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Research Article

A Fuzzy Control Strategy to Synchronize Fractional-Order Nonlinear Systems Including Input Saturation

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One of the most important engineering problems, with numerous uses in the applied sciences, is the synchronization of chaos dynamical systems. This paper introduces a dynamic-free T-S fuzzy sliding mode control (TSFSMC) method for synchronizing the different chaotic fractional-order (FO) systems, when there is input saturation. Using a new definition of fractional calculus and the fractional version of the Lyapunov stability theorem and linear matrix inequality concept, a Takagi–Sugeno fuzzy sliding mode controller is driven to suppress and synchronize the undesired behavior of the FO chaotic systems without any unpleasant chattering phenomenon. Finally, an example of synchronization of complex power grid systems is provided to illustrate the theoretical result of the paper in real-world applications.

1. Introduction

1.1. Introductory Remarks. With a history dating back more than 3 centuries, fractional calculus enables a more in-depth examination of natural phenomena. In FO differential equations, the order of differentiation [1] can be a non-integer. FO systems have garnered a great deal of attention in the past two decades from scientists and engineers who choose for modeling of numerous phenomena using FO Systems. In fields such as electrical systems [2], neural networks [3], quantum mechanics [4], aerospace sciences [5], economical systems [6], medical sciences [7], emotional sciences [8], psychological sciences [9], etc., the role of FO systems is extensively documented.

This is because of their oscillatory properties as well as their extreme sensitivity to the values they start with. In addition, according to several studies, the majority of FOSs display unpredictable behavior because of the oscillatory characteristics they possess and their extreme susceptibility to initial values. As a direct consequence of this, academicians have focused their efforts on the development of alternative methods for synchronizing and maintaining chaotic FO systems [10]. In this context, several different control strategies have been proposed, including the sliding mode controller [11, 12], the adaptive controller [10, 13], the fuzzy controller [14–16], the PID controller [17], the observer controller [18], and the optimal controller [19] to control and synchronize of chaotic FO Systems.

The T-S fuzzy technique has recently become one of the most popular methods for coordinating and regulating nonlinear systems, both in theory and in reality. In fact, the T-S fuzzy method uses fuzzy weight and cost functions as tools to transform the nonlinear system into a linear equivalent. The T-S flexible method has benefits like a good theory study, usability, and durability.

The sliding mode control (SMC) methodology has quickly become one of the most well-liked control methodologies in recent times, both in terms of its theoretical foundations and its practical applications. Typically, the SMC divides into two parts described as follows:

(1) The development of an adequate and stable sliding surface

(2) To generate control signals so that the chaotic trajectories of the FO systems are suppressed and remain on the specified sliding surface

1.2. Survey of the Relevant Literature. The T-S fuzzy method has been adopted by many academics as a means for stabilization and synchronization of the FO nonlinear systems. For instance, in [20], a model free state feedback strategy is designed based on a cost guaranteed function, in the T-S fuzzy space topology. According to [21], a group of T-S fuzzy delayed nonlinear complex networks can be synchronized via a fuzzy memory management method. In [22, 23], fuzzy observer-based state-feedback methods are designed to control uncertain nonlinear MIMO nonlinear systems. In [24], to gain the control of the heart rate of runners, a discrete-time observer-based state feedback technique is proposed. Wu et al. [25] suggest a flexible T-S fuzzy control algorithm to be a solution to the synchronization challenge facing FO fuzzy nonlinear unpredictable systems with delay. A finite-time fuzzy control method to synchronize a collection of T-S fuzzy riotous delayed artificial neural networks has been developed by the authors of [26]. In addition, new data-based fuzzy techniques have been developed in [27, 28] to synchronize and regulate Markovian jump systems as well as complicated distributed systems. In the paper referred to as [29], a novel controller for synchronizing the hyperchaos networks with application in encrypted communication for nonideal networks is suggested. This controller relies on the T-S fuzzy model and the LMI technique. An adaptive T-S fuzzy technique has been given, and the problem of synchronization failure in chaos gyrostat is taken into consideration and discussed in [30]. The method makes use of a wavelet-based model. A switching control strategy is presented as a solution to synchronize problems in the paper referred to as [31]. This approach is employed to synchronize a collection of T-S fuzzy artificial complex networks which include coupled-delayed.

The FO integral fuzzy SMC strategy is suggested in [32] to stabilize uncertain FO nonlinear systems exposed to uncertainties and external. A strong FOSMC is created in [33] using the frequency distribution model to error-control the chaotic FO Jafari–Sprott systems. In [34], a finite-time FO recursive SMC method is developed for unknown linear/nonlinear systems. In [35], homogeneous controllers are proposed to synchronize FO chaotic networks, based on utilizing the properties of the Presnov decomposition. The authors of [36] have addressed adaptive feedback controllers for complete and finite-time synchronization for a class of FO fuzzy neural networks. The challenge of synchronization of FO chaotic systems is considered in [37] using a nonsingular fuzzy-based SMC technique. In [38], a reliable SMC approach based on neural networks is recommended for the finite-time synchronization of uncertain hyperchaotic FO systems. A controller for the synchronization of both transmitter and receiver systems is created in [39]. This control is based on a four-dimensional nonlinear FO hyperchaotic system with an external disturbance. In [40, 41], adaptive terminal SMC methods are designed for the synchronization of FO uncertain chaotic systems in a finite time. In [42], an adaptive neural controller has been presented to stabilize unknown FO delayed systems in the presence of input saturation as nonlinearity. The authors of [43] have considered an intermittent control FT synchronization problem for FO complicated dynamical neural networks.

But generally, these mentioned research works have at least one of the following drawbacks:

(1) The Caputo fractional derivative, which has been used in almost all of these works, has been contested by a number of researchers who find it not adequate for pseudo-state-space developments because it does not permit taking into account the physical behavior of the system (with a fractional-order system, all the system past must be taken into account to define its future, even at time $t = 0$).

(2) The majority of these works have contemplated the synchronization of two identical FOSs; this is despite the fact that in practice, it is extremely uncommon for anyone to have access to two identical FOSs.

(3) The linear/nonlinear components of the systems are completely utilized in the design of the control schemes.

(4) Undesirable chattering phenomena happen when the SMC controllers have operated.

(5) Most of these works have considered a simple definition of the systems without the model of uncertainties, external distributions, and input saturations.

Therefore, the abovementioned disadvantages motivate us to combine the FO edition of Lyapunov-stability-theorem (LST) with the LMI and T-S fuzzy theory for designing a chattering-free T-S fuzzy SMC synchronization technique, which is both dynamic-free and robust against uncertainties.

1.3. Contribution and Motivation. According to the aforementioned talks, it is important to create and propose a no-chatter T-S fuzzy SMC technique (TSFSMC) to synchronize complicated FO chaotic systems in the existence of system uncertainties, extrinsic disturbances, and input saturation. In addition, the no-chatter TSFSMC technique has not been sufficiently studied to synchronize FO chaotic systems until this point, so the primary goal of this research is to take this into consideration. Furthermore, input saturation is another crucial nonlinearity in the construction of practical controllers. Because of input saturation, the control energy does not increase beyond a certain point. Therefore, it significantly lowers energy waste in the control system. Also, utilizing a new definition of the noninteger derivative ensures us about the results.

As a result, this work suggests that constructing a no-chatter TSFSMC approach is the best way to address the synchronization problem of the FO chaos systems in the face of the system's uncertainties, outside disturbances, and input saturation. An easy-to-design and simple-to-use sliding surface is initially offered based on the FO integration concept. Then, the LST's FO version is used to create a suitable dynamic-free T-S fuzzy control strategy to ensure the sliding motion.

It should be made clear that neither linear nor nonlinear phrases of dynamical components of the system were utilized in the construction of this FTSMC technique. To demonstrate the usefulness and effectiveness of the recommended dynamic-free FTSMC approach, certain simulations are also provided as examples.

In conclusion, the following is a summary of the most significant motivations and contributions made by this study:

(1) Developing a no-chatter TSFSMC approach with the goal of synchronizing a large area of complicated and disorganized FO chaotic systems based on a new definition of fractional calculus with the use of an efficient continuous function.

(2) The suggested TSFSMC is resilient and can mitigate the impacts of system uncertainties as well as external disturbances or input saturation.

(3) To obtain trustworthy findings regarding the global and asymptotical stability of the coordinated closed-loop chaotic FO systems, the linear matrix inequality (LMI) and FO type of the LST were used.

(4) When applied to real-world issues, the suggested TSFSMC technique works better than other methodologies.

1.4. The Outline of the Paper's Structure. The structure of this paper can be observed in the following statements: Section 2 provides preliminary and fundamental concepts concerning the FO calculus and the FO systems. The problem description of synchronization of the FO chaotic system is presented in Section 3. Then, a dynamic-free TSFSMC technique is designed to solve the synchronization problem in Section 4. After that, in Section 5, applied examples are illustrated to show the effectiveness and efficiency of the desired TSFSMC method in practice. Finally, the obtained results, consequences, and future plans are covered in detail in Section 6.

2. Basic Concepts

Definition 1 (See [44]). Let $H(t)$ be a continuous function in R. Then, the FO integral defined by Riemann-Liouville for $G(t)$ is represented as follows:

$$\begin{aligned} I_{0,t}G(t) &= D_{0,t}^{-\beta}G(t) \\ &= \frac{1}{\Gamma(\beta)}\int_{t_0}^{t}[zwj]\,G(\vartheta)(t-\vartheta)^{\beta-1}d\vartheta, \end{aligned} \tag{1}$$

where t_0 shows the beginning time, $\Gamma(.)$ is the Gamma function, and β is the order of derivative.

Definition 2 (See [45]). Suppose we have the following function $G(t)$:$[0,\ \infty) \longrightarrow R$, and then the conformable fractional derivative of f order is defined by

$$D^{\beta}G(t) = \lim_{h\longrightarrow 0}\frac{G\left(t+he^{(\beta-1)t}\right)-G(t)}{h}. \tag{2}$$

For any $t>0$, $\beta\in(0,1)$.

Feature 1 [45]: If $w\in R$ is a genuinely constant number, then one gets $D^{\beta}w=0$.

Feature 2 [45]: At any point in $t>0$, let $0<\beta\leq 1$ be true and f and g be differentiable. Then,

$$D^{\beta}(f(t).g(t)) = f(t)D^{\beta}(g(t)) + g(t)D^{\beta}(f(t)). \tag{3}$$

Definition 3 (See [46]). Suppose that $W\in R^{n\times n}$ and I be the identity matrix. Then, the matrix sign function has the following definition:

$$\begin{aligned} sgn_{(l)}(W) = {}&\left[(I_{n\times n}+W)^{l}-(I_{n\times n}-W)^{l}\right] \\ &\cdot\left[(I_{n\times n}+W)^{l}+(I_{n\times n}-W)^{l}\right]^{-1}, \end{aligned} \tag{4}$$

where in l is the order of the approximation and $sgn_{(l)}(W)\in[-1,1]$.

Also, for any sliding surface like $S(t)\in R^{m}$

$$\begin{aligned} \mathrm{sgn}_{(l)}(S(t)) = {}&\left[(I+S(t))^{l}-(I-S(t))^{l}\right] \\ &\cdot\left[(I+S(t))^{l}+(I-S(t))^{l}\right]^{-1}. \end{aligned} \tag{5}$$

Lemma 4 (See [47]). *The following inequality is true for constant matrices G and E and a symmetric constant matrix S of acceptable dimensions:*

$$S + \mathrm{DGE} + E^{T}G^{T}D^{T} < 0. \tag{6}$$

If and only if for any $\varepsilon>0$

$$S + \left[\varepsilon^{-1}E^{T}\quad \varepsilon D\right] + \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} \varepsilon^{-1}E \\ \varepsilon D^{T} \end{bmatrix} < 0, \tag{7}$$

where in G fulfills $G^{T}G\leq R$.

