figure 9 shows the waveform when adaptive frequency tracking control is added. The peaks of  $u_1$  and  $i_1$  are positive negative symmetric, ensuring that  $u_1$  and  $i_1$  are in the same frequency and phase, with no obvious DC component. The adopted adaptive frequency tracking scheme in this experiment effectively adjusts the waveform of the voltage and current at the transmitting end to be completely symmetrical along the axes. This suppression of the generation of reactive power contributes to an improvement in the power transfer efficiency of the WPT system.

4.2. Experiment Results and Analysis. To further validate the proposed adaptive frequency tracking control algorithm, a WPT system experimental platform was designed, and the devices are shown in Figure 10. The system parameters are shown in Table 2. The experimental schematic diagram of the WPT system is showed in Figure 11.

The schematic implementation is briefly analyzed as follows.

The obtaining of the peak of input voltage  $|U_1|$ : first, the instantaneous  $u_1(t)$  is measured through a resistive voltage divider; the peak value of  $U_1$  is measured using an analog peak detector (PD) consisting of a fast recovery diode (SS14) and a capacitor. Then, an analog low-pass filter (LPF) is used to remove the input voltage's ripples.

The measuring of current  $i_1(t)$  using a current Hall sensor (ACS758LCB):  $u_1(t)$  and  $i_1(t)$  are multiplied using an analog multiplier (AD835), then the average value of active power  $P_1(t)$  is extracted by an analog low-pass filter (LPF).

Similarly, an analog circuit which contains an operational amplifier (OPA2134PA) and an RC filter are used to generate a signal  $u_1(t - T/4)$  by performing a phase shift of  $-\pi/2$  on  $u_1(t)$ . Then, an analog multiplier (AD835) is used to multiply it with  $i_1(t)$ , and the average value of reactive power  $Q_1(t)$  is extracted by using an analog LPF.

Combining with the adaptive tracking control strategy in Section 3.2, the collected data are analyzed by FPGA (EP1C3T144). The IR2110 module is selected as the driving circuit to achieve frequency tracking control of the system.

The experimental process and results analysis are presented as follows.

4.2.1. The Experiment of ZVS Realization with Lagging Arm  $Q_C$ . The voltage drop of the inverter MOS transistor is measured by a high-voltage probe, with a 10 times attenuation

Parameter	Value
U <sub>in</sub>	48 V
$R_1$	$0.482\Omega$
$R_2$	0.481 Ω
$C_1$	12.41 nF
ω <sub>c</sub>	6.28 M rad/s
$C_2$	12.42 nF
$L_1$	204.12 µH
$L_2$	203.96 µH
$f_0$	100 KHz
$R_{\rm L}$	50 Ω



FIGURE 7: The open critical waveform of ZVS realization with lagging arm  $Q_{\rm C}$ .



FIGURE 8: The waveform without adaptive frequency tracking algorithm.

factor. The critical waveform for opening the upper tube of the lagging arm  $Q_C$  is shown in Figure 12. It can be seen clearly that before  $Q_C$  is turned on, its pressure drop has basically decreased from 48 V to zero. The waveform of the shutdown process of  $Q_C$  is shown in Figure 13, demonstrating that when  $Q_C$  is turned off, the voltage drop of the tube increases slowly from zero, achieving zero voltage shutdown. Through simulation and experiments, it can be concluded that this design has achieved ZVS soft switching of the inverter, thereby improving the working efficiency of the inverter.

4.2.2. Experimental Verification of Adaptive Frequency Tracking. The experiment process is the same as the simulation, the capacitor  $C_1$  in the system jumps from 12.41 nF





FIGURE 9: The waveform with adaptive frequency tracking algorithm.

to 15.32 nF, and the resonance frequency of the system changes from 100 kHz to 90 kHz. The waveforms of  $u_1$  and  $i_1$  in transmitting ends without and with adaptive frequency tracking control are recorded, as shown in Figures 14 and 15.

From Figures 14 and 15, it is evident that the introduction of an adaptive frequency tracking algorithm results in  $u_1$  and  $i_1$  exhibiting good positive and negative half-axis symmetry, with no obvious DC component, indicating stable system operation. The adaptive frequency tracking algorithm used in this experiment continuously adjusts the voltage and current at the transmitting end to be completely symmetrical, effectively suppressing the generation of frequency deviation and reducing system losses.

Then, the capacitor  $C_1$  is adjusted from 15.32 nF to 12.41 nF, and the resonance frequency of the system changes from 90 kHz to 100 kHz. Figure 16 shows the tracking response curves between fuzzy PI adaptive control and the proposed method in this paper, when the inverter frequency is adjusted from 90 kHz to 100 kHz at t = 0.5 ms.

As shown in Figure 16, under the fuzzy PI adaptive tracking algorithm, the overshoot of the system is 8%, and the response time is about 1.3 ms. In contrast, under the tracking method proposed in this paper, the overshoot of the system is 6% and the response time is about 0.8 ms. Furthermore, it is confirmed that the proposed method can adjust the inverter drive signal in real-time based on the changing of  $\angle Z_1$ , when there is a significant change in the resonant frequency, ensuring that the system maintains a resonant state and exhibits fast dynamic response and good stability.

