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Research Article

A Branch-and-Price Algorithm for an Integrated Online and Offline Retailing Distribution System with Product Return

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This study identifies critical inefficiencies within a dual-channel operation model employed by a fast fashion company, particularly the independent operation of three logistics distribution systems. These systems result in high operational costs and low resource utilization, primarily due to redundant vehicle dispatches to meet the distinct demands of retail store replenishment, online customer orders, and customer return demands, as well as random and scattered return requests leading to vehicle underutilization. To address these challenges, we propose a novel integrated logistics distribution system design and management method tailored for dual-channel sales and distribution businesses. The approach consolidates the three distribution systems into one cohesive framework, thus streamlining the delivery process and reducing vehicle trips by combining retail and customer visits. An optimization algorithm is introduced to factor in inventory and distribution distance, aiming to achieve global optimization in pairing retail store inventory with online customer orders and unifying the distribution of replenishment products, online products, and returned products. The paper contributes to the field by introducing a new variation of the Vehicle Routing Problem (VRP) that arises from an integrated distribution system, combining common VRP issues with more complex challenges. A custom Branch-and-Price (B&P) algorithm is developed to efficiently find optimal routes. Furthermore, we demonstrate the benefits of the integrated system over traditional, segregated systems through real-world data analysis and assess various factors including return rates and inventory conditions. The study also enhances the model by allowing inventory transfers between retail stores, improving inventory distribution balance, and offering solutions for scenarios with critically low inventory levels. Our findings highlight a significant reduction in total operating cost savings of up to 49.9% and vehicle usage when using the integrated distribution system compared to independent two-stage and three-stage systems. The integrated approach enables the utilization of vacant vehicle space and the dynamic selection and combination of tasks, preventing unnecessary mileage and space wastage. Notably, the integration of inventory sharing among retail stores has proven to be a key factor in generating feasible solutions under tight inventory conditions and reducing operational costs and vehicle numbers, with the benefits amplified in large-scale problem instances.

1. Introduction

In recent decades, the development of e-commerce has played an increasingly important role in global commerce. As a result, some new e-commerce models focusing on the sale of best-selling items emerge and become an important part of e-commerce that cannot be ignored. Under these new models, enterprises on the one hand make use of the advantages of online marketing and online shopping orders to enrich sales channels and increase sales volume; on the other hand, they follow the traditional offline retail stores to improve product publicity and customer experience. Compared with the traditional offline retail channels, the retail stores under the new model not only serve the customers directly in the store but also act as the front warehouse. As a direct service customer, retail stores collect the market reaction of best-selling products in different regions and feed back to the headquarters to adjust the replenishment and allocation strategy of subsequent retail stores. As a front-loading warehouse, it follows the unified arrangement of the headquarters to meet the needs of online orders and improve the service level of offline customers and the delivery time of online customers.

As a typical application industry of dual-channel, an increasing number of apparel companies are starting to take the fast fashion route. They have established online direct sales channels in addition to their traditional offline channels. Companies can sell their products through dual channels or even multiple channels, which can further enhance the brand's visibility and influence. The relationship between fast fashion and dual-channel is mutually reinforcing. Fast fashion brands can provide customers with a more convenient and personalized shopping experience by combining online and offline channels. Customers can browse the latest styles online, then try on and purchase in a physical store, or choose items in-store and place an order online. Dual-channel can help fast fashion brands manage inventory more efficiently. Through real-time data analysis, brands can adjust their inventory timely based on online and offline sales data, thus reducing overstock. Through dual-channel sales, fast fashion brands can collect a vast amount of customer behavior data. These data help brands better understand market trends, predict customer needs, respond quickly, and thereby create products that more closely match market demand. Dual-channel provides fast fashion brands with more points of sale, whether to expand geographic coverage or to offer a 24/7 uninterrupted shopping experience, both of which can attract a broader customer base. A successful dual-channel strategy can enhance brand image, making the brand appear more modern and consumer-friendly, especially for fast fashion customers who pursue the latest trends. In summary, a dual-channel strategy provides fast fashion brands with a robust framework that leverages the organic integration of online and offline channels to increase efficiency, expand touch points, optimize customer experience, and ultimately strengthen market competitiveness. However, this emerging dual-channel retail system poses new challenges to enterprises' logistics, requiring them to redesign their logistics distribution systems by taking into account key factors such as customer experience, business cost, and service efficiency. The new logistics distribution system needs to break the original traditional offline fixed range distribution mode but absorb the advantages of online flexible distribution and coordinate and integrate the replenishment distribution of retail stores and online customer distribution [1].

However, while diversified supply chain models drive the rapid development of fast fashion apparel companies, they also bring about some new challenges. The product return process caused by quality problems of the product itself or negligence in the distribution process makes return a common phenomenon in online distribution channels [2]. Since consumers cannot physically interact with products when shopping online, their observation of the products may not be as direct, leading to returns due to issues with color, style, size, quality, etc. In addition, the process of returning goods can lead to a poor customer experience due to long return cycles and inconvenient procedures, which can significantly impact the company's revenue growth. Therefore, how to improve the level of consumer return services and expedite the return process has become a key issue for businesses to address. Therefore, under the new e-commerce model, the distribution system should not only integrate online order distribution and offline replenishment but also deal with customer returns, which have become a new research direction and a new hotspot.

Next, we will describe the dual-channel operation model of a well-known large fast fashion company to illustrate the problems studied in this paper. The company has a central warehouse in the suburbs and a number of scattered retail stores downtown. In order to gain more customers and serve them well, the company has established a dual-channel sales and distribution system that supports both the online e-commerce channel and the offline retail channel. Depending on their preferences, customers can either visit physical retail stores to experience offline purchases or order products online to enjoy home delivery services. In the actual operation process, the headquarters decides the quantity of products to be supplied to the retail stores from the central warehouse according to the sales volume and the existing inventory of the retail stores. Offline orders are purchased and returned directly by customers at retail stores. Online orders are not delivered directly from the central warehouse. Instead, a manual decision is made to select the retail store nearest to the customer with inventory and prioritize the consumption of the old batches of products in inventory. And the product needs further processing and packaging in retail stores before it can be delivered to customers. For products returned by customers, the headquarters arranges door-to-door service to collect them and return them to the central warehouse for further inspection or renovation. Therefore, the company's logistics distribution system consists of three parts, as shown in Figure 1:

(1) Retail store distribution system, replenishing products from the central warehouse to retail stores. The retail replenishment delivery plan is represented by a blue arrow in Figure 1. Each plan includes a central warehouse and multiple retail stores and arranges specific vehicles for distribution based on the actual distribution tasks generated by the replenishment plan. This is a typical Capacitated Vehicle Routing Problem (CVRP), which considers the capacity constraints of vehicles to arrange a reasonable number of vehicles to minimize delivery costs.

(2) Online order distribution system, where products ordered online are picked up from retail stores and delivered to customers. The online order pickup and delivery plan is represented by a green arrow in Figure 1. Each plan consists of multiple retail stores and multiple customers, but a sales order has only one retail store as the pickup point and one customer as the delivery point. Then specific vehicles are arranged for delivery based on the actual delivery tasks generated by the sales orders confirmed by all retail stores. This is a typical one-to-one Pickup and Delivery Problem (PDP), where the order of pickup and delivery is reasonably arranged to minimize the

delivery cost while considering the capacity constraints of the vehicle.

(3) Return collection system, where returned products are collected from customers and transported to the central warehouse. The return collection plan is represented by an orange arrow in Figure 1. Each plan consists of a central warehouse and multiple customer points, and specific vehicles are arranged for collection operations based on the actual collection tasks generated by all current return orders. This is also a typical CVRP, which considers the capacity constraints of vehicles to arrange a reasonable number of vehicles to minimize delivery costs.

Although the fast fashion company adopts a dual-channel operation model, the three logistics distribution systems are currently operated independently, which will lead to some problems of high operating costs and low resource utilization. Firstly, in order to meet retail stores' demand for replenishment, online customer order demand, and customer order return demand, vehicles are arranged for distribution, respectively, which is easy to cause excessive number of vehicles. For example, the customer has the operation demand of purchase and return at the same time, which may need to be visited twice, resulting in lack of flexibility and excessive vehicle operation costs. Secondly, the return demand of customers is relatively random and scattered, and the vehicles sent to pick up the returned products are easy to lead to low load rate and waste of resources. Finally, human decision making is limited in that it does not achieve the optimal pairing of retail store inventory with online customer orders at the global optimization level.

Especially with the fierce competition among fast fashion companies, it has become necessary to redesign a dual-channel logistics distribution system to solve the above problems for improving distribution system efficiency and save total logistics costs. The new system must simultaneously meet the needs of retail replenishment, online customer orders, and customer return demands, while avoiding excessive vehicle numbers, low passenger capacity, and resource waste. In addition, the model matched with the new system can promote the optimal pairing of retail inventory and online orders, generate optimal distribution strategies, reduce operating costs, and increase competitiveness in a dual-channel sales environment.

In order to overcome the above problems, the motivation of this paper is to propose a design and management method of logistics integrated distribution system to meet the needs of dual-channel sales and distribution business. On the one hand, the three distribution systems are integrated into one integrated distribution system. This simplifies the delivery process and reduces the number of vehicles by reducing the number of retail and customer visits. On the other hand, the optimization algorithm is introduced to comprehensively consider inventory and distribution distance and other factors, so as to realize the optimal matching between retail store inventory and online customer orders and realize the unified distribution of replenishment products, online products, and returned products. This can lead to higher load rates and lower overall delivery miles, which can lead to improved transportation efficiency and lower service costs. In addition, from the perspective of management, the integrated logistics distribution system can more easily promote the implementation of the headquarters supply chain strategy and promote the sustainable development of enterprise business. Figure 2 shows an example of this integrated delivery system and illustrates seven possible delivery scenarios: (1) transport replenishment products from central warehouse to retail stores, (2) go to the retail store to pick up the products ordered online and deliver them to customers, (3) collect returned products from customers and finally return them to the central warehouse, (4) combination of scenario (1) and scenario (2), (5) combination of scenario (1) and scenario (3), (6) combination of scenario (2) and scenario (3), and (7) combination of scenario (1), scenario (2), and scenario (3), where vehicles depart from the central warehouse, then can deliver products to retail stores, or pick up online ordered products from retail stores to customers, or collect the return products from customers, and finally return to the central warehouse. In these scenarios, the integrated distribution system includes a variety of PDP scenarios. It has a one-to-many-to-one PDP scenario of picking up goods from the warehouse and delivering them to retail stores and picking up goods from returning customers and returning them to the warehouse. It also has a many-to-many PDP scenario where you select a certain retail store to pick up the goods and then deliver them to a certain online ordering customer. In these scenarios, the integrated distribution system includes multiple VRPs or PDPs. It has a typical CVRP that picks up the goods from the warehouse and delivers them to the retail store, and there is another typical CVRP that picks up the goods from the returning customer and returns them to the warehouse. The combination of these two CVRPs forms a one to many to one PDP. In addition, it also has many-to-many PDPs that select a certain retail store to pick up the goods and then deliver them to a certain online ordering customer. These VRPs or PDPs are mixed together, which leads to a much higher complexity of the problem and poses higher challenges to the design of the algorithm.

In order to make the above new logistics integrated distribution system achieve the expected effect, it also brings us new four research questions that need to be solved urgently from the academic point of view. The first question is how to construct a new variant model that expresses the VRP of a new integrated distribution system based on the traditional VRP model. The second question is how to design a novel algorithm to efficiently solve problems of different scales corresponding to the new model. The third question is in what ways the integrated distribution system outperforms the three individual distribution systems when evaluated with real-world operational data. The final question is how allowing inventory transfers among different retail stores affects the overall balance of inventory levels and the feasibility of solutions, especially under tight inventory conditions.

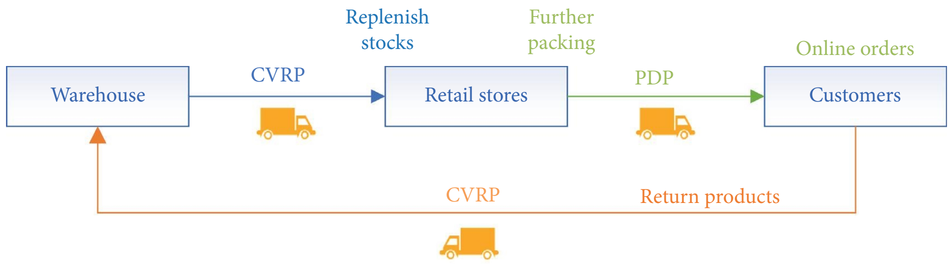

FIGURE 1: The operational procedures of the independent distribution system.

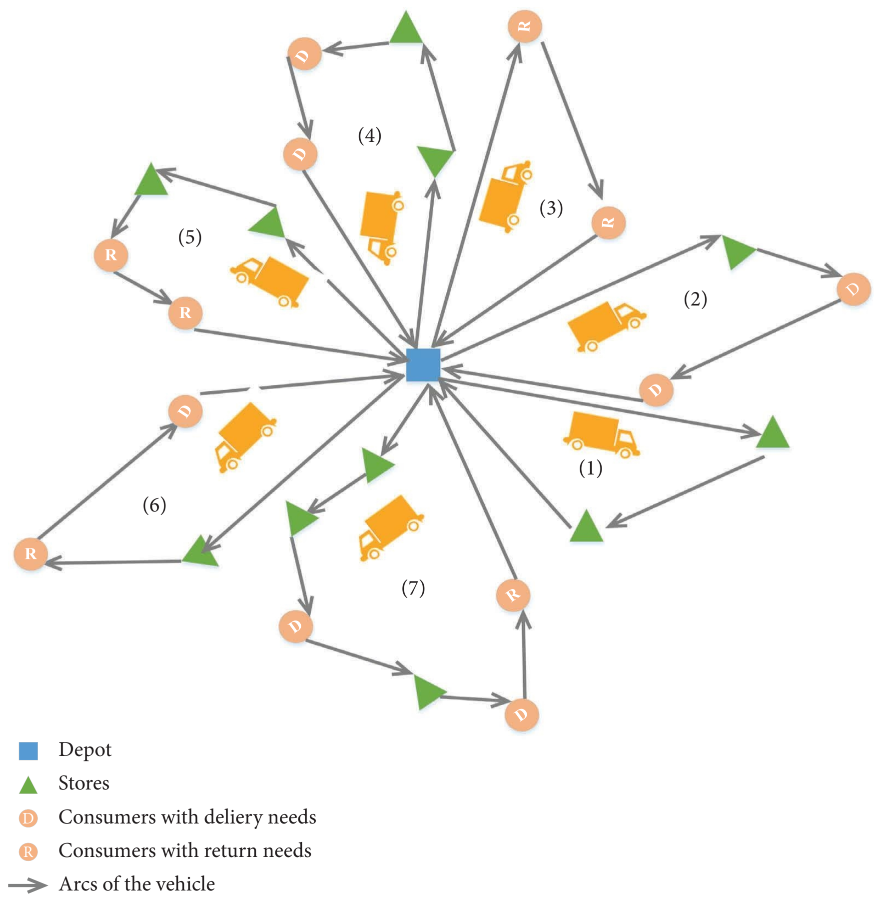

FIGURE 2: An illustrative example of the integrated distribution system.

