Hindawi
International Journal of Intelligent Systems
Volume 2023, Article ID 6686446, 21 pages
https://doi.org/10.1155/2023/6686446

Research Article

Generalized Ordered Intuitionistic Fuzzy C-Means Clustering Algorithm Based on PROMETHEE and Intuitionistic Fuzzy C-Means

Muhammad Adnan Bashir,[1] **Tabasam Rashid**,[1] **and Muhammad Salman Bashir**[2]

[1]*University of Management and Technology, Lahore 54770, Pakistan*
[2]*Department of Computer Science and Information Technology, Virtual University of Pakistan, Lahore 54770, Pakistan*

Correspondence should be addressed to Muhammad Adnan Bashir; s2017265005@umt.edu.pk

Received 14 April 2023; Revised 10 July 2023; Accepted 30 August 2023; Published 22 September 2023

Academic Editor: Riccardo Ortale

The problem of ordered clustering in the context of decision-making with multiple criteria has garnered significant interest from researchers in the field of management science and operational research. In real-world scenarios, the datasets often exhibit imprecision or uncertainty, which can lead to suboptimal ordered-clustering outcomes. However, the intuitionistic fuzzy c-means (IFCM) clustering algorithm enhances the accuracy and effectiveness of decision-making processes by effectively handling uncertain dataset information for clustering. Therefore, we propose a new clustering algorithm, called the generalized ordered intuitionistic fuzzy c-means (G-OIFCM), based on PROMETHEE and the IFCM clustering algorithm. Different from the classical IFCM clustering algorithm, we use positive flow ($\varphi^+(s_i) \in [0, 1]$) and negative flow ($\varphi^-(s_i) \in [0, 1]$) of PROMETHEE to generate ordered clusters within the intuitionistic environment. We define a new objective function based on the positive and negative flow of the PROMETHEE and IFCM clustering algorithm, whose properties are mathematically justified in terms of convergence and optimization. The performance of the proposed algorithm is evaluated using two different real-world datasets to assess both the ordered clustering and the quality of partitioning. To demonstrate the effectiveness of G-OIFCM, a comparison is conducted with three other algorithms: fuzzy c-means (FCM), ordered fuzzy c-means (OFCM), and an adaptive generalized intuitionistic fuzzy c-means (G-IFCM). The results demonstrate the effectiveness of G-OIFCM in enhancing optimal ordered clustering and utility when dealing with uncertainty in datasets.

1. Introduction

The fundamental goal of clustering is to separate samples and assign them to groups that exhibit similarity. Hard and soft clustering are two distinct approaches commonly used in partition-based clustering techniques. In hard clustering, each sample belongs exclusively to a single cluster, with no overlap between clusters. On the other hand, fuzzy clustering allows samples to have partial membership in multiple clusters [1]. The fuzzy c-means (FCM) clustering algorithm [2] is a fuzzy variant of the well-known K-means clustering algorithm [3]. To expand upon the concept of fuzzy sets (FS) [4], intuitionistic FS (IFS) was introduced as a means of managing and interpreting data with varying degrees of uncertainty, utilizing the membership, nonmembership, and hesitation degrees [5]. In order to address this uncertainty, the intuitionistic fuzzy c-means (IFCM) clustering algorithm [6] incorporates a weighted Euclidean distance between IFSs in the objective function of the FCM algorithm. The application of IFCM has proven valuable, including medical image segmentation [7], segmentation of MRI brain tissues [8], decision making [9, 10], pattern and face recognition [11], and intuitionistic fuzzy c-ordered clustering algorithm [12]. The generalized intuitionistic fuzzy c-means (G-IFCM) clustering algorithm utilizes an adaptive intuitionistic fuzzification technique that is based on crisp or fuzzy data sets

[13]. However, there has been limited exploration of IFCM clustering algorithm application in the context of ordered clustering assessed across multiple conflicting criteria.

Within the multicriteria decision-making context, researchers typically consider three fundamental types of problems [14]. The first type is the choice problem, which involves identifying the best or promising solutions that offer a suitable compromise. The second type concerns ranking alternatives based on merit, from the best to the worst. The third type is the sorting problem, which involves placing alternatives into predefined categories. One prominent problem facing multicriteria decision aid (MCDA) is supervised classification, which involves allocating alternatives into a fixed number of clusters. In this scenario, researchers have created innovative and outstanding clustering algorithms in the MCDA, including ELECTRE-SORT [15], Flowsort [16], TODIM-Sort [17], PROAFTN [18], and PAIRCLASS [19]. On the other hand, numerous types of research studies have been conducted to address the problem of undefined classes from the MCDA perspective [20] and their performance evaluation [21]. This area highlights three types of problems in clustering: relational, nonrelational, and ordered clustering (Boujelben [22]; Meyer and Olteanu [23]). Complete ordered clustering is a useful addition to the ranking process since it may help to construct order priority links between a subset of alternates for each cluster.

We are considering a specific multicriteria method called PROMETHEE, which has gained widespread recognition due to its various extensions in clustering and their practical applications. PROMETHEE relies on pairwise preferences, including positive and negative flows and partial and total net outranking, which serve as the primary sources for the expansion of clustering techniques [24], such as P2CLUST [25], hierarchical clustering [26], interval clustering [27], ordered profile clustering [28], performance evaluation [29], uncertainty management [30], and multicriteria dual clustering [31]. There have been several contributions in the field of clustering that specifically address issues related to uncertainty and vagueness through an integrated approach. For instance, some of these contributions include hierarchical multicriteria clustering [32], interval clustering [33], and multicriteria ordered clustering of countries [34]. These approaches aim to incorporate multiple criteria and account for uncertainty or vagueness of information in the clustering process, resulting in more robust and accurate clustering results.

Ordered clustering, as used in MCDA, integrates both the ranking and sorting processes, allowing for the discovery and ordering of clusters from the best to the worst. De Smet et al. [35] present a technique for finding a total ordered partition in MCDA based on pairwise preference relations and the inconsistency matrix. Chen et al. [36] introduced the ordered K-means (OKM) clustering algorithm, which combines the traditional K-means method with the partial net outranking flow of PROMETHEE. OKM clustering algorithms may not give a clear recommendation for the number of clusters to be used. This resulted in the creation of the ordered fuzzy c-means (OFCM) clustering algorithm based on the net outranking flow of PROMETHEE and the fuzzy c-means method [37]. In real-world scenarios, datasets often exhibit imprecision or uncertainty, which can result in suboptimal outcomes in ordered clustering. This highlights the need for caution when interpreting scores for categorization. To address this issue, we propose a generalized ordered intuitionistic fuzzy c-means (G-OIFCM) clustering algorithm. G-OIFCM incorporates the positive flow $(\varphi^{+}(s_i) \in [0, 1])$ and negative flow $(\varphi^{-}(s_i) \in [0, 1])$ of PROMETHEE and the intuitionistic fuzzy c-means (IFCM) clustering algorithm. By operating within an intuitionistic environment, G-OIFCM effectively handles imprecise or uncertain information and constructs optimal ordered clustering. In addition, Kaushal and Lohani [13] introduced the generalized intuitionistic fuzzy c-means (G-IFCM) clustering algorithm, which utilizes an adaptive intuitionistic fuzzification technique for uncertain datasets. However, it is rarely investigated in the context of multicriteria ordered clustering.

In this paper, we present a multicriteria ordered intuitionistic fuzzy c-means clustering technique. Motivated by the positive and negative flow of the PROMETHEE [38] and IFCM, we propose generalized ordered intuitionistic fuzzy c-means (G-OIFCM) to optimize clustering results for imprecise or uncertain information. G-OIFCM first randomly selects K different intuitionistic centroids, rank the centroids, and computes the membership function values based on membership, nonmembership, and hesitance degree. Subsequently, it minimizes the flow objective function and determines new centroids using an optimization model. This novel approach aims to generate reliable and well-ordered clustering results within the field of multicriteria decision analysis (MCDA) in an intuitionistic environment.

2. Preliminaries

This section provides an overview of the PROMETHEE and intuitionistic fuzzy c-means (IFCM) clustering algorithms, focusing on the following aspects.

2.1. PROMETHEE Method. This section briefly describes the PROMETHEE method developed by Brans and Vincke [39], which is primarily employed in ranking problems. The following multicriteria problem is considered:

$$\max\left\{q_1(s_1), q_2(s_2), \cdots, q_j(s_i), \cdots, q_l(s_n) \,\middle|\, s_i \in S\right\}, \tag{1}$$

where S comprises a finite set of alternatives $\{s_1, s_2, \cdots, s_i, \cdots, s_n\}$ and is being evaluated under the criteria $Q = \{q_1(.), q_2(.), \cdots, q_j(.), \cdots, q_l(.)\}$ in multicriteria decision making analysis (MCDMA). Let $W = \{w_1, w_2, \cdots, w_j, \cdots, w_l\}$, where $w_i \in]\ 0, 1\ [\ (i = 1, \cdots, n)$ are the weights assigned to express the relative importance of criteria, which can be either equal or different. The PROMETHEE method assumes that complete knowledge of the performance (scores) of each alternative for every criterion is available. This assumption allows for a comprehensive evaluation and comparison of alternatives based on their performance values across multiple criteria. Let o_{ij} denote the performance value of the i^{th} alternative concerning the j^{th} criterion. In addition, the PROMETHEE method requires the preference function p_i (x), $\forall i = 1, \cdots, n$. This function p_i: $\mathbb{R} \longrightarrow [0, 1]$, stretches the preference degree $P_i \in [0, 1]$ between each pair of alternatives according to the difference in performance. In PROMETHEE, Figure 1 illustrates six different types of preference functions that can be employed.

The preference function for a cost-type and benefit-type criterion is represented by equations (2) and (3), respectively.

$$P_i(s_j, s_k) = p_i(o_{ji} - o_{ki}) \quad \forall i, j, k, \tag{2}$$

$$P_i(s_j, s_k) = p_i(o_{ki} - o_{ji}) \quad \forall i, j, k, \tag{3}$$

if $P_i(s_j, s_k) > 0$, then s_j is preferred over s_k with the preference degree $P_i(s_j, s_k)$. On the other hand, if $P_i(s_j, s_k) = 0$, then s_j is not preferred to s_k at all.

In PROMETHEE, the preference degree p_i: $[0, 1] \longrightarrow [0, 1]$ is computed using the relative difference instead of the absolute one. In that case, benefit and cost-type criteria are shown in equations (4) and (5), respectively.

$$P_i(s_j, s_k) = p_i\left(1 - \frac{o_{ki}}{o_{ji}}\right) \quad \forall i, j, k, \tag{4}$$

$$P_i(s_j, s_k) = p_i\left(1 - \frac{o_{ji}}{o_{ki}}\right) \quad \forall i, j, k. \tag{5}$$

For the criterion q_l, the preference function quantifies the level of preference for alternatives s_j to s_k [39]. Then, the preference degrees $P_i(s_j, s_k)$ are aggregated to obtain the preference indices for each alternative $\pi(s_j, s_k), \forall j, k$. This is accomplished by calculating the weighted sum of preference degrees for each alternative using the following formula, as shown in the following equation:

$$\pi(s_j, s_k) = \sum_{i=1}^{n} w_i P_i(s_i, s_k), \tag{6}$$

where $\pi(s_j, s_k) \in [0, 1]$ is the sum of all preferences of alternatives s_j over the s_k when all criteria are taken into consideration. The PROMETHEE's positive and negative flows provides partial ranking for each alternative as shown in equations (7) and (8), and thus, Vincke and Brans [40] proposed to further aggregate these partial flows to the net flows as shown in equation (10).

$$\varnothing^+(s_i) = \frac{1}{m-1} \sum_{s_k \in K \setminus \{s_i\}} \pi(s_i, s_k) \in [0, 1], \tag{7}$$

$$\varnothing^-(s_i) = \frac{1}{m-1} \sum_{s_k K \setminus \{s_i\} K} \pi(s_k, s_i) \in [0, 1], \tag{8}$$

$$0 \leq \varnothing^+(s_i) + \varnothing^-(s_i) \leq 1, \tag{9}$$

$$\varnothing(s_i) = \varnothing^+(s_i) - \varnothing^-(s_i). \tag{10}$$

The positive flow $\varnothing^+(s_i) \in [0, 1]$ in the context of decision-making indicates the extent to which one alternative is superior to the others, while the negative flow ($\varnothing^-(s_i) \in [0, 1]$) reflects the degree to which the remaining alternatives outperform a specific alternative. Figure 2 provides a visual representation of this concept.

Step-wise implementation of the PROMETHEE method is presented in Algorithm 1.