4.2.3. The Experimental Test of the System Transfer Efficiency. As previously mentioned, the maximum transfer power occurs at the resonant frequency, which is a function of the circuit resonant components  $(L_1, L_2, C_1, \text{ and } C_2)$  and the coupling factor k. [16, 24] When the WPT system is determined, the system transfer efficiency is mainly related to the operating frequency and transmission distance, with a corresponding relationship between the resonant operating frequency and transmission distance of the system [25]. When the distance between the transmitting coil and the receiving coil changes, the coupling coefficient k will change, and the system transfer efficiency will also change. When the transmission distance closely aligns with the distance corresponding to the resonance frequency of the



48VDC Regulated FPGA Controller Apparent Power Full-bridge Load Simulator Power Conversion Module Rectifier

FIGURE 10: The experimental platform of the WPT system.



FIGURE 11: Schematic diagram of the proposed method.



FIGURE 12: The opening waveform of lagging arm  $Q_c$ .



FIGURE 13: The shutdown waveform of lagging arm  $Q_c$ .



FIGURE 14: The waveform of  $u_1$  and  $i_1$  without adaptive control.



FIGURE 15: The waveform of  $u_1$  and  $i_1$  with adaptive control.

system, the transfer efficiency is the highest. By changing the distance between receiving and transmitting coils, the transfer efficiency of the WPT system with and without frequency tracking control is recorded in Figure 17.

The experimental results show that the adaptive frequency tracking control algorithm achieved by applying apparent power detection can adjust the working frequency of the inverter in real-time to ensure consistency with the system resonance frequency and adapt to any changes in coil distance. Among all the methods mentioned in this paper, not only does it exhibit robustness to coupling changes but it can also consistently provide global maximum transfer efficiency at any distance, showing excellent applicability. The adjustment time, resonance frequency, efficiency improvement, system complexity, and maximum transfer efficiency with different frequency tracking control methods in [16–18, 20, 21, 26, 27] and this paper are shown in Table 3.

The similar operating frequencies, system power, and distance ranges have applied to the abovementioned control methods. Compared with the frequency control method in [16], significant advantages in adjusting time and transfer efficiency after frequency detuning have been obtained in this paper. Although the system adjustment time and transfer efficiency in [17, 20, 21] are relatively close to the methods proposed in this paper, but the system design complexity is relatively high and the application is difficult to achieve. The zero crossing



FIGURE 16: Tracking response curves of the frequency tracking algorithms.



FIGURE 17: The transfer efficiency comparison curves.

Reference	Method	Adjustment time (ms)	Resonant frequency (kHz)	Efficiency improvement (%)	Complexity	Maximum efficiency, η (%)		
Zheng et al. [16]	AFTC	~300	85	3.6~4	Medium	86.9		
Hu et al. [17]	VFPS	Not mentioned	85	~4	High	>91		
Kim et al. [18]	PFM	2000	90	Not mentioned	High	~91		
Nasr et al. [20]	DCS	~11	767	~8	High	88		
Hua et al. [21]	APFM	Not mentioned	200	2.5	High	86.4		

Not mentioned

~200

<1

TABLE 3: The summary of various indices for frequency tracking methods.

85

37~161.2

100

comparison and phase detector are the main methods to achieve zero phase angle frequency tracking through inverter frequency modulation in [18, 26, 27], but it is difficult to handle multiple current zero crossing within the same cycle when dealing with over coupling conditions; as a result, there is no

PFM (simulation)

PFM

AFTC

Matsuura et al. [26]

Yang et al. [27]

This paper

significant improvement in system's adjustment time and transfer efficiency. Compared with the mentioned methods, the proposed adaptive frequency tracking method in this paper has faster tracking speed and better correction ability in dealing with the problem of frequency detuning.

High

Medium

Medium

85

~70

91

Not mentioned

2.3

4~5

## 5. Conclusions

Adaptive frequency tracking of a full bridge phase-shifting inverter is achieved by real-time acquisition of apparent power and calculation of resonant circuit impedance angle in this paper, combined with ZVS soft switching technology. The proposed frequency tracking method can not only solve the problem of sampling rate limitation but also has a wide frequency adjustment range, fast tracking response, and high accuracy. The highest transfer efficiency of the WPT system can always be maintained through the theoretical and experimental results analysis.

Although this article has conducted in-depth and systematic research on the frequency tracking control key technologies in the WPT systems, there are still many issues that need further research due to the author's level and time, which can be summarized into the following two aspects:

- (1) This paper adopts the fundamental harmonic analysis method when analyzing inverters and designing frequency control methods. In the situation that the quality factor of the resonant network is low, or output power changes significantly, the accuracy decreases with this analysis method. Therefore, methods based on time-domain analysis with the design of frequency tracking still need to be explored.
- (2) Constant voltage output control has important application value in WPT systems. FPGA soft switching control strategy is combined to achieve efficient frequency tracking control in this paper, but the constant voltage output tracking control has not been considered. Therefore, an improved algorithm with adjustable PWM pulse duty cycle can be the next research object.

## **Data Availability**

The data that support the findings of this study are available from the corresponding author upon reasonable request.

#### **Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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