The novelty of this paper is manifold, as follows:

(i) Initially, it introduces a novel variation of the VRP that emerges from an integrated distribution system. This variant combines common VRP issues such as CVRP, one-to-many-to-one PDP, and many-to-many PDP, as well as various complex issues resulting from the combination of these issues.

(ii) Secondly, to address the proposed model, a branch-and-price (B&P) algorithm is developed, featuring a unique custom label-setting algorithm designed to find optimal routes efficiently.

(iii) Thirdly, the paper evaluates the advantages of the integrated distribution system relative to the traditional, segregated systems by utilizing real-world operational data. It conducts a thorough analysis of various factors, including return rates and different inventory conditions, to assist in the practical application of the integrated system.

(iv) Finally, the model is further enhanced by allowing for inventory transfers between retail stores. This enables a more balanced distribution of inventory across stores and provides feasible solutions in scenarios where inventory levels are critically low.

The remainder of this paper is organized as follows. Section 2 presents the literature review concerning related topics. Section 3 outlines the problem descriptions and mathematical formulations. Section 4 describes the proposed B&P algorithm. The computational experiments are conducted in Section 5. Finally, Section 6 concludes this paper with some further research directions.

2. Literature Review

In this section, we will conduct literature analysis and review from four research aspects related to the research content of this paper. The four aspects are (A) dual-channel, fast fashion and order fulfillment, (B) product return, (C) vehicle scheduling, and (D) solution methods. The corresponding research results are as follows.

2.1. Dual-Channel, Fast Fashion and Order Fulfillment. In recent years, with the vigorous development of e-commerce and the change of consumer behavior, dual-channel, fast fashion and order fulfillment have attracted more and more attention from academia and industry. The fast fashion apparel industry allows consumers to obtain the latest trendy fashion products in real time. And it has distinct industry characteristics, such as high frequency of product updates, numerous distribution occurrences, and small quantities per distribution. Dual-channel can help fast fashion brands manage inventory. The efficient and swift order fulfillment process ensures that fast fashion company's new products are rapidly brought to market and reach consumers. And it is not only about delivering goods but also processing returns and exchanges to help maintain customer satisfaction and brand loyalty. The research on the relationship between dual-channel, fast fashion and order fulfillment is a very broad and hot research area, and there have been many valuable research contents.

The research on how dual-channel affects fast fashion has attracted the attention of a large number of scholars, and there have been a lot of relevant studies. Pentecost et al. [3] discussed how the retail industry, especially the fast fashion sector, adapts to the needs of consumers from different generations and the use of online and offline channels. Frazer et al. [4] explored the impact of the integration of online and offline environments on retail, including the fast fashion industry. Zlatica's article [5] focused on the merger of the omnichannel experience in marketing and digitization, involving case studies related to fast fashion brands. Liu et al. [6] modeled a two-echelon supply chain to optimize retail and wholesale pricing for two substitutable products under stochastic demand and evaluated the impact of channel power structures on pricing decisions and chain member profits. Bhardwaj et al. [7] examined how fast fashion brands respond rapidly to changes in the fashion industry, likely touching upon the implementation of omnichannel strategies. Cao's research [8] focused on the strategic dynamic capabilities for fast fashion and online retailing, discussing the omnichannel model in the Chinese market.

The success of fast fashion relies heavily on an efficient order fulfillment process that ensures speed, flexibility, and accuracy. Several studies have focused on the agility of supply chain management in fast fashion. According to Christopher et al. [9], agility is the cornerstone of fast fashion, enabling brands to respond rapidly to changing consumer trends. Ferdows et al. [10] underscored the importance of a flexible supply chain that can adjust production volume and turnaround times. Additionally, research by Choi [11] found that streamlined supply chains with strategic supplier relationships are critical for minimizing lead times and maximizing responsiveness. Logistics play a critical role in the fast fashion order fulfillment process. A study by Barnes and Lea-Greenwood [12] noted that logistics strategies must be designed to support the high turnover of products characteristic of fast fashion. Hines [13] discussed the importance of efficient transportation and warehousing to ensure that products reach stores and consumers without delay. Caro and Gallien [14] highlighted the use of advanced distribution systems that align with the fast-paced nature of the industry. Cachon and Swinney [15] explored how online platforms complement traditional retail channels, facilitating faster order fulfillment and returns processing. Bhardwaj and Fairhurst [7] emphasized the importance of a seamless omnichannel experience, where inventory visibility and order accuracy are paramount to customer satisfaction. Technological advancements have had a profound impact on order fulfillment. H&M's implementation of RFID technology for inventory management is a prime example, as investigated by Moon et al. [16]. This technology allows for real-time inventory tracking, which aids in accurate stock levels and reduces the risk of stockouts or overstocking. Aviv et al. [17] discussed how predictive analytics can be used to forecast demand and optimize inventory in fast fashion. The environmental impact of fast fashion order fulfillment cannot be ignored. Shen [18] explored the concept of sustainable supply chains and suggested that fast fashion companies need to balance speed with ecological and social responsibility. Companies like Zara have been scrutinized for their sustainability practices, leading to a call for greener logistics and packaging solutions in the industry.

With the development of e-commerce and the diversification of customer needs, cross-channel order fulfillment problems have attracted more and more attention. For example, Hendalianpour et al. [19] explored the effectiveness of various contracts as supply chain coordination mechanisms within a competitive, two-echelon supply chain, finding that revenue-sharing contracts perform best in enhancing coordination between manufacturers and retailers, with ordering decisions being crucial for competitive strategy. Aksen and Altinkemer [20] studied the online order fulfillment problem through retail stores. In particular, retailers have to decide which retail store to serve which order, based on the store operating cost and the last-mile delivery cost. Liu et al. [21] addressed the challenge of creating an integrated shopping experience in dual-channel retail by using a multiobjective optimization model that balances minimum distribution network costs with maximum customer convenience. Hendalianpour [22] introduced a game-theoretic model using Double Interval Grey Numbers to optimize pricing and inventory decisions for perishable goods in the supply chain, demonstrating that consumer preference for product freshness significantly influences demand and optimal retailer strategies. Chen [23] examined a setting with an online retailer and two physical retailers. In addition to serving their in-store customers, the physical retailers act as drop shippers for the online retailer, which carries no inventory of its own. They considered cross-channel operations across independent retailers and each retailer has separate profit functions. Jalilipour Alishah et al. [24] studied the order fulfillment network consisting of only one retail store and one fulfillment center (FC), and online demands are routed to the offline retail store only when the FC runs out of stock. Zhao et al. [25] modeled a dual-channel supply chain in which the manufacturer manages an online retail store and operates an online-to-offline strategy, in which online orders are fulfilled from the retail store inventory of the manufacturer's retail partners, with the possibility of inventory transshipments between the retailer and the manufacturer in case of stockouts. Their findings proved the existence of an optimal inventory policy and an optimal transshipment price. Ishfaq and Raja [26] provided a framework for the online order fulfillment, which includes the use of (1) distribution centers (DCs), in which retailers integrate the fulfillment of retail store and online demands through a unified warehouse; (2) dedicated direct-to-customers FCs to fulfill online demand from dedicated centers direct-to-customer; (3) retail stores, which leverage retail store inventory to fulfill both online and offline demands; and (4) vendors, which directly fulfill online orders without the utilization of retail store inventory. Bayram and Cesaret [27] investigated stochastic dynamic fulfillment decisions that include both online and offline orders. Online orders can be fulfilled either from the FC or from one of the retail stores. They also developed a heuristic policy that maximizes the retailer's total profit from sales across all channels.

From the above, it can be found that the relationships among dual-channel, fast fashion and order fulfillment are complex and multifaceted. However, the abovementioned research mainly focuses on the relationship between two of them, and there is no research on the relationship between these three. As consumer expectations continue to evolve, fast fashion brands based on dual-channel must continuously innovate their order fulfillment strategies to maintain a competitive edge in the marketplace. Fast fashion hopes to fulfill orders faster and more efficiently, which requires opening up dual channels and front-loading and sharing inventory to reduce the delivery distance between goods and customers. Dual channels increase the distribution link of fast fashion and improve the flexibility of service portfolio and the sustainability of revenue. However, the increase in service portfolio will correspondingly increase the types and frequency of order fulfillment and distribution, which will lead to an increase in overall logistics and distribution costs. This requires considering the relationship among dual-channel, fast fashion and order fulfillment to redesign a logistics integrated distribution system that integrates multiple types of distribution links and merges distribution resources to strike a balance between speed, efficiency, flexibility, sustainability, and cost. Among the various design options for customer-centered dual channels and order fulfillment, most choose retail stores as distribution centers to enhance distribution flexibility for various customer needs and reduce distribution time and costs. Therefore, in this study, the inventory of retail stores must not only support direct purchases by offline customers but also fulfill online orders. So, our logistics distribution model needs to consider the matching between retail stores and online customer orders to determine the optimal distribution strategy.

2.2. Product Return. For consumers using online channels, they can only understand product information through text and images before purchasing, and the product received may differ from expectations, leading to a need for returns [28]. Since consumers cannot experience the unique qualities of a product firsthand when shopping online, issues such as discrepancies between the actual item and the merchant's description or product defects often arise. These issues result in a persistently high rate of returns, causing significant inconvenience to consumers and deteriorating their shopping experience. For businesses, this is a major hindrance to their development. To improve consumers' shopping experiences and attract more customers, merchants must place greater emphasis on after-sales investments. Therefore, investments related to the return process have gradually become a crucial consideration in business operations. Scholars from home and abroad have considered various factors and studied the issue of returns within the supply chain. Ofek et al. [29] believe that the application of the Internet has made merchants more flexible and the online channel offers a better shopping experience to consumers. They looked at consumer returns and retail channel services, starting with the utility function to study the dual-channel pricing problem under the risk of returns. Ramanathan [30] investigated online shopping user reviews and found that return handling performance can affect a company's brand loyalty. It was also discovered that the impact of return handling varies for different products. Return strategies significantly influence the pricing and revenue of the supply chain; hence, some explorations into return policies have been undertaken. Mukhopadhyay and others [31] found that it is necessary for manufacturers to provide a return policy in online channels and developed an optimal return policy through a profit maximization model. Jing Chen and colleagues' research indicates that retailers can increase profits by segmenting the market with policies that allow or prohibit returns and considering these returns in pricing and ordering decisions [32]. Some scholars have also studied how the amount of the refund affects consumers' shopping behavior. For example, Suwelack et al. [33] showed that offering a full refund guarantee can resonate with consumers, who are then willing to pay a higher price for products. McWilliams [34] found that in a competitive market with high-quality goods, low-quality retailers attract consumers by offering more attractive return policies to enhance their competitiveness. Most dual-channel return studies focus on no-fault returns, but with changing consumer attitudes, quality issues have become a significant reason for returns. A small number of scholars have also focused on quality decisions as a key research area. Mukhopadhyay [35] initially studied product quality issues. Building on previous research, Li et al. [36] further investigated whether online shopping return strategies are influenced by product pricing and quality.

At present, most studies on dual-channel supply chains consider return services alone in the online channel, without considering linkage with services in other channels. Cross-channel return service can significantly reduce return logistics costs and improve consumer shopping experience. Our research integrates three different types of services: depot-to-store distribution, store-to-customer distribution, and return services, which have not yet appeared in existing studies.

2.3. Vehicle Scheduling. VRP is well acknowledged as the foundation of logistics distribution operations, which involves delivering some items from depots to customers using capacitated vehicles [37]. Over the years, different variants of the VRP have emerged, each addressing specific operational constraints and real-world complexities. The basic VRP seeks to minimize the total route cost for vehicles delivering goods from a depot to a number of customers. However, the classic VRP has been extended to include numerous practical considerations, leading to the development of several well-known variants such as (a) VRP with Time Windows (VRPTW), where deliveries must occur within predefined time intervals [38], (b) Capacitated VRP (CVRP), which introduces vehicle capacity constraints [39], and (c) VRP with Pickup and Delivery (PDP), incorporating scenarios where vehicles must manage both pickups and deliveries [40]. With the increasing complexity of logistics systems, more advanced VRP variants have arisen, including (a) Periodic VRP (PVRP), where customers need to be visited multiple times over a planning horizon [41], (b) Multidepot VRP (MDVRP), involving multiple depots from which vehicles begin and end their routes [42], and (c) Split Delivery VRP (SDVRP), allowing a customer's demand to be split and delivered by multiple vehicles [43]. Emerging concerns about sustainability and social responsibility have led to new VRP formulations: (a) Green VRP (G-VRP), aiming to reduce fuel consumption and CO2 emissions, often through the use of alternative fuel vehicles [44], and (b) VRP in Disaster Relief (VRP-DR), focusing on rapid and efficient logistics in response to humanitarian crises [45]. Among them, PDP constitutes an important branch of CVRP, in which items must be picked up from some original locations firstly and then transported to different destinations. Berbeglia et al. [46] claimed that PDPs could be further classified into three subcategories according to the type of demand and route structure, as many-to-many (M-M) type in which each commodity may have multiple origins and destinations, one-to-many-to-one (1-M-1) type in which commodities are delivered from a depot to many customers and some other commodities are collected from customers and transported back to the depot, and one-to-one (1-1) type in which each commodity has a single origin and a single destination The VRP and its variants represent a rich domain of study with direct applications in the improvement of logistics and transportation systems.

The problem studied in this paper from the macro level is hybrid VRP that simultaneously contains CVRP, one-to-many-to-one PDP, and many-to-one PDP. However, at a micro level, decisions need to be made to identify specific retail stores to meet the distribution needs of specific customers, and the above types of VRPs considered together provide difficulty in decision making. As expected, we note that the existing literature does not study this new VRP variant.