2.2. Some Important Details of the Atanassov Intuitionistic Fuzzy Set (AIFS). An intuitionistic fuzzy set (IFS) is denoted by F in the universe of discourse $H = \{h_1, h_2, \ldots, h_n\}$ is described as follows:

$$F = \{\langle h, \mu_F(h), \nu_F(h)\rangle \mid h \text{ in } H\}. \tag{11}$$

The functions μ_F: $H \longrightarrow [0, 1]$ and ν_F: $H \longrightarrow [0, 1]$ each element h in H are assigned a degree of membership and nonmembership in Atanassov intuitionistic fuzzy set (AIFS), which are represented by membership and nonmembership values, respectively, if

$$0 \leq \mu_F(h) + \nu_F(h) \leq 1, \tag{12}$$

then, it carries intuitionistic fuzzy due uncertainty in data called hesitancy index $\pi_F(h)$, such that

$$\pi_F(h) = 1 - \mu_F(h) - \nu_F(h), \tag{13}$$

where $0 \leq \pi_F(h) \leq 1$ and if $\pi_F(h) = 0, \forall h \in H$, the IFS H is reduced to fuzzy set, whereas when $\mu_F(c) = \nu_F(h) = 0$ then IFS F is completely intuitionistic. Consider the elements $h_i \in H \quad \forall (i = 1, 2, \ldots, n)$ having different weights $w = (w_1, w_2, \ldots, w_n)$ of $h_i (i = 1, 2, \ldots, n)$, with $w_i \geq 0, \sum_{i=1}^{n} w_i = 1$.

The fuzzy set provided by Zadeh has been generalized by the Atanassov intuitionistic fuzzy set (AIFS) [5]. In order to assign a scalar value to each data item based on its membership and nonmembership values, it has been found that AIFS is efficient at estimating uncertainty in imprecise and vague datasets. Effective modelling of uncertainty relies heavily on the AIFS hesitation component. The inclusion of hesitancy in AIFS has made significant contributions to various applications, such as clustering (see [7, 41]), decision-making ([9, 10]), medical image segmentation [42], and pattern recognition [11].

2.3. Intuitionistic Fuzzy C-Means (IFCM) Clustering Algorithm. Intuitionistic fuzzy c-means (IFCM), which combines the ideas of AIFS with the fuzzy c-means algorithm to provide effective partitioning between clusters by exploiting hesitancy in its process, was first presented by Xu and Wu [6]. In IFCM, predetermined number of initial clusters is chosen randomly and used as starting cluster centroids in iterative process. Let $T = \{T_1, T_2, \ldots, T_p\}$ denote p intuitionistic fuzzy sets (IFSs), each with n elements. The parameter c represents the number of clusters, where $(1 \leq c \leq p)$. The set $\widetilde{V} = \{\widetilde{V}_1, \widetilde{V}_2, \ldots, \widetilde{V}_c\}$ contains the prototypical IFSs, $m > 1$ is the fuzzy factor parameter, μ_{ij} is the membership degree of the j^{th} sample T_j to the i^{th} cluster and

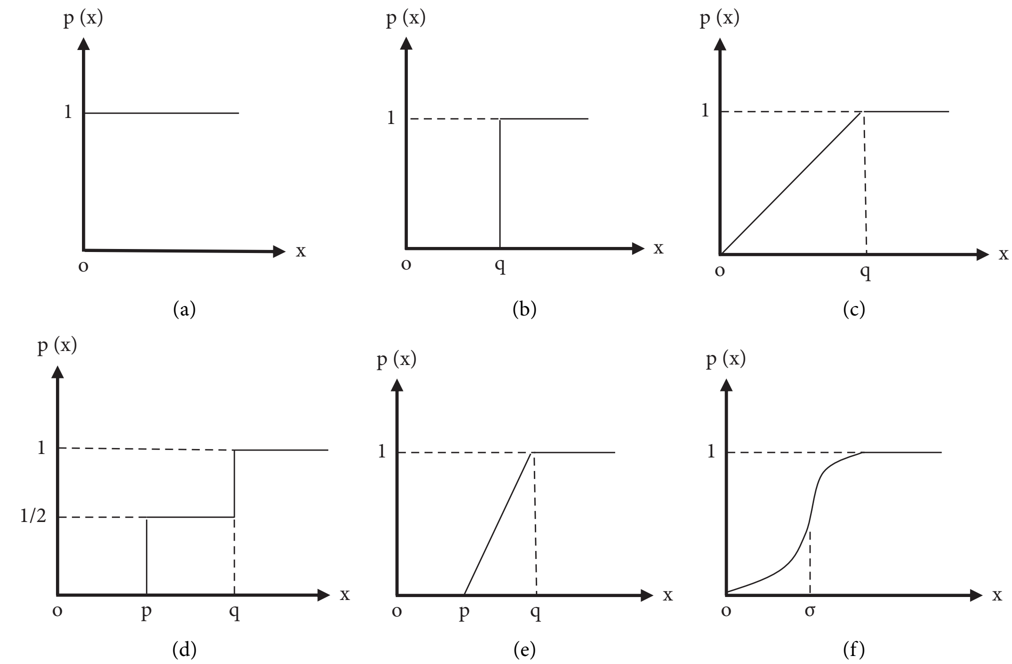

Figure 1: The preference functions employed in PROMETHEE are categorized into six types, namely, (a) usual, (b) U-shape, (c) V-shape, (d) level, (e) linear, and (f) Gaussian.

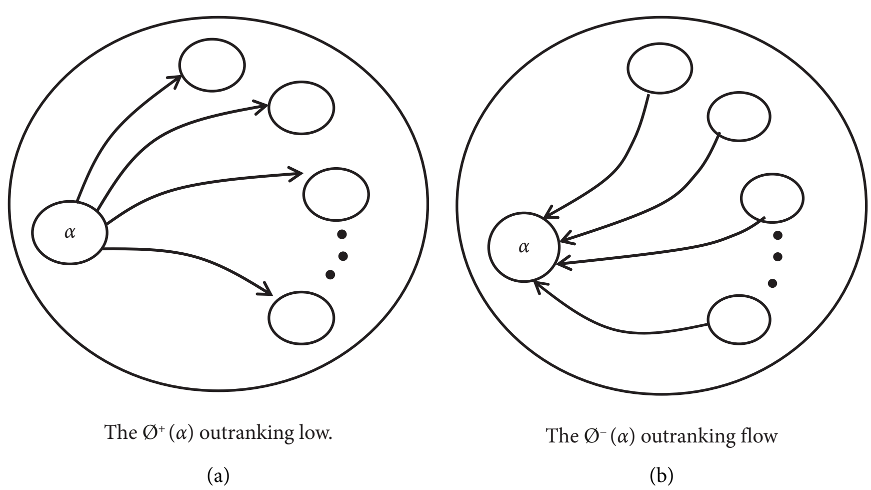

Figure 2: Positive and negative outranking flow of PROMETHEE. (a) The $\phi^{+}(\alpha)$ outranking flow. (b) The $\phi^{-}(\alpha)$ outranking flow.

Input:
(1) Performance matrix P represents the performance values of alternatives for each criterion.
(2) Weights vector W: vector containing the relative weights of each criterion.
Output:
(1) Preference ranking of alternatives
BEGIN
Step 1: Calculate the preference degrees $P_i(s_j, s_k) \in [0, 1]$ for each pair of alternatives by using equations (4) and (5).
Step 2: Aggregate the preference degrees $P_i(s_j, s_k)$ to obtain the preference indices $\pi(s_j, s_k) \in [0, 1]$ by using equation (6).
Step 3: Calculate the positive flow $\varnothing^{+}(s_i) \in [0, 1]$ and negative flow ($\varnothing^{-}(s_i) \in [0, 1]$) by using equations (7) and (8) respectively.
Step-4: Calculate the net outranking flow by using equation (10) and sort the alternatives.
Step-5: Return the preference ranking of alternatives.
END.

Algorithm 1: The PROMETHEE method.

$U = (\mu_{ij})_{c\times p}$ is a matrix has degree $c \times p$. The objective of the IFCM algorithm is to minimize the following function:

$$\text{Minimize } J_m(U,\widetilde{V}) = \sum_{j=1}^{p}\sum_{i=1}^{c}\mu_{ij}^{m}d_1^2\left(T_j,\widetilde{V}_i\right), \tag{14}$$

$$\text{s.t.} \sum_{i=1}^{c}\mu_{ij} = 1, \quad 1 \le j \le p, \tag{15}$$

$$\mu_{ij} \ge 0, 1 \le i \le c,\ 1 \le j \le p, \tag{16}$$

$$\sum_{j=1}^{p}\mu_{ij} > 0, \quad 1 \le i \le c. \tag{17}$$

Xu and Wu [6] formulate IFCM clustering algorithm by taking the basic distance measure $d_1(T_j,\widetilde{V}_i)$ in $J_m(U,\widetilde{V})$ of as a proximity function.

Problem 1. To address the optimization problem presented in equation (14), Xu and Wu [6] employ the Lagrange multiplier method [43]. The following equation is considered:

$$L = \sum_{j=1}^{p}\sum_{i=1}^{c}\mu_{ij}^{m}d_1^2\left(T_j,\widetilde{V}_i\right) - \sum_{j=1}^{p}\lambda_j\left(\sum_{i=i}^{c}\mu_{ij} - 1\right), \tag{18}$$

where

$$d_1^2\left(T_j,\widetilde{V}_i\right) = \left(\frac{1}{2}\sum_{l=1}^{n}w_l\left(\mu_{T_j}(x_l) - \mu_{\widetilde{V}_i}(x_l)\right)^2 + \left(\nu_{T_j}(x_l) - \nu_{\widetilde{V}_i}(x_l)\right)^2 + \left(\pi_{T_j}(x_l) - \pi_{\widetilde{V}_i}(x_l)\right)^2\right), \tag{19}$$

Furthermore, let

$$\frac{\partial L}{\partial \mu_{ij}} = \frac{\partial L}{\partial \lambda_j} = 0, \quad 1 \le i \le c,\ 1 \le j \le p, \tag{20}$$

we obtain

$$\mu_{ij} = \frac{1}{\sum_{r=1}^{c}\left(d_1\left(T_j,\widetilde{V}_i\right)/d_1\left(T_j,\widetilde{V}_r\right)\right)^{2/m-1}}, \quad 1 \le i \le c; 1 \le j \le p. \tag{21}$$

Next, we compute $\widetilde{V} = (\mu_{\widetilde{V}_i}(x_l), \nu_{\widetilde{V}_i}(x_l), \pi_{\widetilde{V}_i}(x_l))$, the prototypical IFSs. Let

$$\frac{\partial L}{\partial \mu_{\widetilde{V}_i}(x_l)} = \frac{\partial L}{\partial \nu_{\widetilde{V}_i}(x_l)} = \frac{\partial L}{\partial \pi_{\widetilde{V}_i}(x_l)} = 0, \quad 1 \le i \le c;\ 1 \le l \le n, \tag{22}$$

we have the following equation:

$$\mu_{\widetilde{V}_i}(x_l) = \frac{\sum_{j=1}^{p}\mu_{ij}^{m}\,\mu_{T_j}(x_l)}{\sum_{j=1}^{p}\mu_{ij}^{m}}, \quad 1 \le i \le c;\ 1 \le l \le n, \tag{23}$$

$$\nu_{\widetilde{V}_i}(x_l) = \frac{\sum_{j=1}^{p}\mu_{ij}^{m}\nu_{T_j}(x_l)}{\sum_{j=1}^{p}\mu_{ij}^{m}}, \quad 1 \le i \le c;\ 1 \le l \le n, \tag{24}$$

$$\pi_{\widetilde{V}_i}(x_l) = \frac{\sum_{j=1}^{p}\mu_{ij}^{m}\,\pi_{T_j}(x_l)}{\sum_{j=1}^{p}\mu_{ij}^{m}}, \quad 1 \le i \le c;\ 1 \le l \le n. \tag{25}$$

The following is a simplified description of a weighted average operator for IFSs provided by Xu and Wu [6]. Let $B = \{B_1, B_2, \ldots, B_p\}$ be a set of IFSs, where each IFSs has n elements. Let $w = \{w_1, w_2, \ldots, w_p\}$ be a set of weights corresponding to the IFSs, such that the sum of all the weights is equal to 1. The weighted average operator f can be defined as follows:

$$f(B,w) = \left\{ < x_l, \sum_{j=1}^{p}w_j\mu_{T_j}(x_l), \sum_{j=1}^{p}\sum_{j=1}^{p}w_j\nu_{T_j}(x_l) > \mid 1 \le l \le n \right\}, \tag{26}$$

According to (23)–(26), if we let

$$w^i = \left\{\frac{\mu_{i1}}{\sum_{j=1}^{p}\mu_{ij}}, \frac{\mu_{i2}}{\sum_{j=1}^{p}\mu_{ij}}, \cdots, \frac{\mu_{ip}}{\sum_{j=1}^{p}\mu_{ij}}\right\}, \quad 1 \le i \le c. \tag{27}$$

Then, the prototypical IFSs $\widetilde{V} = \{\widetilde{V}_1, \widetilde{V}_2, \cdots, \widetilde{V}_c\}$ of the IFCM algorithm can be computed as follows:

$$\widetilde{V}_i = f\left(T, w^{(i)}\right) = \left\{ < x_l, \sum_{j=1}^{p}w_j^{(i)}\mu_{T_j}(x_l), \sum_{j=1}^{p}w_j^{(i)}\nu_{T_j}(x_l) > \mid 1 \le l \le n, 1 \le i \le c \right\}. \tag{28}$$

Step-wise implementation of IFCM is presented in Algorithm 2.