Lemma 5 (See [48]). *Given any three matrices $X, Y,$ and Z of proper dimensions such that $Z>0$, we get the following:*

$$X^{T}Y + XY^{T} \leq X^{T}Z^{-1}X + Y^{T}Z^{-1}Y. \tag{8}$$

Theorem 6 (See [49]). *Let $\beta\in(0,1)$, and assuming that the FO system $D^{\beta}\gamma(t)=g(\gamma,t)$ satisfies the Lipschitz requirement, including $\gamma=0$ for an equilibrium-point. Also, suppose that there are class-K functions $k_1, k_2,$ and k_3 with a Lyapunov-function $V(t,\gamma(t))$ satisfying*

$$\begin{aligned} &\mathrm{k}_1(\|\gamma\|) \leq \mathrm{V}(\mathrm{t}, \gamma) \leq \mathrm{k}_2(\|\gamma\|), \\ &\mathrm{D}^{\mathrm{m}}\mathrm{V}(\mathrm{t}, \gamma(\mathrm{t})) \leq -\mathrm{k}_3(\|\gamma\|), \end{aligned} \tag{9}$$

where $0 < m < 1$. Following that, the equilibrium point of the system $D^{\beta}\gamma(t) = g(\gamma, t)$ will reach a state of asymptotically stable behavior.

2.1. Notation. The symbol * will be utilized for symmetric elements of a symmetric-matrix. Moreover, OT means orthogonal transformation.

3. The Description of the Problem and the T-S Fuzzy Forming

Here, first, an explanation of the issue will be provided, and then T-S fuzzy description of the problem is going to be taken into consideration.

3.1. Description of the Problem. That which follows is the outcome of two n-dimensional nonlinear structures with indeterminate drives and unpredictable FO response structure:

Drive system:

$$D^{\beta}x(t) = (A + \Delta A)(x, t),\ i = 1, \ldots, n. \tag{10}$$

Response system:

$$D^{\beta}y(t) = (B + \Delta B)(y, t) + \phi(u(t)), \tag{11}$$

where $\beta \in (0, 1)$, $x(t)$ and $y(t)$ are in $R^{n\times 1}$ and are the vectors of states of the system, and A and B determine the matrixes of the system structure. Furthermore, ΔA and ΔB show the uncertainties and external disturbances.

Moreover, $u(t)$ denotes the controller and $\phi(u(t)) = [\phi_1(u_1(t)), \ldots, \phi_n(u_n(t))]^T$ is the actuator saturation, which for $i = 1, \ldots, n$ are introduced as

$$\phi_i(u_i(t)) = u_i(t) + \Delta u(u_i(t)),\ i = 1, \ldots, n, \tag{12}$$

where

$$\Delta u(u_i(t)) = \begin{cases} u^- - u_i(t), & \text{if } u_- > u_i(t), \\ (\theta - 1)u_i(t), & \text{if } u^+ < u_i(t) < u^-,\ i = 1, \ldots, n, \\ u^+ - u_i(t), & \text{if } u_i(t) \geq u_+, \end{cases} \tag{13}$$

where $u^+, u_+ \in R^+$, and $u^-, u_- \in R^-$ represent the ranges of the measured-saturation function, while $\theta \in R^+$ represents the measured-saturation slope.

Now, error system's structure will be introduced as follows:

$$\begin{aligned} e(t) &= X(t) - Y(t), \\ e(t) &= X(t) - Y(t) \\ &= [x_1(t) - y_1(t), x_2(t) - y_2(t), \ldots, x_n(t) - y_n(t)]^T \\ &= [e_1(t), e_2(t), \ldots, e_n(t)]^T. \end{aligned} \tag{14}$$

In accordance with this, the error structure to synchronize scenario is carried out as follows:

$$\begin{aligned} D^{\alpha}e(t) &= D^{\alpha}X(t) - D^{\alpha}Y(t) \\ &= (A + \Delta A)(x, t) - (B + \Delta B)(y, t) - \phi(u(t)) \\ &= (\Lambda + \Delta\Lambda)(x, y, t) - \phi(u(t)). \end{aligned} \tag{15}$$

The main objective of this paper is to propose a T-S fuzzy method that is plausible and modifiable in such a way that

$$\lim_{t\longrightarrow\infty}\|e_i(t)\| = \lim_{t\longrightarrow\infty}\|y_i(t) - x_i(t)\| = 0, i = 1, 2, \ldots, n. \tag{16}$$

Constructing equation (14) is essential in order to solve the synchronization problem that occurs between equations (10) and (11).

3.2. Development of the T-S Fuzzy Algorithm to FO Systems. This section will focus on the formulation of the synchronizing problem for the T-S fuzzy uncertain error FO system (15). Suppose that the following rule is a model that can be used for the complex FO error system (15) with saturation input nonlinearities.

Plant Rule j: IF ϑ_1 be ρ_{j1}, ϑ_2 be ρ_{j2}, … and ϑ_k be ρ_{jk}, THEN.

$$D^{\alpha}e(t) = \left(\Lambda_j + \Delta\Lambda_j\right)(x, y, t) - \phi_j(u(t)), \tag{17}$$

where $\vartheta_1, \vartheta_2, \ldots, \vartheta_k$ indicates that as a given hypothesis variable, ρ_{jm} for $j = 1,2, \ldots, k, i = 1,2, \ldots, n$ represents the fuzzy-set, n represents the elements of plant rules, and n represents the total number of plant rules; $\phi_j(u(t))$ means the input saturation, Λ_j shows the matrix of known constants, and $\Delta\Lambda_j$ is the matrix of undetermined uncertainty factors caused by an exogenous perturbation. The complete output of the fuzzy system can be broken down into the following categories:

$$D^{\alpha}e(t) = \frac{\sum_{j=1}^{n}\omega_j(\theta(t))\left\{\left(\Lambda_j + \Delta\Lambda_j\right)(x, y, t) - \phi_j(u(t))\right\}}{\sum_{j=1}^{n}\omega_j(\theta(t))}, \tag{18}$$

$$\Longrightarrow D^{\alpha}e(t) = \sum_{j=1}^{n}h_j(\theta(t))\left\{\left(\Lambda_j + \Delta\Lambda_j\right)(x, y, t) - \phi_j(u(t))\right\}, \tag{19}$$

where $\theta(t) = [\vartheta_1, \vartheta_2, \ldots, \vartheta_k]^T$, $\omega_j(\theta(t)) = \prod_{i=1}^{k}\rho_{ji}(\vartheta_i(t))$, and $h_j(\theta(t)) = \omega_j(\theta(t))/\sum_{j=1}^{n}\omega_j(\theta(t))$.

Scalars $\rho_{ji}(\vartheta_i(t))$ is the fuzzy set of membership of $\vartheta_i(t)$ in ρ_{ji}. Here, it is supposed that $\omega_j(\theta(t)) \geq 0$, thus $\sum_{j=1}^{n}\omega_j(\theta(t)) \geq 0$ and $h_j(\theta(t)) \geq 0, j = 1,2, \ldots, n$ and $\forall t \geq 0, \sum_{j=1}^{n}\omega_j(\theta(t)) = 1$.

4. Main Results

Here, the sliding surface will be introduced based on the TS fuzzy method and LMI inequality approach. Then, the controller methodology is going to design, and the analytical result will be presented.

We consider that the submatrix K_i is a full rank m, and we suppose that there is an OT matrix H in such a way that

$$\mathrm{HK} = \begin{bmatrix} 0_{(n-m)\times m} \\ \overline{K} \end{bmatrix}, \tag{20}$$

where in $H = \mathrm{col}\{\Omega_1^T \quad \Omega_2^T\}$, $\Omega_1 \in R^{n\times m}$, and $\Omega_2 \in R^{n\times(n-m)}$ are unitary matrices and $\Omega = [\Omega_2\,\Omega_2]$. Also, $\overline{K} \in R^{m\times m}$ shows a nonsingular matrix, and let $\overline{K}$ be decomposed into the singular value subsets: $\overline{K} = \Omega \begin{bmatrix} \Upsilon_{m\times m} \\ 0_{(n-m)\times m} \end{bmatrix} W^T$ such that $\Upsilon_{m\times m}$ is a positive diagonal matrix.

Now, if one defines $\sigma(t) = He(t) = \begin{bmatrix} \sigma_1(t) \\ \sigma_2(t) \end{bmatrix}$, then following relation can be derived from (19):

$$D^\alpha \sigma_1(t) = \sum_{j=1}^{n} h_j(\theta(t))\left\{\left(\overline{\Lambda}_{11j} + \Delta\overline{\Lambda}_{11j}\right)\sigma_1(t) - \left(\overline{\Lambda}_{12j} + \Delta\overline{\Lambda}_{12j}\right)\sigma_2(t)\right\}, \tag{21}$$

$$D^\alpha \sigma_2(t) = \sum_{j=1}^{n} h_j(\theta(t))\left\{\left(\overline{\Lambda}_{21j} + \Delta\overline{\Lambda}_{21j}\right)\sigma_1(t) - \left(\overline{\Lambda}_{22j} + \Delta\overline{\Lambda}_{22j}\right)\sigma_2(t) + \overline{K}F_j\phi_j(u(t))\right\}, \tag{22}$$

where in $\sigma_1(t) \in R^{n-m}$, $\sigma_1(t) \in R^m$, $\overline{\Lambda}_{11j} = \Omega_2^T\Lambda_j\Omega_2$, $\overline{\Lambda}_{12j} = \Omega_2^T\Lambda_j\Omega_1$, $\overline{\Lambda}_{21j} = \Omega_1^T\Lambda_j\Omega_2$, $\overline{\Lambda}_{22j} = \Omega_1^T\Lambda_j\Omega_1$, $\Delta\overline{\Lambda}_{11j} = \Omega_2^T P_j L_j(t)\mathrm{Q}_j\Omega_2$, $\Delta\overline{\Lambda}_{12j} = \Omega_2^T P_j L_j(t)\mathrm{Q}_j\Omega_1$, $\Delta\overline{\Lambda}_{21j} = \Omega_1^T P_j L_j(t)\mathrm{Q}_j\Omega_2$, and $\Delta\overline{\Lambda}_{22j} = \Omega_1^T P_j L_j(t)\mathrm{Q}_j\Omega_1$.

Based on (21) and (22), the sliding surface is designed as

$$S(t) = \mathfrak{S}\sigma_1(t) + \sigma_2(t). \tag{23}$$

It is clear that the requirement $s(t) = 0$ will be satisfied whenever there is sliding motion; consequently,

$$\begin{aligned} S(t) = 0 \longrightarrow \sigma_2(t) \\ = -\mathfrak{S}\sigma_1(t). \end{aligned} \tag{24}$$

So, in (21), one gets.

$$\begin{aligned} D^\alpha \sigma_1(t) = \sum_{j=1}^{n} h_j(\theta(t))\big\{\big[\left(\overline{\Lambda}_{11j} + \Delta\overline{\Lambda}_{11j}\right) \\ +\left(\overline{\Lambda}_{12j} + \Delta\overline{\Lambda}_{12j}\right)\mathfrak{S}\big]\sigma_1(t)\big\}. \end{aligned} \tag{25}$$

Theorem 7. *The sliding surface (23) ensures that the sliding-mode dynamic motion described in equation (25) is asymptotically stable if for any constant $p > 0$, there exist positive symmetric matrices $\overline{M}$ and N that fulfill the following LMI:*

$$\begin{bmatrix} \Xi & \left(\overline{M}\Omega_2^T - N^T\Omega_1^T\right)\mathrm{Q}_j^T & \Omega_2^T P_j \\ * & -\overline{\xi}^{-1}I & 0 \\ * & * & -\overline{\xi}^{-1}I \end{bmatrix} < 0, \; j = 1, 2, \ldots, n. \tag{26}$$

This is such that $\overline{\xi}_j = \xi_j^2$ and $N = \mathfrak{S}\overline{M}$.

$$\begin{aligned} \Xi &= \left(\overline{\Lambda}_{11j}\overline{M} - \overline{\Lambda}_{12j}N\right) + \left(M\overline{\Lambda}_{11j}^T - N^T\overline{\Lambda}_{12j}^T\right) \\ &= \left(\overline{\Lambda}_{11j}\overline{M} - \overline{\Lambda}_{12j}N\right) + \left(\overline{\Lambda}_{11j}\overline{M} - \overline{\Lambda}_{12j}N\right)^T. \end{aligned} \tag{27}$$