2.4. Solution Methods. The difficulty of solving VRPs is NP-hard, and numerous effects have been devoted to solve these problems using different methodologies or techniques. The solution methods for solving VRPs would encompass various studies that explore heuristic, exact, and metaheuristic approaches to tackle the complexity of routing and scheduling vehicles. The issues related to the research questions of this paper are mainly PDP, so we look into different solution methods that have been proposed and developed to solve VRPs and PDP. Exact methods, such as the branch-and-bound algorithm and its variations, have traditionally been used to find the optimal solution to VRPs. Cordeau and Laporte [47] were among the first to apply branch-and-cut algorithms to VRPs, demonstrating their effectiveness for small problem instances. However, as noted by Toth and Vigo [39], the computational complexity of these methods limits their application to larger, real-world instances. Dumas et al. [48] proposed a branch-and-cut algorithm specifically designed for the PDP, while Cordeau and Laporte [47] extended these techniques to handle more complex constraints, such as time windows and multiple vehicles. Rousseau et al. [49] and Azi et al. [50] claimed that column generation (CG) or Dantzig–Wolfe (DW) decomposition-based algorithms can accommodate complex constraints in VRPs. Some review papers concerning the exact approach for VRPs have been conducted [51–53]. Heuristic approaches are designed to find good, but not necessarily optimal, solutions to VRPs in a reasonable amount of time. Laporte and Semet [54] and Laporte et al. [55] once surveyed the commonly used heuristic approaches. Clarke and Wright [56] introduced the Savings algorithm, an intuitive and straightforward heuristic that has inspired numerous variations. Fisher and Jaikumar [57] further developed insertion heuristics, which iteratively build routes by inserting the most appropriate customer into an existing route. Savelsbergh and Sol [58] developed an insertion heuristic that constructs routes by iteratively adding the most cost-effective pickup and delivery requests. Bent and Van Hentenryck [59] introduced an adaptive large neighborhood search (ALNS) that systematically explores large portions of the search space through a series of destroy and repair operators. Liu et al. [60] proposed a heuristic algorithm with a robust optimization approach to minimize the total costs of the inventory-routing in a supply chain.

Metaheuristics provide a framework for developing heuristic strategies capable of escaping local optima and exploring the solution space more effectively. Since then, a variety of metaheuristic methods have been applied, including simulated annealing [61], tabu search [62], and ant colony optimization [63]. Nanry and Barnes [64] applied a simulated annealing algorithm to the PDP, demonstrating its effectiveness in finding high-quality solutions. Parragh et al. [40] explored a hybrid variable neighborhood search (VNS) and tabu search algorithm, which dynamically adjusts its parameters based on the problem instance. Recognizing the strengths and weaknesses of different methodologies, researchers have proposed hybrid approaches. Pisinger and Ropke [65] combined elements of exact and heuristic methods in an adaptive large neighborhood search algorithm, showing improved performance on benchmark problems. Vidal et al. [66] further explored hybrid metaheuristics, integrating various techniques to balance intensification and diversification strategies. Ropke and Pisinger [67] introduced a hybrid heuristic that combines elements of ALNS with local search procedures, effectively balancing exploration and exploitation. Adewumi et al. [68] presented a cooperative approach that integrates a genetic algorithm with local search heuristics, allowing for a diverse exploration of the solution space. More recently, the focus has shifted towards incorporating machine learning and artificial intelligence into VRP solutions. Kontoravdis and Bard [69] pioneered the use of neural networks for vehicle routing, and more recent studies have explored reinforcement learning [70] as a means to adaptively learn routing policies. Vidal et al. [71] have investigated the application of machine learning techniques to learn and improve heuristic rules for vehicle routing. The VRP solution methods reveal a continuous evolution of techniques to address the inherent complexity of the problem.

While exact methods offer optimality for smaller instances, heuristic and metaheuristic approaches extend the problem-solving capabilities to larger and more realistic scenarios. The advancement of hybrid and AI-based methods reflects the field's ongoing innovation, striving for greater efficiency and adaptability in vehicle routing. However, for the new VRP variant we proposed, no algorithm that can be used directly has been found in existing research. It is still necessary to redesign and transform the existing algorithm according to the characteristics of the problem. As industries and societies continue to evolve, it is expected that new VRP variants will emerge. So, there is an ongoing quest for more efficient and adaptive solution techniques, requiring continued innovation in optimization algorithms and computational methods.

In summary, our study addresses a novel and complex VRP variant containing one CVRP and two PDPs in a fast fashion dual-channel supply chain. It merges depot-to-store and store-to-customer distribution with cross-channel return services. The biggest difficulty here is to comprehensively consider the constraints required by the three independent distribution services, establish a unified data model, and design an algorithm to solve the problem. From the modeling perspective, we first study the integrated online and offline retailing distribution problem in the context of real-world application. And we need to generate the vehicle routing plan to satisfy the requirements from retail stores, online customer orders, and product return requirements. From the algorithmic perspective, we need to design a new algorithmic approach, which can exploit the structural information of the proposed model and find the good quality solutions using reasonable computational time.

3. Model Formulation

3.1. Problem Description. The problem investigated in this paper is motivated by a giant fast fashion company, which owns a central warehouse and a group of retail stores. The company operates a dual-channel sales and distribution system, i.e., offline sales and online sales. In case of online ordering, the online orders placed by customers indeed are served by retail stores. In this research, we assume that the available inventory of each retail store is known, and the matching between online orders and retail stores is treated as decision variables. Moreover, products are delivered from the central warehouse to retail stores in a daily pace so as to replenish the inventory of retail stores. In addition, product return needs to be properly processed as well. In this research, the company collects the return products from the customers and then delivers them to the central warehouse for further inspection and refurbishment.

Therefore, the distribution network is composed of three delivery components, as (a) to deliver products from the central warehouse to retail stores to replenish their stocks, (b) to pick up products from retail stores and deliver to customers, and (c) to collect return products from customers and deliver to the central warehouse. In this research, we aim to synthesize the three independent delivery systems as one integrated distribution system so as to improve the operational efficiency and flexibility. Hence, we propose an integrated distribution model using a fleet of homogeneous vehicles to serve the requirements of three tasks. The first task is to meet the demand of $r \in R$ stores with product delivery from the central warehouse. The second task is to meet the demand of $c \in C_D$ online customers with product delivery from retail stores. The third task is to process the product return from $c \in C_R$ customers with product delivery from customers to the central warehouse.

3.2. Model Formulation. The symbols and notations used in the mathematical formulation are listed as follows (Table 1).

In order to elaborate our mathematical model so that it is better understood, we specifically define the routes around the characteristics of the problem we solve. The problem we solve is a single time period problem. During this period, in the route plan output by our model, each route corresponds to only one vehicle, and each vehicle can only take one route. The number of routes in the current route plan mainly depends on the problem size and optimization space during the current solution period. The mathematical problem can be formally formulated as follows:

Table 1: The symbols and notations.

	Explanations
Sets	
V	Set of all nodes (central warehouse, retail stores, and customers)
R	Set of retail stores
C_D	Set of customers ordering products online
C_R	Set of customers with product returns
K	Set of available vehicles
Parameters	
Q	Vehicle capacity
d_i	Demand of node i
r_i	Number of returned products from customer i
I_i	Inventory level at retail store i
c_{ij}	Delivery cost with traversing arc (i, j)
T_{ij}	Delivery time with traversing arc (i, j)
S_i	Service time at the node i
Decision variables	
$p_i^k \in Z_+$	The number of products needed to be picked up at retail store i by vehicle k
$q_i^k \in Z_+$	The product quantity carried by vehicle k after leaving node i
$t_i^k \in Z_+$	The arrive time at node i by vehicle k
$x_{ij}^k \in \{0, 1\}$	1 if the arc (i, j) is traversed by vehicle k, and 0 otherwise
$y_{ij} \in \{0, 1\}$	1 if customer j is served by store i and 0 otherwise

$$\min \sum_{k\in K}\sum_{i\in V}\sum_{j\in V} c_{ij}^k x_{ij}^k, \tag{1}$$

subject to

$$\sum_{k\in K}\sum_{j\in V} x_{ij}^k = 1, \quad \forall i \in R \cup C_D \cup C_R, \tag{2}$$

$$\sum_{j\in R} x_{0j}^k = \sum_{i\in V} x_{i,v+1}^k \leq 1, \quad \forall k \in K, \tag{3}$$

$$\sum_{i\in V} x_{ij}^k - \sum_{i\in V} x_{ji}^k = 0, \quad \forall j \in V, k \in K, \tag{4}$$

$$\sum_{l\in V} x_{il}^k - \sum_{l\in V} x_{lj}^k \leq \left(1 - y_{ij}\right)M, \quad \forall i \in R, j \in C_D, k \in K, \tag{5}$$

$$\sum_{i\in R} y_{ij} = 1, \quad \forall j \in C_D, \tag{6}$$

$$\sum_{j\in C_D} y_{ij} d_j \leq I_i, \quad \forall i \in R, \tag{7}$$

$$t_i^k + T_{ij} + s_i \leq t_j^k + \left(1 - x_{ij}^k\right)M, \quad \forall i, j \in V, k \in K, \tag{8}$$

$$t_i^k + T_{ij} + s_i \leq t_j^k + \left(1 - y_{ij}\right)M, \quad \forall i \in R, j \in C_D, k \in K, \tag{9}$$

$$q_0^k = \sum_{i\in V}\sum_{j\in R} d_j x_{ij}^k, \quad \forall k \in K, \tag{10}$$

$$q_{n+1}^k = \sum_{i\in C_R}\sum_{j\in V} r_i x_{ij}^k, \quad \forall k \in K, \tag{11}$$

$$p_i^k = \sum_{j\in C_D} d_i y_{ij}, \quad \forall i \in R, k \in K, \tag{12}$$

$$q_j^k \le p_i^k - d_j + Q\left(1 - x_{ij}^k\right), \quad \forall i \in R, j \in V, k \in K, \tag{13}$$

$$q_j^k \le q_i^k - d_j + Q\left(1 - x_{ij}^k\right), \quad \forall i \in C_D, j \in V, k \in K, \tag{14}$$

$$q_j^k \ge q_i^k + r_j - Q\left(1 - x_{ij}^k\right), \quad \forall i \in C_R, j \in V, k \in K, \tag{15}$$

$$0 \le q_i^k \le Q \quad \forall i \in V, k \in K, \tag{16}$$

$$x_{ij}^k \in \{0, 1\} \quad \forall i, j \in V, k \in K, \tag{17}$$

$$y_{ij} \in \{0, 1\} \quad \forall i \in R, j \in C_D. \tag{18}$$

Objective function (1) minimizes the total traveling cost of all arcs traversed by all vehicles. Constraint (2) ensures that each store and each customer are visited only once. Constraint (3) confirms that each vehicle starts its route from the depot and ends its route at the depot. Constraint (4) represents the flow conservation for all nodes and vehicles. Constraint (5) means that if a retail store serves a specific customer, both the retail store and customer are visited by the same vehicle. Constraint (6) indicates that each customer is served by only one retail store. Constraint (7) denotes that the inventory level of retail stores cannot be violated. Constraint (8) indicates that if $x_{ij}^k = 1$, the time for the vehicle k to reach a node j is greater than or equal to the time it departs from the node i plus the time consumed on the way from the node i to the node j. If $x_{ij}^k = 0$, this constraint (8) is relaxed. Constraint (9) indicates that if $y_{ij} = 1$ that represents the customer is served by the retail store, the time for the vehicle k to reach a customer j is greater than or equal to the time it departs from the retail store i plus the time consumed on the way from the node i to the node j. If $y_{ij} = 0$, this constraint (9) is relaxed. Constraint (10) calculates the load of each vehicle when it leaves the central warehouse. Constraint (11) calculates the load of each vehicle when it returns to the central warehouse. Constraint (12) calculates the number of products that need to be picked up at a store. Constraints (13), (14), and (15) indicate the vehicle load consistency when it leaves a store, a customer with demand, and a customer with returned goods, respectively. Constraint (16) ensures the load of a vehicle cannot exceed its capacity. Constraints (17) and (18) are binary constraints.

3.3. Model Extension. The model described above is based on the hypothesis that the inventory cannot share among multiple retail stores. In some cases, there may be no feasible solutions when the inventory is tight. For example, in Figure 3, the inventory of retail stores A and B is 10, and the demand of customers C and D is 9 and 11, respectively. In this case, customer D cannot be served by any of the retail stores as the inventory between A and B cannot be shared. Hence, if the inventory can be shared among multiple retail stores, more feasible solutions, such as $0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow D \longrightarrow 0$, can be obtained.

Therefore, in order to improve the operational flexibility of this integrated distribution system and share the inventories at different retail stores, constraints (6), (7), (9), and (12) can be replaced by constrains (19)–(21). Constraint (19) ensures that the number of products to be delivered cannot exceed the inventory of the retail store, in which z_{ij} is an integer variable representing the number of products delivered from a store i to a customer j. Constraint (20) requires that the total number of products to be delivered to customers cannot exceed the total available inventories of all retail stores. Constraint (21) ensures that $z_{ij} = 0$ if $y_{ij} = 0$ and $z_{ij} \le I_i$ if $y_{ij} = 1$.

$$\sum_{j\in C_D} z_{ij} \le I_i \quad \forall i \in R, \tag{19}$$

$$\sum_{k\in K}\sum_{i\in V}\sum_{j\in C_D} x_{ij}^k d_j = \sum_{i\in R}\sum_{j\in C_D} z_{ij}, \tag{20}$$

$$z_{ij} \le I_i y_{ij} \quad \forall i \in R, j \in V, k \in K. \tag{21}$$

4. Solution Approaches

The branch-and-price (B&P) algorithm is an advanced optimization method that combines two powerful techniques: branch-and-bound (B&B) and column generation. It is often applied to solve large-scale and complex combinatorial optimization problems. B&B is a general algorithmic method for finding optimal solutions to various optimization problems, especially in integer programming. The goal of B&B is to prune the search tree and reduce the number of feasible solutions to be examined. Column generation (CG) is a mathematical optimization technique used to solve large-scale linear programming (LP) problems that have a large number of variables. It starts with a smaller, more manageable subset of the variables (the "restricted master problem" or RMP) and iteratively adds new variables

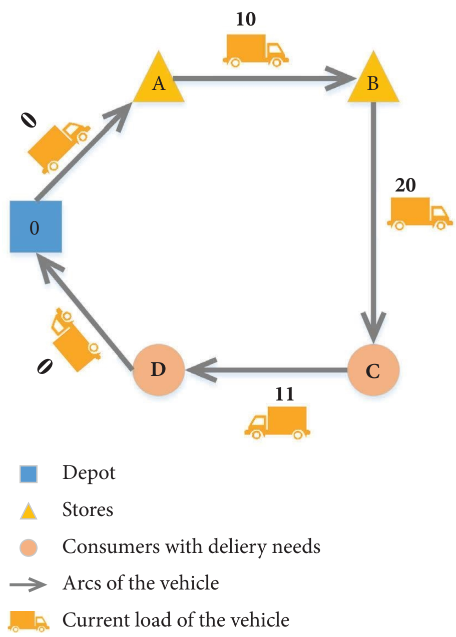

Figure 3: An example of inventory sharing of multiple retail stores.

