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*Research Article*

# Optimal Maintenance Decision Method for a Sensor Network Based on Belief Rule Base considering Attribute Correlation

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Optimal maintenance decision for a sensor network aims to intelligently determine the optimal repair time. The accuracy of the optimal maintenance decision method directly affects the reliability and safety of the sensor network. This paper develops a new optimal maintenance decision method based on belief rule base considering attribute correlation (BRB-c), which is designed to address three challenges: the lack of observation data, complex system mechanisms, and characteristic correlation. This method consists of two sections: the health state assessment model and the health state prediction model. Firstly, the former is accomplished through a BRB-c-based health assessment model that considers characteristic correlation. Subsequently, based on the current health state, a Wiener process is used to predict the health state of the sensor network. After predicting the health state, experts are then required to establish the minimum threshold, which in turn determines the optimal maintenance time. To demonstrate the proposed method is effective, a case study for the wireless sensor network (WSN) of oil storage tank was conducted. The experimental data were collected from an actual storage tank sensor network in Hainan Province, China. The experimental results validate the accuracy of the developed optimal maintenance decision model, confirming its capability to efficiently predict the optimal maintenance time.

## 1. Introduction

Sensor networks comprise distributed, autonomous nodes that sense environmental information and transmit it via communication channels for processing and analysis [1]. Serving as essential tools for information collection and transmission, sensor networks are capable of sensing meteorological data, water quality information, and so on in environmental monitoring, thereby providing robust support for disaster early warning and resource management [2]. In the realm of agriculture, sensor networks can monitor parameters such as soil moisture and temperature to optimize crop cultivation strategies [3]. In the industrial domain, sensor networks have also become indispensable tools for monitoring complex systems, such as oil tanks, liquid-fueled launch vehicles, and traffic control [4]. By sensing various environmental parameters, these networks provide crucial data support to ensure the safety, reliability, and efficiency of the systems. However, as the size of the sensing network continues to expand, the number of nodes and the node failure rate increase, the stability and reliability of the network are also affected [5]. Therefore, to ensure the smooth operation of the network, it is crucial to maintain the sensing network. Maintenance of a sensing network helps to ensure that it can consistently provide high quality data, thus avoiding faulty decisions and analyses. Since sensor networks usually consist of a large number of nodes, regular maintenance of each node can be costly, so exploring optimal maintenance decisions for sensor networks can help reduce maintenance costs. Most importantly, making optimal maintenance decisions can help extends the lifetime of the network as well as ensures its proper operation in complex environments [6].

Many researchers have made remarkable contributions in various aspects of sensor network prediction and health management, as well as the optimization of sensor network performance. For instance, K. S. Hoong Ong et al. [7] proposed a model-independent deep reinforcement learning-based approach for predictive maintenance in edge sensor networks, which addresses the challenge of handling a large amount of sensor data and device state prediction. Iqbal et al. [8] focused on underwater WSN, addressing energy-efficient optimization and reliable data transmission in underwater environments. Saeed et al. [9] applied extra trees for fault diagnosis in WSN. Through this ensemble learning method, the accuracy and efficiency of fault free diagnosis were improved, and the training time was shorter. Rajan et al. [10] introduced a defect-tolerant WSN fault detection and node classification method using an adaptive neuro-fuzzy inference system (ANFIS). This method improved traditional fault detection strategies through a distributed adaptive mechanism, enhancing the robustness and reliability of WSN. Chawra et al. [11] proposed a hybrid metaheuristic-based wake-up scheduling scheme based on the Memtic algorithm and Tabu search. This approach aims to optimize coverage and connectivity in 3D wireless sensor networks (3D WSNs), thereby enhancing their overall efficiency. Although the above studies have made significant progress in the prediction, health management, and performance optimization of sensor networks, these methods are often limited to specific application scenarios and face challenges in dealing with data scarcity, uncertainty, system complexity, and environmental dynamics. This may result in existing methods being unable to effectively address the increased node failure rate and network stability issues in large-scale networks.

Specifically, in the current optimization of maintenance decisions for sensor networks, researchers face two primary challenges: a lack of sufficient observation data and complex system mechanisms [8, 9]. With ongoing improvements in the manufacturing industry, the reliability of sensors has also been enhanced, leading to a decrease in instances of sensor failures. However, this also implies that failure data available for accurate maintenance decisions are relatively scarce. In such cases, it is necessary to leverage additional information from the sensor network to enhance the accuracy and reliability of maintenance decisions. Sensor networks bear the responsibility of monitoring complex systems, distributed across different locations and exhibiting various characteristics. The intricate system mechanisms introduce interdependencies among sensor nodes and susceptibility to external environmental changes, potentially resulting in complex correlations and nonlinear relationships among sensor data. Due to the broad and heterogeneous nature of monitoring, traditional maintenance schedules based solely on expert recommendations often prove overly conservative, leading to unnecessary increases in maintenance costs. Furthermore, the uncertainty and vagueness of expert knowledge pose challenges in its application to maintenance decision-making. To simultaneously reduce maintenance costs and enhance the reliability of sensor networks, it is crucial to address the aforementioned issues, particularly in effectively utilizing limited data and uncertain expert knowledge in optimizing maintenance decisions. By comprehensively considering multiple sources of information, such as sensor data and system states, a more comprehensive and accurate foundation can be provided for decision-making. Therefore, there is an urgent need for a comprehensive approach that effectively utilizes limited data and uncertain expert knowledge to address these challenges in optimizing maintenance decisions for sensor networks.

To address the outlined challenges, the belief rule base (BRB) system is explored as a potential solution in the subsequent discussion. Developed by Yang et al. [12, 13], the BRB expert system integrates fuzzy theory, IF-THEN rules, and evidence theory, allowing it to handle indeterminate, unclear, or incomplete information effectively [14]. Its superior performance has led to applications across various domains, including safety assessment, fault diagnosis, and health state prediction. For instance, Xu et al. [15] designed a new fault diagnosis model for marine diesel engines using multiple BRB subsystems. He et al. [16] addressed the challenge of belief rule combination explosion through an interval construction BRB, and Feng et al. [17] enhanced the interpretability of the BRB-based assessment model with a new optimization approach. Incorporating observation data and expert knowledge, BRB extend the information source of sensor networks and aid in solving problems related to optimal maintenance decision-making in engineering practice [18–20]. Furthermore, as a typical prediction method, the Wiener process can predict the system's future state based on the current one, making it widely used in life prediction, health state prediction, and other fields due to its high adaptability in engineering applications [21]. However, influenced by the complex environment and the sensor mounting model, there are correlations between the characteristics of the Wiener process. That is to say, the characteristic has some redundant information that is represented by the other characteristics, and it will influence the accuracy of the BRB model.