2.4. Davies–Bouldin (DB) Index for Cluster Validity Index. The metric used to evaluate clustering algorithms is known as the cluster validity index [44]. In order to achieve optimal clustering, it is important to minimize the DB scale, which quantifies the ratio of within-cluster separation to between-cluster separation. To calculate the scatter within the i^{th} cluster, denoted as Z_i, we calculate it as follows:

$$Z_i = \frac{1}{|C_i|} \sum_{x \in C_i} \left\{ \left\| x - s_i \right\| \right\}. \tag{29}$$

The distance between cluster C_i to another cluster C_j is denoted as e_{ij}, is defined as $e_{ij} = \|s_i - s_j\|$. Here, s_i represents the i^{th} center. DB scale is defined as follows:

$$\text{DB} = \frac{1}{M} \sum_{i=1}^{M} L_{i,qt}, \text{where } L_{i,qt} = \max_j, j \neq i \left\{ \frac{Z_{i,q} + Z_{j,q}}{e_{ij,t}} \right\}. \tag{30}$$

2.5. Dunn Index (DI). Dunn index (DI) is a clustering validation measure that is used to evaluate the quality of clustering results. It measures the ratio of the minimum distance between clusters to the maximum diameter of clusters [45]. The goal of DI is to minimize intracluster distances and maximize intercluster distances. A high value of DI suggests that the clusters are well-separated and compact, indicating that the clustering algorithm effectively separates noise from the clusters.

2.6. Ranking of Intuitionistic Fuzzy Values. Szmidt and Kacprzyk [46] proposed a method for comparing the intuitionistic fuzzy values of $\varnothing^+(s_i)$ and $\varnothing^-(s_i)$ and provide a quantitative measure of the similarity between two intuitionistic fuzzy values.

$$\rho\Big(\varnothing(s_i) = 0.5\left(1 + \pi_{\varnothing(s_i)}\right)\left(1 - \mu_{\varnothing(s_i)}\right). \tag{31}$$

3. Multicriteria Ordered Clustering Algorithm Combining with IFCM and PROMETHEE

We address a unique type of clustering problem called multicriteria ordered clustering in an intuitionistic environment. The objective of ordered clustering is to assist the decision maker (DM) in sorting the possibilities in a specific order. Unlike traditional clustering problems, ordered clustering involves dividing the alternatives into a predefined number of groups while maintaining a complete ordering relationship among these groups.

Let $S = \{s_1, s_2, \ldots, s_i, \ldots, s_n\} \subseteq \mathbb{R}^m$ be a set of alternatives assessing through a set of multicriteria $Q = \{q_1, q_2, \ldots, q_i, \ldots, q_l\}$. We call a partition an ordered partition if it meets the following criteria:

(i) $S = \cup_{i=1,2,\ldots,K} \mathbb{C}_i$

(ii) $\forall_{i \neq j}, \mathbb{C}_i \cap \mathbb{C}_j = \varphi$

(iii) $\mathbb{C}_1 \succ \mathbb{C}_2 \succ \ldots \succ \mathbb{C}_K$

Here, $\mathbb{C}_i$ represents the i^{th} ordered cluster and $\succ$ denotes a priority relation among the clusters. For instance if $\mathbb{C}_i \succ \mathbb{C}_j$, it means that the elements in $\mathbb{C}_i$ are superior to those in $\mathbb{C}_j$.

The PROMETHEE preference indices are aggregated to negative flows ($\varnothing^-(s_i) \in [0, 1]$) and positive ($\varnothing^+(s_i) \in [0, 1]$) of each alternative to estimate the priority degree for each pair of alternatives. As a result, we present generalized ordered intuitionistic fuzzy c-means (G-OIFCM), a novel supervised clustering technique based on PROMETHEE and IFCM. This algorithm will generate an optimal partition of alternative for uncertain and vague data based on the PROMETHEE and IFCM clustering algorithm.

3.1. Minimum Partial Outranking Flow Objective Function. Equations (7) and (8) provide the means to calculate the positive flow ($\varphi^+(s_i) \in [0, 1]$) and negative flow ($\varphi^-(s_i) \in [0, 1]$) of a specific alternative in the PROMETHEE method. The positive flow represents the average preference indices indicating how much better the alternative is compared to the others in a multicriteria context. Conversely, the negative flow represents the average preference indices comparing the remaining alternatives to the evaluated option, indicating how much worse it is in comparison. The sum of the positive and negative flows of PROMETHEE for a given alternative, i.e., ($0 \leq \varphi^+(s_i) + \varphi^-(s_i) \leq 1$), lies between 0 and 1. These positive and negative flows of PROMETHEE are the relative optimized membership ($\mu^{\varphi^+(s_i)} = \varphi^+(s_i)$) and nonmembership ($\nu^{\varphi^-(s_i)} = \varphi^-(s_i)$) values of the alternatives building a natural intuitionistic environment for uncertain or vague data. The hesitancy degree between preference indices can be calculated as follows:

$$\pi^{\varphi(s_i)} = 1 - \mu^{\varphi^+(s_i)} - \nu^{\varphi^-(s_i)}. \tag{32}$$

So, ($\mu^{\varphi^+(s_i)}, \nu^{\varphi^-(s_i)}, \pi^{\varphi(s_i)}$) is a relative optimized intuitionistic fuzzy values for uncertain or vague data.

Let $S = \{s_1, s_2, \ldots, s_j, \ldots, s_n\}$ be a finite set of alternatives are being evaluated under the criteria $Q = \{q_1(.), q_2(.), \ldots, q_j(.), \ldots, q_l(.)\}$ in multicriteria decision making analysis (MCDMA). $V = \{V_1, V_2, \ldots\ldots, V_c\}$ are the centroids of the ordered clusters and m is the fuzzy factor $m > 1$, μ_{ij} is the membership degree of the j^{th} sample S_j to the i^{th} ordered cluster, $U = (\mu_{ij})_{c \times p}$ is a matrix of $c \times p$.

Motivated by [47], we can calculate the weighted Euclidean distance between the optimized intuitionistic fuzzy values ($\mu^{\varphi^+(s_i)}, \nu^{\varphi^-(s_i)}, \pi^{\varphi(s_i)}$) of PROMETHEE as follows:

Input:
(1) Data matrix X: matrix representing the dataset
(2) Number of clusters c: the desired number of clusters
(3) Fuzzifier m: parameter controlling the fuzziness of the clusters
(4) Maximum number of iterations max_iter: the maximum number of iterations allowed
Output:
(1) Fuzzy cluster centers
(2) Fuzzy membership matrix U
(3) Objective function
BEGIN
(1) Step-1: Initialize seeds $\widetilde{V}(0)$, let $k = 0$ and $\epsilon > 0$.
(2) Step-2: Calculate $U(k) = (\mu_{ij}(k))_{c\times p}$, where
If $\forall j, r, d_1(T_j, \widetilde{V}_r(k)) > 0$, then
$\mu_{ij}(k) = 1/\sum_{r=1}^{c}(d_1(T_j, \widetilde{V}_j(k))/d_1(T_j, \widetilde{V}_r(k)))^{2/m-1}$ $1 \le i \le c$; $1 \le j \le p$
If there exist j, r such that $d_1(T_j, \widetilde{V}_r(k)) = 0$, then let $\mu_{rj}(k) = 1$ and $\mu_{ij}(k) = 0$ for all $i \ne r$.
Step-3: Calculate $\widetilde{V}(k+1) = \{\widetilde{V}_1(k+1), \widetilde{V}_2(k+2), \ldots, \widetilde{V}_c(k+1)\}$, where
$\widetilde{V}_i(k+1) = f(T, w^{(i)}(k+1)), 1 \le i \le c$
where, $w^{(i)}(k+1) = \{\mu_{i1}(k)/\sum_{j=1}^{p}\mu_{ij}, \mu_{i2}(k)/\sum_{j=1}^{p}\mu_{ij}, \ldots, \mu_{ip}(k)/\sum_{j=1}^{p}\mu_{ij}\}$ $1 \le i \le c$
Step-4: If $\sum_{i=1}^{c} d_1(\widetilde{V}_i(k), \widetilde{V}_i(k+1))/c < \epsilon = 10^{-5} > 0$, then proceed step 5.
Otherwise, let $k = k+1$, and then return to the Step 2.
END.

Algorithm 2: Intuitionistic fuzzy c-means (IFCM) clustering.

$$d_1(S_j, V_i) = \left(\frac{1}{2}\sum_{i=1}^{c} w_i\left(\mu^{\varphi^+}(s_j) - \mu^{\varphi^+}(v_i)\right)^2 + \left(\nu^{\varphi^-}(s_j) - \nu^{\varphi^-}(v_i)\right)^2 + \left(\pi^{\varphi}(s_j) - \pi^{\varphi}(v_i)\right)^2\right)^{1/2}, \quad (33)$$

where we take $w = (1/n, 1/n, \ldots, 1/n)$, then

$$d_2(S_j, V_i) = \left(\frac{1}{2n}\sum_{i=1}^{c}\left(\mu^{\varphi^+}(s_j) - \mu^{\varphi^+}(v_i)\right)^2 + \left(\nu^{\varphi^-}(s_j) - \nu^{\varphi^-}(v_i)\right)^2 + \left(\pi^{\varphi}(s_j) - \pi^{\varphi}(v_i)\right)^2\right)^{1/2}. \quad (34)$$

Similar to IFCM clustering algorithm, we introduce a new objective function $J_{\text{OIFCM}}(U, V)$ based on PROMETHEE to minimize:

$$J_{G\text{-OIFCM}}(U, V) = \sum_{j=1}^{n}\sum_{i=1}^{c}\mu_{ij}^{m} d_1^2(S_j, V_i), \quad (35)$$

$$\text{s.t.} \sum_{i=1}^{c}\mu_{ij} = 1, \quad 1 \le j \le n, \quad (36)$$

$$\mu_{ij} \ge 0, \quad 1 \le i \le c;\ 1 \le j \le n, \quad (37)$$

$$\sum_{j=1}^{p}\mu_{ij} > 0, \quad 1 \le i \le c. \quad (38)$$

To address the optimization problem presented in equation (35), Xu and Wu [6] employ the Lagrange multiplier method [43]. The following equation is considered:

$$L(U, V) = \sum_{j=1}^{n}\sum_{i=1}^{c}\mu_{ij}^{m} d_1^2(S_j, V_i) - \sum_{j=1}^{n}\lambda_j\left(\sum_{i=1}^{c}\mu_{ij} - 1\right), \quad (39)$$

where

$$d_1(S_j, V_i) = \left(\frac{1}{2}\sum_{i=1}^{c} w_i\left(\mu^{\varphi^+}(s_j) - \mu^{\varphi^+}(v_i)\right)^2 + \left(\nu^{\varphi^-}(s_j) - \nu^{\varphi^-}(v_i)\right)^2 + \left(\pi^{\varphi}(s_j) - \pi^{\varphi}(v_i)\right)^2\right)^{1/2}, \quad (40)$$

$$d_2(S_j, V_i) = \left(\frac{1}{2n}\sum_{i=1}^{c}\left(\mu^{\varphi^+}(s_j) - \mu^{\varphi^+}(v_i)\right)^2 + \left(\nu^{\varphi^-}(s_j) - \nu^{\varphi^-}(v_i)\right)^2 + \left(\pi^{\varphi}(s_j) - \pi^{\varphi}(v_i)\right)^2\right)^{1/2}. \quad (41)$$

