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A DISTRIBUTIONAL REPRESENTATION OF STRIP ANALYTIC FUNCTIONS

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ABSTRACT. A strip analytic function converging in the $\mathcal{D}'$ topology to certain boundary values (from the interior of the strip) is represented as the difference of two generalized Cauchy integrals.

KEY WORDS AND PHRASES. Analytic function, distribution in $\mathcal{D}'$, distribution in $\mathcal{E}'$, convergence of distributions, Cauchy representation of a distribution (generalized Cauchy integral), Plemelj distributional formulas.

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1. INTRODUCTION.

In the theory of distributional behavior of analytic functions, two following topics are central: (1) the representation of distributions in terms of boundary values of analytic functions; (2) the representation of analytic functions in terms of distributions.

The present paper, influenced by [1, Theorem 97, p. 130] via [2, Theorem 3.6, p. 68], continues the note [3] and contributes to the second topic. In the cited theorem of Beltrami and Wohlers, there is established a decomposition of strip analytic functions into the difference of two Cauchy distributional representations concerning the S' topology. Here, a version of this boundary value theorem is proved involving the $\mathcal{D}'$ topology.

2. NOTATION AND PRELIMINARIES.

Throughout this paper the following symbols will be used:

t: the real coordinate of a point of $\mathbb{R}$;

z, ζ: the complex coordinates of points of $\mathbb{C}$, $z = x + iy$;

Δ^+, Δ^-: the open upper half-plane $\{z \in \mathbb{C}: \operatorname{Im}(z) > 0\}$ and the open lower half-plane $\{z \in \mathbb{C}: \operatorname{Im}(z) < 0\}$ respectively;

$C^\infty = C^\infty(\mathbb{R})$: the vector space of all infinitely differentiable complex valued functions defined on $\mathbb{R}$;

$\mathcal{D} = \mathcal{D}(\mathbb{R})$: the vector space of all C^∞-function with a compact support;

$\mathcal{D}' = \mathcal{D}'(\mathbb{R})$: the space of all continuous linear functionals (Schwartz distributions) on $\mathcal{D}$.

For the completeness we recall a few basic definitions and facts on the spaces $\mathcal{G}_\alpha = \mathcal{G}_\alpha(\mathbb{R})$ and $\mathcal{G}'_\alpha = \mathcal{G}'_\alpha(\mathbb{R})$.

Let α be a real number. We say that a function $\phi \in \mathcal{G}_\alpha$ if $\phi \in C^\infty$ and for each non-negative integer p there exists a constant M_p such that $|D^p \phi(t)| \leq M_p (1 + |t|)^\alpha$ for all $t \in \mathbb{R}$. A sequence $(\phi_n) = (\phi_n)_{n \in \mathbb{N}}$ is said to converge to zero in $\mathcal{G}_\alpha$ if the following are satisfied: (1) each $\phi_n \in \mathcal{G}_\alpha$; (2) for each p the sequence $(D^p \phi_n)$ converges uniformly to zero on every compact subset of $\mathbb{R}$; (3) for each p there exists a constant M_p, independent of n, such that $|D^p \phi_n(t)| \leq M_p(1 + |t|)^\alpha$ for all $t \in \mathbb{R}$. The space $\mathcal{D}$ is dense in $\mathcal{G}_\alpha$ (that is, for each $\phi \in \mathcal{G}_\alpha$ there exists a sequence (ϕ_n) in $\mathcal{D}$ which converges to ϕ in $\mathcal{G}_\alpha$). A linear functional T on $\mathcal{G}_\alpha$ into $\mathbb{C}$ is continuous if $\lim_{n \to \infty} \langle T, \phi_n \rangle = \langle T, \lim_{n \to \infty} \phi_n \rangle = \langle T, \phi \rangle$ for any sequence (ϕ_n) that converges to ϕ in $\mathcal{G}_\alpha$. The space $\mathcal{G}'_\alpha$ is the space of all continuous linear functionals (distributions) on $\mathcal{G}_\alpha$. Finally, note the proper inclusions $\mathcal{D} \subset \mathcal{G}_\alpha$ and $\mathcal{G}'_\alpha \subset \mathcal{D}'$.

In the following we shall use the same expression to denote a regular distribution and a function that generates it (when no confusion is possible).

3. AUXILIARY RESULTS.

In order to establish the main result, we shall need the following three simple lemmas.

LEMMA 3.1: If $h^+(z)$ is a function analytic in Δ^+ with $h^+(z) = O(\frac{1}{|z|})$ as $|z| \to \infty$ in Δ^+, and if $h^+(x + i\varepsilon)$ converges to h^+_x in the $\mathcal{D}'$ topology as $\varepsilon \to +0$, that is,

$$\langle h_x^+, \phi \rangle = \lim_{\varepsilon \to +0} \langle h^+(x + i\varepsilon), \phi \rangle = \lim_{\varepsilon \to +0} \int_{-\infty}^{\infty} h^+(x + i\varepsilon)\, \phi(x)\, dx$$

for each $\phi \in \mathcal{D}$, then $h_x^+ \in \mathfrak{S}_\alpha'$ for all $\alpha < 0$.