Therefore, this study introduces a new optimal maintenance decision model based on BRB-c and the Wiener process. The model consists of two key components: a BRB-c-based health state assessment for evaluating the current health of the sensor network and a Wiener process-based prediction model for forecasting future health conditions. The initial structure and parameters of the assessment model are decided determined with expert input [22]. However, due to the indeterminacy of expert knowledge, an optimization model employing the projection covariance matrix adaptation evolution strategy (P-CMA-ES) is utilized to simultaneously train the health state assessment and prediction models, improving estimation accuracy.

The key contributions of this research are as follows:

(1) The proposed model leverages limited failure data more effectively by integrating BRB-c for health state assessment, which is adept at processing uncertain data. This approach allows for a more detailed and accurate analysis, enhancing the model's predictive performance and decision-making quality.

(2) By combining the BRB-c-based assessment with the Wiener process for prediction, the model integrates multiple sources of information, including sensor data, historical maintenance records, and expert knowledge. This facilitates a comprehensive view of the network's health, enabling more informed and reliable maintenance decisions.

(3) The model explicitly accounts for the interdependencies among sensor characteristics, utilizing the BRB-c to capture and analyse these correlations. This leads to a more nuanced understanding of the network's behaviour, significantly improving the accuracy and reliability of maintenance predictions and decisions.

In addition, for sensor networks, a significant amount of normal observation data can be collected, which can provide enough information to understand the correlation between characteristics. Compared to existing methods such as fault tree analysis (FTA), reliability-centered maintenance (RCM), and data-driven approaches, the developed method based on correlation coefficient can handle the constraints of the optimization model and reduce model complexity. Therefore, the optimal maintenance decision model incorporates the correlation coefficient.

The rest of this article is arranged in the following parts. In Section 2, the problems in optimal maintenance decision are formulated and a new one based on BRB-c is built. Section 3 presents the inference process of the model. The optimization model for the developed optimal maintenance decision model is proposed in Section 4. A case study is carried out to demonstrate the effectiveness of the model in Section 5. Section 6 provides a conclusion.

## 2. Problem Formulation

In the optimal maintenance decision of the sensor networks, there are two problems: the shortage of observation data and the complicacy of the sensor network system. It should be noted that the complexity of the sensor network can be divided into two aspects: complex system mechanism and characteristic correlation. These problems are formulated in Subsection 2.1, and then the construction of the optimal maintenance decision model based on BRB-c and Wiener process is presented in Subsection 2.2.

*2.1. Problem Formulation of Optimal Maintenance Decision for Sensor Network.* The three problems in the optimal maintenance decision for a sensor network can be listed as follows:

*Problem 1.* In the health management of a sensor network, the availability of observation data is crucial for constructing an effective maintenance model. As electronic products, including sensors, continue to improve in quality, their reliability has significantly increased. Consequently, the probability of sensor failures has decreased. However, even with enhanced reliability, it is inevitable for sensors to experience failures over prolonged operation periods. As a result, the amount of failure data that can be collected for analysis remains limited. This scarcity of failure data poses a significant challenge in accurately establishing an optimal maintenance decision framework. This is the first issue that should be resolved.

*Problem 2.* The primary function of a sensor network is to monitor the state of a complex system. In the case of such systems, the sensor deployment often spans a wide range, leading to strong nonlinearity and interdependencies among the observed information from different sensors. Consequently, relying solely on expert knowledge to provide accurate information for optimal maintenance decision-making in sensor networks becomes nearly impossible. Furthermore, expert knowledge itself is prone to uncertainties, incompleteness, and vagueness, which further complicates its practical application. Therefore, it becomes necessary to incorporate additional sources of information alongside expert knowledge to tackle these challenges. Addressing this second problem is essential to enhance the effectiveness and robustness of the optimal maintenance decision-making process in sensor networks.

*Problem 3.* In engineering practice, the characteristics of a sensor network are often influenced by the complex environment in which they operate. Consequently, redundant system information can be observed within these characteristics, indicating that they are not independent but rather exhibit correlations among themselves. This characteristic correlation poses challenges to accurately represent system information and, subsequently, affects the accuracy of the optimal maintenance decision model. This paper highlights the significance of considering characteristic correlation in order to enhance the accuracy of optimal maintenance decisions. By considering the interdependencies among the characteristics, the aim is to improve the fidelity of the model and enable more precise maintenance decision-making in sensor networks.

Thus, to solve these problems, an optimal maintenance decision model can be constructed as follows:

$$\mathrm{T}_{\mathrm{opt}}(t) = \Xi\left(\Psi\left(x_1(t), x_2(t), \ldots, x_M(t)\right), \mathrm{K}_{\mathrm{thre}}, E, C\right), \quad (1)$$

where $\mathrm{T}_{\mathrm{opt}}(t)$ is the optimal maintenance time of the sensor network at time instant $t$. $\Psi(\cdot)$ and $\Xi(\cdot)$ are the nonlinear assessment model and prediction model, respectively. $\mathrm{K}_{\mathrm{thre}}$ is the repair threshold that is determined by experts. $E$ means the expert knowledge used in the optimal maintenance decision model. $x_1(t), x_2(t), \ldots, x_M(t)$ denote the observation data of the $M$ characteristics of a sensor network. $C$ denotes the attribute correlation between the characteristics.

*2.2. Construction of Optimal Maintenance Decision for Sensor Networks.* There are two components in the optimal maintenance decision for the sensor network: the health state assessment model and the health prediction model.