Proof. According to Theorem 6, a Lyapunov function is proposed as follows:

$$\mathrm{V}(\mathrm{t}) = \sigma_1^T(t)M\sigma_1(t), M > 0. \tag{28}$$

Using (3) in feature 2 and (27), one obtains

$$D^\alpha \mathrm{V}(\mathrm{t}) = \sigma_1^T(t)MD^\alpha\sigma_1(t) + \left(D^\alpha\sigma_1(t)\right)^T M\sigma_1(t). \tag{29}$$

According to (25),

$$\sigma_1^T(t)MD^\alpha\sigma_1(t) = \sigma_1^T(t)M\left[\sum_{j=1}^{n} h_j(\theta(t))\big\{\big[\left(\overline{\Lambda}_{11j} + \Delta\overline{\Lambda}_{11j}\right) - \left(\overline{\Lambda}_{12j} + \Delta\overline{\Lambda}_{12j}\right)\mathfrak{S}\big]\sigma_1(t)\big\}\right]. \tag{30}$$

Therefore,

$$D^{\alpha}\mathrm{V}(\mathrm{t}) = \sigma_1^T(t)M\left[\left[\sum_{j=1}^{n} h_j(\theta(t))\left\{\left[\left(\overline{\Lambda}_{11j} + \Delta\overline{\Lambda}_{11j}\right) - \left(\overline{\Lambda}_{12j} + \Delta\overline{\Lambda}_{12j}\right)\mathfrak{S}\right]\sigma_1(t)\right\}\right] + \left[\sum_{j=1}^{n} h_j(\theta(t))\left\{\left[\left(\overline{\Lambda}_{11j} + \Delta\overline{\Lambda}_{11j}\right) - \left(\overline{\Lambda}_{12j} + \Delta\overline{\Lambda}_{12j}\right)\mathfrak{S}\right]\sigma_1(t)\right\}\right]^T\right]M\sigma_1(t) < 0. \tag{31}$$

Then,

$$D^{\alpha}\mathrm{V}(\mathrm{t}) = \sigma_1^T(t)\mu\,\sigma_1(t) < 0, \tag{32}$$

where in

$$\mu = M\left(\overline{\Lambda}_{11j} - \overline{\Lambda}_{12j}\mathfrak{S}\right) + \left(\overline{\Lambda}_{11j} - \overline{\Lambda}_{12j}\mathfrak{S}\right)^T M + M\left(\Delta\overline{\Lambda}_{11j} - \Delta\overline{\Lambda}_{12j}\mathfrak{S}\right) + \left(\Delta\overline{\Lambda}_{11j} - \Delta\overline{\Lambda}_{12j}\mathfrak{S}\right)^T M. \tag{33}$$

I is the case of $D^{\alpha}\mathrm{V}(\mathrm{t})$, and for any $\sigma_1(t) \neq 0, \mu < 0$ must be fulfilled.

Here, by multiplying M^{-1} to both sides of μ and symbolizing $\overline{M} = M^{-1}MM^{-1}$, we can get

$$\begin{aligned}\mu &= \left(\overline{\Lambda}_{11j} - \overline{\Lambda}_{12j}\mathfrak{S}\right)\overline{M} + \overline{M}\left(\overline{\Lambda}_{11j} - \overline{\Lambda}_{12j}\mathfrak{S}\right)^T + \left(\Delta\overline{\Lambda}_{11j} - \Delta\overline{\Lambda}_{12j}\mathfrak{S}\right)\overline{M} + \overline{M}\left(\Delta\overline{\Lambda}_{11j} - \Delta\overline{\Lambda}_{12j}\mathfrak{S}\right)^T,\\ &= \left(\overline{\Lambda}_{11j} - \overline{\Lambda}_{12j}\mathfrak{S}\right)\overline{M} + \overline{M}\left(\overline{\Lambda}_{11j} - \overline{\Lambda}_{12j}\mathfrak{S}\right)^T + \left(\Omega_2^T P_j L_j(t) Q_j \Omega_2 - \Omega_2^T P_j L_j(t) Q_j \Omega_1 \mathfrak{S}\right)\overline{M} + \overline{M}\left(\Omega_2^T P_j L_j(t) Q_j \Omega_2 - \Omega_2^T P_j L_j(t) Q_j \Omega_1 \mathfrak{S}\right)^T,\\ &= \left(\overline{\Lambda}_{11j} - \overline{\Lambda}_{12j}\mathfrak{S}\right)\overline{M} + \overline{M}\left(\overline{\Lambda}_{11j} - \overline{\Lambda}_{12j}\mathfrak{S}\right)^T + \Omega_2^T P_j L_j(t)\left[\left(\overline{M}\Omega_2^T - N^T\Omega_1^T\right)Q_j^{\ T}\right]^T + \left[\Omega_2^T P_j L_j(t)\left[\left(\overline{M}\Omega_2^T - N^T\Omega_1^T\right)Q_j^{\ T}\right]^T\right]^T < 0.\end{aligned} \tag{34}$$

According to Lemma 4, the LMI (32) satisfies for all $L_j(t)$ holding $L_j^T(t)L_j(t) \leq I$ if there exists a positive number ξ_j^{-1} so that

$$\left(\overline{\Lambda}_{11j} - \overline{\Lambda}_{12j}\mathfrak{S}\right)\overline{M} + \overline{M}\left(\overline{\Lambda}_{11j} - \overline{\Lambda}_{12j}\mathfrak{S}\right)^T + \left[\xi_j^{-1}\left(\overline{M}\Omega_2^T - N\Omega_1^T\right)Q_j^{\ T} \quad \xi_j\left(\Omega_2^T P_j\right)\right] \cdot \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} \xi_j Q_j\left(\Omega_2 M - \Omega_1 N\right) \\ \xi_j\left(P_j^T \Omega_2\right) \end{bmatrix} < 0. \tag{35}$$

Now, if $S = (\overline{\Lambda}_{11j} - \overline{\Lambda}_{12j}\mathfrak{S})\overline{M} + \overline{M}(\overline{\Lambda}_{11j} - \overline{\Lambda}_{12j}\mathfrak{S})^T$, $D = \Omega_2^T P_j$ and $[(\overline{M}\Omega_2^T - N^T\Omega_1^T)Q_j^{\ T}]^T$. Then, the condition of Lemma 4 is satisfied and when the Schur complement is applied to (33), the result is (26). This concludes the proof.

After constructing the sliding surface such that the sliding dynamic systems have the proper response, the second phase in the SMC design methodology is to develop a switched control rule that ensures the ease of access of the defined sliding surface (22). In this case, the conditions are written as follows:

$$\begin{aligned} D^{\alpha}S(\mathrm{t}) &< b\,S(\mathrm{t}) - q\,\mathrm{sgn}_{(l)}(S(\mathrm{t})), \text{if } S(\mathrm{t}) > 0,\\ D^{\alpha}S(\mathrm{t}) &> -b\,S(\mathrm{t}) - q\,\mathrm{sgn}_{(l)}(S(\mathrm{t})), \text{if } S(\mathrm{t}) < 0.\end{aligned} \tag{36}$$

Now, the FSMC rule is presented as

$$u_i(t) = -\left(\overline{\wp}_j(\sigma(t)) + \overline{\mathcal{Q}}(S(t)) + \overline{\varrho}(S(t))\right), \tag{37}$$

where

$$\begin{aligned}\overline{\wp}_j(\sigma(t)) &= \sum_{j=1}^{n} h_j(\theta(t))\left[\overline{K}^{-1}\left(\mathfrak{S}\left(\overline{\Lambda}_{11j}\sigma_1(t)+\overline{\Lambda}_{12j}\sigma_2(t)\right)+\overline{\Lambda}_{21j}\sigma_1(t)+\overline{\Lambda}_{22j}\sigma_2(t)\right)\right] = \sum_{j=1}^{n} h_j(\theta(t))\overline{K}^{-1}Z_s,\\ \overline{\mathcal{G}}(S(t)) &= \sum_{j=1}^{n} h_j(\theta(t))\left[\overline{K}^{-1}\varphi_{\min}^{-1}\left(2\left\|\mathfrak{S}\Omega_2^TP_j\right\|^2+2\left\|Q_j\Omega_2\sigma_1(t)\right\|^2+2\left\|Q_j\Omega_1\sigma_2(t)\right\|^2+2\left\|\Omega_1^TP_j\right\|^2\right)\right]sgn_l(S(t)),\\ \overline{\varrho}(S(t)) &= \sum_{j=1}^{n} h_j(\theta(t))\left[\overline{K}^{-1}\varphi_{\min}^{-1}\left(FS(t)+q\,sgn_l(S(t))\right)\right],\end{aligned} \tag{38}$$

where, $F>0$ and $q>0$ and $\begin{cases} S\Lambda_s \geq 0 \longrightarrow \mu_i = \varphi_{\min}^{-1} \\ S\Lambda_s < 0 \longrightarrow \mu_i = \varphi_{\max}^{-1}\end{cases}$. In which, $\varphi_{\min}$ and $\varphi_{\max}$ are the minimum and maximum eigenvalues of F_i, respectively.

In addition, Figure 1 demonstrates the structure of the suggested TSFSMC control mechanism in a block diagram as follows. □

Theorem 8. *We consider the TS fuzzy chaotic error system (19). We suppose that the LMIs (26) are practicable and the sliding surface (23) be provided, where $\mathfrak{S}$ may be determined by (26). In that case, all the error states in the closed-loop T-S fuzzy chaotic error system with the control rule (37) will be asymptotically stable.*

Proof. Based on $S^T(t)D^{\alpha}S(t)$ and the sliding surface (23), one can get the following results:

$$\begin{aligned}D^{\alpha}S(t) &= \sum_{j=1}^{n} h_j(\theta(t))\left[\mho_1+\mho_2-h_i(\theta(t))(\mho_3+\mho_4+\mho_5)\right]\\ &= \sum_{j=1}^{n} h_j(\theta(t))h_i(\theta(t))\left[\mho_1-\mho_3+\mho_2-\mho_4-\mho_5\right].\end{aligned} \tag{39}$$

In which

$$\begin{aligned}\mho_1 &= \left[\mathfrak{S}\left(\overline{\Lambda}_{11j}\sigma_1(t)+\overline{\Lambda}_{12j}\sigma_2(t)\right)+\overline{\Lambda}_{21j}\sigma_1(t)+\overline{\Lambda}_{22j}\sigma_2(t)\right],\\ \mho_2 &= \mathfrak{S}\left(\Omega_2^TP_jL_j(t)Q_j\Omega_2\sigma_1(t)+\Omega_2^TP_jL_j(t)Q_j\Omega_1\sigma_2(t)\right)+\left(\Omega_1^TP_jL_j(t)Q_j\Omega_2\sigma_1(t)+\Omega_1^TP_jL_j(t)Q_j\Omega_1\sigma_2(t)\right),\\ \mho_3 &= F_j\mu_i\left[\mathfrak{S}\left(\overline{\Lambda}_{11j}\sigma_1(t)+\overline{\Lambda}_{12j}\sigma_2(t)\right)+\overline{\Lambda}_{21j}\sigma_1(t)+\overline{\Lambda}_{22j}\sigma_2(t)\right] = F_j\mu_j\mho_1,\\ \mho_4 &= F_j\varphi_{\min}^{-1}\left[2\left(\left\|\mathfrak{S}\Omega_2^TP_j\right\|^2+\left\|Q_j\Omega_2\sigma_1(t)\right\|^2+\left\|Q_j\Omega_1\sigma_2(t)\right\|^2+\left\|\Omega_1^TP_j\right\|^2\right)\right]\\ &\quad\cdot \mathrm{sgn}_l(S(t)) = \widetilde{\mho}_4\mathrm{sgn}_l(S(t)), \mho_5 = F_j\varphi_{\max}^{-1}\left[FS(t)+q\mathrm{sgn}_l(S(t))\right],\end{aligned} \tag{40}$$

where $F>0$ and $q>0$.