PROOF. For each $\varepsilon > 0$ the function $x \mapsto h^+(x + i\varepsilon)$ is continuous on $\mathbb{R}$. Therefore for each $\varepsilon > 0$ the linear functional on $\mathcal{D}$ into $\mathbb{C}$ defined by the integral

$$\langle h^+(x + i\varepsilon), \phi \rangle = \int_{-\infty}^{\infty} h^+(x + i\varepsilon)\, \phi(x)\, dx$$

is a regular distribution in $\mathcal{D}'$. By the hypothesis on the behavior of $h^+(z)$ there exist the constants $R > 0$ and $A > 0$ such that for each $\varepsilon > 0$ and all $|x| > R$ the inequality

$$|h^+(x + i\varepsilon)| \le \frac{A}{\sqrt{x^2 + \varepsilon^2}} < \frac{A}{|x|}$$

holds. Then for all $\phi \in \mathcal{D}$ with a support contained in the set $E = \{x \in \mathbb{R}: |x| \ge r > R\}$ it follows

$$|\langle h_x^+, \phi \rangle| = \lim_{\varepsilon \to +0} \left| \int_{-\infty}^{\infty} h^+(x + i\varepsilon)\, \phi(x)\, dx \right| \le A \int_{-\infty}^{\infty} |x|^{-1} |\phi(x)| dx .$$

Thus the distribution h_x^+ has the asymptotic bound $|x|^{-1}$. Hence, by Theorem [4, p. 54] it can be extended from $\mathcal{D}'$ to $\mathfrak{S}_\alpha'$ for all $\alpha < 0$. In other words, $h_x^+ \in \mathfrak{S}_\alpha'$ $(\alpha<0)$.

Also, since

$$|\langle h^+(x + i\varepsilon), \phi \rangle| \le A \int_{-\infty}^{\infty} |x|^{-1} |\phi(x)| dx$$

for each $\varepsilon > 0$ and all ϕ with Supp $\phi \subset E$, we conclude that $h^+(x + i\varepsilon)$ is a regular distribution in $\mathfrak{S}_\alpha'$ $(\alpha < 0)$.

REMARK 3.1. Perhaps it may be of interest to prove the above result directly. Consider a linear functional on $\mathfrak{S}_\alpha (\alpha < 0)$ defined by means of

$$\langle h^+(x + i\varepsilon), \phi \rangle = \int_{-\infty}^{\infty} h^+(x + i\varepsilon)\, \phi(x)\, dx, \ \phi \in \mathfrak{S}_\alpha. \tag{3.1}$$

For each $\varepsilon > 0$ the integral (3.1) exists because the integrand is equal to

$O(|x|^{-1+\alpha})$. Let (ϕ_n) be any sequence which converges to zero in $\mathfrak{G}_\alpha$ as $n \to \infty$. We must show that

$$\lim_{n \to \infty} \langle h^+(x + i\varepsilon), \phi_n \rangle = 0 .$$

Let r denote a positive real number. Then we can write

$$\left| \int_{-\infty}^{\infty} h^+(x + i\varepsilon)\, \phi_n(x)\, dx \right| \le \left| \int_{|x| \le r} h^+(x + i\varepsilon)\, \phi_n(x)\, dx \right| + \int_{|x|>r} |h^+(x + i\varepsilon)\, \phi_n(x)|\, dx . \tag{3.2}$$

Letting δ be an arbitrarily small positive real number, we may choose the number r so large $(r > R)$ that

$$\int_{|x|>r} |h^+(x + i\varepsilon)\, \phi_n(x)|\, dx \le A\, M_o \int_{|x|>r} |x|^{-1+\alpha}\, dx < \delta \tag{3.3}$$

for all n. The closed interval $[-r, r]$ being now fixed, it follows from the convergence of (ϕ_n) to zero in $\mathfrak{G}_\alpha$ and the Lebesgue dominated convergence theorem that

$$\lim_{n \to \infty} \int_{|x| \le r} h^+(x + i\varepsilon)\, \phi_n(x)\, dx = 0 . \tag{3.4}$$

The bound (3.3) and the limit (3.4) together show that the estimate (3.2) can be made arbitrarily small for large enough n. Consequently, the linear functional (3.1) is a regular distribution in $\mathfrak{G}'_\alpha$ $(\alpha < 0)$.

The previous results suggest the following lemma.

LEMMA 3.2. If the function $h^+(z)$ satisfies the conditions of Lemma 3.1, then $h^+(x + i\varepsilon)$ converges to h^+_x in the $\mathfrak{G}'_\alpha$ $(\alpha < 0)$ topology as $\varepsilon \to +0$, that is,

$$\langle h^+_x, \phi \rangle = \lim_{\varepsilon \to +0} \langle h^+(x + i\varepsilon), \phi \rangle = \lim_{\varepsilon \to +0} \int_{-\infty}^{\infty} h^+(x + i\varepsilon)\, \phi(x)\, dx$$

for each $\phi \in \mathfrak{G}_\alpha$ $(\alpha < 0)$.