As an expert system, BRB-c can combine observation data and expert knowledge simultaneously. The health state assessment model contains more than one belief rules, and the $k$th belief rule can be shown as follows:

$$\begin{aligned}
&B_k(t)\text{: If } x_1(t) \text{ is } A_1^k \wedge x_2(t) \text{ is } A_2^k \cdots \wedge x_M(t) \text{ is } A_M^k,\\
&\text{Then } H(t) \text{ is } \left\{\left(D_1,\beta_{1,k}\right),\ldots,\left(D_N,\beta_{N,k}\right)\right\},\\
&\text{With rule weight } \theta_k, \text{characteristic weight } \delta_1,\delta_2,\ldots,\delta_M,\\
&\text{and the correlation matrix } C,
\end{aligned} \tag{2}$$

where $H(t)$ is the estimated health state of the sensor network at time instant $t$. $A_1^k, A_2^k, \cdots, A_M^k$ are the reference points of $M$ characteristics that are used to transform the different observation information to a unified framework [23]. $\left\{(D_1,\beta_{1,k}),\ldots,(D_N,\beta_{N,k})\right\}$ denote the output of the health state assessment model, where $(D_1,\ldots,D_N)$ are the $N$ reference degree of the assessment model and $(\beta_{1,k},\ldots,\beta_{N,k})$ are the belief degrees that correspond to them. $\theta_k$ denotes the $k$th weight of the rule, and $\delta_1,\delta_2,\ldots,\delta_M$ are the weights of the $M$ characteristics [24]. Unlike the BRB model, the BRB-c model includes a new parameter, which is the correlation matrix $C$ used to handle related attributes. The matrix $C$ can be represented as follows:

$$C = \begin{bmatrix} 1 & c(x_1,x_2) & \cdots & c(x_1,x_M) \\ c(x_2,x_1) & 1 & \cdots & c(x_2,x_M) \\ \vdots & \vdots & \ddots & \vdots \\ c(x_M,x_1) & c(x_M,x_2) & \cdots & 1 \end{bmatrix}. \tag{3}$$

The BRB-c model enhances the consideration of attribute correlations by incorporating a correlation matrix. Based on system mechanisms and observational data, it employs the correlation coefficient method for calculation of interattribute correlations. This integration aims to improve the accuracy and efficiency of the BRB model, with the specific method of calculating attribute correlations detailed in (5) of Section 3.1.

Then, based on the estimated health state, the Wiener process-based health state prediction model of the sensor network in the future can be obtained by

$$X\Phi = H(t) + \psi(t)\Xi(t) + \varphi(t)t, \tag{4}$$

where $\Phi$ is the predicted health state in the future. $\psi(t)$ and $\varphi(t)$ are the parameters of the Wiener process-based at time instant $t$. $\Xi(t)$ is the normal process of Brownian motion.

The optimal maintenance time can be determined by the repair threshold $\mathrm{K}_{\mathrm{thre}}$ and the predicted health state $\Phi$.

# 3. Inference of the Optimal Maintenance Decision for Sensor Networks

This section presents the inference process of the developed optimal maintenance decision model. There are two parts in the optimal maintenance decision model: the health state assessment model and the health state prediction model, presented in Subsection 3.1 and 3.2, respectively.

*3.1. Health State Assessment Model Based on BRB-c.* In the assessment model, there are five steps: attribute correlation calculation, observation data transformation, belief rule activation, belief rule combination, and health state calculation.

Step 1: To address the influence of the correlation between the attributes in the BRB-c model, the attribute correlation can be first calculated by the correlation coefficient method, and it can be shown as follows:

$$c\left(x_i,x_j\right) = \begin{cases} \dfrac{\operatorname{Cov}\left(x_i,x_j\right)}{\sqrt{\operatorname{Var}(x_i)\operatorname{Var}\left(x_j\right)}}, & i \neq j \\ 1, & i = j, \end{cases} \tag{5}$$
$$(i,j = 1,\cdots,M),$$

where $x_i$ and $x_j$ are the two attributes of the BRB-c model. $\operatorname{Cov}(x_i,x_j)$ denotes the covariance and $\operatorname{Var}(x_i)$ and $\operatorname{Var}(x_j)$ represent the variance.

Step 2: In engineering practice, different characteristics have different data forms. Thus, the following formula can be used to transform the observation data of the network characteristics into a uniform form.

$$m_j^i(t) = \begin{cases} \dfrac{A_{i(k+1)} - x_i^*(t)}{A_{i(k+1)} - A_{ik}}, & j = k \text{ if } A_{\mathrm{ik}} \leq x_i^*(t) \leq A_{i(k+1)}, \\ \dfrac{x_i^*(t) - A_{ik}}{A_{i(k+1)} - A_{ik}}, & j = k+1, \\ 0, & j = 1,2,\cdots,|x_i|, j \neq k, j \neq k+1, \end{cases} \tag{6}$$

where $m_j^i(t)$ is the degree of matching between the $i$th characteristic and the $j$th reference point. $A_{\mathrm{ik}}$ is the reference point of the $i$th characteristic in the $k$th belief rule and $A_{i(k+1)}$ is the $(k+1)$th belief rule. $|x_i|$ denotes the quantity of the belief rule that comprises the $i$th characteristic [25].

Step 3: Following the obtaining of the matching degree of every reference point, the degree of matching between the input information of network characteristics and the belief rule is calculated by the following:

$$\overline{\delta}_i = \frac{\delta_i\left(1 - \sum_{j=1,j\neq i}^{T_k} c\left(x_i, x_j\right)\right)}{\max_{i=1,\ldots,T_k}\{\delta_i\}} \tag{7}$$

$$m_k = \prod_{i=1}^{T_k}\left(m_k^i\right)^{\overline{\delta}_i} \tag{8}$$

where $m_k$ is the matching degree of the input information to the $k$th belief rule. $T_k$ represents the number of the characteristic in the $k$th belief rule. $\overline{\delta}_i$ refers to the relative weight of the $i$th characteristic, representing the importance of the characteristic in the $T_k$ characteristics [26]. It should be noted that the characteristic correlation represents its objective aspect and the characteristic weight denotes its subjective aspect. Thus, the relative weight $\overline{\delta}_i$ is a combined coefficient.

Then, based on the matching degree of the input information, the activation of the belief rules in the rule base can be carried out in different degrees. The degree of matching of the belief rule can be obtained by

$$w_k = \frac{\theta_k m_k}{\sum_{l=1}^{L}\theta_l m_l}, \quad k = 1, 2, \cdots, L, \tag{9}$$

where $w_k$ is the activation weight of the $k$th belief rule. $L$ refers to the number of the belief rule in the assessment model of health state on the basis of BRB-c, and $\theta_k$ denotes the rule weight that shows the relative importance of the belief rule in the rule base [27].