Theorem 2. *Let* ψ: $\mathbb{U}_{ij} \times \mathbb{V}_{ik} \longrightarrow \mathbb{R}$, *such that* $\psi(U, V) = J(U, V)$, *where* $V \in \mathbb{V}_{ik}$ *is fixed. Then* $\mathbb{U}^* = \mu_{ij}$ *is a strict local minima if it is derived from the (47). Here,* $\mathbb{U}_{ij}$ *and* $\mathbb{V}_{ik}$ *are a collection of matrices of membership and cluster center, respectively.*

Proof. Let $L(U, V)$ be a Lagrangian of criterion function under the constraints is defined with the help of λ_j $(1 \le j \le p)$. The parameter $m \in (0, 1) \cup (1, \infty)$ is weighting exponent for a memberships. Now,

$$L(U, V) = \sum_{j=1}^{n} \sum_{i=1}^{c} \mu_{ij}^{m} d_1^2(S_j, V_i) - \sum_{j=1}^{n} \lambda_j \left(\sum_{i=i}^{c} \mu_{ij} - 1 \right). \tag{42}$$

By using equation (40), we have the following equation:

$$L(U, V) = \sum_{j=1}^{n} \sum_{i=1}^{c} \mu_{ij}^{m} \left(\frac{1}{2} \sum_{i=1}^{c} w_i \left(\mu^{\varphi^+}(s_j) - \mu^{\varphi^+}(v_i) \right)^2 + \left(\nu^{\varphi^-}(s_j) - \nu^{\varphi^-}(v_i) \right)^2 + \left(\pi^{\varphi}(s_j) - \pi^{\varphi}(v_i) \right)^2 \right) - \sum_{j=1}^{n} \lambda_j \left(\sum_{i=i}^{c} \mu_{ij} - 1 \right). \tag{43}$$

The Lagrangian condition (42) is solved by setting the derivatives with respect to Lagrangian multiplier λ_j equal to zero:

$$\frac{\partial L(U, V)}{\partial \lambda_j} = 0 - \left(\sum_{i=i}^{c} \mu_{ij} - 1 \right) = 0. \tag{44}$$

Similarly, the Lagrangian condition (42) is also solved by setting the derivatives with respect to membership parameter μ_{ij} equal to zero, where $1 \le i \le c$; $1 \le j \le n$.

$$\frac{\partial L(U, V)}{\partial \mu_{ij}} = \sum_{j=1}^{n} \sum_{i=1}^{c} m \mu_{ij}^{m-1} \left(\frac{1}{2} \left(\mu^{\varphi^+}(s_j) - \mu^{\varphi^+}(v_i) \right)^2 + \left(\nu^{\varphi^-}(s_j) - \nu^{\varphi^-}(v_i) \right)^2 + \left(\pi^{\varphi}(s_j) - \pi^{\varphi}(v_i) \right)^2 \right) = 0, \tag{45}$$

on combining (44) and (45), we have the following equation:

$$\mu_{ij} = \frac{1}{\sum_{r=1}^{c} \left(\left(1/2 \sum_{i=1}^{c} w_i \left(\mu^{\varphi^+}(s_j) - \mu^{\varphi^+}(v_i) \right)^2 + \left(\nu^{\varphi^-}(s_j) - \nu^{\varphi^-}(v_j) \right)^2 + \left(\pi^{\varphi}(s_j) - \pi^{\varphi}(v_j) \right)^2 \right) / \left(1/2 \sum_{i=1}^{c} w_i \left(\mu^{\varphi^+}(s_j) - \mu^{\varphi^+}(v_r) \right)^2 + \left(\nu^{\varphi^-}(s_j) - \nu^{\varphi^-}(v_r) \right)^2 + \left(\pi^{\varphi}(s_j) - \pi^{\varphi}(v_r) \right)^2 \right) \right)^{2/m-1}}, \tag{46}$$

$$\mu_{ij} = \frac{1}{\sum_{r=1}^{c} \left(d_1^2(S_j, V_i) / d_1^2(S_j, V_r) \right)^{2/m-1}}. \tag{47}$$

Equation (42) is formulated as an optimization problem to find the optimal solution that maximizes or minimizes a given objective function while satisfying certain constraints. □

Theorem 3. *The optimal minima of the problem* $L(U, V)$ *is obtained at a point* $V_i(k) = V_i(k+1)$. *Here, the point* $V_i(k) = V_i(k+1)$ *is derived based upon (51)–(53).*

Proof. Let $L(U, V)$ be a Lagrangian of criterion function under the constraints is defined with the help of λ_j $(1 \le j \le p)$. The parameter $m \in (0,1) \cup (1, \infty)$ is weighting exponent for a memberships. Now,

$$L(U, V) = \sum_{j=1}^{n} \sum_{i=1}^{c} \mu_{ij}^{m} d_1^2(S_j, V_i) - \sum_{j=1}^{n} \lambda_j \left(\sum_{i=i}^{c} \mu_{ij} - 1 \right). \tag{48}$$

By using (40), we have the following equation:

$$L(U, V) = \sum_{j=1}^{n} \sum_{i=1}^{c} \mu_{ij}^{m} \left(\frac{1}{2} \sum_{i=1}^{c} w_i \left(\mu^{\varphi^+}(s_j) - \mu^{\varphi^+}(v_i) \right)^2 + \left(\nu^{\varphi^-}(s_j) - \nu^{\varphi^-}(v_i) \right)^2 + \left(\pi^{\varphi}(s_j) - \pi^{\varphi}(v_i) \right)^2 \right) - \sum_{j=1}^{n} \lambda_j \left(\sum_{i=i}^{c} \mu_{ij} - 1 \right). \tag{49}$$

The derivatives of (48) are set equal to zero with respect to $V_i(k+1) = (\mu^{\varphi^+(v_i)}, \nu^{\varphi^-(v_i)}, \pi^{\varphi(v_i)})$ as follows:

$$\frac{\partial L}{\partial \mu^{\varphi^+(v_i)}} = \frac{\partial L}{\partial \nu^{\varphi^-(v_i)}} = \frac{\partial L}{\partial \pi^{\varphi(v_i)}} = 0, \quad 1 \le i \le c. \tag{50}$$

We have the following equation:

$$\mu^{\varphi^+(v_i)} = \frac{\sum_{j=1}^{n} \mu_{ij}^{m} \mu^{\varphi^+(s_j)}}{\sum_{j=1}^{n} \mu_{ij}^{m}}, \quad 1 \le i \le c, \tag{51}$$

$$\nu^{\varphi^-(v_i)} = \frac{\sum_{j=1}^{p} \mu_{ij}^{m} \nu^{\varphi^-(s_j)}}{\sum_{j=1}^{n} \mu_{ij}^{m}}, \quad 1 \le i \le c, \tag{52}$$

$$\pi^{\varphi(v_i)} = \frac{\sum_{j=1}^{p} \mu_{ij}^{m} \pi^{\varphi(s_j)}}{\sum_{j=1}^{n} \mu_{ij}^{m}}, \quad 1 \le i \le c, \tag{53}$$

So, $V_i(k+1) = (\mu^{\varphi^+(v_i)}, \nu^{\varphi^-(v_i)}, \pi^{\varphi(v_i)})$ is a required centroid for optimization of $L(U, V)$.

By using equations (51)–(53), the membership matrix U is updated as a result of the centroid being updated. The objective function $J_{\mathrm{OIFCM}}(U, V)$ is optimized continuously updating the centroid and membership function until $\sum_{i=1}^{c} d_1(V_i(k), V_i(k+1))/c < \epsilon = 10^{-5}$ is satisfied.

Step wise implementation of G-OIFCM is presented in Algorithm 3.

The primary framework of the G-OIFCM is derived from the IFCM clustering algorithm, as illustrated in Figure 3. The implementation of G-OIFCM may result in only a slightly increased computational load compared to the traditional IFCM. □

4. Case Studies of Generalized Ordered Intuitionistic Fuzzy C-Means (G-OIFCM) Clustering Algorithm

In this section, we will delve into two distinct case studies that demonstrate the process of ordered regrouping of countries based on their performance within an intuitionistic environment. Our focus is not on determining the precise ranking of nations but rather on categorizing countries into predefined labels based on their performance in specific criteria. Our objective is to employ a targeted rank-based approach to effectively regroup countries in an intuitionistic environment, taking into account uncertainty crisp data by considering the positive and negative flow of PROMETHEE.

4.1. Case Study-1: Regrouping the Countries in Global Health Security Index (GHSI). In the first case study, we are focusing on a particular class of data sets: uncertain crisp data sets from the Global Health Security Index (GHSI). The Nuclear Threat Initiative, the Johns Hopkins University Center for Health Security, and The Economist Intelligence Unit (EIU) created the GHSI in 2019 as a benchmark to help nations improve their capacity to control infectious disease outbreaks that could have significant global repercussions [48]. To assess a country's ability to prevent and mitigate epidemics and pandemics in 2019-2020, the GHSI employs six categories, 34 indicators and 85 subindicators, with 171 questions. These indicators and subindicators are grouped into six categories or criteria, namely, $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ = {prevention of the emergence, detection and reporting, rapid response, health system, compliance with international norms, and risk environment}.

Recently, there have been emerging criticisms surrounding the Global Health Security Index (GHSI). These criticisms encompass the inverse correlation observed between certain indicators and subindicators during the COVID-19 pandemic, as well as the subjective implementation of weights in the scoring system [49]; the questionable validity of some indicators and subindicators [50]; and the incomplete definition of relevant categories (most prepared, more prepared, and least prepared [51]). In addition, during pandemics, the ranking based on results from indicators and subindicators is not directly comparable [52]. Kaiser et al. [53] have identified two main criticisms of composite indices in GHSI. The first criticism is related to the hierarchical structure of criteria, indicators, and subindicators. This structure includes issues of utility, bias, and reliability that require further study. The second criticism is the uncertainty arising due to missing data, normalization of data, aggregation of normalized indicators, and the difficulty of assigning weights to criteria, indicators, and subindicators.

4.1.1. Regrouping the Countries in Global Health Security Index Based on the G-OIFCM Algorithm. This subsection demonstrates the utilization of G-OIFCM to cluster countries in GHSI, considering their performance across six criteria during the period of 2019-20. Let $A = \{a_i \mid i = 1, 2, \ldots, n\}$ represent the alternatives or countries across the six criteria. Let s_i denote the ranking of the i^{th} country in the GHSI ranking. The goal is to cluster the countries into ordered groups based on their scores. The stepwise execution process of Algorithm 3 for ordered clustering is as follows:

Step 1: first, we calculate the preference indices for each country as $\pi(s_j, s_k), \forall j, k$ and then obtain positive, negative, and hesitancy flow $(\mu^{\varphi^+(s_i)}, \nu^{\varphi^-(s_i)}, \pi^{\varphi(s_i)}) \in [0, 1]$ of PROMETHEE by using the equations (7), (8), and (32), respectively, as illustrated in Figure 4. We select the same linear preference function for each criterion, as mentioned in the following equation [34]:

$$f_k(\nu) = \begin{cases} 0, \ \nu \le 0, \\ \dfrac{\nu}{p_l}, \ 0 \le \nu \ge p_l, & l = 1, 2, 3. \\ 1, \ \nu \ge p_l, \end{cases} \tag{54}$$

Input:
$S = \{s_1, s_2, \ldots, s_j, \ldots, s_n\}$, K clusters, $V = \{V_1, V_2, \ldots, V_i, \ldots, V_c\}$ centroids, $t = 0, T_{\max} = 1000$.
Output:
Matrix U, set of centroids V^*, Objective function $J_{\text{OIFCM}}(U, V)$.
BEGIN
Step-1: Compute $(\mu^{\varphi^+(s_i)}, \nu^{\varphi^-(s_i)}, \pi^{\varphi(s_i)})$ by using the Equations (7), (8) and (32).
Step-2: Initialized K different centroids $(V_r(k))$ from $(\mu^{\varphi^+(v_i)}, \nu^{\varphi^-(v_i)}, \pi^{\varphi(v_i)})$ and rank the centroids by using the (31), i.e., if $(\rho(\varnothing(s_1)) > (\rho(\varnothing(s_2))$ then $\mathbb{C}_1 \succ \mathbb{C}_2$.
Step-3: Compute membership function $U(k) = (\mu_{ij}(k))_{c\times p}$ by using (47) and let $m=2$, where
(a) If $\forall j, r, d_1(S_j, V_r(k)) > 0$, then
$\mu_{ij}(k) = 1/\sum_{r=1}^{c}(d_1^2(S_j, V_i)/d_1^2(S_j, V_r))^{2/m-1}$ $1 \le i \le c;\ 1 \le j \le p$
(b) If there exist j, r such that $d_1(S_j, V_r(k)) = 0$, then let $\mu_{rj}(k) = 1$ and $\mu_{ij}(k) = 0$ for all $i \ne r$.
Step-4: Update the centroids $V(k+1) = \{V_1(k+1), V_2(k+2), \ldots, V_c(k+1)\}$, where
$V_i(k+1) = f(Z, w^{(i)}(k+1)), 1 \le i \le c$
Where, $w^{(i)}(k+1) = \{\mu_{i1}(k)/\sum_{j=1}^{n}\mu_{ij}, \mu_{i2}(k)/\sum_{j=1}^{n}\mu_{ij}, \ldots, \mu_{ij\,(k)}/\sum_{j=1}^{n}\mu_{ij}\}$ $1 \le i \le c$
Step-5: If $\sum_{i=1}^{c} d_1(V_i(k), V_i(k+1))/c < \epsilon = 10^{-5} > 0$, then proceed step 6.
Otherwise, let $k = k+1$, and then return to the Step 3.
END.