(columns) that can potentially improve the objective function (these are found by solving a "pricing problem" or "subproblem"). The process continues until no more improving columns exist, at which point the current solution is optimal for the LP relaxation of the problem. In B&P, column generation is used to solve the LP relaxation of the branch-and-bound nodes more efficiently. At each node of the branch-and-bound tree, instead of considering all possible variables, the column generation approach is employed to focus on a smaller subset and to generate new columns only as needed. The branching decisions often involve choosing a variable to "branch" on, splitting the problem into two new subproblems where the chosen variable takes on different integer values in each subproblem. The B&P algorithm continues this process, navigating through the search tree, solving the LP relaxation at each node, adding new columns as needed, and applying bounds to eliminate suboptimal branches.

Mix PDPs have a vast and complex solution space due to the combination of pickup and delivery tasks. And B&P is a sophisticated optimization method that takes advantage of both the systematic exploration of solutions in branch-and-bound and the efficient handling of numerous variables in column generation, making it an effective tool for tackling mix PDPs. The B&P algorithm can handle this complexity effectively by breaking down the problem into smaller, more manageable subproblems using CG and then systematically exploring potential solutions with B&B. The routing decisions for mix PDPs involve many binary variables that indicate whether a vehicle travels a certain arc or whether a customer is served by a particular route. B&P can manage the large set of variables by generating columns only as needed, which reduces the computational burden compared to considering all the variables simultaneously. Vehicles in mix PDPs often have capacity limits, and routes must be planned to avoid exceeding these limits while considering the mix of pickups and deliveries. B&P allows for the integration of these constraints into the subproblems, making it possible to generate feasible routes that respect vehicle capacities and other constraints. The column generation component of B&P creates routes dynamically, which is ideal for mix PDPs where the combination of pickups and deliveries may lead to a wide variety of potential routes. New columns (routes) generated can be highly specific to the problem's constraints, thereby improving the quality of the solution. At each node in the branch-and-bound tree, B&P solves the LP relaxation of the problem using column generation, which can quickly find the optimal fractional solution that guides the branching decisions. This approach ensures that the integer constraints of the problem are respected, which is particularly important for mix PDPs, where pickup and delivery pairings may have to adhere to strict sequencing. B&P algorithms can be adapted to different variants of mix PDPs, including those with additional constraints like time windows, multiple depots, or heterogeneous fleets. The method scales relatively well for larger instances of the problem, which is a common challenge in real-world applications. Overall, the B&P algorithm's capability to manage a large number of variables, generate feasible and efficient routes dynamically, and systematically explore the solution space while incorporating complex constraints makes it a highly suitable choice for solving mix PDPs.

The positive points of using the B&P algorithm for solving mix PDPs in comparison to other solution approaches are highlighted by its ability to effectively handle a variety of challenges inherent in these types of problems. B&P seamlessly integrates complex routing constructs such as pickup and delivery pairings, precedence relations, and vehicle capacities, which might be cumbersome for traditional exact methods like branch-and-bound or heuristic approaches. The CG process in B&P focuses on generating only those routes (columns) that are likely to improve the solution, making the computational process more efficient than methods that attempt to evaluate all possible routes. Although mix PDPs rapidly increase in complexity with the number of customers and vehicles, B&P is relatively scalable, making it suitable for larger instances where purely heuristic methods may not guarantee optimality. B&P is an exact method that can provide optimal or near-optimal solutions and stronger lower bounds on the objective value compared to heuristics, which offer limited guarantees regarding solution quality. The B&P algorithm can be adapted to address stochastic demand and dynamic routing changes, which are characteristic of real-world logistics scenarios and may not be as effectively managed by simpler heuristics or classical optimization techniques. B&P can be modified to account for various extensions of mix PDPs, such as time windows, heterogeneous fleets, and multidepot scenarios, more readily than many other approaches, which might require significant adaptation. The combination of branch-and-bound with column generation allows for continuous refinement of the solution space, providing tight bounds and ensuring that nonpromising branches are pruned early in the search process. While the B&P algorithm has many advantages, it is worth noting that its performance can depend on the specific instance of the mix PDPs and the quality of the implementation.

In this section, a new B&P algorithm is designed for solving the proposed model. B&P algorithm indeed is one of well-acknowledged algorithms for solving large-scale integer programming problems. In this research, in accordance with the B&P solution framework, we develop a customized B&P approach, which can effectively and efficiently exploit the characteristics of the proposed model. Figure 4 illustrates the framework of our proposed B&P approach, which comprises four subcomponents, as data preprocessing, initial solution generation, column generation (CG), and branch and pruning strategies.

The implementation rationale of the proposed B&P approach is described as follows. Initially, the input data are preprocessed to rule out some unlikely arcs and distribution combinations so as to narrow down the search space. Then, we design a simple and efficient way to generate the initial feasible routes for starting the CG algorithm. Within the CG algorithm, a master problem can be constructed based on the initialized feasible routes, and then both primal and dual variables can be obtained by solving its relaxed linear counterpart. The dual variables are utilized to generate the subsequent subproblem to search for more promising feasible routes. After that, the integrality of the solutions of the master problem is evaluated. If necessary, branch and pruning strategies are executed in case of fractional results. The above process is iteratively conducted until the termination criterion is met. The pseudocode of the proposed B&P algorithm is presented in Algorithm 1.

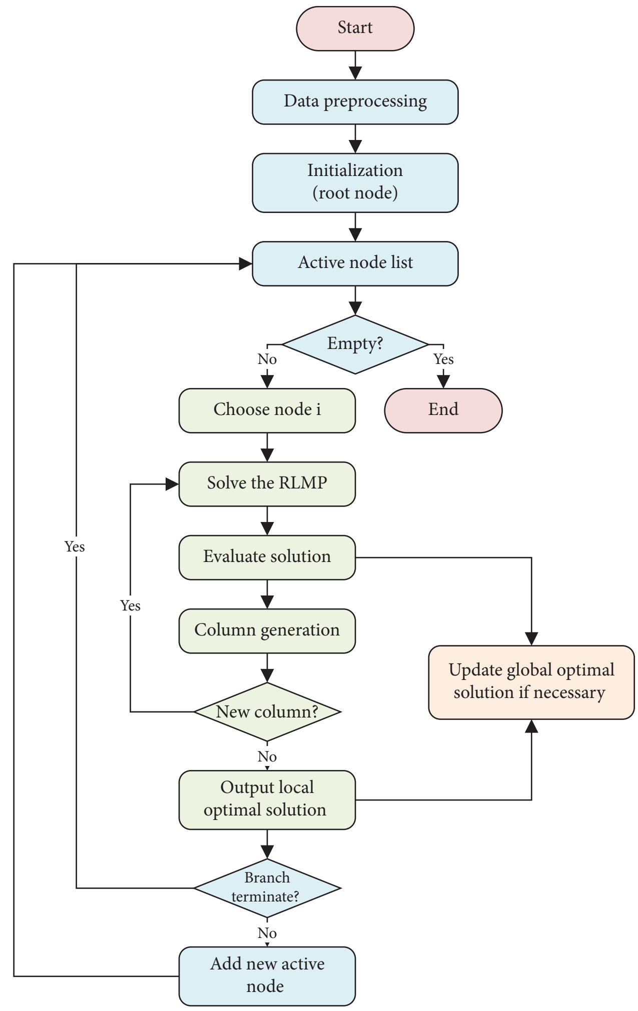

Figure 4: The B&P algorithm framework.

4.1. Data Preprocessing. Unlike the theoretical complete network graph, the network design for practical applications needs to be tailored taking into account a variety of realistic considerations. In this research, considering the proper matching of inventory and demand, we design the following data preprocessing logic to exclude some impossible distribution combinations so as to reduce the solution space and facilitate the searching process.

(1) $Q > d_i + d_j, \forall i, j \in R \Rightarrow x_{ij}^k = 0$

If the sum of the demands of store i and j is greater than the vehicle capacity Q, then the two stores cannot be in the same distribution route.

(2) $I_i < d_j, \forall i \in R, j \in C_D \Rightarrow y_{ij} = 0$

If the inventory of store i is less than the demand of the customer j, the store i cannot serve the customer j.

(3) $I_i < d_j + d_k, \forall i \in R, j, k \in C_D \Rightarrow y_{ij} + y_{ik} \leq 1$

If the inventory of store i is less than the sum of demand of customers j and k, then the store i cannot serve both customers j and k simultaneously.

(4) $\text{Rank}\,(i, j) > e, \forall i \in R, j \in C_D \cup C_R$

If the distance between store i and customer j is below the top e in ascending order, we believe that the probability of their distribution combination can be ignored.

4.2. Initial Solution Generation. Prior to the execution of column generation algorithm, it is necessary to acquire an initial solution as an input to the restricted LP master problem so as to obtain its corresponding dual variables. The most intuitive and simplest way to generate the initial solution is to dispatch individual vehicle to serve each requirement of retail stores and customers. However, in this case, a unique feature setting is that customer requirements must be served by retail stores. Hence, retail stores and customers must be combined to form distribution routes. In this research, we adopt a special strategy to generate the possible route combinations by constructing an assignment-based mathematical model. The factors, such as the vehicle capacity, the accumulated demands from customers, and the inventory level of retail stores, are evaluated so as to determine the combinations of retail stores and customers. The promising matching combinations of retail stores and customers can be quickly calculated using the CPLEX solver as described below.

Require: The initial solution S^0
Ensure:
Generate an initial route set R^0 based on the data of S^0;
Create a search stack $\mathrm{SS} \leftarrow \varnothing$, $\mathrm{SS} \leftarrow \mathrm{SS} \cup \{R^0\}$;
Set the optimal route set $R^* \leftarrow \varnothing$;
Set an upper bound limit $\mathrm{UB} \leftarrow$ MAX_VALUE;
While $\mathrm{SS} \neq \varnothing$ **do**
  $R \leftarrow \mathrm{pop}(\mathrm{SS})$
  repeat
    Solve the RLPM with the basis of R using simplex algorithm;
    Calculate the dual values π and use π to construct the subproblem;
    Call the dynamic programming algorithm to search the route set denoted by R';
    $R \leftarrow R \cup \{R'\}$
  until $R' = \varnothing$;
  Obtain the cost $C(R)$;
  if $C(R) \leq \mathrm{UB}$ **then**
    if $\exists$ fractional arc (i, j) **then**
      Branch on arc (i, j);
      Obtain two branching route sets R_0 and R_1 from R;
      $\mathrm{SS} \leftarrow \mathrm{SS} \cup \{R_0, R_1\}$;
    else
$R^* \leftarrow R$, $\mathrm{UB} \leftarrow C(R)$;
    end if
  end if
end while
Return R^*

Algorithm 1: B&P algorithm.

In accordance with the previous analysis, the initial solution S_0 consists of two parts: S_0^1 for the routes that collect return products from customers to the central warehouse, and S_0^2 for the routes that deliver products from the central warehouse to retail stores and customers. S_0^1 is constructed by $0 \longrightarrow C_{R_i} \longrightarrow 0, C_{R_i} \in C_R$. S_0^2 is constructed by $0 \longrightarrow R_i \longrightarrow C_{D_{R_i}} \longrightarrow 0, R_i \in R, C_{D_{R_i}} \in C_D$, and $C_{D_{R_i}}$ represents the customers served by R_i. The route of $C_{D_{R_i}}$ can be constructed by solving the following model.

Initial solution generation model:

$$\min \sum_{i \in R} \sum_{j \in C_D} c_{ij} x_{ij}, \tag{22}$$

subject to

$$\sum_{i \in R} x_{ij} = 1, \quad \forall j \in C_D, \tag{23}$$

$$\sum_{j \in C_D} d_i x_{ij} \leq \min\{I_i, Q\} \quad \forall i \in R, \tag{24}$$

$$x_{ij} \in \{0, 1\}, \quad \forall i \in R, j \in C_D. \tag{25}$$

Constraint (23) indicates that one customer can only be served by one retail store. Constraint (24) requires that the sum of customer demands cannot exceed the inventory of retail stores. The mathematical model above is a typical assignment model with small scale and low complexity, which can be solved easily and quickly using the CPLEX solver.

4.3. Column Generation Algorithm. Column generation (CG) algorithm was initially invented by Dantzig and Wolfe [72] with the purpose of solving large-scale optimization problems, especially with a huge amount of decision variables. For a detailed overview of column generation algorithms, readers may refer to the work of [73, 74]. Initially, an optimization problem was formulated as a set-partitioning model (SPM) with nearly infinite variables (columns). Then, SPM was linearly relaxed as a linear master problem (LMP), in which the columns still cannot be enumerated. After that, a certain number of columns are utilized to form a restricted linear master problem (RLMP). The RLMP can be easily solved using the simplex method, and its corresponding dual variables can be obtained as well. Meanwhile, there might be some other promising columns with negative reduced cost, which can be added to the RLMP to improve its objective value. Therefore, a subproblem is constructed to find the promising columns, which exploits the benefit of dual feasibility. The above processes are iteratively conducted until no more promising columns can be detected, in which case, the optimal solution of RLMP is also the optimal solution of LMP. After that, a branch scheme is conducted to obtain the integral solution based on the factional solution of LMP.

4.3.1. Set-Partitioning Model-Based RLMP. Let Ω be the set of all feasible routes, $a_{i\omega}$ be a binary constant, 1 iff node $i \in V$ is covered by route $\omega \in \Omega$, and c_ω be the cost of route ω. Let θ_ω be a binary decision variable equal to 1 iff route ω is used.