Step 4: With the activation weight calculated, the health state can be obtained in the rule output. As the output of belief rules takes different forms, the health state cannot be obtained directly and the final output can be derived from the analytical format of the evidential reasoning (ER) algorithm, which can be shown as follows:

$$\beta_n = \frac{\mu\left[\prod_{k=1}^{L}\left(w_k\beta_{n,k} + 1 - w_k\sum_{j=1}^{N}\beta_{j,k}\right) - \prod_{k=1}^{L}\left(1 - w_k\sum_{j=1}^{N}\beta_{j,k}\right)\right]}{1 - \mu\left[\prod_{k=1}^{L}\left(1 - w_k\right)\right]} \tag{10}$$

$$\mu = \left[\sum_{n=1}^{N}\prod_{k=1}^{L}\left(w_k\beta_{n,k} + 1 - w_k\sum_{j=1}^{N}\beta_{j,k}\right) - (N-1)\prod_{k=1}^{L}\left(1 - w_k\sum_{j=1}^{N}\beta_{j,k}\right)\right]^{-1}, \tag{11}$$

where $\beta_n$ is the combined output belief degree of $n$th output reference degree $D_n$, $L$ is the number of the belief rule, and $N$ the output reference degree [17]. Note that in this modeling process, the health state of the sensor network varies, and the optimal maintenance time should be determined by the health state at current time.

Step 5: The combined belief degree of the output reference degree represents the probability of the health state at different reference degree. The formula below can be used to calculate the final health state.

$$e(x(t)) = \sum_{n=1}^{N} u(D_n)\beta_n, \tag{12}$$

where $e(x(t))$ is the estimation of health state at time instant $t$ by the gathered observation information of sensor characteristics. $u(D_n)$ represents the utility of the reference degree that is given by experts and used to measure the impact of different reference degrees on the final estimated health state [28].

*3.2. Wiener Process-Based Health State Prediction Model.* Wiener process is a traditional prediction method that has been used widely in engineering practice [29, 30]. Wiener process, a stochastic process, is widely used in various fields such as engineering, finance, and physics. It can be used to predict the health state of a sensor network. The Wiener process estimates the future state of a sensor network based on historical data and statistical analysis. By analysing patterns and trends in the data, it is possible to predict the evolution of the state of health, identify potential problems or failures, and optimize maintenance strategies. In this study, the Wiener process is used for the prediction of the health state of the sensor network.

On the basis of the estimated health state under the assessment model, a prediction can be made about the future health state with the model as follows:

$$\Phi(t_i) = \Theta(t_{i-1}) + \varphi(t_i)\Delta t + \psi(t_i)\Xi(\Delta t), \tag{13}$$

where $\Phi(t_i)$ and $\Theta(t_{i-1})$ are the health states of the network at time instant $t_i$ and $t_{i-1}$, respectively. $\Delta t = t_i - t_{i-1}$. $\Xi(\Delta t)$ denotes the Brownian motion and $\Xi(\Delta t) \sim N(0, \Delta t)$. $\varphi(t_i)$ and $\psi(t_i)$ represent the degradation coefficient and the diffusion coefficient of the health state prediction model, respectively.

The change of the health state can be obtained by the following theorem.

**Theorem 1.** *The degradation coefficient and the diffusion coefficient of the health state prediction model can be estimated as follows:*

$$\widehat{\varphi}(t_i) = \frac{\sum_{j=1}^{T(t_i)} \Delta\Phi_j}{\sum_{j=1}^{T(t_i)} \Delta t_j}, \tag{14}$$

$$\widehat{\psi}(t_i) = \sum_{j=1}^{T(t_i)} \frac{\left(\Delta\Phi_j - \varphi(t_i)\Delta t_j\right)^2}{\Delta t_j}, \tag{15}$$

*where $\Delta\Phi_j$ is the jth change of the health state and $T(t_i)$ is the number of the available health state of the sensor network at time instant $t_i$.*

*Proof.* The change of the state transition can be obtained by

$$\begin{aligned}\Delta\Phi &= \Phi(t_i) - \Phi(t_{i-1}) \\ &= \varphi(t_i)\Delta t + \psi^2(t_i)\Xi(\Delta t).\end{aligned} \tag{16}$$

Then, the mean and the variance of the change of the state transition can be obtained by the following formula:

$$E(\Delta\Phi) = \varphi(t_i)\Delta t, \tag{17}$$

$$D(\Delta\Phi) = \psi^2(t_i)\Delta t. \tag{18}$$

Thus, the change of the state transition obeys normal distribution $N(\varphi(t_i)\Delta t, \psi^2(t_i)\Delta t)$. The probability distribution of $\Delta\Phi$ can be profiled as follows:

$$f(\Delta\Phi) = \frac{1}{\sqrt{2\pi\psi^2(t_i)\Delta t}} \exp\left\{-\frac{(\Delta\Phi - \varphi(t_i)\Delta t)^2}{2\psi^2(t_i)\Delta t}\right\}. \tag{19}$$

Then, the maximum likelihood function can be constructed and shown as follows:

$$L(\Delta\Phi) = \prod_{j=1}^{T(t_i)} \frac{1}{\sqrt{2\pi\psi^2(t_i)\Delta t_j}} \exp\left\{-\frac{\left(\Delta\Phi_j - \varphi(t_i)\Delta t_j\right)^2}{2\psi^2(t_i)\Delta t_j}\right\}, \tag{20}$$

$$\begin{aligned}\ln L(\Delta\Phi) = &-\frac{1}{2}\ln 2\pi\psi^2(t_i) + \sum_{j=1}^{T(t_i)}\left(-\frac{1}{2}\ln \Delta t_j\right) \\ &- \sum_{j=1}^{T(t_i)} \frac{\left(\Delta\Phi_j - \varphi(t_i)\Delta t_j\right)^2}{2\psi^2(t_i)\Delta t_j}.\end{aligned} \tag{21}$$

The estimated degradation coefficient and the diffusion coefficient of the health state prediction model can be calculated by

$$\begin{cases} \dfrac{\partial \ln L(\Delta\Phi)}{\partial\varphi(t_i)} = 0, \\[2ex] \dfrac{\partial \ln L(\Delta\Phi)}{\partial\psi(t_i)} = 0. \end{cases} \tag{22}$$

Therefore, the estimated degradation coefficient $\widehat{\varphi}(t_i)$ and the diffusion coefficient $\widehat{\psi}(t_i)$ of the prediction model can be acquired as the theorem.