Also, we have the following conditions:

$$\begin{cases} S\Lambda_s \geq 0 \longrightarrow \mu_i = \varphi_{\min}^{-1}, \\ S\Lambda_s < 0 \longrightarrow \mu_i = \varphi_{\max}^{-1}.\end{cases} \tag{41}$$

They lead to

$$\begin{aligned}\mho_1-\mho_3 &= \mho_1 - F_j\mu_i\psi_1 \leq 0, \text{when } S(t)>0\mho_1-\mho_3\\ &= \mho_1 - F_j\mu_i\psi_1 \geq 0, \text{when } S(t)<0.\end{aligned} \tag{42}$$

Using Lemmas 4 and 5, it is clear that $\mho_2 \leq \widetilde{\mho}_4$ and one obtains

$$D^{\alpha}S(t) \leq \mho_5, \text{when } S(t)>0D^{\alpha}S(t) \geq \mho_5, \text{when } S(t)<0. \tag{43}$$

Therefore, when the control law (37) is operated, the FO dynamic systems (21) and (22) will be converging to the sliding surface (23).

Now, according to the abovementioned results and Theorem 7, the FO dynamic systems (21) and (22) are asymptotically stable. Therefore, since $\sigma(t) = He(t)$, if $\sigma(t) \longrightarrow 0$, then $e(t)$ will converge to zero asymptotically. Therfore, the proof is completed. □

Remark 9. It is difficult to establish a combination of SMC and TS fuzzy techniques to develop a dynamic-free approach that takes saturation nonlinearities into account. Despite the fact that the suggested method may appear to be traditional, such a combination does not exist very often. In addition, making use of the definition of the sign function (formulas (4) and (5)) for one of the other advances that this approach possesses is the design of the method that is being sought after.

Remark 10. Recently, in [50, 51], the problems of synchronization/control of discrete nonhomogeneous Markovian switching systems are considered via H_{∞} performance and finite-time control methods. However, the use and combination of the T-S fuzzy method with the described controllers in these papers show that the combination of the concept of T-S fuzzy and nonlinear controllers can be very reliable and popular.

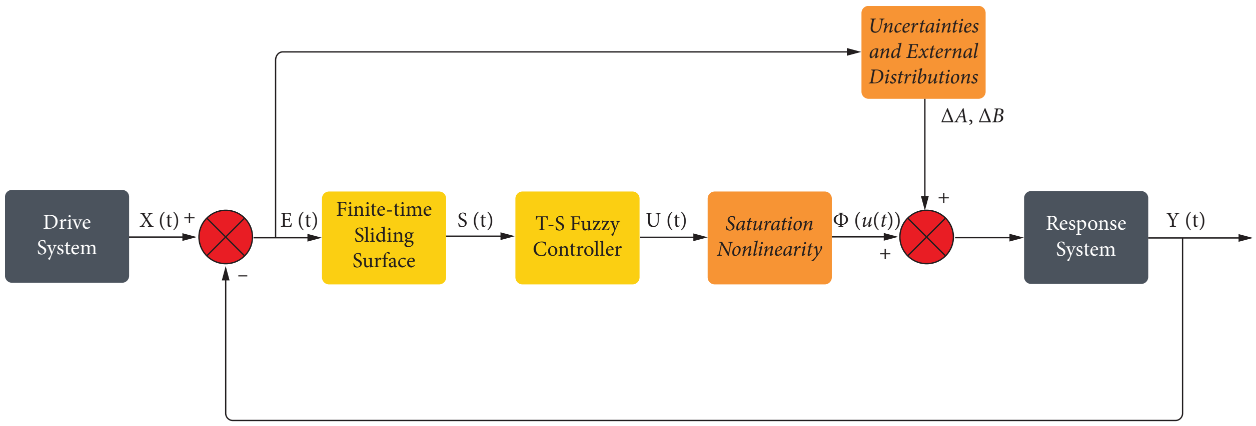

FIGURE 1: The proposed framework of the TSFSMC in the block schematic form.

Remark 11. Research on synchronization and stabilized switched systems in the FO nonlinear chaotic system is an emerging field. Although it has obtained very acceptable results, there are still many problems and developments, such as providing some conservativeness analysis of the proposed stability methods (Refs. [52, 53]), which should be considered as a future research issue in this field of study.

Remark 12. Research on the proposed TSFSMC approach for synchronization of an FO chaotic system is a growing field with many limits; although it has yielded extremely satisfactory results, there are still many restrictions that we hope will be eliminated in the future. For instance, we have (I) the restriction on the order of derivation between 0 and 1. (II) Complicated system modeling using the T-S fuzzy approach. (III) Choosing the controller settings by trial and error.

5. Numerical Simulations

Here, to demonstrate the applicability and effectiveness of the designed TSFSMC approaches, two examples are provided. In the first scenario, the real situation involving the synchronization of two distinct chaotic FO power grid systems is studied. Furthermore, in the second scenario, the synchronization of PMSM and BLDC motors is taken into account. In addition, simulations are programmed using a revolutionary numerical technique in [54]. In addition, it should be mentioned that the controller starts operating after $t = 3$ seconds.

5.1. Scenario 1. Here, the synchronization of the FO chaotic systems Lu and Chen, which exhibit chaotic behaviors for = 0.94, demonstrates the efficiency of the recommended TSFSMC (37).

The chaotic Lü system is one of the dynamical systems employed most often in complicated power grid systems [55]. Figure 2 is a real-world example of a complex power grid system. Furthermore, one of the intriguing aspects of the fractional-order Lü system is that it exhibits chaotic behavior for orders lower than three when we know that is not the case for orders lower than three in comparable integer-order situations [56]. The following introduces FO chaotic Lu system's dynamics as a driving system [56].

$$\begin{aligned} D^{\beta} x_1 &= 35\left(x_2 - x_1\right), \\ D^{\beta} x_2 &= -x_1 x_3 + 28\, x_2, \\ D^{\beta} x_3 &= -x_1 x_2 - 3\, x_3. \end{aligned} \tag{44}$$

This is with $\left(x_1(0), x_2(0), x_3(0)\right) = (4, 2, -3)$ for initial conditions. Moreover, the uncertain FO response system Chen has been presented as [57]

$$\begin{aligned} D^{\beta} y_1 &= 35\left(y_2 - y_1\right), \\ D^{\beta} y_2 &= -7 y_1 + 28\, y_2 - y_1 y_3, \\ D^{\beta} y_3 &= -y_1 y_2 - 3\, y_3. \end{aligned} \tag{45}$$

This is with $\left(y_1(0), y_2(0), y_3(0)\right) = (-2, -2, 3)$ for initial conditions.

Let $e_1(t) \in (-d, d)$ and $d = 15$, the membership functions of fuzzy modeling to synchronize (44) and (45) generate as

$$\begin{aligned} \daleth_1\left(e_1(t)\right) &= \frac{1}{2}\left(1 + \frac{x_1(t)}{15}\right), \daleth_2\left(e_1(t)\right) = 1 - \daleth_1\left(e_1(t)\right) \\ &= \frac{1}{2}\left(1 - \frac{x_1(t)}{15}\right), \\ \aleph_1\left(e_1(t)\right) &= \frac{1}{2}\left(1 + \frac{y_1(t)}{15}\right), \aleph_2\left(e_1(t)\right) = 1 - \aleph_1\left(e_1(t)\right) \\ &= \frac{1}{2}\left(1 - \frac{y_1(t)}{15}\right). \end{aligned} \tag{46}$$

Therefore, the following fuzzy dynamics will be achieved:

(i) Plant Rule 1: If $e_1(t)$ is $\daleth_1\left(e_1(t)\right)$ Then $D^{\alpha} e(t) = \left(\Lambda_1 + \Delta\Lambda_1\right)\left(e(t)\right) - \phi\left(u(t)\right)$

(ii) Plant Rule 2: If $e_1(t)$ is $\daleth_2\left(e_1(t)\right)$ Then $D^{\alpha} e(t) = \left(\Lambda_2 + \Delta\Lambda_2\right)\left(e(t)\right) - \phi\left(u(t)\right)$

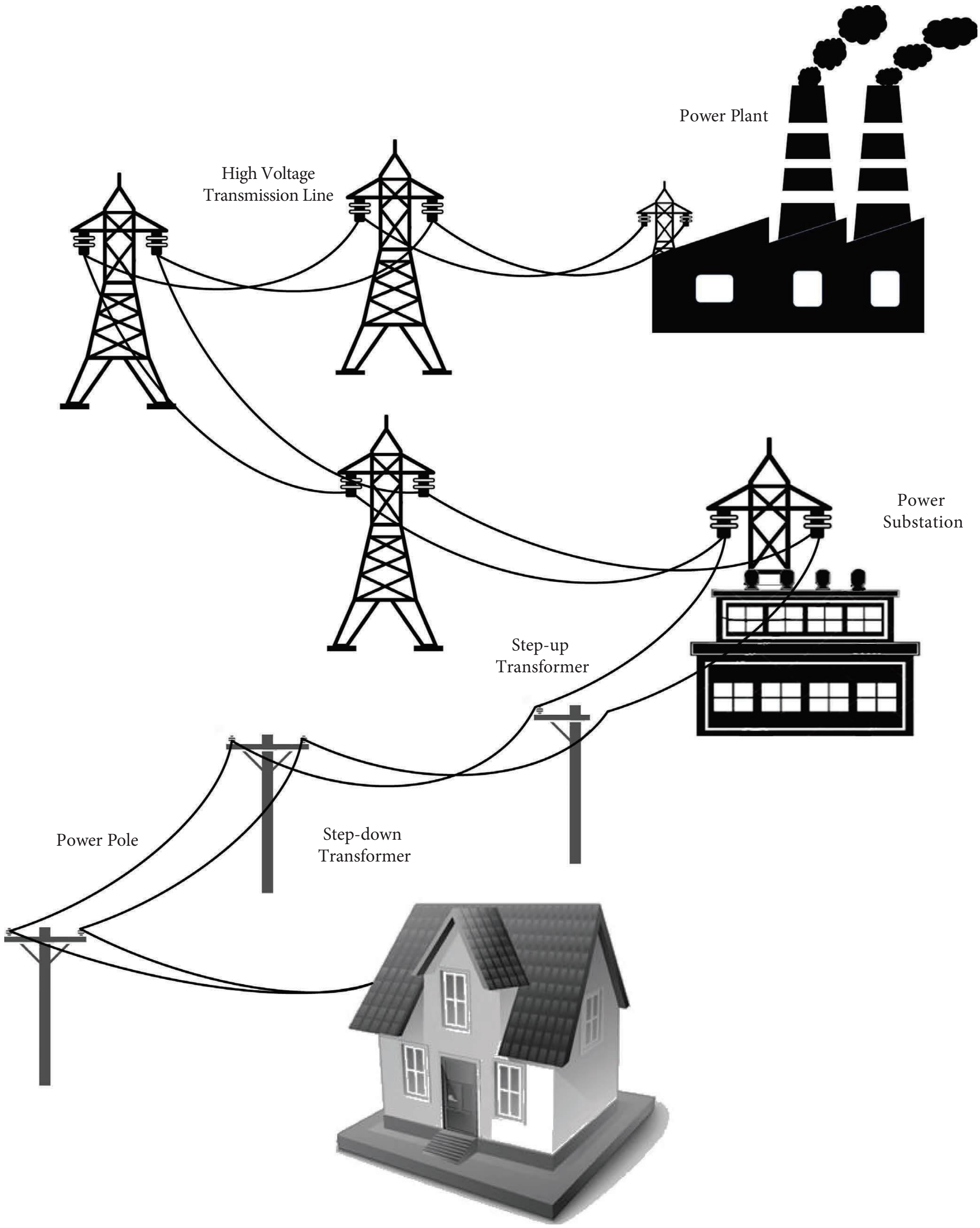

FIGURE 2: An illustration of a complex electrical power grid system.