Algorithm 3: Generalized ordered intuitionistic fuzzy c-means clustering (G-OIFCM).

The threshold value (p_l) and the corresponding weights $(w^{(i)})$ for each criterion have been presented in Table 1.

Step-2: initialized K different centroids $(V_r(k))$ from $(\mu^{\varphi^+(v_i)}, \nu^{\varphi^-(v_i)}, \pi^{\varphi(v_i)})$ and rank the centroids by using the equation (31), i.e., if $(\rho(\varnothing(s_1)) > (\rho(\varnothing(s_2))$ then $\mathbb{C}_1 \succ \mathbb{C}_2$, i.e., countries in $\mathbb{C}_1$ are better than the $\mathbb{C}_2$.

Step 3: compute membership function $U(k) = (\mu_{ij}(k))_{c\times p}$ by using equation (47) and let $m=2$, where

(a) If $\forall j, r, d_1(S_j, V_r(k)) > 0$, then

$$\mu_{ij}(k) = \frac{1}{\sum_{r=1}^{c}\left(d_1^2\left(S_j, V_i\right)/d_1^2\left(S_j, V_r\right)\right)^{2/m-1}}, \quad 1 \le i \le c;\ 1 \le j \le p. \tag{55}$$

(b) If there exists j, r such that $d_1(S_j, V_r(k)) = 0$, then let $\mu_{rj}(k) = 1$ and $\mu_{ij}(k) = 0$ for all $i \ne r$.

Step 4: update the centroids $V(k+1) = \{V_1(k+1), V_2(k+2), \ldots, V_c(k+1)\}$, where

$$V_i(k+1) = f\left(Z, w^{(i)}(k+1)\right), \quad 1 \le i \le c, \tag{56}$$

where $w^{(i)}(k+1) = \{\mu_{i1}(k)/\sum_{j=1}^{n}\mu_{ij}, \mu_{i2}(k)/\sum_{j=1}^{n}\mu_{ij}, \ldots, \mu_{ij\,(k)}/\sum_{j=1}^{n}\mu_{ij}\}$ $1 \le i \le c$

Step 5: if $\sum_{i=1}^{c} d_1(V_i(k), V_i(k+1))/c < \epsilon = 10^{-5} > 0$, then go to step 6.

Otherwise, let $k = k+1$, and return to Step 3.

Step 6: end

We propose an ordered classification of countries in a manner similar to the Human Development Index (HDI). In order to assess the ordering in the Global Health Security Index (GHSI) problem, we divide countries' performance into five tiers, which can be found on http://www.ghsindex.org. Due to uncertainty in the data, i.e., measurement and parameterization, we employ the IFCM clustering algorithm on the optimized values of PROMETHEE, i.e., positive, negative, and hesitance flow for the year 2019. This allows us to identify countries in five levels of GHSI: (1) very high level, (2) high level, (3) medium level, (4) low level, and (5) lowest level. In Table 2, it is shown that the first 33 countries belong to the cluster of very high level Global Health Security Index, 32 countries belong to high level, 43 countries belong to medium level, 46 countries belong to low level, and 41 countries belong to the cluster of lowest level Global Health

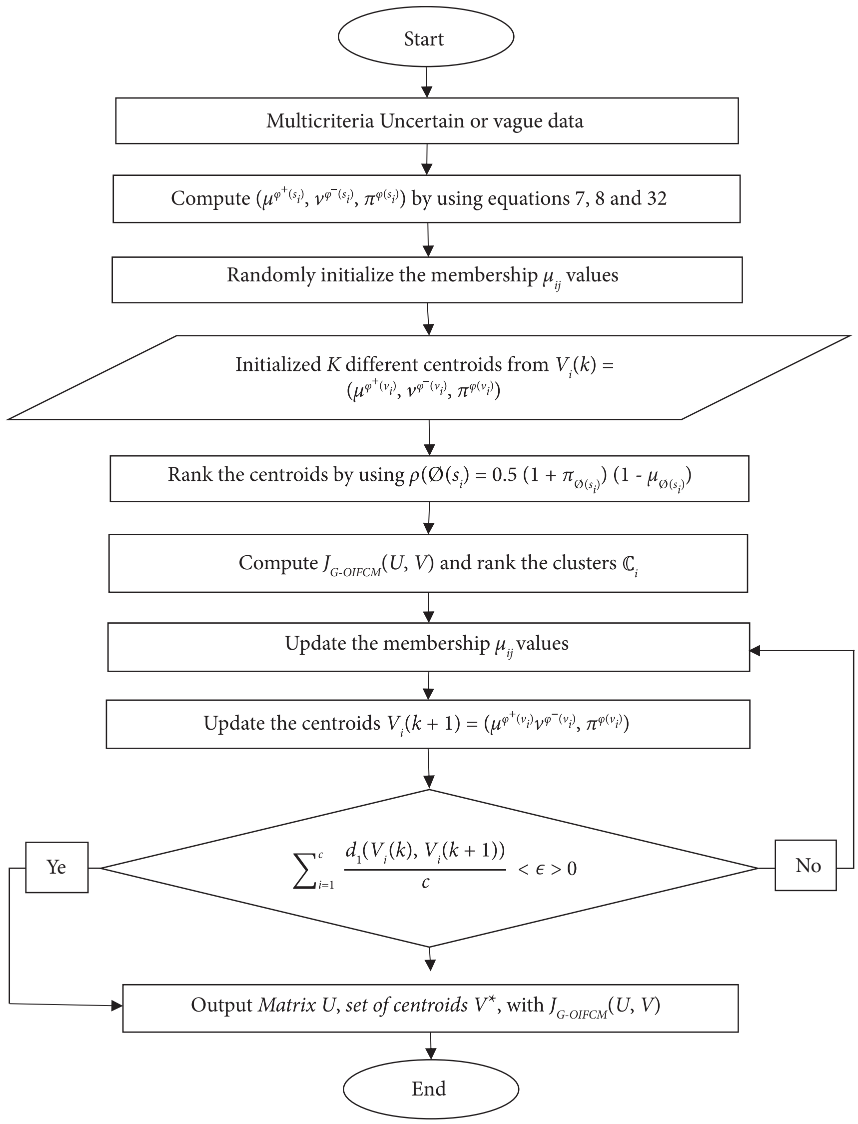

Figure 3: Process of G-OIFCM clustering algorithm.

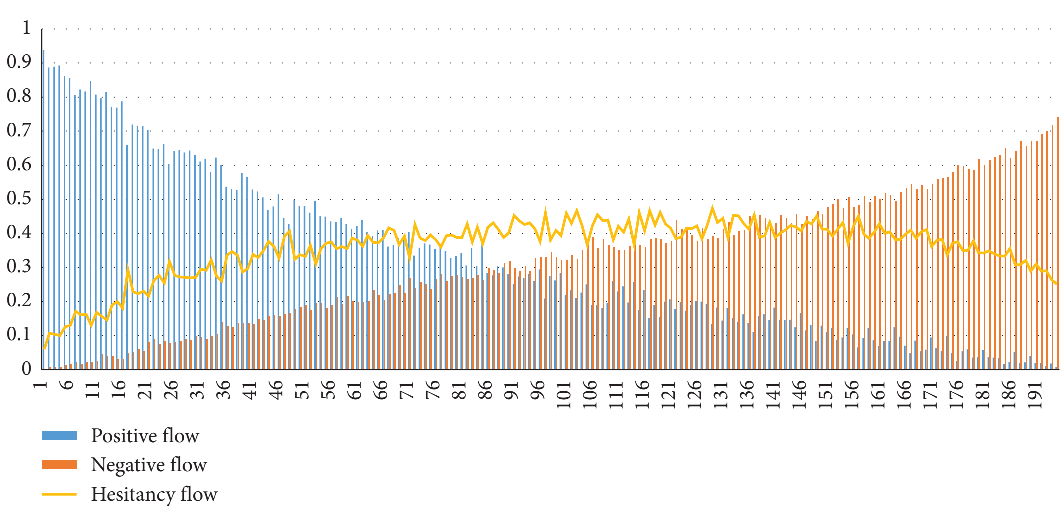

Figure 4: The x-axis denotes the label of 195 countries and y-axis denotes positive, negative, and hesitancy flow $(\mu^{\varphi^+(s_i)}, \nu^{\varphi^-(s_i)}, \pi^{\varphi(s_i)}) \in [0, 1]$.

Table 1: The preference (p_l), indifference (q_l), thresholds, and weight (w_l) of each criterion.

Parameters	C_1	C_2	C_3	C_4	C_5	C_6
Strict preference threshold: p_l	14.8	20.5	13.3	15.0	10.9	13.9
Indifference threshold: q_l	7.4	10.2	6.7	7.5	5.4	6.9
Weight of criteria: w_l	0.167	0.167	0.167	0.167	0.167	0.167

Security Index. The boundaries separating the five clusters are considered appropriate as they effectively divide countries into very high, high, medium, low, and lowest level countries in proportions that are appropriate.

To verify the number of tiers with the clusters, we employ the elbow method on $(\mu^{\varphi^+(\mathbf{s}_i)}, \mathbf{v}^{\varphi^-(\mathbf{s}_i)}, \pi^{\varphi(\mathbf{s}_i)})$ of PROMETHEE as shown in Figure 5. This method aids in identifying the point of maximum curvature and verifies the appropriate number of clusters.

Next, we present the extended results of ordered clustering for the year 2019, categorized into 1-very high, 2-high, 3-medium, 4-low, and 5-lowest level GHSI, as depicted in Figure 6. The x-axis represents GHSI ranks for 195 countries, while the y-axis indicates i^{th} cluster for $i = 1, 2, 3, 4, 5$. Furthermore, the clusters are not evenly distributed. Cluster C_1 contains 17% of the countries, C_2 contains 16%, C_3 contains 22%, C_4 contains 24%, and C_5 contains 21%. This observation aligns with common sense as it is expected that the number of countries in the very high level GHSI category would be smaller than the number of countries in the lowest level GHSI category. It is evident that the ordered grouping aligns closely with the GHSI ranks provided by EIU, which can be accessed at http://www.ghsindex.org.

4.1.2. Comparison of G-OIFCM with Other Clustering Algorithms. We compared the clustering results of G-OIFCM with the FCM, OFCM, and G-IFCM clustering algorithms to address the same GHSI issue mentioned earlier. The aim was to further validate our proposed clustering technique, as shown in Table 3. The clustering results obtained from FCM and G-IFCM were deemed inadequate in aligning with the GHSI ranking. This inadequacy can be attributed to the observed heterogeneity among countries resulting from overlapping. In contrast, G-OIFCM consistently produced smooth results, presenting groups of homogeneous countries, as depicted in Figure 7. Notably, the classification results from OFCM exhibited fewer inconsistencies compared to the other clustering algorithms. Details of comparison are further included for soling GHSI problem as mentioned above.

(1) Regrouping the Countries by Using FCM. We applied the conventional FCM clustering technique to preprocessed data with respect to six different criteria of GHSI to group the countries into five predefined clusters. Figure 7 shows the results derived by FCM that reveal apparent inconsistencies between the partitioning results and GHSI ranking. The

Table 2: A Ranking comparison of GHSI countries based on G-OIFCM clustering.