With the predicted health state, the optimal maintenance time of the sensor network can be provided using the following formula:

$$T_{\text{optimal}} = \begin{cases} t_p, & \text{if } \Phi\left(t_p\right) < \Phi_{\text{thre}}, \\ t_p + t_c, & \text{if } \Phi\left(t_p\right) \geq \Phi_{\text{thre}}, \end{cases} \tag{23}$$

where $T_{\text{optimal}}$ is the optimal maintenance time and $t_c$ is the time adjusted when the health status reaches or exceeds the threshold. $\Phi_{\text{thre}}$ is the maintenance threshold that is determined by experts with both considering sensor network reliability and maintenance cost. □

*Remark 1.* It is worth noting that predicted health states below the maintenance threshold may be affected by ambient noise. Therefore, in order to improve the accuracy of determining the optimal maintenance time, the modeling study involves the simultaneous monitoring of the predicted health state at multiple consecutive moments. This approach helps to mitigate the effects of ambient noise and provides more reliable maintenance predictions. For example, only when the health states at time instant $t$, $t+1$, and $t+2$ lower than $\Phi_{\text{thre}}$, the optimal maintenance time is determined as $t$.

## 4. Optimization Model for Optimal Maintenance Decision Model

On account of the indeterminacy of the expert knowledge, the original optimal maintenance decision model cannot provide accurate maintenance time. Thus, this section elaborates the construction of the optimization model for the developed optimal maintenance decision model and concludes its modeling process.

*4.1. Optimization Model Based on P-CMA-ES.* As an expert system, BRB-c model is used to construct the assessment model for health state. The initial model is built by domain experts, who provide reference points and values, output belief degrees for the belief rules, rule weights, and attribute weights. However, in the real environment, the initial model cannot accurately assess the health state of the sensor network due to the uncertainty and ambiguity of the expert knowledge. Therefore, the model parameters need to be adjusted according to the monitoring data to improve its accuracy.

In addition, one of the greatest advantages of the BRB-c model is that it is highly interpretable. This means that it is possible to understand how the model arrived at the assessment results, thus increasing the level of confidence in the results. In order to maintain the interpretability of the model, certain constraints can be introduced into the optimization model. These constraints can include limiting the range of attribute weights, controlling the sum of rule weights, and so on to ensure that the output of the model is in line with expert knowledge and the actual situation.

Through the analysis mentioned above, a conclusion is reached that the optimization model of the established health assessment model has a single optimization objective, which makes it a constrained one. In this study, the P-CMA-ES algorithm, which is the foundation of the suggested optimization model, is an intelligent optimization algorithm that will solve the problem of gradient diffusion.

For the health assessment model, the optimization objective is to minimize the gap between the real health state and the estimated health state. Therefore, the mean square error (MSE) between the two can represent the accuracy of the health assessment model. By using formula below, the MSE can be computed.

$$\mathrm{MSE} = \frac{1}{T}\sum_{t=1}^{T}\left(\mathrm{output}_{\mathrm{model}} - \mathrm{output}_{\mathrm{actual}}\right)^2, \tag{24}$$

where $\mathrm{output}_{\mathrm{model}}$ and $\mathrm{output}_{\mathrm{actual}}$ are the output of the developed model and the actual health state of the sensor network, respectively. $T$ denotes the amount of the observation data in the developed model.

In the health state assessment model based on BRB-c, the parameters have physical meaning. Thus, in order to guarantee their physical meaning in the modeling process, some constraints need to be satisfied as (26)–(29).

Then, the optimization model is presented as

$$\begin{gathered}\min \mathrm{MSE}\left(\theta_k, \beta_{n,k}, \delta_i\right),\\ \text{subject to the constraints:}\end{gathered} \tag{25}$$

$$0 \le \theta_k \le 1, \quad k = 1, 2, \cdots L, \tag{26}$$

$$0 \le \beta_{n,k} \le 1, \quad n = 1, 2, \cdots, N, k = 1, 2, \cdots, L, \tag{27}$$

$$0 \le \delta_i \le 1, \quad i = 1, 2, \cdots, t-1, \tag{28}$$

$$\sum_{n=1}^{N}\beta_{n,k} \le 1, \quad k = 1, 2, \ldots, L, \tag{29}$$

where the above parameters are the optimization parameters in the optimization model. It should be noted that the constraints can be adjusted according to the sensor network status.

In the optimization model, it is worth noting that the parameters in the health state prediction model are not optimized, and they are calculated by the maximum likelihood estimation method as shown in Subsection 3.2 of Section 3.

*Remark 2.* In the modeling process, the BRB-c based optimal maintenance decision method is a recursive process. The P-CMA-ES algorithm is an optimization method that can research the optimal result intelligently. It aims to search the optimal parameters of the BRB-c model based on the constructed optimization model. The modeling process of the BRB-c-based optimal maintenance decision method is determined by the optimization model and it has nothing to do with the optimization algorithm. Thus, P-CMA-ES has not influenced the recursive process of the BRB-c-based optimal maintenance decision.

*4.2. Modeling Process of the Developed Optimal Maintenance Decision Model.* According to the above inference of the optimal maintenance decision model, the modeling process of the optimal maintenance decision model is profiled in Figure 1.

The modeling process of the BRB-c-based optimal maintenance decision model can be concluded as follows:

Step 1: Differentiate the two types of observation data, namely, training data and testing data, which are used to train and test the optimal maintenance decision model, respectively.

Step 2: Train the developed optimal maintenance decision model by the training data. The observation data of the characteristics of the sensor network are used to optimize the health assessment model, and then the obtained health states are utilized for calculating the parameters of the health state prediction model.

Step 3: Test the optimized model by the testing data. The present health state of the sensor network can be obtained by the BRB-c-based health assessment model. And with the estimated health state at the current time, the future health state can be obtained by the health state prediction model.

Step 4: The repair threshold is determined by experts, and based on the predicted health state, the optimal maintenance time can be obtained.

## 5. Case Study

To demonstrate the effectiveness of the developed optimal maintenance decision model, a case study was conducted on a WSN for an oil storage tank located in Hainan province, China.

*5.1. Problem Formulation for the Optimal Maintenance Decision of WSN.* Optimizing the maintenance decision for the sensor network is a valuable approach to enhance its reliability. However, two main challenges need to be addressed in this context. On the one hand, the high reliability of the sensor leads to a low sensor failure rate. Thus, the failure data of the sensor that can be obtained is relatively small, and there is not sufficient information to construct an accurate optimal maintenance decision model. On the other side, the sensors in the WSN are distributed in different positions of the monitored object, and its monitored characteristics are different. Moreover, the noise of the environment of the WSN also influences the accuracy of the observation data. Therefore, experts cannot provide accurate knowledge of the WSN, and the indeterminacy of the expert knowledge also improves the difficulty of using the knowledge. Thus, the above two factors lead to the inaccuracy of the optimal maintenance time of the WSN.