(iii) Plant Rule 3: If $e_1(t)$ is $\aleph_1(e_1(t))$ Then $D^{\alpha}e(t) = (\Lambda_3 + \Delta\Lambda_3)(\mathrm{e}(t)) - \phi(u(t))$

(iv) Plant Rule 4: If $e_1(t)$ is $\aleph_2(e_1(t))$ Then $D^{\alpha}e(t) = (\Lambda_4 + \Delta\Lambda_4)(\mathrm{e}(t)) - \phi(u(t))$

where $e(t) = [e_1(t), e_2(t), e_3(t)]^T$, $u(t) = [u_1(t), u_2(t), u_3(t)]^T$ and

$$\phi(u(t)) = \begin{cases} 50, & \text{if } u_i(t) > 50, \\ 0.98\,u_i(t), & \text{if } -50 \le u_i(t) \le 50,\ i = 1,\ 2,\ 3. \\ -50, & \text{if } u_i(t) < -50, \end{cases} \tag{47}$$

Also, Λ_1, Λ_2, Λ_3, and Λ_4 are the system's unknown parameters; also, for $i = 1, \ldots, 4$, $\Delta\Lambda_i = P_i L_i(t) Q_i$ shows uncertainty terms of the system, defining them as the following matrixes.

$$\Lambda_1 = \begin{bmatrix} -35 & 35 & 0 \\ 0 & 28 & -d \\ 0 & d & -3 \end{bmatrix}, \Lambda_2 = \begin{bmatrix} -35 & 35 & 0 \\ 0 & 28 & d \\ 0 & -d & -3 \end{bmatrix},$$
$$\Lambda_3 = \begin{bmatrix} -35 & 35 & 0 \\ -7 & 28 & -d \\ 0 & -d & -3 \end{bmatrix}, \Lambda_4 = \begin{bmatrix} -35 & 35 & 0 \\ 0 & 28 & d \\ 0 & d & -3 \end{bmatrix},$$
$$\mathrm{P}_1 = \mathrm{P}_2 = \begin{bmatrix} -0.05 & 0 & 0.1 \\ 0.05 & 0 & 0.05 \\ 0.1 & 0.05 & 0 \end{bmatrix}, \mathrm{P}_3 = \mathrm{P}_4 = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0.05 & 0 & -0.05 \\ 0.1 & 0 & 0.05 \end{bmatrix},$$
$$L_i = \begin{bmatrix} -0.1 & 0 & 0 \\ 0 & 0.07 & 0 \\ 0 & 0 & -0.13 \end{bmatrix}, Q_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{48}$$

In order to demonstrate the practicability of the suggested TSFSMC strategy and the correctness of the proposed stabilization condition, the system parameters are arbitrarily selected as $\overline{K}_i = \begin{bmatrix} 1.2 \\ 1.3 \\ -1.5 \end{bmatrix}$, $F_1 = F_2 = 4$, $F_3 = F_4 = F_5 = 6$, $F = 7$, $\varsigma = 5.3$, and $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Also, for LMI (26), $\overline{M} = 4.5$ and $\mathfrak{S} = 7.8$. In addition, it should be mentioned that the controller starts operating after $t = 3$ seconds.

Figure 3 is displayed in order to demonstrate how T-S fuzzy FO nonlinear systems Lu and Chen are synchronized in equations (44) and (45). In addition, the error structure among T-S fuzzy FO chaotic systems (44) and (45), which has approximated the value of zero, is depicted in Figure 4. The synchronization between two distinct chaotic FO systems (44) and (45) is clearly visible, as can be seen from the pictures in the article. In addition, Figure 5 illustrates the saturated control signals (37) and Figure 6 shows the sliding surface (23), respectively. Based on Figure 5, there are no indications of the chattering occurrence in the controller impulses at all.

In addition, as shown in Figure 5, the saturation condition suppresses the control laws as they near the saturation limits, resulting in the leaping occurrence. Therefore, transitioning and leaping states are easily applied, particularly when switches and predetermined saturation conditions are employed. From Figure 6, one can observe that the sliding surface (23) is approached to its origin. This means that the proposed TSFSMC can successfully synchronize the T-S fuzzy FO chaotic systems, Lu and Chen, in equations (44) and (45).

Now, in order to compare the TSFSMC technique, the FO chaotic systems Lu and Chen are synchronized with $\beta = 0.92$ by utilizing an adaptive controller suggested in [58]. Research by Luo et al. has resulted in the development of an adaptive control strategy, which has recently been used to the synchronizing of FO systems. In this context, the operating of the control technique in [58] may be seen as follows:

$$u_i(t) = k_i \operatorname{sgn}(e_i), \tag{49a}$$

$$D^{\beta} k_i = |e_i|, i = 1, 2, 3. \tag{49b}$$

After operating the designed adaptive control approach (49a) and (49b), Figure 7 displays the state trajectories of the error between equations (44) and (45), synchronized via both of the controllers (37) and (49a) and (49b). As can be observed, even though the error states converges to zero, but the suggested TSFSMC (37) has a better convergence in comparison with the adaptive method (49a) and (49b).

5.2. Scenario 2. In this part, the synchronization of two FO electrical motors including the permanent magnet synchronous motor (PMSM) and brushless DC motor (BLDCM) systems, which exhibit chaotic behaviors for $\beta = 0.99$, illustrates the effectiveness of the designed TSFSMC (37). (20).

The following introduces FO chaotic PMSM system's dynamics as a driving system [59].

$$\begin{aligned} D^{\beta} x_1 &= -x_1 + x_2 x_3, \\ D^{\beta} x_2 &= -x_2 + 100 x_3 - x_1 x_3, \\ D^{\beta} x_3 &= 10(x_2 - x_3), \end{aligned} \tag{50}$$

where $(x_1(0), x_2(0), x_3(0)) = (-1, -0.5, 1)$ are the initial conditions. Moreover, the uncertain FO BLDCM system has been defined as a response system [60].

$$\begin{aligned} D^{\beta} y_1 &= -0.875\, y_1 + y_2 y_3, \\ D^{\beta} y_2 &= -y_2 + 55\, y_3 - y_1 y_3, \\ D^{\beta} y_3 &= 4(y_2 - y_3), \end{aligned} \tag{51}$$

with $(y_1(0), y_2(0), y_3(0)) = (0.6, 1, -1)$ as the initial values.

Let $e_3(t) \in (-d, d)$ and $d = 20$, the following equations are used to determine the membership functions in fuzzy structure in order to synchronize the FO PMSM and FO BLDCM:

$$\begin{aligned} \gimel_1(e_3(t)) &= \frac{1}{2}\left(1 + \frac{x_3(t)}{20}\right), \gimel_2(e_1(t)) = 1 - \gimel_1(e_3(t)) \\ &= \frac{1}{2}\left(1 - \frac{x_3(t)}{20}\right), \\ \aleph_1(e_3(t)) &= \frac{1}{2}\left(1 + \frac{y_3(t)}{20}\right), \aleph_2(e_3(t)) = 1 - \aleph_1(e_3(t)) \\ &= \frac{1}{2}\left(1 - \frac{y_3(t)}{20}\right). \end{aligned} \tag{52}$$

Then, the fuzzy relations will be achieved as follows:

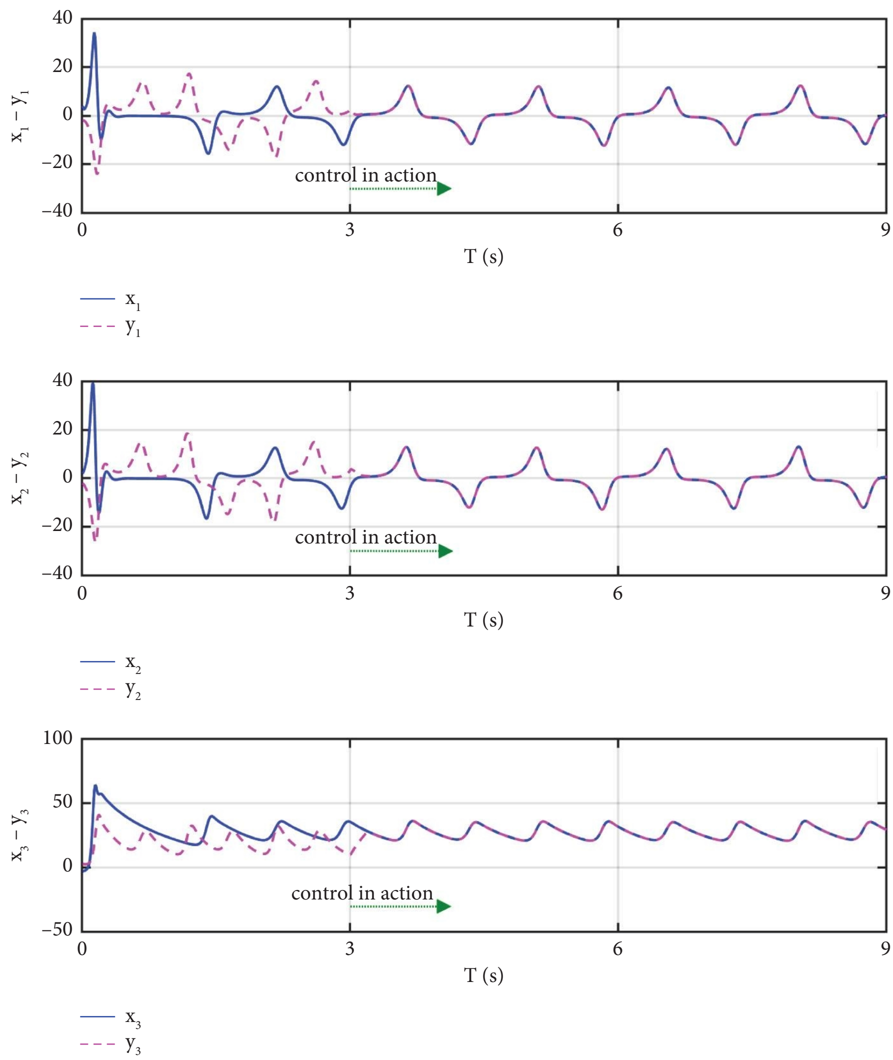

Figure 3: Time history of synchronizing FO systems Lu and Chen in (44) and (45).

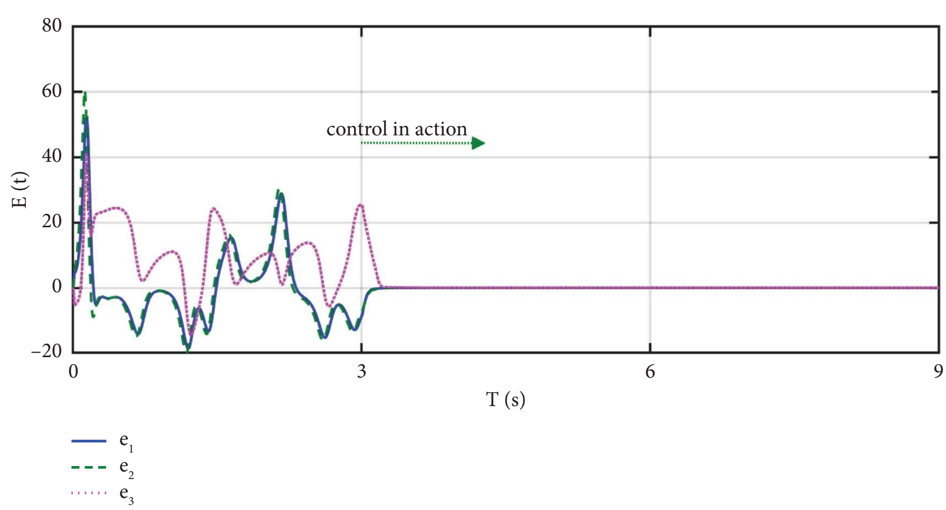

Figure 4: Synchronization errors between FO systems Lu and Chen in (44) and (45).