The initial SPM cannot be solved due to both the nearly infinite set of feasible routes as Ω and the binary integer setting $\theta_\omega \in \{0,1\}$. In order to overcome these problems, firstly an LMP is constructed by linearly relaxing the binary constraints, and then the complete set Ω is replaced by an initial finite set of routes R. The set-partitioning model-based RLPM can be presented as follows.

Set partitioning model:

$$\min \sum_{\omega\in R} c_\omega \theta_\omega, \tag{26}$$

subject to

$$\sum_{\omega\in\Omega} a_{i\omega}\theta_\omega \geq 1, \quad \forall i \in V, \tag{27}$$

$$\theta_\omega \geq 0, \quad \forall \omega \in \Omega. \tag{28}$$

The objective function (26) minimizes the total route cost. Constraint (27) ensures that every node is visited by one route. Constraint (28) indicates the relaxed binary constraints. The RLMP model can be solved using CPLEX solver efficiently. Meanwhile, the corresponding dual values can be obtained simultaneously using the inner function provided by the CPLEX solver, which is used to build the subsequent pricing problem model.

4.3.2. Pricing Subproblem. The pricing problem is formulated to find a specific feasible route with the minimum reduced cost. Let μ_i be the dual variable for constraint (27) for node $i \in V$. The pricing problem is an elementary shortest path problem with resource constraints (SPPRC) with the objective of finding the feasible routes with negative reduced cost. The constraints of the pricing subproblem are similar as constraints (2)–(18) without the index of vehicle k.

Pricing subproblem:

$$\min \sum_{i\in V}\sum_{j\in V}\left(c_{ij}-\mu_i\right)x_{ij}. \tag{29}$$

Subject to

Constraints (2)–(18) without superscript k.

4.3.3. Label-Setting Algorithm. Label-setting algorithms, as one category of dynamic programming algorithms, are commonly used to solve the elementary shortest path problem with resource constraints (SPPRC) [75, 76]. In a label-setting algorithm, a label or a state represents a partial route from the source node to a certain node where the consumption of each resource does not exceed its limit. In each iteration, a label is extended along all the feasible arcs to create a set of new labels. These new labels can be further extended until there exist no feasible extensions for all the unextended labels due to the consumption of resources. In this way, a label-setting algorithm is guaranteed to enumerate all the feasible routes and hence to find an optimal route. A complete label-setting algorithm is composed of three aspects as label definition, label extension, and dominance rules, which are explained in following sections.

Algorithm 2 presents the pseudocode of the label-setting algorithm, in which UL_i and TL_i denote the unextended and extended labels, respectively. The algorithm first creates an initial label L_0 and adds it to UL_0. In each iteration, the algorithm selects an unextended label and extends it along all the feasible arcs. The resulting labels are added to UL_j if they are not dominated by the existing labels. Meanwhile, the existing labels which are dominated by the new labels are discarded. The process terminates when all the labels have been extended. Finally, the optimal route can be found among the labels in UL_{n+1}.

To further speed up the search of the label-setting algorithm, two optimization techniques are used. The first one is to sort the searched labels for subsequent extensions or to avoid impossible tags in advance. The search route must visit retail stores firstly and then access customers with delivery requirements. The sorting operations of labels adopt the minimum weight method commonly used in the shortest path algorithm. And the second one is to record all searched labels during the search. Based on the recorded labels, we only need to reupdate the parameter value of the existing labels and then start searching the last labels of the unfinished line in case of iterative searching after reupdating the weight c_{ij}.

(1) Label Definition. Let $L_i = (\overline{C}_i, t_i, q_i, q_i^p, q_i^c, q_i^s, Q_i, V_i, W_i, T_i, D_i, S_i, F_i)$ be a label associated with node i, where

(i) $\overline{C}_i$ is the reduced cost;

(ii) t_i is the arrive time at node i;

(iii) q_i^t is the total quantity carried by the vehicle after leaving node i, which is equal to the value of the last node of Q_i;

(iv) q_i^p is the amount of products that L_i has picked up at visited return customers;

(v) q_i^c is the amount of products that L_i has delivered at visited delivery customers;

(vi) q_i^s is the amount of inventory that L_i has traveled at visited store nodes;

(vii) Q_i is the total quantity set carried by the vehicle after leaving visited node;

(viii) V_i is the node set that records whether a node is visited;

(ix) W_i is the ordered list of nodes that have been visited by the label;

(x) T_i is the node type of i;

(xi) D_i is the dominated flag of node i. If L_i is dominated, $D_i = 1$;

(xii) S_i is a mapping set that records which retail store serves which customers;

(xiii) F_i is the visited flag that records whether a store node is visited. If L_i has visited a store node, $F_i = 1$.

(2) Label Extension. If node j is a retail store node, add the delivery quantity of node j to the amount of the first node (depot) of Q_i and then update the values of subsequent

Require: updated c_{ij} based on due cost μ_i and μ_0
Ensure: The solution S1;
Generate an initial label $L_0 = (\overline{C}_0, t_0, q_0, q_0^p, q_0^c, q_0^s, Q_0, V_0, W_0, T_0, D_0, S_0, F_0)$;
Set $\mathrm{UL}_0 = \{L_0\}$ and $\mathrm{TL}_i = \varnothing$;
for all $i \in V\backslash\{0\}$ **do**;
  Set $\mathrm{UL}_i = \varnothing$ and $\mathrm{TL}_i = \varnothing$;
end for
while $\cup_{i\in V\backslash\{n+1\}}\mathrm{UL}_i = \varnothing$ **do**
  Choose a label $L_i = (\overline{C}_i, t_i, q_i, q_i^p, q_i^c, q_i^s, Q_i, V_i, W_i, T_i, D_i, S_i, F_i) \in \mathrm{UL}_i\,(\mathrm{UL}_i \neq \varnothing)$;
  for all $(i, j) \in A$ and extension along (i, j) is feasible **do**
    Extend L_i along arc (i, j) and create label L_j;
    if L_j is not dominated by any label in $\mathrm{TU}_j \cup \mathrm{TL}_j$ **then**
      Set $\mathrm{UL}_j = \mathrm{UL}_j \cup \{L_j\}$;
      Discard the labels in $\mathrm{UL}_j \cup \mathrm{TL}_j$ which are dominated by L_j;
    end if
    Set $\mathrm{UL}_i = \mathrm{UL}_i\backslash\{L_i\}$ and $\mathrm{TL}_i = \mathrm{TL}_i\backslash\{L_i\}$;
  end for
end while
Return the route with minimum reduced cost defined by labels in UL_{n+1}.

Algorithm 2: Label-setting algorithm.

nodes in sequence. Let $Q_i \leq Q$ to ensure that the load of each node that a vehicle has visited does not exceed its capacity. Let $q_i^t - I_j \leq Q$ to ensure that the load of a vehicle does not exceed its capacity after the products are delivered to the store j.

If node j is a delivery customer node, from the retail stores visited, select the retail stores k with surplus inventory to meet customer j requirements, then add the delivery quantity of node j to the amount of the node k (retail store) of Q_i, and update the values of subsequent nodes in sequence. If there are multiple retail stores with surplus inventory to meet the customer demand, multiple extension labels are performed. Let $Q_i \leq Q$ to ensure that the load of a vehicle does not exceed its capacity after the products are delivered and picked up from the store j. Let $q_i - d_j \leq Q$ to ensure that the load of a vehicle does not exceed its capacity after the products are delivered to the customer j.

If node j is a return customer node, let $q_i + d_j \leq Q$ to ensure that the load of a vehicle does not exceed its capacity after the products are picked up at the customer j.

Then, a new label $L_j = (\overline{C}_j, t_j, q_j^t, q_j^p, q_j^c, q_j^s, Q_j, V_j, W_j, T_j, D_j, S_j, F_j)$ can be created along arc (i, j) if the following conditions are satisfied (Table 2).

(3) Dominance Rules. The dominance rules proposed here are inspired from the work of Dumas et al. [48] and Ropke and Cordeau [74]. The design of dominance rules is critical towards the convergence of the search process. Dominance rules are exploited to identify a small subset of labels from which at least an optimal route can be generated. The labels not in this subset are referred to as dominated labels. The dominated labels are discarded once they are generated. Therefore, dominance rules can reduce the number of labels extended in a label-setting algorithm to accelerate the algorithm.

In this research, taking into account the distribution sequence between depot, retail stores, delivery customers, and return customers, there are certain circumstances in which the dominance rules take effect. Given two labels associated with node i as follows:

$$\begin{aligned} L_i^1 &= \left(\overline{c}_i^1, t_i^1, q_i^1, q_i^{p1}, q_i^{c1}, q_i^{s1}, Q_i^1, V_i^1, W_i^1, T_i^1, D_i^1, S_i^1, F_i^1\right),\\ L_i^2 &= \left(\overline{c}_i^2, t_i^2, q_i^2, q_i^{p2}, q_i^{c2}, q_i^{s2}, Q_i^2, V_i^2, W_i^2, T_i^2, D_i^2, S_i^2, F_i^2\right), \end{aligned} \tag{30}$$

L_i^1 dominates L_i^2 if these constraints: $\overline{c}_i^1 \leq \overline{c}_i^2$, $t_i^1 \leq t_i^2$, $V_i^1 \supseteq V_i^2$, $q_i^1 \leq q_i^2$, and the following conditions are true:

(1) All nodes accessing these two labels are return customers
(2) All nodes accessing these two labels are retail stores
(3) These two labels remove the previous and same part; the nodes accessing the remaining part are return customers
(4) These two labels remove the previous and same part; the nodes accessing the remaining part are retail stores
(5) These two labels remove the previous and same part; the nodes accessing the remaining part are delivery customers which are serviced by the same retail store.

4.4. Branching and Upper Bound Optimization. Branching occurs when the optimal solution of the SPM is fractional and no violated capacity inequalities can be found. In the proposed solution approach, it is not proper to branch on column variables θ_ω, as setting θ_ω to 0 means preventing the pricing subproblem from generating corresponding columns and results in significant increment of

Table 2: The conditions for label extension.

Retail store node j	Delivery customer node j	Return customer node j
$c_j = c_i + c_{ij}$	$c_j = c_i + c_{ij}$	$c_j = c_i + c_{ij}$
$t_j = t_i + s_i + t_{ij}$	$t_j = t_i + s_i + t_{ij}$	$t_j = t_i + s_i + t_{ij}$
$q_j^t = q_i^t - I_j$	$q_j{}^t = q_i{}^t - d_j$	$q_j{}^t = q_i{}^t + d_j$
$q_j^p = q_i^p$	$q_j^p = q_i^p$	$q_j^p = q_i^p + d_j$
$q_j^c = q_i^c$	$q_j^c = q_i^c + d_j$	$q_j^c = q_i^c$
$q_j^s = q_i^s + I_j$	$q_j^s = q_i^s$	$q_j^s = q_i^s$
Update $Q_i, Q_j = Q_i \cup \{q_j\}$	Update $Q_i, Q_j = Q_i \cup \{q_j\}$	Update Q_i, $Q_j = Q_i \cup \{q_j\}$
$V_j = V_i \cup \{j\}$	$V_j = V_i \cup \{j\}$	$V_j = V_i \cup \{j\}$
$W_j = W_i \cup \{j\}$	$W_j = W_i \cup \{j\}$	$W_j = W_i \cup \{j\}$
$T_j = 1$	$T_i = 2$	$T_i = 3$
$D_j = 0$	$D_j = 0$	$D_j = 0$
$S_j = S_i$	$S_j = S_i \cup \{(k, j)\}$	$S_j = S_i$
$F_j = 1$	$F_j = F_i$	$F_j = F_i$

the computational complexity of the pricing subproblem [77]. Therefore, we need to design proper branching strategies which do not change the structures of the pricing subproblem.

In this research, we adopt an arc-based branching scheme, which was introduced by Ryan and Foster [78]; we design our arc-based branching strategies. At first, the fractional solution based on the route is transformed into a fractional value of its corresponding arc. b_{ij} denotes the value of arc (i, j). Then, we select the fractional value b_{ij} with max $\{b_{ij} - \lfloor b_{ij} \rfloor\}$ and branch it to 0 and 1. When b_{ij} is branched to 0, delete routes associated with arc (i, j) from the current route set R to form the new path set R_0. Then, in the next route search of the labelling algorithm, the cost of the arc c_{ij} is set to a very large value to make sure that routes that travel on arc (i, j) will not be generated. When b_{ij} is branched to 1, the current route set R needs to delete other routes that include arc $(i, h), h \in V/\{j\}$ and $(l, j), l \in V/\{j\}$ to form the new route set R_1. Moreover, in the next route search of the labelling algorithm, the cost of the arcs c_{ih} and c_{lj} is set to a very large value to make sure that routes that travel on arcs (i, h) and (i, h) will not be generated.

If an integer solution is obtained during the search process in the B&P algorithm framework, it can be set as the upper bound to facilitate the pruning operation. When the linear result of the current branch is greater than the known minimum upper bound, the branch can be cut off directly without any further branching operations. In the branching process, some unnecessary branch searches can be avoided if the integer solution can be obtained in advance. The following are three trigger criteria for the upper bound optimization in order to acquire the integer solution of the master problem in advance.

(1) When the initial solution is generated, the integer solution value will be obtained necessarily.

(2) When the root node of the B&P search tree is formed, the master problem can directly calculate the integer solution to update the upper bound value. As the root node is derived from the initial solutions using CG procedures, it is likely that better upper bound can be obtained.

(3) When the linear result of the master problem is around 10% less than the current upper bound, the master problem can directly calculate the integer solution to update the upper bound value.

5. Results and Discussion

In this section, instance generation mechanism is firstly introduced. Then we examine the performance of the proposed approach using both small-scale and large-scale instances. For small-scale test instances, CPLEX solver is used as the comparable counterpart for evaluating the performance of the proposed approach. After that, the benefit of the proposed integrated distribution system is presented in comparison with the three individual delivery systems. Moreover, the product return rate and the inventory scenarios of retail stores are discussed. In addition, the inventory sharing among multiple retail stores is explored as well. All the computational experiments are conducted on a PC with Intel Core i7-4770, 8 Duo 3.4 GHZ.

5.1. Instance Generation. Test instances are generated referring to a real-world fast fashion company in China. This company owns 1 central warehouse and 50 physical retail stores in Shanghai city. Around 1500 customers place their orders online on a daily basis. Public records can be accessed from the open database of this company, and each record contains the location information of customers, retail stores and the central warehouse, and the demand forecast of online customers for each retail store. We generate the test instances based on these public records using the following generation mechanism.