In this particular case study, the WSN is deployed near the sea to monitor the condition of an oil storage tank. Two key characteristics, namely the failure rate (FR) and coverage range (CR), are selected for the WSN. These characteristics serve as the basis for constructing a health state assessment

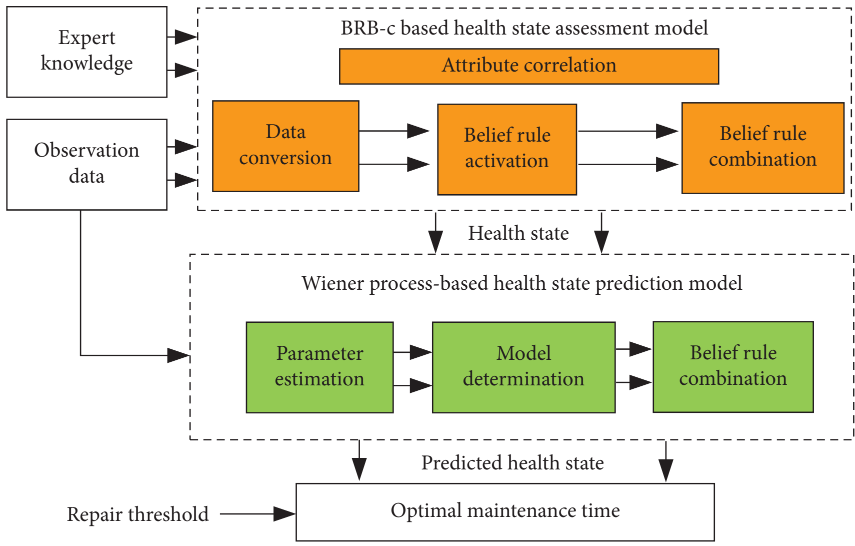


Figure 1: Modeling process of the developed optimal maintenance decision model.

model, which is then utilized to develop a Wiener-based health state prediction model using both the reference health state and estimated health state.

*5.2. Construction of the Optimal Maintenance Decision Model.* In this subsection, experts construct the optimal maintenance decision model, where 4 and 5 reference points are selected for CR and FR, respectively, and L, SL, M, SH, and H represent low, slight low, medium, slight high, and high, respectively. Tables 1 and 2 showing the reference values of these two characteristics are decided by experts. And Table 3 shows the reference degrees of the health state in the output of the assessment model, where L, SL, M, SH, and H denote low, slight low, medium, slight high, and high health state, respectively. Therefore, the belief rule in this health assessment model of the sensor network is as follows:

$$\begin{aligned}&B_k(t)\text{: If } CR(t) \text{ is } A_1^k \wedge FR(t) \text{ is } A_2^k,\\&\text{Then } H(t) \text{ is } \left\{\begin{matrix}(L,\beta_{1,k}),(SL,\beta_{2,k}),(M,\beta_{3,k}),\\(SH,\beta_{4,k}),(H,\beta_{5,k})\end{matrix}\right\},\\&\text{With rule weight } \theta_k, \text{and characteristic weight } \delta_1\delta_2,\end{aligned}\tag{30}$$

where the initial rule weight and the initial characteristic weight are assumed to be 1.

Based on the structure of the belief rule, there are 20 belief rules in the BRB-c-based health state assessment model. Table 4 shows the initial assessment model. Then, the initial Wiener process-based health state prediction model can be obtained by the historical health state, and it can be shown as the following equation:

$$\Phi(t_i) = \Theta(t_{i-1}) + 0.0116\cdot\Delta t + 0.0112\cdot\Xi(\Delta t),\tag{31}$$

where $\Delta t = 1$. Thus, $\Xi(1) \sim N(0,1)$.

Table 1: The referential points and values for CR.

| Referential point | L | M | SH | H |
|---|---|---|---|---|
| Referential value | 5.3998 | 5.5 | 5.7 | 5.815 |

Table 2: The referential points and values for FR.

| Referential point | L | SL | M | SH | H |
|---|---|---|---|---|---|
| Referential value | 0.3699 | 0.4 | 0.42 | 0.44 | 0.4505 |

Table 3: The reference point of output degree.

| Referential point | L | SL | M | SH | H |
|---|---|---|---|---|---|
| Referential value | 0 | 0.25 | 0.5 | 0.75 | 1 |

*5.3. Training and Testing of the Constructed Optimal Maintenance Decision Model.* In training and testing of the optimal maintenance decision model for the WSN, 250 sets of observation data of two characteristics are randomly selected as training data and the rest are used for testing. Among them, there are 64 sets of fault data, which indicate that the storage tanks have experienced problems or failures.

In the P-CMA-ES algorithm, the optimization iteration is set to 300. The optimization iteration process of the BRB-c-based health assessment model is shown in Figure 2. The iterations were set to 50, 100, 200, and 300, respectively. It can be seen that as the number of optimization iterations increased, the actual value and the estimated value gradually approached each other. When the number of iterations reached 300, it can meet the requirements of assessment accuracy and also avoid the occurrence of overfitting.

In Figure 3, the blue and red lines represent the estimation errors of the health status evaluation model based on BRB-c and BRB for the actual health status of WSN, respectively. Obviously, the health status assessment model based on BRB-c can more accurately estimate the health status of WSN.

Table 4: Initial health assessment model for WSN.

| No. | Rule weight | Characteristic CR | Characteristic FR | Output distribution {L, SL, M, SH, and H} |
|---|---|---|---|---|
| 1 | 1 | L | L | (1 0 0 0 0) |
| 2 | 1 | L | SL | (0.7 0.3 0 0 0) |
| 3 | 1 | L | M | (0.6 0.4 0 0 0) |
| 4 | 1 | L | SH | (0.5 0.5 0 0 0) |
| 5 | 1 | L | H | (0.6 0.3 0 0 0) |
| 6 | 1 | M | L | (0.3 0.7 0 0 0) |
| 7 | 1 | M | SL | (0.2 0.8 0 0 0) |
| 8 | 1 | M | M | (0.1 0.9 0 0 0) |
| 9 | 1 | M | SH | (0 0.7 0 0 0.3) |
| 10 | 1 | M | H | (0 0.5 0 0 0.5) |
| 11 | 1 | SH | L | (0.2 0.8 0 0 0) |
| 12 | 1 | SH | SL | (0 1 0 0 0) |
| 13 | 1 | SH | M | (0 0.8 0 0 0.2) |
| 14 | 1 | SH | SH | (0 1 0 0 0) |
| 15 | 1 | SH | H | (0 0.8 0 0 0.2) |
| 16 | 1 | H | L | (0 0.9 0 0 0.1) |
| 17 | 1 | H | SL | (0 1 0 0 0) |
| 18 | 1 | H | M | (0 0.8 0 0 0.2) |
| 19 | 1 | H | SH | (0 0.6 0 0 0.4) |
| 20 | 1 | H | H | (0 0 0 0.4 0.6) |