Country	G-OIFCM
The United States	1
Australia	1
Canada	1
The United Kingdom	1
The Netherlands	1
Sweden	1
Finland	1
Denmark	1
Slovenia	1
South Korea	1
France	1
Thailand	1
Switzerland	1
Norway	1
Germany	1
Spain	1
Malaysia	1
Belgium	1
Portugal	1
Japan	1
Latvia	1
Singapore	1
Ireland	1
Austria	1
Estonia	1
Chile	1
Argentina	1
Mexico	1
Poland	1
Indonesia	1
New Zealand	1
Italy	1
Hungary	1
Czech Republic	1
Brazil	1
Lithuania	1
Turkey	2
South Africa	2
Serbia	2
Greece	2
Croatia	2
Peru	2
Georgia	2
Vietnam	2
The United Arab Emirates	2
Slovakia	2
Kyrgyz Republic	2
Armenia	2
Ecuador	2
China	2
Mongolia	2
Israel	2
Iceland	2
Romania	2
The Philippines	2
Bulgaria	2
Saudi Arabia	2
Liechtenstein	2
Kuwait	2

Table 2: Continued.

Country	G-OIFCM
India	2
Russia	2
Colombia	2
Kenya	2
Costa Rica	2
Oman	2
Luxembourg	2
Cyprus	2
Panama	2
Montenegro	2
Uganda	2
Moldova	2
Nicaragua	2
Uruguay	2
Jordan	2
Bosnia	2
Ethiopia	2
Kazakhstan	2
Qatar	2
Morocco	2
Myanmar	2
Lebanon	3
Bhutan	3
Egypt	3
Laos	3
Trinidad and Tobago	3
North Macedonia	3
Madagascar	3
Bahrain	3
Malta	3
Senegal	3
Ukraine	3
Dominican Republic	3
Cuba	3
Sierra Leone	3
Cambodia	3
Iran	3
Nigeria	3
St Lucia	3
Suriname	3
Tanzania	3
Bolivia	3
Uzbekistan	3
Liberia	3
Belarus	3
Paraguay	3
Nepal	3
Zimbabwe	3
St Vincent and the Grenadines	3
The Maldives	3
Namibia	3
Pakistan	3
Cameroon	3
Ghana	3
C"te d'Ivoire	3
Mauritius	3
Tunisia	3
Barbados	3
The Gambia	3
Bangladesh	3

Table 2: Continued.

Country	G-OIFCM
The Seychelles	3
Rwanda	3
Sri Lanka	3
Micronesia	3
Guyana	3
Azerbaijan	3
Belize	3
Tajikistan	3
Afghanistan	3
Botswana	3
Bahamas	3
eSwatini (Swaziland)	3
San Marino	4
Antigua and Barbuda	4
Andorra	4
Niger	4
Guatemala	4
Togo	4
Guinea	4
Cabo Verde	4
Lesotho	4
Haiti	4
Turkmenistan	4
Burkina Faso	4
Jamaica	4
Benin	4
Mali	4
Grenada	4
Chad	4
St Kitts and Nevis	4
Malawi	4
Central African Republic	4
Zambia	4
Mozambique	4
Samoa	4
Comoros	4
Papua New Guinea	4
Vanuatu	4
Congo (Democratic Republic)	4
Honduras	4
Papua New Guinea	4
Sudan	5
Fiji	5
Dominica	5
Congo (Brazzaville)	5
Tonga	5
Mauritania	5
Angola	5
Timor-Leste	5
Algeria	5
Libya	5
Venezuela	5
Iraq	5
Tuvalu	5
Palau	5
Burundi	5
Djibouti	5
Niue	5
Eritrea	5
Nauru	5

Table 2: Continued.

Country	G-OIFCM
The Solomon Islands	5
The Cook Islands	5
Syria	5
South Sudan	5
Gabon	5
The Marshall Islands	5
Kiribati	5
Guinea-Bissau	5
Yemen	5
São Tomé and Príncipe	5
North Korea	5
Equatorial Guinea	5
Somalia	5

main reason for this is that the classical FCM calculates the similarity between any two countries using the Euclidean distance. In other words, due to the symmetric nature of the Euclidean distance, the conventional FCM is unable to capture preference relationships between objects and clusters.

(2) Regrouping the Countries by Using OFCM. We employed the FCM clustering technique to group countries into five predefined clusters based on the net outranking of the PROMETHEE in GHSI. However, in GHSI scenarios, the datasets frequently demonstrate imprecision or uncertainty, which can result in suboptimal outcomes when applying ordered clustering. Figure 7 presents the OFCM classification results, highlighting apparent less inconsistencies as compare to the FCM. The primary reason for this discrepancy may be the need for caution when interpreting scores for categorization during the partitioning process.

(3) Regrouping the Countries by Using IFCM. To establish five specified clusters, we utilized the adaptive generalized intuitionistic fuzzy c-means (G-IFCM) clustering algorithm [13] on preprocessed data derived from six distinct GHSI criteria. Figure 7 displays the results obtained from G-IFCM, revealing notable inconsistencies between the partitioning outcomes and GHSI ranking. These inconsistencies primarily stem from the fact that traditional G-IFCM relies on the Euclidean distance to measure similarity between any two countries. In other words, the symmetry of the Euclidean distance restricts the conventional G-IFCM's capacity to represent preference relations between objects and clusters.

In conclusion, the G-OIFCM algorithm distinguishes itself from classical clustering algorithms by incorporating the objective function as the sum of all alternatives' optimized values, including the positive and negative flow of PROMETHEE. In addition, it introduces a comprehensive ordered relationship among the clusters, addressing the limitations of the traditional approach. The boundaries between the various clusters are adequate because they split very high, high, medium, low, and lowest level countries in proportions that are appropriate.

4.1.3. Clustering Validation Measure. To evaluate the quality of the preference structure generated by G-OIFCM, we conducted a comparative analysis of its clustering results with those obtained from classical and other ordered clustering algorithms. We employed the Davies–Bouldin index (DB) as a cost criterion and the Dunn index (DI) as a benefit criterion to evaluate the performance of these algorithms. The GHSI data were classified into five distinct categories: very high, high, medium, low, and the lowest level of global health security. Our analysis revealed that the G-OIFCM algorithm produced more accurate outcomes compared to the OFCM, FCM, and G-IFCM algorithms, as demonstrated in Table 4. Upon reviewing the evaluation results of the assessed algorithms, it became evident that G-OIFCM exhibited superior performance by improving intercluster distance and reducing cluster diameter.

4.2. Case Study-2: Regrouping the Countries in Human Development Index (HDI). The United Nations Development Program (UNDP) ranks the 179 countries in the Human Development Index (HDI) ranking based on three criteria, i.e., $G = \{t_{1,}t_2, t_3\} = \{\text{life expectancy, education, income index}\}$. Literature has also explored various types of uncertainty in the HDI data [54], which emphasizes the need for caution when interpreting scores for categorization [55]. To validate the efficiency of our proposed approach, we apply the G-OIFCM on HDI problems adapted from [35]. Our objective is to apply a specific rank-based clustering methodology that considers the positive and negative flow of all three criteria, followed by a comparison of the outcomes with those generated by the FCM, OFCM, and G-IFCM techniques.

4.2.1. Regrouping the Countries in a Human Development Index (HDI) Based on the G-OIFCM Algorithm. We first compute preference indices $\pi(s_j, s_k), \forall j, k$ for each country in HDI and then obtain positive, negative, and hesitancy flow $(\mu^{\varphi^+(s_i)}, \nu^{\varphi^-(s_i)}, \pi^{\varphi(s_i)})$ of PROMETHEE by using the equations (7), (8), and (32), respectively, which are showing in Figure 8. We select the same linear preference function for each criterion as mentioned in [35].

We employed Algorithm 3 for ordered clustering in an intuitionistic environment based on positive, negative, and

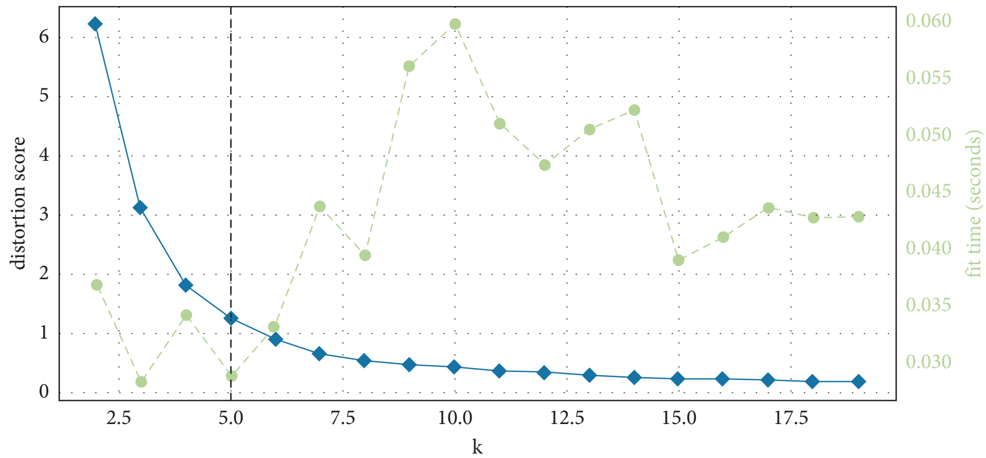

FIGURE 5: Elbow graph of $(\mu^{\varphi^{+}(s_i)}, v^{\varphi^{-}(s_i)}, \pi^{\varphi(s_i)})$ indicates the five clusters.

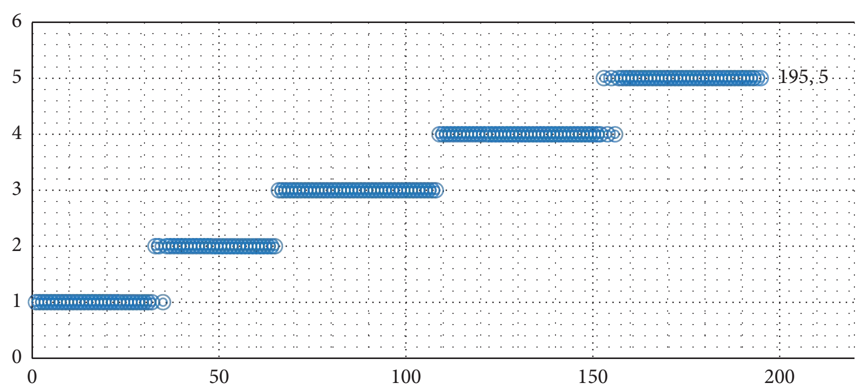

FIGURE 6: The ordered clustering results of the G-OIFCM for five clusters for the year 2019-20. The x-axis denotes the label of countries and y-axis denotes the cluster number.

TABLE 3: Partitioning comparison of FCM, OFCM, G-IFCM, and G-OIFCM clustering algorithms into five clusters of GHSI.

Proposed classifications	Number of countries			
	FCM	OFCM	G-IFCM	G-OIFCM
Very high level global health security index	21	20	33	33
High level global health security index	37	30	42	32
Medium level global health security index	41	45	49	43
Low-level global health security index	51	58	40	46
Lowest level global health security index	45	42	31	41

hesitancy flow $(\mu^{\varphi^{+}(s_i)}, v^{\varphi^{-}(s_i)}, \pi^{\varphi(s_i)}) \in [0, 1]$ for 179 countries and results are compared. In evaluating the ordering of the HDI problem, we establish four prerequisite clusters: very high, high, medium, and low human-developed index countries. The first 50 countries fall within the cluster of very high human development index, while 75 countries are

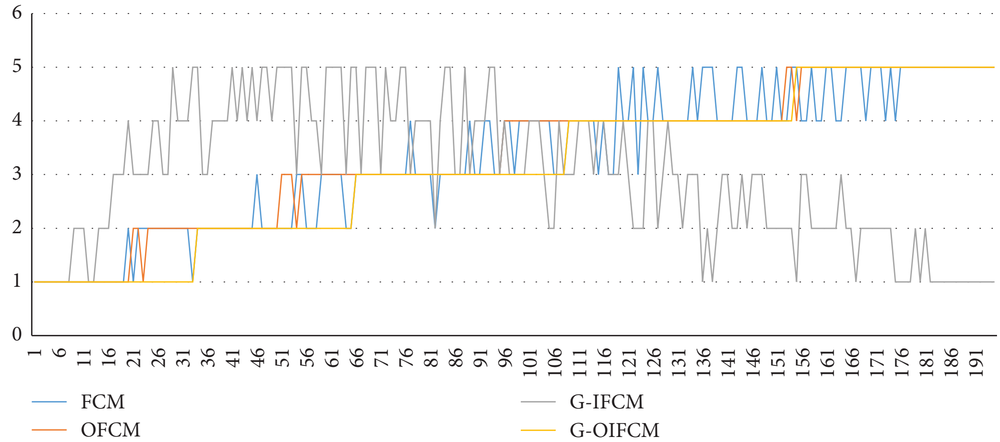

FIGURE 7: Comparison of clustering results and boundaries between the objects and clusters obtained from FCM, OFCM, and G-IFCM with those of G-OIFCM.