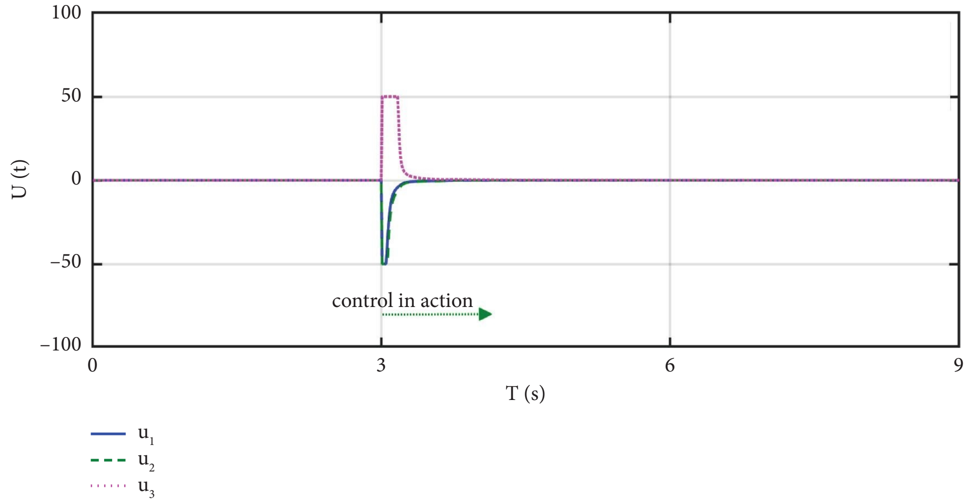

Figure 5: Time history of control inputs (37) for synchronization of the FO systems Lu and Chen in (44) and (45).

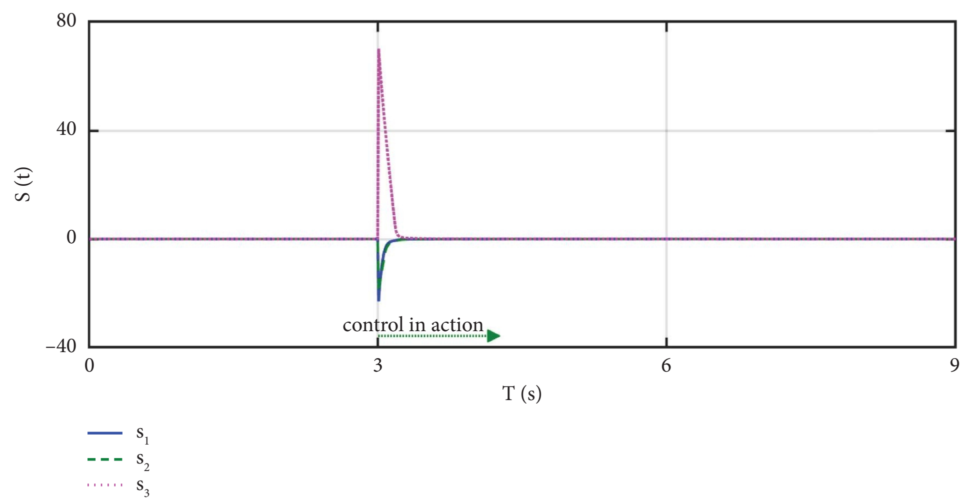

Figure 6: The sliding surface (23) operated for synchronization of the FO systems Lu and Chen in (44) and (45).

(i) Plant Rule 1: If $e_3(t)$ is $\gimel_1(e_3(t))$, then $D^{\alpha}e(t) = (\Lambda_1 + \Delta\Lambda_1)(e(t)) - \phi(u(t))$

(ii) Plant Rule 2: If $e_3(t)$ is $\gimel_2(e_3(t))$, then $D^{\alpha}e(t) = (\Lambda_2 + \Delta\Lambda_2)(e(t)) - \phi(u(t))$

(iii) Plant Rule 3: If $e_3(t)$ is $\aleph_1(e_3(t))$, then $D^{\alpha}e(t) = (\Lambda_3 + \Delta\Lambda_3)(e(t)) - \phi(u(t))$

(iv) Plant Rule 4: If $e_3(t)$ is $\aleph_2(e_3(t))$, then $D^{\alpha}e(t) = (\Lambda_4 + \Delta\Lambda_4)(e(t)) - \phi(u(t))$

where $e(t) = [e_1(t), e_2(t), e_3(t)]^T$, $u(t) = [u_1(t), u_2(t), u_3(t)]^T$, and

$$\phi(u(t)) = \begin{cases} 25, & \text{if } u_i(t) > 25, \\ 0.98\, u_i(t), & \text{if } -25 \leq u_i(t) \leq 25, i = 1, 2, 3. \\ -25, & \text{if } u_i(t) < -25, \end{cases} \tag{53}$$

Also, Λ_1, Λ_2, Λ_3, and Λ_4 elucidate the system's unidentified parameters and characteristics. Furthermore, for $i = 1, \ldots, 4$, $\Delta\Lambda_i = P_i L_i(t) Q_i$ indicates uncertainty terms of the systems, introduced as follows:

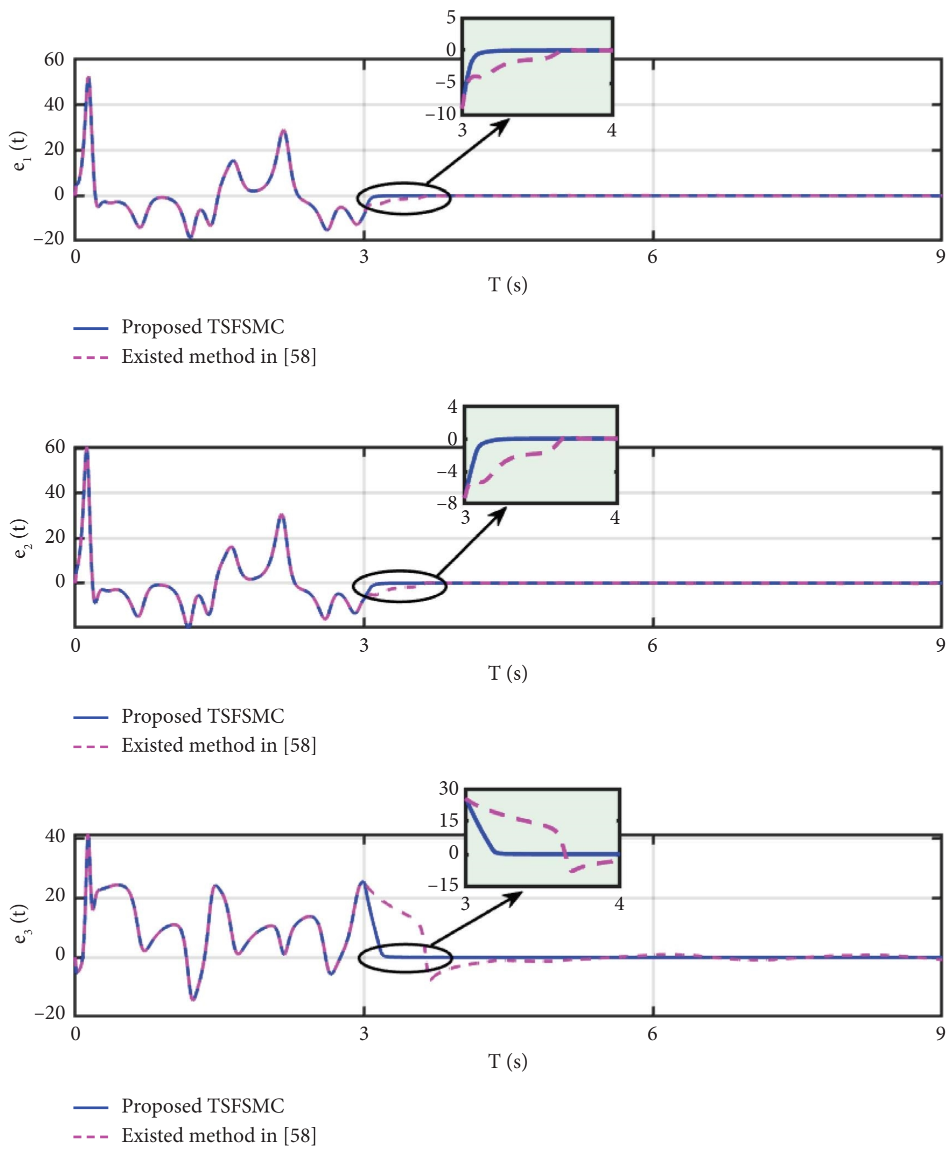

Figure 7: Comparison of the FO error systems (44) and (45), controlled with designed TSFSMC (37) and active control method (49a) and (49b) in [58].

$$\Lambda_1=\begin{bmatrix}-1 & d & 0\\ -d & -1 & 100\\ 0 & 10 & -10\end{bmatrix},\Lambda_2=\begin{bmatrix}-1 & -d & 0\\ d & -1 & 100\\ 0 & 10 & -10\end{bmatrix},$$
$$\Lambda_3=\begin{bmatrix}-1 & d & 0\\ d & -1 & 100\\ 0 & 10 & -10\end{bmatrix},\Lambda_4=\begin{bmatrix}-1 & d & 0\\ -d & -1 & 100\\ 0 & 10 & -10\end{bmatrix},$$
$$\mathrm{P}_1=\mathrm{P}_2=\begin{bmatrix}0.05 & 0 & 0.1\\ -0.05 & 0 & 0.05\\ 0.1 & -0.05 & 0\end{bmatrix},\mathrm{P}_3=\mathrm{P}_4=\begin{bmatrix}0.1 & 0 & 0\\ 0.05 & 0.1 & -0.05\\ -0.1 & 0 & 0.05\end{bmatrix},$$
$$L_i=\begin{bmatrix}0.1 & 0 & 0\\ 0 & -0.07 & 0\\ 0 & 0 & 0.15\end{bmatrix},Q_i=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}. \tag{54}$$

In order to demonstrate the practicability of the suggested TSFSMC strategy and the correctness of the proposed stabilization condition, the system parameters are arbitrarily selected as $\overline{K}_i=\begin{bmatrix}-1.5\\ 1.8\\ 2\end{bmatrix}$, $F_1=F_2=5$, $F_3=F_4=F_5=5$, $F=6$, $\varsigma=4.4$, and $H=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$. Also for LMI (26), $\overline{M}=6$ and $\mathfrak{S}=6$.

Figure 8 is shown to demonstrate the synchronization in the T-S fuzzy FO chaotic systems proposed in equations (50) and (51). Then, the error function in the T-S fuzzy FO chaotic systems developed by PMSM and BLDCM (50) and (51), which is shown in Figure 9, has also almost reached zero. In addition, Figures 10 and 11, respectively, display the saturated control signals (37) and sliding surface (23), respectively. Figure 10 shows that the control signals show no

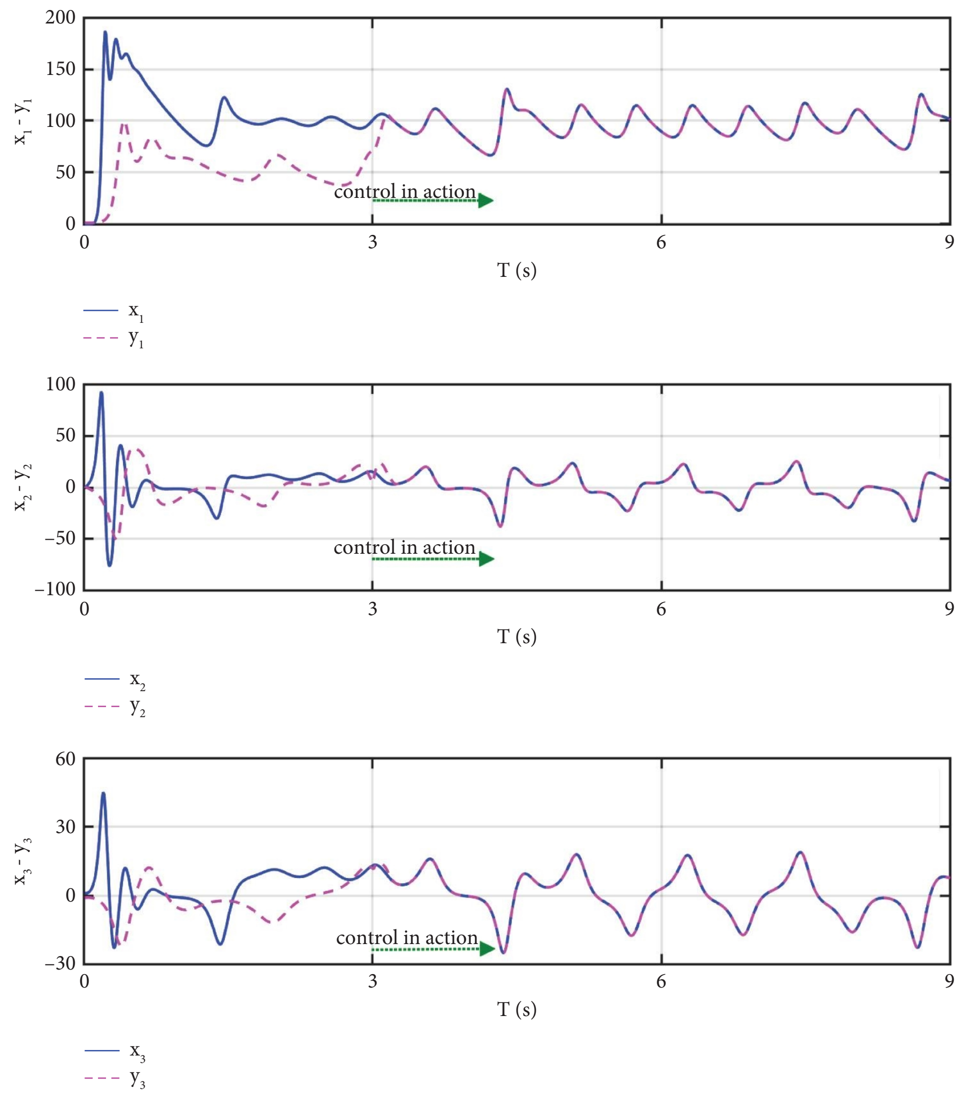

Figure 8: Time history of synchronization of FO systems in (50) and (51).