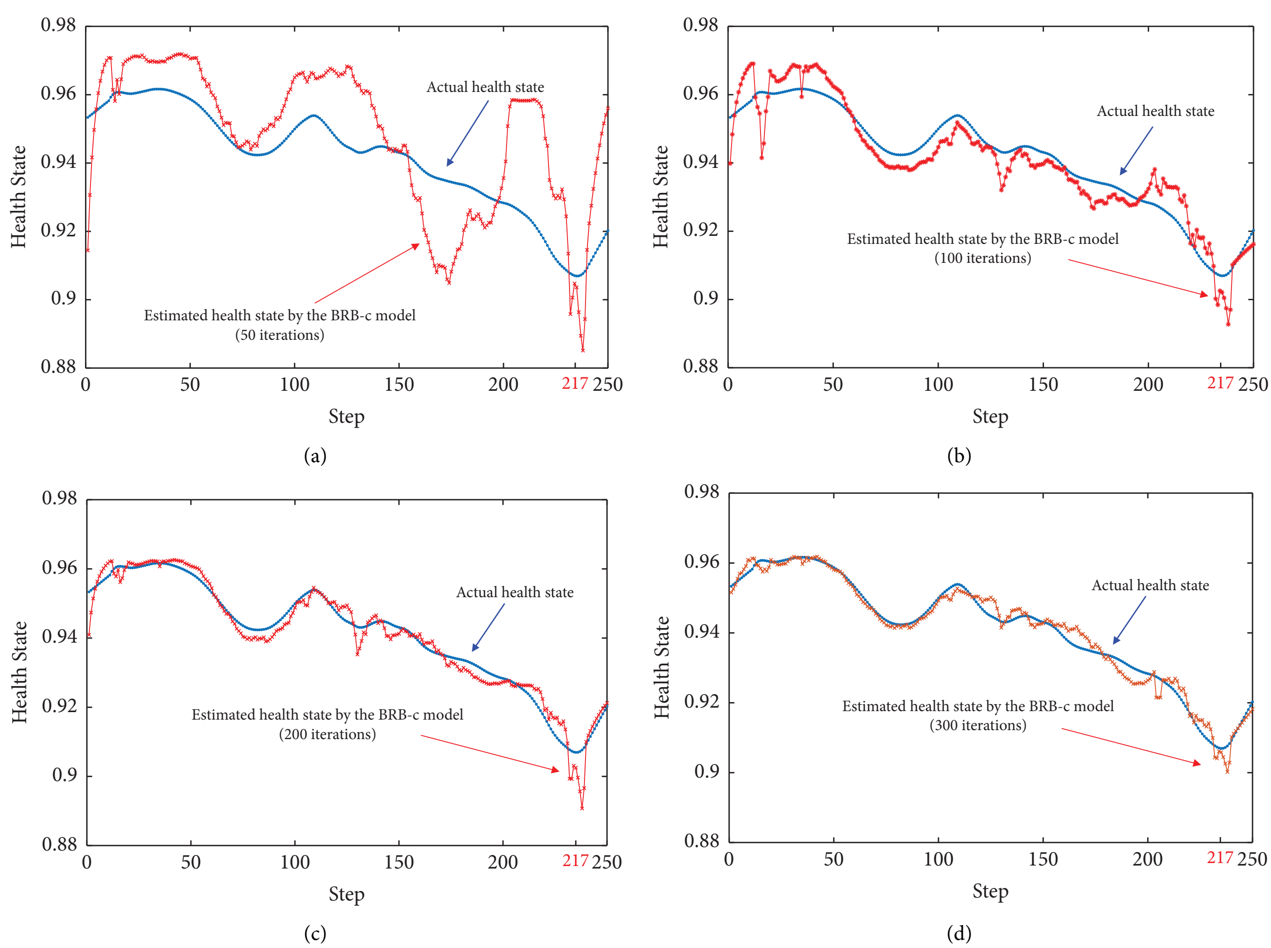


Figure 2: Output of the health assessment model: (a) comparison of actual and estimated values at 50 iterations, (b) comparison of actual and estimated values at 100 iterations, (c) comparison of actual and estimated values at 200 iterations, and (d) comparison of actual and estimated values at 300 iterations.

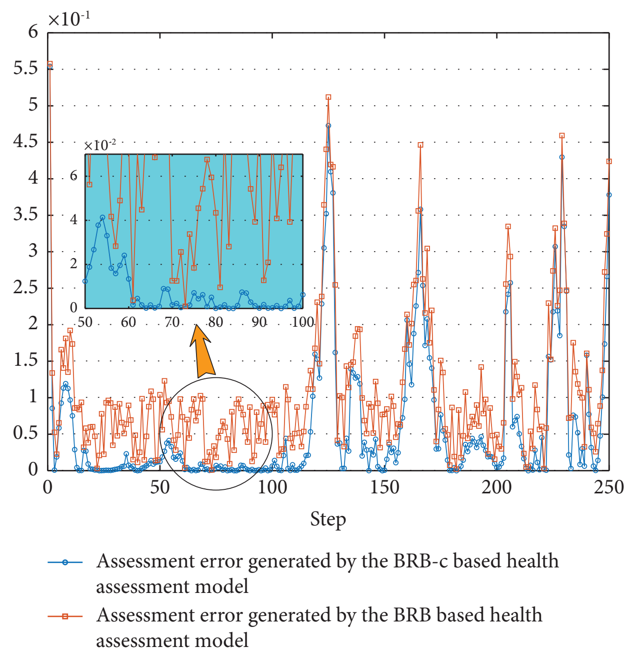


Figure 3: Errors generated by BRB-c and BRB.