The original three independent logistics distribution systems will lead to an increase in the number of vehicles, a low vehicle full load rate, and an increase in overall transportation costs. The original commonly used arrangement of nearby retail stores to meet customer needs will lead to a situation where some retail stores have inventory, but some customer needs cannot be met. When the inventory level becomes more abundant, although customer demand can be fully satisfied, there are still problems such as low vehicle occupancy rate and the overall transportation cost cannot be reduced. However, our model provides a matching rate of demand through global decision making on matching retail store inventory and customer demand. At the same time, by integrating vehicle resources in the three logistics and distribution systems for unified distribution, the vehicle utilization rate is improved, the distance of vehicles from multiple trips to and from the retail store and the central warehouse is reduced, and the distance cost caused by the long distance matching between the retail store and the customer is offset. Considering that the inventory of retail stores will have a very large impact on the experimental results, we consider three inventory scenarios in designing the experimental data generation mechanism: tight inventory scenarios, relaxed inventory scenarios, and abundant inventory scenarios. In each scenario, it is ensured that the total inventory of retail stores is greater than the total demand of customer stores. In the tight inventory scenarios, the setting formula for the inventory of each retail store is $I_i = \sum_{j\in C} d_j + U[0.1, 0.2]\sum_{j\in C} d_j/|R|$. The relaxed inventory scenario is $I_i = \sum_{j\in C} d_j + U[0.5, 1]\sum_{j\in C} d_j/|R|$, and the abundant inventory scenario is $I_i = \sum_{j\in C} d_j$.

Totally, 4 sets of instances are generated, namely, Set 1, Set 2, Set 3, and Set 4. Set 1 is with small-sized test instances with all inventory scenarios of retail stores. Sets 2–4 are medium-large test instances with different inventory scenarios of retail stores. The number of retail stores (N_S) of Set 1 is set as 3 to 4, and the number of retail stores of Sets 2–4 is set as 5 to 40. For each traveling path, the travel cost is set to the travel distance and the travel time is set to the travel distance divided by the vehicle speed of 60 km/h. The service time at each retail store is set to 15 minutes. The number of available vehicles is set to 30. Regarding the inventory scenarios, Set 2 is set as tight inventory instances. Set 3 is set as relaxed inventory instances. Set 4 is set as abundant inventory instances. Moreover, for each set of instances, two types of customers, i.e., customers with delivery requirement and return requirement, are randomly chosen from the database. The number of online customers with delivery requirement (N_{C_D}) is set as three times the number of retail stores, and the number of customers with return requirement (N_{C_R}) is set as $\lceil 0.1 N_{C_D} \rceil$ to $\lceil 0.5 N_{C_D} \rceil$. Totally, 150 test instances are generated. The instances are named in the scheme of "X-Sn-Dn-Rn," where "X" represents the inventory scenario, "Sn" represents the number of retail stores, "Dn" represents the number of the customers with deliver needs, and "Rn" represents the number of the customers with return needs.

5.2. Algorithm Performance on Small-Sized Instances. Set 1 with 30 small-sized instances is firstly employed to examine and validate the performance of the proposed approach in comparison with the solutions obtained from the CPLEX solver. The test results are presented in Table 3. The column "Sum (Inv)" denotes the total inventory of retail stores. The column "Sum (Sd)" represents the total demand of retail stores. The column "Sum (Cd)" indicates the total demand of customers. The column "Sum (Re)" denotes the total amount of returned goods. The column "Time (s)" denotes the computational time (in seconds) of CPLEX and the proposed algorithm separately. The column "Best" and "Avg. 10" provides the best results obtained with CPLEX and the average result found in 10 runs using the proposed algorithm, respectively. The best results of CPLEX are obtained either from the optimal solution or from the best upper bound (given in bold) found within 300 seconds. The column "%dev" denotes the percentage deviation of algorithmic performance between CPLEX and the proposed approach.

Table 3 shows that for each test instance, the solution result obtained by the proposed algorithm is not worse than that obtained from CPLEX solver, which verifies the correctness of the proposed algorithm. More importantly, when the scale of the problem grows larger, the calculation time of CPLEX will increase accordingly and gradually reach the maximum calculation time limit. The average calculation time of CPLEX solver requires 289.9 seconds. In contrast, the proposed algorithm is efficient and stable in small-scale examples. The average solving time for all test instances using the proposed algorithm is around 19.6 seconds, which is 14.8 times faster than the CPLEX time. Especially when the problem size becomes larger, the difference in calculation time becomes larger and larger, and the maximum difference can be 35.3 times. This is because the CPLEX solver uses a general mathematical method to solve the problem. When the problem size becomes larger, the decision variables and constraints increase exponentially and the calculation time becomes longer. However, our algorithm is specifically designed for this problem, reducing a large amount of ineffective calculations, such as some invalid route searches. Hence, the proposed algorithm significantly outperforms the commercial CPLEX solver in terms of both solution quality and solving efficiency, and it can solve large-scale problems more efficiently than the solver.

5.3. Algorithm Performance on Large-Sized Instances. Tables 4–6 present the computational results on the large-sized instances of Sets 2–4, respectively. As shown in Tables 4–6, the proposed algorithm can generally find good quality solutions within reasonable computational time. The instances without solutions are marked using "—". These "no solution" instances will be dealt in the following part.

5.3.1. Analysis of the Inventory Scenarios. As shown in Tables 4 and 5, some test instances with tight and relaxed inventories have no solution. It is noted that there are 16 out of 40 instances in Tables 4,and 6 out of 40 instances in Table 5 have no solutions. But there are no unsolvable

Table 3: Performance comparison of CPLEX solver and the proposed algorithm with Set 1.

Instance	Sum (Inv)	Sum (Sd)	Sum (Cd)	Sum (Re)	CPLEX			The proposed algorithm			%dev
					Best	k	Time (s)	Avg. 10	k	Time (s)	
T-S3-D9-R1	39	197	36	1	188368	1	195	188368	1	7.2	0.0
R-S3-D9-R1	63	197	36	1	174472	1	227	174472	1	7.1	0.0
A-S3-D9-R1	108	197	36	1	174472	1	176	174472	1	9.5	0.0
T-S3-D9-R2	42	197	36	10	**184656**	1	300	180688	1	8.5	−2.2
R-S3-D9-R2	66	197	36	10	**180395**	1	300	176669	1	8.5	−2.1
A-S3-D9-R2	108	197	36	10	**180395**	1	300	176669	1	10.0	−2.1
T-S3-D9-R3	48	197	36	5	193772	1	300	193772	1	12.8	0.0
R-S3-D9-R3	60	197	36	5	191047	1	300	191047	1	10.0	0.0
A-S3-D9-R3	108	197	36	5	191047	1	300	191047	1	19.7	0.0
T-S3-D9-R4	48	197	36	12	191583	1	300	191583	1	10.0	0.0
R-S3-D9-R4	66	197	36	12	188858	1	300	188858	1	10.5	0.0
A-S3-D9-R4	108	197	36	12	188858	1	300	188858	1	13.4	0.0
T-S3-D9-R5	42	197	36	14	203714	1	300	203714	1	13.4	0.0
R-S3-D9-R5	57	197	36	14	**201916**	1	300	200312	1	15.5	−0.8
A-S3-D9-R5	108	197	36	14	200312	1	300	200312	1	34.6	0.0
T-S4-D12-R1	56	206	52	10	192523	2	300	192523	2	17.7	0.0
R-S4-D12-R1	100	206	52	10	192523	2	300	192523	2	14.7	0.0
A-S4-D12-R1	208	206	52	10	192523	2	300	192523	2	21.1	0.0
T-S4-D12-R3	56	206	52	21	211098	2	300	211098	2	18.9	0.0
R-S4-D12-R3	84	206	52	21	209391	2	300	209391	2	10.9	0.0
A-S4-D12-R3	208	206	52	21	**209573**	2	300	207960	2	14.0	−0.8
T-S4-D12-R4	72	206	52	23	**255768**	2	300	254073	2	17.5	−0.7
R-S4-D12-R4	80	206	52	23	**235864**	2	300	230901	2	33.0	−2.1
A-S4-D12-R4	208	206	52	23	**246404**	2	300	223166	2	13.5	−9.4
T-S4-D12-R5	68	206	52	21	**260354**	2	300	254570	2	27.7	−2.2
R-S4-D12-R5	80	206	52	21	**259689**	2	300	254580	2	24.5	−2.0
A-S4-D12-R5	208	206	52	21	**264646**	2	300	258683	2	60.0	−2.3
T-S4-D12-R6	68	206	52	31	**256079**	2	300	255026	2	37.1	−0.4
R-S4-D12-R6	100	206	52	31	**255035**	2	300	254482	2	39.5	−0.2
A-S4-D12-R6	208	206	52	31	**291126**	2	300	260144	2	46.5	−10.6
Avg.	95.8	201.5	44.0	14.8	212215.4	1.5	289.9	209082.8	1.5	19.6	−1.3

instances in Table 6. It was initially judged that there was no solution due to the "tight" settings of inventory levels of retail stores. We further examine the impact of inventory scenarios on these test instances. Figure 5 shows the cost changes along with the change of inventory scenarios in each test instance, in which the instances initially with no solutions, such as S10-D30-R9 and S15-D45-R5 of tight inventory, are marked a sufficiently large value like 350,000.

As expected, when more inventories are available in the network, the number of instances with no solutions and the total cost tend to decrease. For example, instances S25-D75-R23 and S25-D75-R38 become feasible when their inventory scenarios are changed from tight to relaxed. Moreover, it is noted that instances S35-D105-R32, S40-D120-R24, S40-D120-R36, and S40-D120-R60 are still infeasible with relaxed inventory scenarios and become feasible only when the inventory scenarios are changed to abundant. The increase of the available inventory in the network enlarges more flexibility and possibility when assigning online customer orders to retailer stores, which further reflects the decrease of the total operational cost, with the highest decrease of 172.2% and the lowest decrease of 3.4%. In each of our instances, the total inventory in the retail store is greater than the total order demand. However, there are still unsolvable situations when inventory levels are tight and relaxed. This is because the demand for retail stores to meet orders is met in full at one time, resulting in some retail stores remaining with sporadic inventory. In the end, this cannot satisfy any of the remaining order requirements at once. When inventory levels are abundant, each retail store's inventory can meet all order requirements, so there are no unsolvable instances. However, in real scenarios, such abundant inventory levels are rare. Most of them are tight and relaxed inventory levels. The existing way of single retail stores meeting the demand of orders at one time will still lead to inventory waste and unfulfilled order demand. Therefore, the best way to avoid inventory waste and unsatisfied order demands is to realize inventory sharing in retail stores and connect these inventories through vehicles to meet order demands globally. Relevant experiments in this regard are described in detail later.

Interestingly, the total operational cost does not vary much with the instances of less than 10 retail stores (the instances with no solutions are excluded). An in-depth examination of the vehicle routes for these instances shows that that the number of retail stores is relatively small, resulting in fewer split combinations to meet order demand, so there is not much room for optimization of operating costs. However, with the increase of retail stores, the total operational cost changes substantially, which is due to the

Table 4: Computational results for tight inventory instances (Set 2).

Instance	Sum (Inv)	Sum (Sd)	Sum (Cd)	Sum (Re)	The proposed algorithm		
					Avg. 10	K	Time (s)
T-S5-D15-R2	65	399	64	14	347974	5	277.7
T-S5-D15-R3	85	399	64	20	315086	5	413.0
T-S5-D15-R5	80	399	64	24	362652	5	90.7
T-S5-D15-R6	95	399	64	26	354474	5	469.6
T-S5-D15-R8	70	399	64	35	429973	5	399.4
T-S10-D30-R3	230	415	178	28	440946	3	315.3
T-S10-D30-R6	260	415	178	40	461574	3	193.3
T-S10-D30-R9	180	415	178	50	—	—	—
T-S10-D30-R12	190	415	178	75	546840	3	249.7
T-S10-D30-R15	190	415	178	100	859817	5	209.7
T-S15-D45-R5	225	844	225	19	—	—	—
T-S15-D45-R9	270	844	225	62	866002	5	275.7
T-S15-D45-R14	300	844	225	73	835588	5	267.7
T-S15-D45-R18	270	844	225	103	784298	5	565.3
T-S15-D45-R23	285	844	225	126	1060574	5	328.2
T-S20-D60-R6	340	950	305	30	1421386	6	195.8
T-S20-D60-R12	380	950	305	50	1040002	5	425.9
T-S20-D60-R18	400	950	305	95	1053547	5	237.2
T-S20-D60-R24	420	950	305	125	1530828	6	416.6
T-S20-D60-R30	440	950	305	153	1886193	6	370.5
T-S25-D75-R8	550	847	425	25	1120225	5	440.1
T-S25-D75-R15	550	847	425	72	1219248	5	371.1
T-S25-D75-R23	425	847	425	125	—	—	—
T-S25-D75-R30	500	847	425	131	1721168	5	303.4
T-S25-D75-R38	500	847	425	183	—	—	—
T-S30-D90-R9	690	1593	483	51	2624073	5	401.9
T-S30-D90-R18	570	1593	483	117	—	—	—
T-S30-D90-R27	540	1593	483	144	—	—	—
T-S30-D90-R36	660	1593	483	198	2592743	6	315.2
T-S30-D90-R45	660	1593	483	267	—	—	—
T-S35-D105-R11	700	1390	528	57	2566190	8	389.4
T-S35-D105-R21	525	1390	528	156	—	—	—
T-S35-D105-R32	700	1390	528	207	—	—	—
T-S35-D105-R42	735	1390	528	274	—	—	—
T-S35-D105-R53	700	1390	528	321	—	—	—
T-S40-D120-R12	880	1770	633	83	—	—	—
T-S40-D120-R24	960	1770	633	153	—	—	—
T-S40-D120-R36	880	1770	633	223	—	—	—
T-S40-D120-R48	720	1770	633	240	—	—	—
T-S40-D120-R60	640	1770	633	340	—	—	—
Avg.	446.5	1026.0	355.1	115.4	1101725.0	5.0	331.3

fact that online customers can be fulfilled by more optional retail stores, and the nearby retail store with stock can be allocated.