TABLE 4: Davies–Bouldin (DB) index and Dunn index scores for classical and other ordered clustering algorithms.

Partitioning measures	Clustering algorithms			
	G-IFCM	FCM	OFCM	G-OIFCM
Davies–Bouldin index score	2.8812	1.4357	0.1288	**0.1242**
Dunn index	0.0860	0.0906	2.8443	**3.2507**

The best results in the ranking of clustering algorithms are shown in bold.

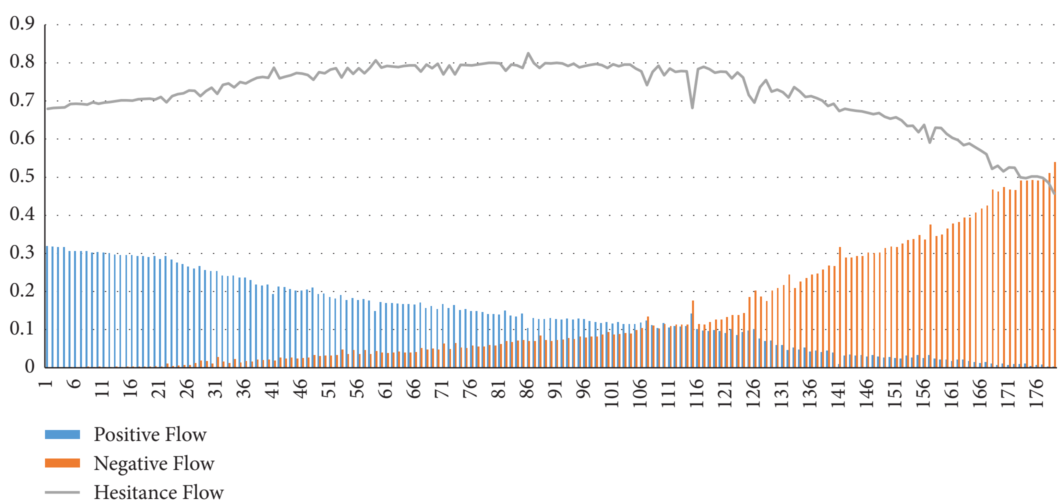

FIGURE 8: The x-axis denotes the label of 179 countries and y-axis denotes positive, negative, and hesitancy flow $(\mu^{\varphi^{+}(s_i)}, v^{\varphi^{-}(s_i)}, \pi^{\varphi(s_i)}) \in [0, 1]$.

Table 5: A ranking comparison of HDI countries based on G-OIFCM clustering is given below.

Countries	G-OIFCM
Iceland	1
Norway	1
Canada	1
Australia	1
Ireland	1
Netherlands	1
Sweden	1
Japan	1
Luxembourg	1
Switzerland	1
France	1
Finland	1
Denmark	1
Austria	1
United States	1
Spain	1
Belgium	1
Greece	1
Italy	1
New Zealand	1
United Kingdom	1
Hong Kong	1
Germany	1
Israel	1
Korea, Rep	1
Slovenia	1
Brunei Darussalam	1
Singapore	1
Kuwait	1
Cyprus	1
United Arab Emirates	1
Bahrain	1
Portugal	1
Qatar	1
Czech Republic	1
Malta	1
Barbados	1
Hungary	1
Poland	1
Chile	1
Slovak Republic	1
Estonia	1
Lithuania	1
Latvia	1
Croatia	1
Argentina	1
Uruguay	1
Cuba	1
The Bahamas	1
Costa Rica	1
Dominican Republic	2
St. Vincent and the Grenadines	2
Georgia	2
China	2
Tunisia	2
Samoa	2
Azerbaijan	2
Paraguay	2
Mexico	2

Table 5: Continued.

Countries	G-OIFCM
Libya	2
Oman	2
Seychelles	2
Saudi Arabia	2
Bulgaria	2
Trinidad and Tobago	2
Panama	2
Antigua and Barbuda	2
Saint Kitts and Nevis	2
Guatemala	2
Kyrgyz Republic	2
Vanuatu	2
Tajikistan	2
South Africa	2
Venezuela, RB	2
Romania	2
Malaysia	2
Montenegro	2
Serbia	2
St. Lucia	2
Belarus	2
North Macedonia	2
Albania	2
Brazil	2
Kazakhstan	2
Ecuador	2
Russian Federation	2
Mauritius	2
Bosnia	2
Turkey	2
Dominican Republic	2
Lebanon	2
Peru	2
Colombia	2
Thailand	2
Ukraine	2
Armenia	2
Iran, Islamic Rep	2
Tonga	2
Grenada	2
Jamaica	2
Belize	2
Suriname	2
Jordan	2
Maldives	2
Algeria	2
El Salvador	2
Philippines	2
Fiji	2
Sri Lanka	2
Syrian Arab Republic	2
Palestinian	2
Gabon	2
Turkmenistan	2
Indonesia	2
Guyana	2
Bolivia	2
Mongolia	2
Moldova	2
Vietnam	2

Table 5: Continued.

Countries	G-OIFCM
Equatorial Guinea	2
Egypt, Arab Rep	2
Honduras	2
Cabo Verde	2
Uzbekistan	2
Nicaragua	2
Botswana	3
Morocco	3
Sao Tome and Principe	3
Djibouti	3
Tanzania	3
Senegal	3
Nigeria	3
Lesotho	3
Uganda	3
Angola	3
Timor-Leste	3
Namibia	3
Congo, Dem. Rep	3
Bhutan	3
India	3
Lao PDR	3
Solomon Islands	3
Myanmar	3
Cambodia	3
Comoros	3
Yemen, Rep	3
Pakistan	3
Mauritania	3
Eswatini	3
Ghana	3
Madagascar	3
Kenya	3
Nepal	3
Sudan	3
Bangladesh	3
Haiti	3
Papua New Guinea	3
Cameroon	3
Togo	3
Gambia, the	3
Benin	4
Malawi	4
Zambia	4
Eritrea	4
Rwanda	4
Cote d'Ivoire	4
Guinea	4
Mali	4
Ethiopia	4
Chad	4
Guinea-Bissau	4
Burundi	4
Burkina Faso	4
Niger	4
Mozambique	4
Liberia	4
Congo, Rep	4
Central African	4
Republic Sierra Leone	4

classified as high human development index, 35 countries fall into the medium level of human development index, and 19 countries fall into the low-level human development index, as indicated in Table 5. The boundaries delineating the various clusters demonstrate adequacy, effectively partitioning developing, developed, and undeveloped countries in suitable proportions, as depicted in Figure 9.

4.2.2. Comparison of G-OIFCM with Other Clustering Algorithms. In order to further validate our suggested clustering technique for addressing the identical HDI issue mentioned earlier, we conduct a comparison of the results obtained from the G-OIFCM algorithm with those from the FCM, OFCM, and G-IFCM clustering algorithms. The outcomes of this comparison are presented in Table 6. We apply the conventional FCM and adaptive G-IFCM clustering technique on preprocessing data based on three different HDI criteria to group the homogeneous countries, results are showing that the clustering and HDI ranking are incompatible as shown in Figure 10. The fundamental reason for this might be that these clustering algorithms are uses the Euclidean distance to assess the level of similarity among any two alternatives and the dissimilarity between the clusters.

In other words, the conventional methods are incapable of providing preference relations between alternatives and clusters due to the symmetric nature of the Euclidean distance. The G-OIFCM algorithm differs from classical clustering algorithms by calculating the objective function as the sum of all alternatives' optimized values, i.e., positive and negative flow of PROMETHEE and incorporating a complete ordered relationship between the clusters.

In conclusion, the G-OIFCM algorithm distinguishes itself from classical clustering algorithms by incorporating the objective function as the sum of all alternatives' optimized values, including the positive and negative flow of PROMETHEE. In addition, it introduces a comprehensive ordered relationship among the clusters, addressing the limitations of the traditional approach. The divisions between the different ordered clusters are suitable as they accurately separate countries into the categories of very high, high, medium, and low levels in appropriate proportions. While G-OIFCM demonstrates superior performance compared to other classical clustering algorithms, the ranking outcomes produced by the classic algorithm do not align with common sense, as depicted in Figure 10. Moreover, it inadequately leverages the complete data structure of alternatives.

In summary, we propose a novel generalized ordered intuitionistic fuzzy c-means (G-OIFCM) clustering algorithm based on positive flow ($\varphi^{+}(s_i) \in [0, 1]$) and negative flow ($\varphi^{-}(s_i) \in [0, 1]$) of PROMETHEE and the intuitionistic fuzzy c-means (IFCM) clustering algorithm. G-OIFCM clustering algorithm deals with crisp and fuzzy datasets that have imprecise or uncertain information and builds the ordered clustering in an intuitionistic environment. G-OIFCM provides a clear and structured way to compare and

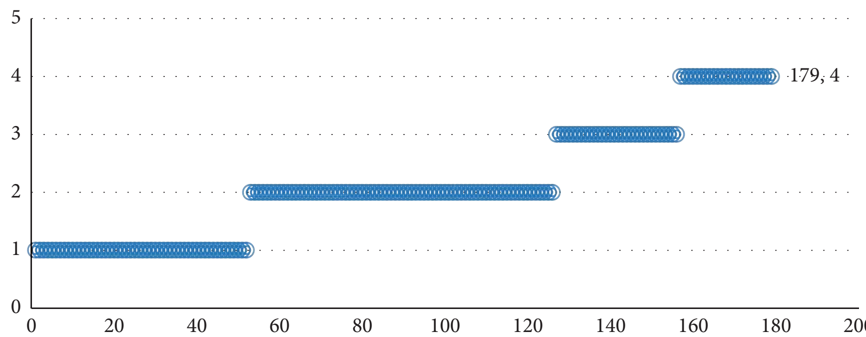

FIGURE 9: The ordered clustering results of the G-OIFCM for four clusters for the year 2008. The x-axis denotes the label of countries and y-axis denotes the cluster number.

TABLE 6: Partitioning comparison of FCM, OFCM, G-IFCM, and G-OIFCM clustering algorithms into five clusters of HDI.

Proposed classifications	Number of countries			
	FCM	OFCM	G-IFCM	G-OIFCM
Very high human development index	54	49	45	50
High human development index	72	75	43	75
Medium level human development index	30	35	50	35
Low human development index	23	20	41	19

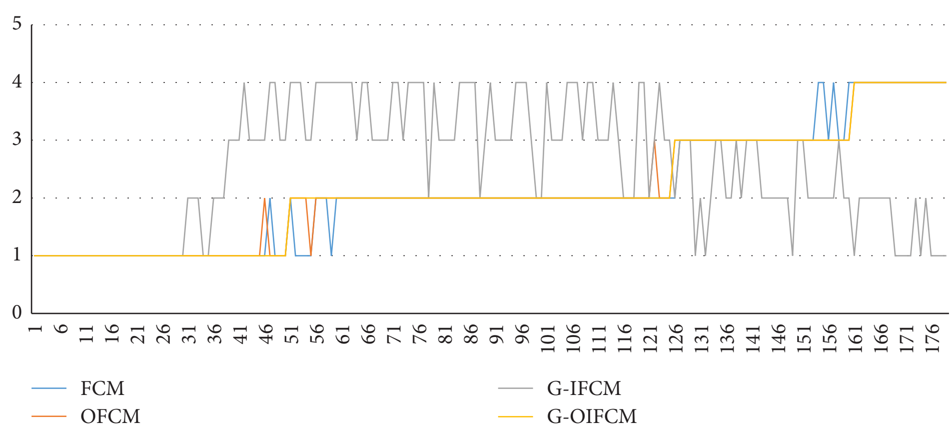

FIGURE 10: Comparison of boundaries between the objects and the clusters of FCM with G-OIFCM.

rank countries based on their performance in the GHSI and HDI problem which can be useful for decision-making and policy planning in the area of global health security.

5. Conclusion

In this article, we present a novel approach to address the multicriteria ordered clustering problem for uncertain or vague information. Our proposed algorithm, known as the generalized ordered intuitionistic fuzzy c-means (G-OIFCM) clustering algorithm, is based on the intuitionistic fuzzy c-means (IFCM) clustering algorithm and PROMETHEE. Different from the classical IFCM using Euclidean norms, we established a new generalized objective function based on the positive ($\varphi^+(s_i) \in [0, 1]$) and negative flow ($\varphi^-(s_i) \in [0, 1]$) of PROMETHEE to build the ordering of clusters. Several important properties of G-OIFCM are also mathematically justified in terms of convergence and optimization. The efficiency of G-OIFCM has been illustrated by the uncertain information of the Global Health Security Index (GHSI) and Human Development Index (HDI) problem. This approach provides a clear and structured way to compare and rank clusters based on their performance in the GHSI and HDI problem, which can be valuable for decision-making and policy planning in the field of ordered classification.