Table 5: MSEs generated by BRB-c and other models.

| Model | BRB-c | Feng et al. (BRB) | Zhou et al. (BRB) | ELM | BP neural network | Fuzzy inference |
|---|---|---|---|---|---|---|
| MSE | 0.0356 | 0.0442 | 0.0389 | 0.0714 | 0.1012 | 0.1225 |

Table 6: Optimized health assessment model for WSN.

| No. | Rule weight | Characteristic | | Output distribution of belief rule {L, SL, M, SH, and H} |
|---|---|---|---|---|
| | | CR | FR | |
| 1 | 0.2237 | L | L | (0.1941 0.3806 0.0839 0.0605 0.2809) |
| 2 | 0.0340 | L | SL | (0.0366 0.2194 0.4384 0.0619 0.2438) |
| 3 | 0.4825 | L | M | (0.2308 0.1866 0.2243 0.2737 0.0848) |
| 4 | 0.9214 | L | SH | (0.2901 0.2480 0.0365 0.0285 0.3968) |
| 5 | 0.1094 | L | H | (0.1499 0.4173 0.0806 0.2005 0.1517) |
| 6 | 0.0213 | M | L | (0.0278 0.4044 0.3744 0.0823 0.1112) |
| 7 | 0.8192 | M | SL | (0.9246 0.0103 0.0564 0.0000 0.0226) |
| 8 | 0.0745 | M | M | (0.3313 0.2423 0.2751 0.0995 0.0519) |
| 9 | 0.1110 | M | SH | (0.3338 0.1885 0.2150 0.0755 0.1872) |
| 10 | 0.3228 | M | H | (0.4458 0.1847 0.1215 0.1488 0.0992) |
| 11 | 0.4260 | SH | L | (0.1800 0.1473 0.0343 0.4604 0.1780) |
| 12 | 0.2566 | SH | SL | (0.5815 0.1738 0.1383 0.0733 0.0332) |
| 13 | 0.8864 | SH | M | (0.8368 0.0646 0.0500 0.0424 0.0063) |
| 14 | 0.3489 | SH | SH | (0.4968 0.0901 0.1494 0.1831 0.0806) |
| 15 | 0.4101 | SH | H | (0.1739 0.2406 0.3724 0.0390 0.1741) |
| 16 | 0.1775 | H | L | (0.0580 0.3663 0.2701 0.2254 0.0802) |
| 17 | 0.9738 | H | SL | (0.1383 0.5759 0.1463 0.0329 0.1066) |
| 18 | 0.0559 | H | M | (0.1947 0.0924 0.1375 0.0333 0.5422) |
| 19 | 0.0907 | H | SH | (0.2482 0.1992 0.2872 0.1637 0.1017) |
| 20 | 0.8838 | H | H | (0.1363 0.4430 0.1582 0.1333 0.1292) |

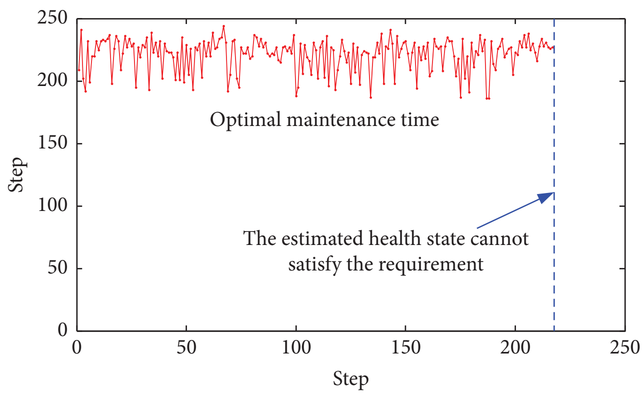


Figure 4: Calculated optimal maintenance time of the network.

*5.4. Comparison Experiments.* Moreover, the effectiveness of the developed health state assessment model in this paper is illustrated through comparison experiments with Feng et al. [28], Zhou et al. [31], extreme learning machine (ELM), fuzzy inference, and BP neural network approaches. The mean squared errors (MSEs) obtained from these experiments are presented in Table 5.

Comparing the BRB-c assessment model with other models, significant improvements in accuracy are observed. Specifically, the developed assessment model shows an improvement of 19.46% over the BRB model by Feng et al., 8.53% over the BRB model by Zhou et al., 50.14% over ELM, 64.82% over fuzzy inference, and an impressive 70.94% over the BP neural network. These results clearly demonstrate the superiority of our developed assessment model in accurately evaluating health states.

After the optimal maintenance decision model has been optimized, the optimized health assessment model and prediction model can be obtained. The optimized health assessment model can be seen in Table 6. And it is shown clearly in Figure 2 that the health state of the WSN can be accurately estimated by the developed model, and the error between the model output and the actual health state is within an acceptable range. On the other side, when the sampling time reaches 217, the health state of the WSN will be lower than 0.92, which means it needs repairing. Thus, based on the developed Wiener process health state prediction model, the predicted health state of the WSN can be obtained.

In addition, Figure 4 illustrates that the optimal maintenance time fluctuates around 217. This suggests that the obtained optimal maintenance time aligns with the requirement of maintaining the health state of the WSN above 0.92. Consequently, it can be concluded that the developed optimal maintenance decision model provides an accurate optimal maintenance time. In summary, the optimized health assessment model, prediction model, and optimal maintenance decision model collectively contribute to accurately estimating the health state of the WSN and determining the appropriate maintenance time.

## 6. Conclusions

In response to the challenges of limited observational data, complex system mechanisms, and the necessity for incorporating attribute correlations in sensor network health management, this study introduces an optimal maintenance decision model. Grounded in the BRB-c and the Wiener process, the model enhances the precision of health state assessments by integrating complex attribute correlations. The Wiener process is then leveraged for predictive modeling, forecasting the sensor network's future health states. Utilizing maximum likelihood estimation, the model computes the degradation and diffusion coefficients, which are instrumental in determining the optimal maintenance timing. A significant strength of this approach is its comprehensive utilization of both quantitative monitoring data and qualitative expert insights, facilitated by the BRB-c. This synergy allows for the adept handling of uncertainties and incomplete data, thus fortifying the maintenance decision-making process. Furthermore, the Wiener process-based prediction model enhances forecast accuracy by factoring in both the current health state and the interdependencies among attributes.

However, it must be emphasized that the current health state assessment model does not address the potential challenge of rule combination explosion, which may affect its ability to effectively assess the health status of sensor networks. This limitation underscores the need for further research and advancements in developing sophisticated models capable of navigating the complexity and diversity of sensor networks. Moreover, this study operates under the assumption that sensor network observational data are completely reliable, disregarding the impact of environmental noise. Such an assumption might not fully reflect the real-world conditions where noise and uncertainties are common. Enhancing the model's practical applicability will require future research efforts to integrate noise modeling and formulate robust strategies to mitigate the impact of these uncertainties [32–35].

## Data Availability

The data used to support the findings of this study are available from the corresponding authors upon reasonable request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Acknowledgments

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