5.3.2. Analysis of the Product Return. In addition to retail store inventory levels and customer order demand, the number of customers returning products also affects vehicle routing and total operating costs. Figure 6 shows the changes in total operating costs under different number of product return customers. The following subfigures, respectively, represent the example result data under different number of returning customers at 10, 20, 30, and 35 retail stores. It is worth noting that total operating costs fluctuate with the number of customers returning products. But overall, as the number of returns increases, the total operating costs show an overall increased trend. This trend is due to an increased need to recycle returned products, resulting in the need to use more vehicles to meet the demand for product returns. But this trend occasionally has some fluctuations, resulting in results that are opposite to the overall trend. This is because the vehicle has the opportunity to use its unoccupied vehicle space to collect more returned products on its way back to the warehouse, thereby reducing the cost of traveling to and from the warehouse repeatedly.

5.4. Effect of the Integrated Distribution System. In order to evaluate the effect of our proposed integrated distribution system, we specially designed experiments to compare it with a two-stage independent distribution system and a three-stage

Table 5: Computational results for relaxed inventory instances (Set 3).

Instance	Sum (Inv)	Sum (Sd)	Sum (Cd)	Sum (Re)	The proposed algorithm		
					Avg. 10	k	Time (s)
R-S5-D15-R2	115	399	64	14	268727	5	296.2
R-S5-D15-R3	100	399	64	20	294628	5	252.5
R-S5-D15-R5	115	399	64	24	306169	5	260.0
R-S5-D15-R6	115	399	64	26	308894	5	356.5
R-S5-D15-R8	105	399	64	35	297996	5	70.4
R-S10-D30-R3	290	415	178	28	417151	3	343.7
R-S10-D30-R6	270	415	178	40	465840	3	258.4
R-S10-D30-R9	270	415	178	50	399700	3	388.9
R-S10-D30-R12	330	415	178	75	492783	3	240.9
R-S10-D30-R15	280	415	178	100	499015	3	253.7
R-S15-D45-R5	435	844	225	19	546485	5	172.8
R-S15-D45-R9	375	844	225	62	666934	5	246.4
R-S15-D45-R14	390	844	225	73	565611	5	275.4
R-S15-D45-R18	450	844	225	103	620721	5	356.7
R-S15-D45-R23	450	844	225	126	675427	5	321.5
R-S20-D60-R6	540	950	305	30	700707	5	142.2
R-S20-D60-R12	500	950	305	50	675616	5	304.3
R-S20-D60-R18	500	950	305	95	697121	5	525.4
R-S20-D60-R24	540	950	305	125	792628	5	536.7
R-S20-D60-R30	540	950	305	153	799752	6	265.1
R-S25-D75-R8	650	847	425	25	903027	6	388.9
R-S25-D75-R15	800	847	425	72	938299	3	363.3
R-S25-D75-R23	825	847	425	125	897028	5	297.3
R-S25-D75-R30	725	847	425	131	943608	8	344.1
R-S25-D75-R38	800	847	425	183	1085692	5	281.4
R-S30-D90-R9	780	1593	483	51	1079256	5	232.3
R-S30-D90-R18	900	1593	483	117	1133485	6	686.8
R-S30-D90-R27	780	1593	483	144	1112044	8	424.9
R-S30-D90-R36	870	1593	483	198	1303971	5	681.9
R-S30-D90-R45	750	1593	483	267	1278208	6	295.8
R-S35-D105-R11	980	1390	528	57	1645316	5	442.1
R-S35-D105-R21	945	1390	528	156	1396845	5	544.1
R-S35-D105-R32	1015	1390	528	207	—	—	—
R-S35-D105-R42	1015	1390	528	274	1245501	8	506.4
R-S35-D105-R53	1050	1390	528	321	1574616	6	520.4
R-S40-D120-R12	960	1770	633	83	2376704	6	348.5
R-S40-D120-R24	1080	1770	633	153	—	—	—
R-S40-D120-R36	1120	1770	633	223	—	—	—
R-S40-D120-R48	1040	1770	633	240	2157401	8	671.4
R-S40-D120-R60	960	1770	633	340	—	—	—
Avg.	618.9	1026.0	355.1	115.4	876747.4	5.2	358.3

independent distribution system. For the two-stage independent distribution system, two groups of vehicles from different fleets are used for distribution. The first group of fleets is tasked with transporting products via vehicles from the central warehouse to retail stores and taking customer returned products back to the central warehouse. The second fleet is tasked with picking up goods from retail stores via vehicles and delivering them to customers buying online. For the three-stage independent distribution system, three groups of vehicles from different fleets are used for distribution. The first fleet is tasked with transporting products via vehicles from central warehouses to retail stores. The second fleet is tasked with picking up products from retail stores via vehicles and delivering them to customers. The third fleet is tasked with picking up returned products from customers via vehicles to the central warehouse.

Table 7 gives the results of large-scale test instances of our proposed integrated distribution system, two-stage independent distribution system, and three-stage independent distribution system. It can be noted that the total operating cost and the number of vehicles increase significantly when a two-stage independent distribution system and a three-stage independent distribution system are adopted. The statistics of the example results show that compared with the integrated distribution system, the total operating cost of the two-stage independent distribution system increased by 15.1% on average, and the vehicles increased by 137.0% on average. The total operating cost of the three-stage independent distribution system increased by 49.9% on average, and the vehicles increased by 183.4% on average. This fully confirms that an integrated distribution system has sufficient advantages over multiple independent distribution

Table 6: Computational results for the abundant inventory instances (Set 4).

Instance	Sum (Inv)	Sum (Sd)	Sum (Cd)	Sum (Re)	The proposed algorithm		
					Avg. 10	k	Time (s)
A-S5-D15-R2	320	399	64	14	260058	5	338.9
A-S5-D15-R3	320	399	64	20	260208	5	348.5
A-S5-D15-R5	320	399	64	24	302714	5	251.0
A-S5-D15-R6	320	399	64	26	313295	5	266.0
A-S5-D15-R8	320	399	64	35	320981	5	280.5
A-S10-D30-R3	1070	415	178	28	411955	3	451.9
A-S10-D30-R6	1070	415	178	40	446610	3	314.6
A-S10-D30-R9	1070	415	178	50	421957	5	168.2
A-S10-D30-R12	1070	415	178	75	466601	3	308.7
A-S10-D30-R15	1070	415	178	100	462565	3	271.6
A-S15-D45-R5	1575	844	225	19	482881	5	219.0
A-S15-D45-R9	1575	844	225	62	618172	5	445.7
A-S15-D45-R14	1575	844	225	73	614723	5	443.6
A-S15-D45-R18	1575	844	225	103	574182	3	142.4
A-S15-D45-R23	1575	844	225	126	571501	5	408.9
A-S20-D60-R6	2440	950	305	30	771561	5	296.9
A-S20-D60-R12	2440	950	305	50	767060	5	541.8
A-S20-D60-R18	2440	950	305	95	724580	5	565.4
A-S20-D60-R24	2440	950	305	125	783674	5	166.8
A-S20-D60-R30	2440	950	305	153	784584	5	388.5
A-S25-D75-R8	3825	847	425	25	801754	5	313.2
A-S25-D75-R15	3825	847	425	72	774379	5	333.9
A-S25-D75-R23	3825	847	425	125	1014909	5	318.0
A-S25-D75-R30	3825	847	425	131	927728	3	365.1
A-S25-D75-R38	3825	847	425	183	892230	6	336.1
A-S30-D90-R9	4830	1593	483	51	964142	5	505.0
A-S30-D90-R18	4830	1593	483	117	966768	6	331.3
A-S30-D90-R27	4830	1593	483	144	1124662	6	732.7
A-S30-D90-R36	4830	1593	483	198	1294804	5	530.9
A-S30-D90-R45	4830	1593	483	267	1231195	8	779.0
A-S35-D105-R11	5810	1390	528	57	1033103	5	518.3
A-S35-D105-R21	5810	1390	528	156	1082917	6	523.3
A-S35-D105-R32	5810	1390	528	207	1139400	5	291.4
A-S35-D105-R42	5810	1390	528	274	1197973	5	358.0
A-S35-D105-R53	5810	1390	528	321	1415709	6	663.4
A-S40-D120-R12	7600	1770	633	83	1728800	5	363.8
A-S40-D120-R24	7600	1770	633	153	1736493	6	429.6
A-S40-D120-R36	7600	1770	633	223	1550196	8	591.6
A-S40-D120-R48	7600	1770	633	240	1630600	6	412.6
A-S40-D120-R60	7600	1770	633	340	1886100	6	377.7
Avg.	3433.8	1026.00	355.1	115.4	868843.1	5.1	392.3

systems in reducing total operating costs and the number of vehicles. The integrated distribution system makes full use of vehicle space through the use of shared vehicles, dynamically selects tasks from three types of tasks, and combines multiple tasks to be executed together to avoid unnecessary vehicle mileage and waste of vehicle space.

5.5. Discussion of Inventory Sharing among Multiple Retail Stores. As discussed in the model extension section, order fulfillment flexibility and feasibility can be increased through inventory sharing among multiple retail stores. It was observed through the above experiments that when inventory cannot be shared between multiple retail stores, there are some test instances with no solution in the case of tight and loose inventory. There are no solutions for 16 of the 40 instances where inventory is tight, and there are no solutions for 4 of the 40 instances where inventory is relaxed. Therefore, we further investigated the impact of inventory sharing among multiple retail stores on operating costs and vehicle usage by comparing the results of an integrated distribution system with and without inventory sharing. The calculation results of the example test are shown in Table 8. The 20 examples that originally had no solutions can now find feasible solutions. This shows that when the total inventory of all retail stores is greater than the total customer order demand, when the inventory between retail stores can also be shared, there is enough flexibility to ensure that all order demands are met by the retail stores. The demand is all met by retail stores. Then, when the integrated distribution system allows retail stores to share inventory to meet

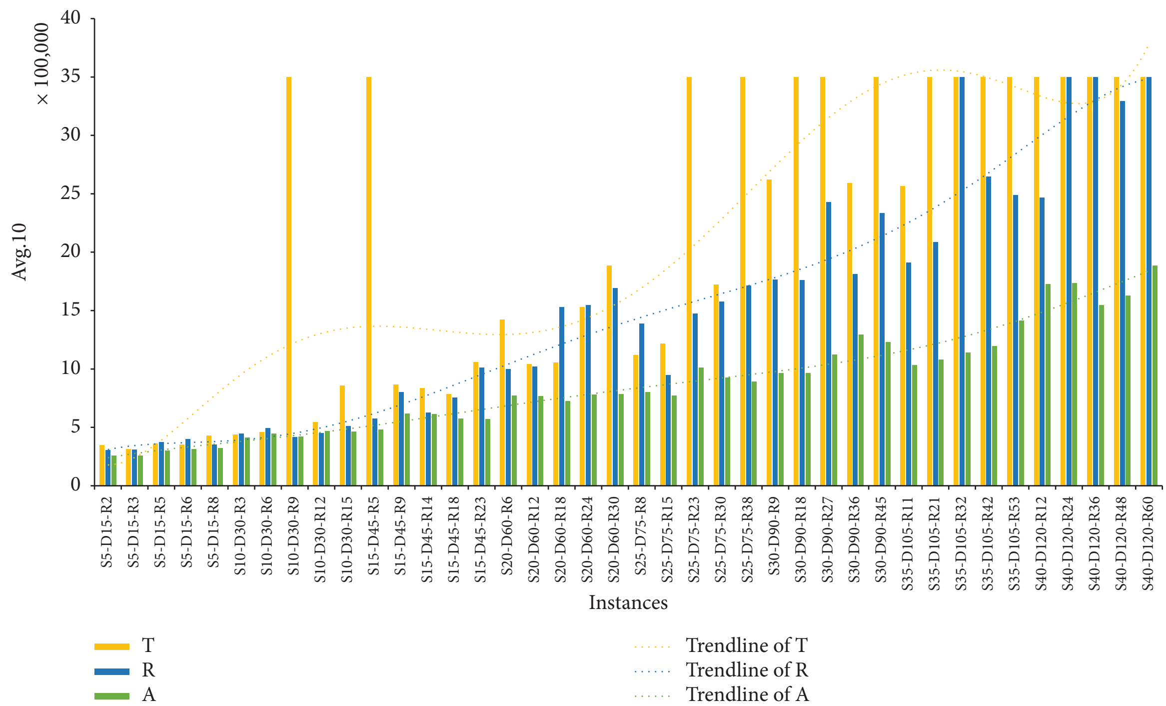

Figure 5: The impact of different inventory scenarios.

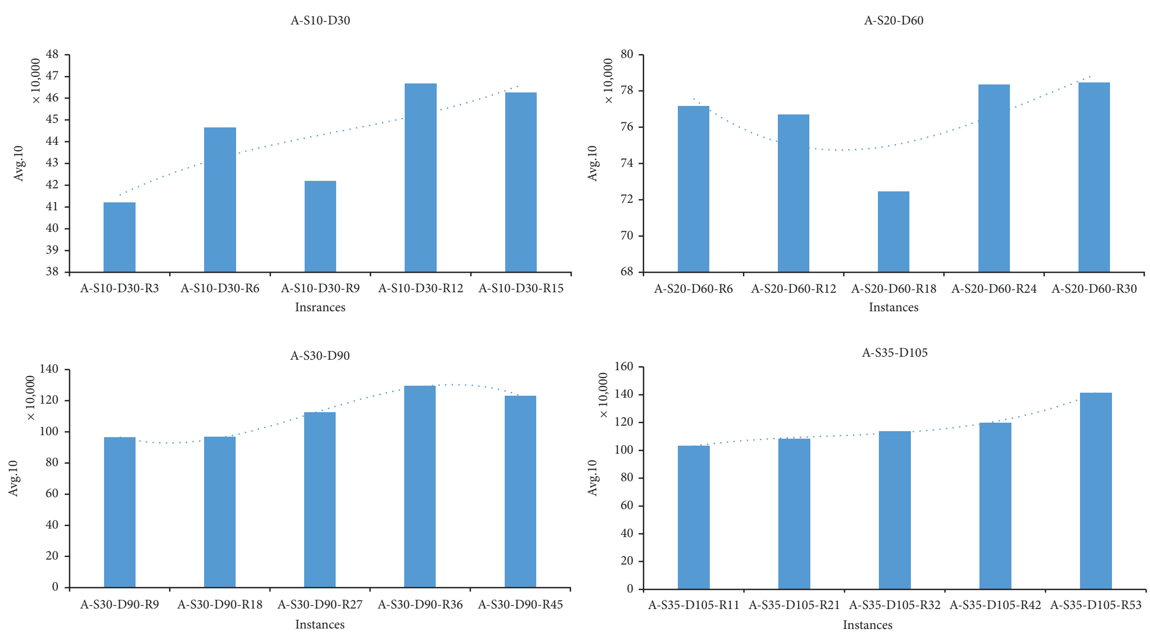

Figure 6: The impact of the product return ratio.

customer order needs, there is no need for high-level inventory, and the total inventory only needs to be greater than or equal to the total order demand. This once again verifies that when the total inventory of all retail stores is greater than or equal to the total demand for the order and the inventory level is not high, order fulfillment can be well achieved through the retail store inventory sharing method.