Meanwhile, the classical FCM, G-IFCM, and ordered fuzzy c-means (OFCM) clustering algorithms are included for comparison. The results of our analysis show that the G-OIFCM algorithm outperforms other classical and ordered clustering algorithms, making it a promising tool for clustering data with uncertain or vague information.

In the future, we plan to utilize G-OIFCM for efficient and rapid iterative clustering of big data. In addition, we intend to conduct further research to explore the potential benefits of incorporating nonlinear preference functions into PROMETHEE to enhance its performance.

Data Availability

The Global Health Security Index (GHSI) and Human Development Index (HDI) data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] D. J. Bora, D. Gupta, and A. Kumar, "A comparative study between fuzzy clustering algorithm and hard clustering algorithm," 2014, https://arxiv.org/abs/1404.6059.

[2] J. C. Bezdek, R. Ehrlich, and W. Full, "FCM: the fuzzy c-means clustering algorithm," *Computers & Geosciences*, vol. 10, no. 2, pp. 191–203, 1984.

[3] A. K. Jain and R. C. Dubes, *Algorithms for Clustering Data*, Prentice-Hall, Inc, Hoboken, NJ, USA, 1988.

[4] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[5] K. T. Atanassov, "More on intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 33, no. 1, pp. 37–45, 1989.

[6] Z. Xu and J. Wu, "Intuitionistic fuzzy C-means clustering algorithms," *Journal of Systems Engineering and Electronics*, vol. 21, no. 4, pp. 580–590, 2010.

[7] T. Chaira, "A novel intuitionistic fuzzy C means clustering algorithm and its application to medical images," *Applied Soft Computing*, vol. 11, no. 2, pp. 1711–1717, 2011.

[8] S. V. Aruna Kumar, E. Yaghoubi, and H. Proença, "A fuzzy consensus clustering algorithm for MRI brain tissue segmentation," *Applied Sciences*, vol. 12, no. 15, p. 7385, 2022.

[9] S. Liu, W. Yu, F. T. Chan, and B. Niu, "A variable weight-based hybrid approach for multi-attribute group decision making under interval-valued intuitionistic fuzzy sets," *International Journal of Intelligent Systems*, vol. 36, no. 2, pp. 1015–1052, 2021.

[10] A. P. Wei, D. F. Li, P. P. Lin, and B. Q. Jiang, "An information-based score function of interval-valued intuitionistic fuzzy sets and its application in multiattribute decision making," *Soft Computing*, vol. 25, no. 3, pp. 1913–1923, 2021.

[11] A. Singh and S. Kumar, "A novel dice similarity measure for IFSs and its applications in pattern and face recognition," *Expert Systems with Applications*, vol. 149, Article ID 113245, 2020.

[12] M. Kaushal and Q. M. D. Lohani, "Intuitionistic fuzzy c-ordered means clustering algorithm," *IEEE Access*, vol. 10, pp. 26271–26281, 2022.

[13] M. Kaushal and Q. M. D. Lohani, "Generalized intuitionistic fuzzy c-means clustering algorithm using an adaptive intuitionistic fuzzification technique," *Granular Computing*, vol. 7, no. 1, pp. 183–195, 2022.

[14] M. Ehrgott, *Multicriteria Optimization*, Springer Science & Business Media, Berlin, Germany, 2005.

[15] A. Ishizaka and P. Nemery, "Assigning machines to incomparable maintenance strategies with ELECTRE-SORT," *Omega*, vol. 47, pp. 45–59, 2014.

[16] P. Nemery and C. Lamboray, "$\mathcal{F}$low $\mathcal{S}$ ort: a flow-based sorting method with limiting or central profiles," *Top*, vol. 16, no. 1, pp. 90–113, 2008.

[17] L. Wang, Z. X. Zhang, A. Ishizaka, Y. M. Wang, and L. Martínez, "TODIMSort: a TODIM based method for sorting problems," *Omega*, vol. 115, Article ID 102771, 2023.

[18] N. Belacel, "Multicriteria assignment method PROAFTN: methodology and medical application," *European Journal of Operational Research*, vol. 125, no. 1, pp. 175–183, 2000.

[19] M. Doumpos and C. Zopounidis, "A multicriteria classification approach based on pairwise comparisons," *European Journal of Operational Research*, vol. 158, no. 2, pp. 378–389, 2004.

[20] S. Eppe, J. Roland, and Y. D. Smet, "On the use of valued action profiles for relational multi-criteria clustering," *International Journal of Multicriteria Decision Making*, vol. 4, no. 3, pp. 201–233, 2014 Jan 1.

[21] J. Rosenfeld, Y. D. Smet, O. Debeir, and C. Decaestecker, "Assessing partially ordered clustering in a multicriteria comparative context," *Pattern Recognition*, vol. 114, Article ID 107850, 2021 Jun 1.

[22] M. A. Boujelben, "A unicriterion analysis based on the PROMETHEE principles for multicriteria ordered clustering," *Omega*, vol. 69, pp. 126–140, 2017.

[23] P. Meyer and A. L. Olteanu, "Formalizing and solving the problem of clustering in MCDA," *European Journal of Operational Research*, vol. 227, no. 3, pp. 494–502, 2013.

[24] Y. De Smet, J. Rosenfeld, and D. Van Assche, "Extensions of PROMETHEE to multicriteria clustering: recent developments," *Welcome to ORBEL*, vol. 32, p. 149, 2010.

[25] Y. De Smet, "P2Clust: An extension of PROMETHEE II for multicriteria ordered clustering," in *Proceedings of the 2013 IEEE International Conference on Industrial Engineering and Engineering Management*, pp. 848–851, IEEE, Bangkok, Thailand, December 2013.

[26] J. Rosenfeld and Y. D. Smet, "An extension of PROMETHEE to hierarchical multicriteria clustering," *International Journal of Multicriteria Decision Making*, vol. 8, no. 2, pp. 133–150, 2019.

[27] R. Sarrazin, Y. De Smet, and J. Rosenfeld, "An extension of PROMETHEE to interval clustering," *Omega*, vol. 80, pp. 12–21, 2018.

[28] M. A. Bashir, G. Muhiuddin, T. Rashid, and M. S. Sardar, "Multicriteria ordered the profile clustering algorithm based on PROMETHEE and fuzzy c-means," *Mathematical Problems in Engineering*, vol. 2023, Article ID 5268340, 13 pages, 2023.

[29] I. Beg and T. Rashid, "An improved clustering algorithm using fuzzy relation for the performance evaluation of humanistic systems," *International Journal of Intelligent Systems*, vol. 29, no. 12, pp. 1181–1199, 2014.

[30] Y. Tang, S. Tan, and D. Zhou, "An improved failure mode and effects analysis method using belief jensen–shannon divergence and entropy measure in the evidence theory," *Arabian Journal for Science and Engineering*, vol. 48, no. 5, pp. 7163–7176, 2022.

[31] Y. De Smet, "Extension of the PROMETHEE method to the multicriteria dual clustering problem," in *Proceedings of the 2022 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*, December 2022.

[32] A. Ishizaka, B. Lokman, and M. Tasiou, "A stochastic multicriteria divisive hierarchical clustering algorithm," *Omega*, vol. 103, Article ID 102370, 2021.

[33] R. Díaz, E. Fernández, J. R. Figueira, J. Navarro, and E. Solares, "A new hierarchical multiple criteria ordered clustering approach as a complementary tool for sorting and ranking problems," *Omega*, vol. 117, Article ID 102820, 2023.

[34] J. Pereira, P. Contreras, D. C. Morais, and P. Arroyo-López, "Multi-criteria ordered clustering of countries in the global health security index," *Socio-Economic Planning Sciences*, vol. 84, Article ID 101331, 2022.
[35] Y. De Smet, P. Nemery, and R. Selvaraj, "An exact algorithm for the multicriteria ordered clustering problem," *Omega*, vol. 40, no. 6, pp. 861–869, 2012.
[36] L. Chen, Z. Xu, H. Wang, and S. Liu, "An ordered clustering algorithm based on K-means and the PROMETHEE method," *International Journal of Machine Learning and Cybernetics*, vol. 9, no. 6, pp. 917–926, 2018.
[37] C. Bai, R. Zhang, L. Qian, L. Liu, and Y. Wu, "An ordered clustering algorithm based on fuzzy c-means and PROMETHEE," *International Journal of Machine Learning and Cybernetics*, vol. 10, no. 6, pp. 1423–1436, 2019.
[38] S. Greco, J. Figueira, and M. Ehrgott, *Multiple Criteria Decision Analysis*, Springer, New York, NY, USA, 2016.
[39] J. Clímaco and J. Craveirinha, "Multiple criteria decision analysis–state of the art surveys," *International Series in Operations Research and Management Science*, pp. 899–951, Springer, Berlin, Germany, 2005.
[40] J. P. Brans and P. Vincke, "Note—a preference ranking organisation method: (the PROMETHEE method for multiple criteria decision-making)," *Management Science*, vol. 31, no. 6, pp. 647–656, 1985.
[41] D. K. Iakovidis, N. Pelekis, E. Kotsifakos, and I. Kopanakis, "Intuitionistic fuzzy clustering with applications in computer vision," in *Proceedings of the Advanced Concepts for Intelligent Vision Systems: 10th International Conference, ACIVS 2008*, pp. 764–774, Springer Berlin Heidelberg, Juan-les-Pins, France, October 2008.
[42] P. Amsini and R. U. Rani, "Interval type 2 intuitionistic FCM cluster with spatial information algorithm applied for histopathology images," *Malaya Journal of Matematik*, vol. 9, no. 1, pp. 52–56, 2021.
[43] K. Ito and K. Kunisch, *Lagrange Multiplier Approach to Variational Problems and Applications*, Society for Industrial and Applied Mathematics, Philadelphia, Pennsylvania, USA, 2008.
[44] D. L. Davies and D. W. Bouldin, "A cluster separation measure," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, no. 2, pp. 224–227, 1979.
[45] J. C. Dunn, "Well-separated clusters and optimal fuzzy partitions," *Journal of Cybernetics*, vol. 4, no. 1, pp. 95–104, 1974.
[46] E. Szmidt and J. Kacprzyk, "Distances between intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 114, no. 3, pp. 505–518, 2000.
[47] Z. Xu, "Some similarity measures of intuitionistic fuzzy sets and their applications to multiple attribute decision making," *Fuzzy Optimization and Decision Making*, vol. 6, no. 2, pp. 109–121, 2007.
[48] Url, "Building Collective Action and Accountability," 2019, https://www.ghsindex.org/wp-content/uploads/2019/10/2019-Global-Health-Security-Index.pdf.
[49] M. J. Boyd, N. Wilson, and C. Nelson, "Validation analysis of global health security index (GHSI) scores 2019," *BMJ Global Health*, vol. 5, no. 10, Article ID e003276, 2020.
[50] A. Razavi, N. A. Erondu, and E. Okereke, "The global health security index: what value does it add?" *BMJ Global Health*, vol. 5, no. 4, Article ID e002477, 2020.
[51] E. J. Abbey, B. A. Khalifa, M. O. Oduwole et al., "The Global Health Security Index is not predictive of coronavirus pandemic responses among Organization for Economic Cooperation and Development countries," *PLoS One*, vol. 15, no. 10, Article ID e0239398, 2020.
[52] B. A. Khalifa, E. J. Abbey, S. K. Ayeh et al., "The Global Health Security Index is not predictive of vaccine rollout responses among OECD countries," *International Journal of Infectious Diseases*, vol. 113, pp. 7–11, 2021.
[53] M. Kaiser, A. T. Chen, and P. Gluckman, "Should policy makers trust composite indices? A commentary on the pitfalls of inappropriate indices for policy formation," *Health Research Policy and Systems*, vol. 19, no. 1, pp. 40-41, 2021.
[54] H. Wolff, H. Chong, and M. Auffhammer, "Classification, detection and consequences of data error: evidence from the human development index," *The Economic Journal*, vol. 121, no. 553, pp. 843–870, 2011.
[55] C. A. Comes, "HSDSI analysis for EU 28 countries," *Procedia Economics and Finance*, vol. 32, pp. 154–159, 2015.