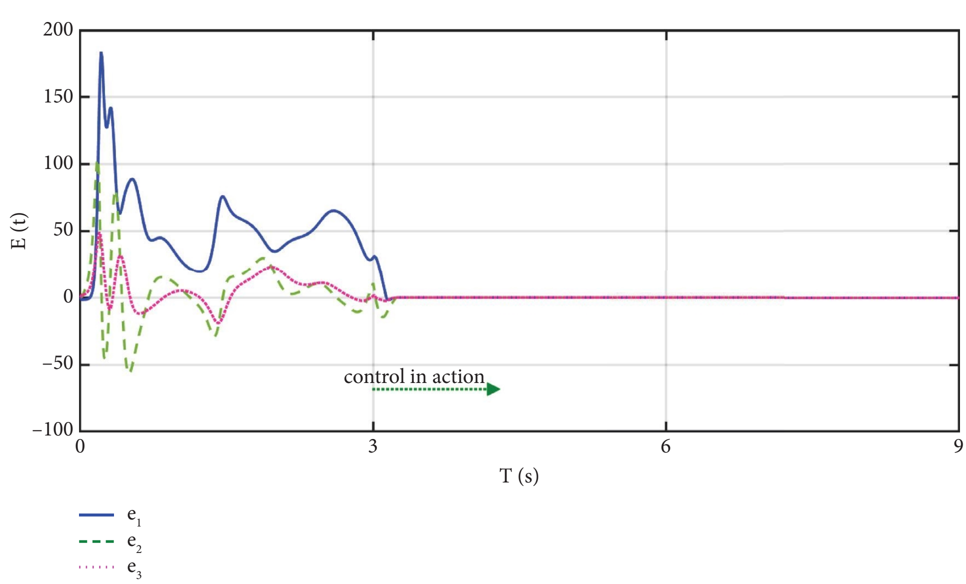

Figure 9: Time history of synchronization errors in FO systems in (50) and (51).

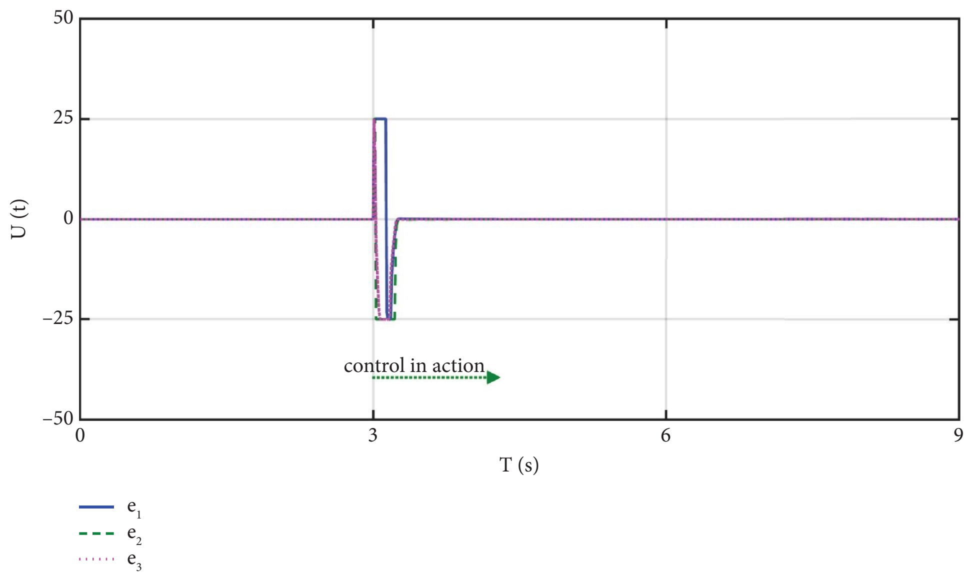

Figure 10: Time history of control laws (37) for synchronization of the FO systems in (50) and (51).

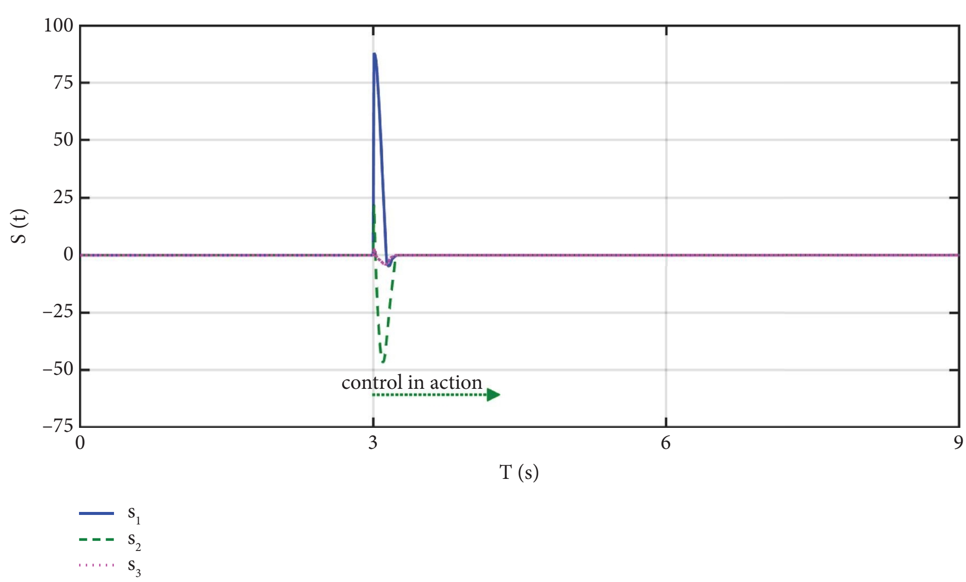

Figure 11: The sliding surface (23) operated for synchronization of the FO systems in (50) and (51).

evidence of the chattering phenomena. In addition, the saturation condition suppresses the control laws as they approach the saturation boundaries, resulting in the leaping phenomena. Therefore, switching and jumping states are easily applicable, especially when relays and predetermined saturation conditions are employed. In Figure 11, the sliding surface (23), which is approaching its origin, can be seen. This suggests that the T-S fuzzy FO chaotic systems PMSM and BLDCM in equations (50) and (51) can be successfully synchronized using the proposed TSFSMC (37).

6. Conclusion

In this research, a dynamic-free T-S fuzzy sliding mode control approach is proposed for the purpose of synchronizing the various chaotic fractional-order systems in the presence of input saturation. A Takagi–Sugeno fuzzy sliding mode controller is driven to suppress and synchronize the undesirable behavior of FO chaotic systems without any unpleasant chattering phenomenon by utilizing a new definition of fractional calculus, the fractional version of the Lyapunov stability theorem, and the linear matrix inequality concept. The design of this method is completely independent of the system's dynamics, and it can synchronize a variety of FO chaotic systems despite the presence of model errors, external disturbances, and input saturation. An example of the synchronization of two different real-world application chaotic systems is illustrated to show the applicability of the theoretical result presented in the paper. Also, for future work, the problem of synchronizing FO delayed chaotic systems in a finite time can be considered. Also, for finding the best parameters of the controller, the machine/deep learning method can be utilized to select optimal control parameters. Moreover, work on the unlimited nature of β is an essential issue. Finally, experimental results will be added to increase the quality and impact of the studies.

Data Availability

No data were used to support the findings of this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] D. Baleanu, K. Diethelm, E. Scalas, and J. J. Trujillo, *Fractional Calculus: Models and Numerical Methods*, World Scientific Publishing Company, Singapore, 2012.

[2] S. S. Majidabad, H. T. Shandiz, and A. Hajizadeh, "Nonlinear fractional-order power system stabilizer for multi-machine power systems based on sliding mode technique," *International Journal of Robust and Nonlinear Control*, vol. 25, no. 10, pp. 1548–1568, 2015.

[3] M. Roohi, C. Zhang, and Y. Chen, "Adaptive model-free synchronization of different fractional-order neural networks with an application in cryptography," *Nonlinear Dynamics*, vol. 100, no. 4, pp. 3979–4001, 2020.

[4] M. Al-Raeei, "Applying fractional quantum mechanics to systems with electrical screening effects," *Chaos, Solitons & Fractals*, vol. 150, Article ID 111209, 2021.

[5] M. Pouzesh and S. Mobayen, "Event-triggered fractional-order sliding mode control technique for stabilization of disturbed quadrotor unmanned aerial vehicles," *Aerospace Science and Technology*, vol. 121, Article ID 107337, 2022/02/01/2022.

[6] Y.-L. Wang, H. Jahanshahi, S. Bekiros, F. Bezzina, Y.-M. Chu, and A. A. Aly, "Deep recurrent neural networks with finite-time terminal sliding mode control for a chaotic fractional-order financial system with market confidence," *Chaos, Solitons & Fractals*, vol. 146, Article ID 110881, 2021/05/01/2021.

[7] K. Diethelm, "A fractional calculus based model for the simulation of an outbreak of dengue fever," *Nonlinear Dynamics*, vol. 71, no. 4, pp. 613–619, 2013/03/01 2013.

[8] W. M. Ahmad and R. El-Khazali, "Fractional-order dynamical models of love," *Chaos, Solitons & Fractals*, vol. 33, no. 4, pp. 1367–1375, 2007.

[9] L. Song, S. Xu, and J. Yang, "Dynamical models of happiness with fractional order," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 3, pp. 616–628, 2010.

[10] M. Roohi, M. P. Aghababa, and A. R. Haghighi, "Switching adaptive controllers to control fractional-order complex systems with unknown structure and input nonlinearities," *Complexity*, vol. 21, no. 2, pp. 211–223, 2015.

[11] M. Roohi, M.-H. Khooban, Z. Esfahani, M. P. Aghababa, and T. Dragicevic, "A switching sliding mode control technique for chaos suppression of fractional-order complex systems," *Transactions of the Institute of Measurement and Control*, vol. 41, no. 10, pp. 2932–2946, 2019.

[12] Y. Chen, C. Tang, and M. Roohi, "Design of a model-free adaptive sliding mode control to synchronize chaotic fractional-order systems with input saturation: an application in secure communications," *Journal of the Franklin Institute*, vol. 358, no. 16, pp. 8109–8137, 2021.

[13] M. P. Aghababa, A. R. Haghighi, and M. Roohi, "Stabilisation of unknown fractional-order chaotic systems: an adaptive switching control strategy with application to power systems," *IET Generation*, vol. 9, no. 14, pp. 1883–1893, 2015.

[14] S. Song, J. H. Park, B. Zhang, X. Song, and Z. Zhang, "Adaptive command filtered neuro-fuzzy control design for fractional-order nonlinear systems with unknown control directions and input quantization," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 11, pp. 7238–7249, 2021.