In addition, to explore the differences between inventory sharing scenarios and abundant inventory scenarios, we compared some instance result data. Figure 7 shows the

Table 7: The performance of the integrated distribution system.

Instance	Integrated distribution		Two-stage distribution				Three-stage distribution			
	Avg. 10	k	Sum	k	dev_sum	dev_k	Sum	k	dev_sum	dev_k
A-S5-D15-R2	260058	5	313174	11	20.4	120.0	384430	13	47.8	160.0
A-S5-D15-R3	260208	5	313526	11	20.5	120.0	420459	13	61.6	160.0
A-S5-D15-R5	302714	5	343287	11	13.4	120.0	462769	13	52.9	160.0
A-S5-D15-R6	313295	5	360946	11	15.2	120.0	475590	13	51.8	160.0
A-S5-D15-R8	320981	5	361050	11	12.5	120.0	485331	13	51.2	160.0
A-S10-D30-R3	411955	3	472996	9	14.8	200.0	610556	11	48.2	266.7
A-S10-D30-R6	446610	3	491147	9	10.0	200.0	762265	11	70.7	266.7
A-S10-D30-R9	421957	5	490570	9	16.3	80.0	720179	11	70.7	120.0
A-S10-D30-R12	466601	3	516536	9	10.7	200.0	792341	11	69.8	266.7
A-S10-D30-R15	462565	3	541966	9	17.2	200.0	808850	11	74.9	266.7
A-S15-D45-R5	482881	5	576815	11	19.5	120.0	736925	13	52.6	160.0
A-S15-D45-R9	618172	5	666216	11	7.8	120.0	816591	13	32.1	160.0
A-S15-D45-R14	614723	5	634794	11	3.3	120.0	792252	13	28.9	160.0
A-S15-D45-R18	574182	3	694774	11	21.0	266.7	842595	13	46.7	333.3
A-S15-D45-R23	571501	5	702305	11	22.9	120.0	921314	13	61.2	160.0
A-S20-D60-R6	771561	5	785842	11	1.9	120.0	1045620	13	35.5	160.0
A-S20-D60-R12	767060	5	842671	11	9.9	120.0	1110011	13	44.7	160.0
A-S20-D60-R18	724580	5	841804	11	16.2	120.0	1153125	13	59.1	160.0
A-S20-D60-R24	783674	5	859361	11	9.7	120.0	1230674	13	57.0	160.0
A-S20-D60-R30	784584	5	918010	11	17.0	120.0	1297467	13	65.4	160.0
A-S25-D75-R8	801754	5	961978	11	20.0	120.0	1177346	15	46.8	200.0
A-S25-D75-R15	774379	5	958638	11	23.8	120.0	1269953	15	64.0	200.0
A-S25-D75-R23	1014909	5	1057780	13	4.2	160.0	1386456	15	36.6	200.0
A-S25-D75-R30	927728	3	1057435	11	14.0	266.7	1348111	15	45.3	400.0
A-S25-D75-R38	892230	6	1105663	11	23.9	83.3	1435711	15	60.9	150.0
A-S30-D90-R9	964142	5	1161818	13	20.5	160.0	1420735	15	47.4	200.0
A-S30-D90-R18	966768	6	1188167	13	22.9	116.7	1509819	15	56.2	150.0
A-S30-D90-R27	1124662	6	1264361	13	12.4	116.7	1533090	15	36.3	150.0
A-S30-D90-R36	1294804	5	1330690	13	2.8	160.0	1748175	15	35.0	200.0
A-S30-D90-R45	1231195	8	1389614	13	12.9	62.5	1768060	15	43.6	87.5
A-S35-D105-R11	1033103	5	1237921	13	19.8	160.0	1517020	15	46.8	200.0
A-S35-D105-R21	1082917	6	1325832	13	22.4	116.7	1661474	15	53.4	150.0
A-S35-D105-R32	1139400	5	1419934	13	24.6	160.0	1759917	15	54.5	200.0
A-S35-D105-R42	1197973	5	1484693	13	23.9	160.0	1837625	15	53.4	200.0
A-S35-D105-R53	1415709	6	1546269	13	9.2	116.7	1872045	15	32.2	150.0
A-S40-D120-R12	1728800	5	1818183	13	5.2	160.0	2142273	15	23.9	200.0
A-S40-D120-R24	1736493	6	1912449	13	10.1	116.7	2223528	15	28.0	150.0
A-S40-D120-R36	1550196	8	1914480	13	23.5	62.5	2344440	15	51.2	87.5
A-S40-D120-R48	1630600	6	1975590	13	21.2	116.7	2577642	15	58.1	150.0
A-S40-D120-R60	1886100	6	2041272	13	8.2	116.7	2617581	15	38.8	150.0
Avg.	868843.1	5.1	997013.9	11.6	15.1	137.0	1275508.6	13.8	49.9	183.4

results of the comparison between the two scenarios. It is worth noting that in most cases, the number of vehicles used and operating costs in the inventory sharing scenario are smaller than those in the abundant inventory scenario, and this difference increases with the size of the instance. It follows that inventory sharing not only facilitates the generation of feasible solutions when retail store inventory is tight but also plays an important role in reducing total operating costs and vehicle numbers, especially when the problem scale is large.

5.6. Managerial Insights. Based on the findings of the paper, several managerial insights can be drawn to guide the operation of logistics and distribution systems in dual-channel retail environments, particularly in the fast fashion industry:

(1) *Long-Term Strategic Alignment*. The adoption of an integrated distribution system aligns with the broader supply chain strategy and supports sustainable business growth. Managers should view logistics integration as part of their long-term strategic planning. The integration of distribution channels can substantially reduce operational costs and the number of vehicles needed. The integrated distribution system performs well across different scales, but the benefits are particularly pronounced in larger operations. As businesses scale up, the integrated approach becomes even more critical to maintaining cost-efficiency. Managers should consider restructuring their logistics operations to create

Table 8: The computational results with inventory sharing.

Instance	Sum (Inv)	Sum (Sd)	Sum (Cd)	Sum (Re)	The integrated distribution		
					Avg. 10	k	Time (s)
T-S10-D30-R9	180	415	178	50	391097	3	166.4
T-S15-D45-R5	225	844	225	19	434856	5	226.8
T-S25-D75-R23	425	847	425	125	790550	5	533.2
T-S25-D75-R38	500	847	425	183	852372	5	208.6
T-S30-D90-R18	570	1593	483	117	905431	5	496.6
T-S30-D90-R27	540	1593	483	144	1029556	5	628.9
T-S30-D90-R45	660	1593	483	267	1094534	5	542.2
T-S35-D105-R21	525	1390	528	156	1023014	5	665.2
T-S35-D105-R32	700	1390	528	207	1127998	5	501.6
R-S35-D105-R32	1015	1390	528	207	1127998	5	492.0
T-S35-D105-R42	735	1390	528	274	1160386	5	519.2
T-S35-D105-R53	700	1390	528	321	1277712	5	570.2
T-S40-D120-R12	880	1770	633	83	1132749	5	696.4
T-S40-D120-R24	960	1770	633	153	1205853	5	682.7
R-S40-D120-R24	1080	1770	633	153	1226793	5	323.1
T-S40-D120-R36	880	1770	633	223	1227468	5	526.4
R-S40-D120-R36	1120	1770	633	223	1226655	5	316.6
T-S40-D120-R48	720	1770	633	240	1341189	5	500.9
T-S40-D120-R60	640	1770	633	340	1339551	5	589.0
R-S40-D120-R60	960	1770	633	340	1347777	5	481.7

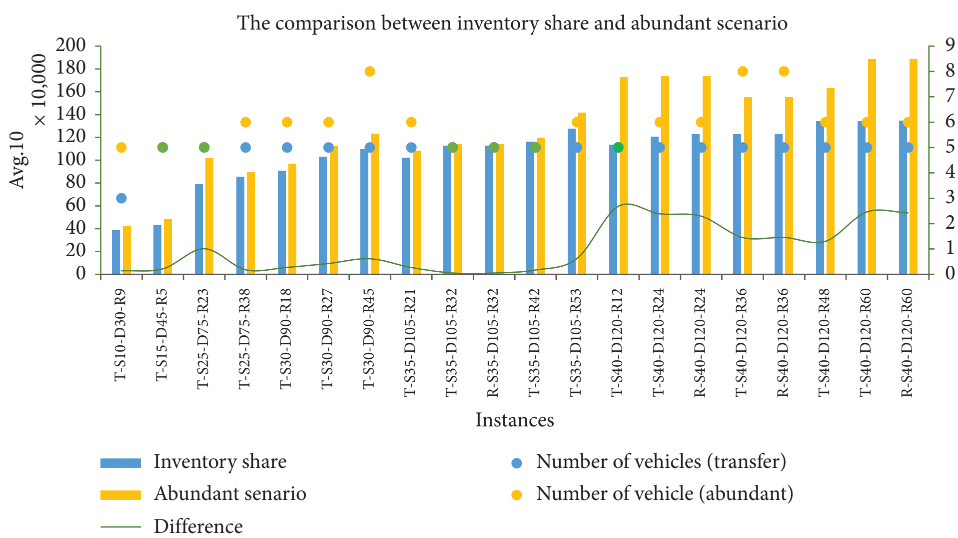

Figure 7: The comparison between inventory sharing scenario and abundant inventory scenario.

a more unified system that combines replenishment, delivery, and returns.

(2) *Importance of Flexible Inventory*. Understanding the impact of different inventory levels on operational costs can help managers in strategic planning. Scenarios with tight, relaxed, and abundant inventories should be modeled to anticipate challenges and create robust distribution strategies. Allowing inventory levels to be more flexible, particularly through sharing inventory between stores, can significantly increase the feasibility of meeting customer demands and reduce operational costs. This flexibility should be built into the logistics strategy, possibly through improved inventory management systems.

(3) *Efficient Management*. An integrated system can better adapt to fluctuations in demand, including the randomness of returns. Managers should consider systems that can dynamically adjust to these changes to prevent resource waste. The implementation of advanced optimization algorithms like the B&P algorithm can lead to significant improvements in route efficiency and distribution strategy. Managers should invest in developing or acquiring these optimization tools to enhance their decision-making processes.

By implementing these managerial insights, companies can expect improved efficiency, customer satisfaction, and competitive advantage in the fast-paced and often unpredictable fast fashion market. The integrated distribution system provides a framework for addressing current logistics challenges while positioning the company for future growth and adaptation to market changes.

However, from the perspective of management and operation, although the integrated distribution system can bring down the overall cost, it also increases the cost of management and the difficulty of operation. Multiple independent distribution systems will be simpler and clearer in management and operation. Each type of task has a dedicated distribution system and operation mechanism. Therefore, there are still many great challenges in implementing an integrated logistics and distribution system.

6. Conclusions

In this research, we design an integrated dual-channel retailing distribution to facilitate the company development and customer requirements in fast fashion industry. The integration of online and offline sales and distribution, especially with product returns, can improve the systematic performance and operational flexibility significantly. The operations of this system are formally formulated using mathematical programming models, which are derived from VRP and PDP. In order to solve the introduced model, we further propose a B&P-based algorithm, in which CG algorithm is exploited to find promising and feasible solutions and label-setting algorithm is designed to solve the pricing subproblem. The performance of the proposed B&P algorithm is examined and validated on a set of instances generated referring to practical operational data. Computational results indicate that the proposed B&P algorithm can produce equivalent solutions when compared to the optimal solutions from CPLEX solver in case of small-scale instances. And the proposed B&P algorithm maintains a high performance with large-scale instances. Moreover, this research illustrates the benefit of using the proposed integrated distribution system with the operational cost saving up to 49.9%. In addition, inventory sharing among multiple retail stores is discussed, which could be a further exploration towards better performance.

Although much effort has been conducted to explore the benefits of the proposed integrated system and the effectiveness of the proposed B&P algorithm, there are still some possible extensions for future research. For instance, from the modeling perspective, the demand and return goods of customers in the proposed model are static and deterministic. It could be of interest that if such settings are relaxed to dynamic and stochastic scenarios. From the solving approach perspective, although the proposed B&P algorithm can solve large-scale instances using computational time around hundred seconds, it is acceptable in static scenarios. However, in case of dynamic customer requirements, such a solving efficiency should be further enhanced. Thus, exploiting more powerful convergent techniques, such as cutting plane constraints, to facilitate the algorithmic searching process could be another promising future research direction.

Data Availability

Our test instances are generated referring to a real-world fast fashion company in China. This company owns 1 central warehouse and 50 physical retail stores in Shanghai city. Around 1500 customers place their orders online on a daily basis. We collaborated with this company and obtained data. However, the ownership of the data belongs to this company. They do not agree to disclose the data for commercial purposes.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Each author's contribution to the research and manuscript is as follows. Wanchen Jie (first author) was mainly responsible for the conception and design of the study, data acquisition, data analysis and interpretation, and drafting the first draft. She was also instrumental in revising the manuscript for important content. Cheng Pei (second author) made significant contributions to the study's methodological design and implementation, experimental design, data collection, and interpretation of experimental results. He assisted in drafting the manuscript and also maintained good communication with the editorial office as the corresponding author. Jiating Xu (third author) provided specific technical expertise in statistical analysis and contributed to the interpretation of the data. In particular, he critically revised the Results and Discussion sections of the manuscript. Hong Yan (fourth author), as senior author and research supervisor, participated in the conceptualization of the initial ideas and supervised the execution of the project. He provided guidance throughout the research process and provided funding and resources needed for the research. And all authors actively participated in responding to reviewers' comments. Wanchen Jie and Cheng Pei led the revision process, including additional analysis and rewriting of portions of the manuscript as suggested by the reviewers. Jiating Xu provided additional data analysis and interpretation and revised the corresponding parts of the manuscript. Hong Yan coordinated the team's responses to reviewers and supervised the finalization of the revised manuscript. All authors reviewed and approved the final revised manuscript before resubmission. In addition, all authors agree to be accountable for all aspects of the work and for ensuring that questions related to the accuracy or integrity of any part of the work are appropriately investigated and resolved.

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