[15] M. Shahsadeghi, M. H. Khooban, and T. Niknam, "A robust and simple optimal type II fuzzy sliding mode control strategy for a class of nonlinear chaotic systems," *Journal of Intelligent and Fuzzy Systems*, vol. 27, no. 4, pp. 1849–1859, 2014.

[16] M. S. Sadeghi, N. Vafamand, and M. H. Khooban, "LMI-based stability analysis and robust controller design for a class of nonlinear chaotic power systems," *Journal of the Franklin Institute*, vol. 353, no. 13, pp. 2835–2858, 2016/09/01/2016.

[17] K. Rajagopal, H. Jahanshahi, S. Jafari, R. Weldegiorgis, A. Karthikeyan, and P. Duraisamy, "Coexisting attractors in a fractional order hydro turbine governing system and fuzzy PID based chaos control," *Asian Journal of Control*, vol. 23, no. 2, pp. 894–907, 2021.

[18] O. Mofid, S. Mobayen, and M. H. Khooban, "Sliding mode disturbance observer control based on adaptive synchronization in a class of fractional-order chaotic systems," *International Journal of Adaptive Control and Signal Processing*, vol. 33, no. 3, pp. 462–474, 2019.

[19] Z. Esfahani, M. Roohi, M. Gheisarnejad, T. Dragičević, and M.-H. Khooban, "Optimal non-integer sliding mode control for frequency regulation in stand-alone modern power grids," *Applied Sciences*, vol. 9, no. 16, p. 3411, 2019.

[20] Z. Rasooli Berardehi, C. Zhang, M. Taheri, M. Roohi, and M. H. Khooban, "Implementation of t-s fuzzy approach for the synchronization and stabilization of non-integer-order complex systems with input saturation at a guaranteed cost," *Transactions of the Institute of Measurement and Control*, vol. 8, Article ID 01423312231155273, 2023.

[21] H. Yang, L. Shu, S. Zhong, T. Zhan, and X. Wang, "Pinning synchronization of stochastic T-S fuzzy delayed complex dynamical networks with heterogeneous impulsive delays," *International Journal of Control, Automation and Systems*, vol. 18, no. 10, pp. 2599–2608, 2020/10/01 2020.

[22] L. Merazka, F. Zouari, and A. Boulkroune, "Fuzzy state-feedback control of uncertain nonlinear MIMO systems," in *Proceedings of the 2017 6th International Conference on Systems and Control (ICSC)*, pp. 103–108, Batna, Algeria, May 2017.

[23] L. Merazka, F. Zouari, and A. Boulkroune, "High-gain observer-based adaptive fuzzy control for a class of multi-variable nonlinear systems," in *Proceedings of the 2017 6th International Conference on Systems and Control (ICSC)*, pp. 96–102, Batna, Algeria, May 2017.

[24] A. Ibeas, A. Esmaeili, J. Herrera, and F. Zouari, "Discrete-time observer-based state feedback control of heart rate during treadmill exercise," in *Proceedings of the 2016 20th*

International Conference on System Theory, Control and Computers, pp. 537–542, Sinaia, Romania, October 2016.
[25] X. Wu, Q. Ai, and Y. Wang, "Adaptive and exponential synchronization of uncertain fractional-order T-S fuzzy complex networks with coupling time-varying delays via pinning control strategy," *IEEE Access*, vol. 9, pp. 2007–2017, 2021.
[26] C. Liu, Y. Guo, H. Rao, M. Lin, and Y. Xu, "Finite-time synchronization for periodic T–S fuzzy master-slave neural networks with distributed delays," *Journal of the Franklin Institute*, vol. 358, no. 4, pp. 2367–2381, 2021/03/01/2021.
[27] X. Wang, J. H. Park, H. Yang, and Z. Yu, "Sampled-Data-Based $\mathcal{H}_{\infty}$ fuzzy pinning synchronization of complex networked systems with adaptive event-triggered communications," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 7, pp. 2254–2265, 2022.
[28] X. Wang, J. H. Park, H. Yang, and S. Zhong, "An improved fuzzy event-triggered asynchronous dissipative control to T–S FMJSs with nonperiodic sampled data," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 10, pp. 2926–2937, 2021.
[29] N. Vafamand, S. Khorshidi, and A. Khayatian, "Secure communication for non-ideal channel via robust TS fuzzy observer-based hyperchaotic synchronization," *Chaos, Solitons & Fractals*, vol. 112, pp. 116–124, 2018.
[30] Y. Farid and A. Ramezani, "A wavelet-based robust adaptive T-S fuzzy controller design for synchronization of faulty chaotic gyrostat systems," *Journal of Control, Automation and Electrical Systems*, vol. 32, no. 1, pp. 57–69, 2021.
[31] X. Wang, Y. Yu, S. Zhong et al., "Novel heterogeneous mode-dependent impulsive synchronization for piecewise T-S fuzzy probabilistic coupled delayed neural networks," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 7, pp. 2142–2156, 2022.
[32] Y. Xie, X. Zhang, W. Meng et al., "Coupled fractional-order sliding mode control and obstacle avoidance of a four-wheeled steerable mobile robot," *ISA Transactions*, vol. 108, pp. 282–294, 2021.
[33] G. Li, X. Zhang, and H. Yang, "Complexity analysis and synchronization control of fractional-order Jafari-Sprott chaotic system," *IEEE Access*, vol. 8, pp. 53360–53373, 2020.
[34] M. Labbadi, M. Defoort, G. P. Incremona, and M. Djemai, "Fractional-order integral terminal sliding-mode control for perturbed nonlinear systems with application to quadrotors," *International Journal of Robust and Nonlinear Control*, vol. 10, 2023.
[35] O. Martínez-Fuentes, A. J. Muñoz-Vázquez, G. Fernández-Anaya, and E. Tlelo-Cuautle, "Synchronization of fractional-order chaotic networks in Presnov form via homogeneous controllers," *Integration*, vol. 90, pp. 71–80, 2023.
[36] H.-L. Li, C. Hu, L. Zhang, H. Jiang, and J. Cao, "Complete and finite-time synchronization of fractional-order fuzzy neural networks via nonlinear feedback control," *Fuzzy Sets and Systems*, vol. 443, pp. 50–69, 2022.
[37] X. Song, S. Song, I. T. Balsera, L. Liu, and L. Zhang, "Synchronization of two fractional-order chaotic systems via nonsingular terminal fuzzy sliding mode control," *Journal of Control Science and Engineering*, vol. 2017, Article ID 9562818, 11 pages, 2017.
[38] K. Shao, Z. Xu, and T. Wang, "Robust finite-time sliding mode synchronization of fractional-order hyper-chaotic systems based on adaptive neural network and disturbances observer," *International Journal of Dynamics and Control*, vol. 9, no. 2, pp. 541–549, 2021.
[39] N. R. Babu, M. Kalpana, and P. Balasubramaniam, "A novel audio encryption approach via finite-time synchronization of fractional order hyperchaotic system," *Multimedia Tools and Applications*, vol. 80, no. 12, pp. 18043–18067, 2021/05/01 2021.
[40] M. D. Johansyah, A. Sambas, S. Mobayen et al., "Dynamical analysis and adaptive finite-time sliding mode control approach of the financial fractional-order chaotic system," *Mathematics*, vol. 11, no. 1, p. 100, 2022.
[41] A. Modiri and S. Mobayen, "Adaptive terminal sliding mode control scheme for synchronization of fractional-order uncertain chaotic systems," *ISA Transactions*, vol. 105, pp. 33–50, 2020.
[42] F. Zouari and A. Boubellouta, "Adaptive neural control for unknown nonlinear time-delay fractional-order systems with input saturation," in *Advanced Synchronization Control and Bifurcation of Chaotic Fractional-Order Systems*, pp. 54–98, IGI Global, Hershey, PA, USA, 2018.
[43] L. Zhang, J. Zhong, and J. Lu, "Intermittent control for finite-time synchronization of fractional-order complex networks," *Neural Networks*, vol. 144, pp. 11–20, 2021/12/01/2021.
[44] I. Podlubny, *Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications*, Elsevier Science, Amsterdam, The Netherlands, 1998.
[45] A. Kajouni, A. Chafiki, K. Hilal, and M. Oukessou, "A new conformable fractional derivative and applications," *International Journal of Differential Equations*, vol. 2021, Article ID 6245435, 5 pages, 2021.
[46] L. S. Shieh, Y. T. Tsay, and R. Yates, "Some properties of matrix sign functions derived from continued fractions," *IEE Proceedings D-Control Theory and Applications*, vol. 130, no. 3, pp. 111–118, 1983.
[47] H. J. Lee, J. B. Park, and G. Chen, "Robust fuzzy control of nonlinear systems with parametric uncertainties," *IEEE Transactions on Fuzzy Systems*, vol. 9, no. 2, pp. 369–379, 2001.
[48] S. Xu and J. Lam, "Robust H/sub/spl infin//control for uncertain discrete-time-delay fuzzy systems via output feedback controllers," *IEEE Transactions on Fuzzy Systems*, vol. 13, no. 1, pp. 82–93, 2005.
[49] Y. Li, Y. Chen, and I. Podlubny, "Stability of fractional-order nonlinear dynamic systems: Lyapunov direct method and generalized Mittag–Leffler stability," *Computers & Mathematics with Applications*, vol. 59, no. 5, pp. 1810–1821, 2010.
[50] J. Cheng, W. Huang, H. K. Lam, J. Cao, and Y. Zhang, "Fuzzy-model-based control for singularly perturbed systems with nonhomogeneous markov switching: a dropout compensation strategy," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 2, pp. 530–541, 2022.
[51] J. Cheng, W. Huang, J. H. Park, and J. Cao, "A hierarchical structure approach to finite-time filter design for fuzzy markov switching systems with deception attacks," *IEEE Transactions on Cybernetics*, vol. 52, no. 8, pp. 7254–7264, 2022.
[52] J. Cheng, L. Liang, J. H. Park, H. Yan, and K. Li, "A dynamic event-triggered approach to state estimation for switched memristive neural networks with nonhomogeneous sojourn probabilities," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 68, no. 12, pp. 4924–4934, 2021.
[53] J. Cheng, L. Liang, H. Yan, J. Cao, S. Tang, and K. Shi, "Proportional-integral observer-based state estimation for markov memristive neural networks with sensor saturations,"

IEEE Transactions on Neural Networks and Learning Systems, vol. 17, pp. 1–12, 2022.

[54] M. S. Asl and M. Javidi, "An improved PC scheme for nonlinear fractional differential equations: error and stability analysis," *Journal of Computational and Applied Mathematics*, vol. 324, pp. 101–117, 2017.

[55] X. Gong, D. Liu, and B. Wang, "Chaotic system synchronization with tridiagonal structure and its initial investigation in complex power systems," *Journal of Vibration and Control*, vol. 20, no. 3, pp. 447–457, 2014.

[56] H. Y. Jia, Z. Q. Chen, and G. Y. Qi, "Topological horseshoe analysis and circuit realization for a fractional-order Lü system," *Nonlinear Dynamics*, vol. 74, no. 1-2, pp. 203–212, 2013.

[57] C. Li and G. Chen, "Chaos in the fractional order Chen system and its control," *Chaos, Solitons & Fractals*, vol. 22, no. 3, pp. 549–554, 2004.

[58] R. Luo, H. Su, and Y. Zeng, "Synchronization of uncertain fractional-order chaotic systems via a novel adaptive controller," *Chinese Journal of Physics*, vol. 55, no. 2, pp. 342–349, 2017.

[59] W. Xue, Y. Li, S. Cang, H. Jia, and Z. Wang, "Chaotic behavior and circuit implementation of a fractional-order permanent magnet synchronous motor model," *Journal of the Franklin Institute*, vol. 352, no. 7, pp. 2887–2898, 2015.

[60] P. Zhou, R. J. Bai, and J. M. Zheng, "Stabilization of a fractional-order chaotic brushless DC motor via a single input," *Nonlinear Dynamics*, vol. 82, no. 1-2, pp. 519–525, 2015.