PROOF. Let α be a negative real number and let r be as in the proof of Lemma 3.1. To consider the limit we write

$$\int_{-\infty}^{\infty} h^+(x + i\varepsilon)\, \phi(x)\, dx = \int_{|x| \le r} h^+(x + i\varepsilon)\, \phi(x)\, dx + \int_{|x|>r} h^+(x + i\varepsilon)\, \phi(x)\, dx ,$$

where $\phi \in \mathcal{G}_\alpha$. As each function ϕ is a C^∞-function, for any given compact of $\mathbb{R}$ there exists a function in $\mathcal{D}$ that is identical to ϕ over this compact [6, p. 4]. So, by the hypothesis the first limit exists. Since $h^+(z)$ is analytic and bounded in the domain $\{z \in \Delta^+ : |\mathrm{Re}(z)| \geq r > R\}$ it follows that $h^+(x + i\varepsilon) \to h^+(x)$ for almost all $|x| \geq r$ as $\varepsilon \to +0$. At the same time $|h^+(x + i\varepsilon)| < \frac{A}{|x|}$ for all $\varepsilon > 0$. Therefore, using the Lebesgue dominated convergence theorem,

$$\lim_{\varepsilon \to +0} \int_{|x|>r} h^+(x + i\varepsilon)\, \phi(x)\, dx = \int_{|x|>r} h^+(x)\, \phi(x)\, dx \in \mathbb{C} .$$

Consequently, there exists a distribution $H \in \mathcal{G}_\alpha'$ $(\alpha < 0)$ such that $\langle H_x, \phi \rangle = \lim_{\varepsilon \to +0} \langle h^+(x + i\varepsilon), \phi \rangle$ for each $\phi \in \mathcal{G}_\alpha$. This implies $h_x^+ = H_x$ over $\mathcal{D}$. But $\mathcal{D}$ is dense in $\mathcal{G}_\alpha$. Hence, $h_x^+ = H_x$ over $\mathcal{G}_\alpha$.

Obviously, the obtained results can be transposed bodily for a function $h^-(z)$ analytic in Δ^- with $h^-(z) = 0\,(\frac{1}{|z|})$ as $|z| \to \infty$ and generating a regular distribution in $\mathcal{D}'$ by the integral

$$\langle h^-(x - i\varepsilon), \phi \rangle = \int_{-\infty}^{\infty} h^-(x - i\varepsilon)\, \phi(x)\, dx.$$

LEMMA 3.3. If the function $h^+(z)$ satisfies the condition of Lemma 3.1, then

$$\frac{1}{2\pi i} \left\langle h_t^+ , \frac{1}{t - z} \right\rangle = h^+(z) \quad \text{for} \quad z \in \Delta^+ , \tag{3.5}$$
$$= 0 \quad \text{for} \quad z \in \Delta^- .$$

PROOF. From Lemma 3.1 we know, in particular, that the distribution h_t^+ acts on the space $\mathcal{G}_\alpha$ with $\alpha = -1$. Since the function $t \mapsto \frac{1}{t - z}$ belongs to this space $(\mathrm{Im}(z) \neq 0)$, the Cauchy representation of h_t^+ is well defined. To prove the lemma, we shall first evaluate the limit of the integral

$$\frac{1}{2\pi i} \left\langle h^+(t + i\varepsilon), \frac{1}{t - z} \right\rangle$$

as $\varepsilon \to +0$ (observe that this integral exists for each $\varepsilon > 0$). Let z be any point in Δ^+. By the Cauchy integral formular applied to the function

$$\zeta \mapsto \frac{h^+(\zeta + i\varepsilon)}{\zeta - z}$$

along the closed path consisting of a sufficiently large semicircle in Δ^+ of radius r and the segment [-r, r], we get

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{h^+(t+i\varepsilon)}{t-z}\,dt = h^+(z+i\varepsilon) \quad \text{for} \quad z\in\Delta^+ .$$

For $z \in \Delta^-$ this integral vanishes. Thus, letting $\varepsilon \to +0$, we have

$$\frac{1}{2\pi i}\lim_{\varepsilon\to+0}\langle h^+(t+i\varepsilon), \frac{1}{t-z}\rangle = h^+(z) \quad \text{for} \quad z\in\Delta^+ ,$$
$$= 0 \quad \text{for} \quad z\in\Delta^- .$$

Now by Lemma 3.2 the representation (3.5) follows.

For a function $h^-(z)$ analytic in Δ^- and satisfying here the conditions similar to ones of $h^+(z)$, we infer by the same procedure that

$$-\frac{1}{2\pi i}\langle h_t^-, \frac{1}{t-z}\rangle = h^-(z) \quad \text{for} \quad z\in\Delta^- ,$$
$$= 0 \quad \text{for} \quad z\in\Delta^+ .$$

4. THE MAIN RESULT.

We are now prepared to prove the main result of this paper.

THEOREM 4.1. Let $f(z)$ be a function analytic in the strip $\Delta = \{z \in \mathbb{C}: y_1 < \mathrm{Im}(z) < y_2\}$ with $f(z) = O\left(\frac{1}{|z|^{1+\lambda}}\right)$ for some $\lambda > 0$ as $|z| \to \infty$ in Δ. Suppose that $f_1 = \lim_{\varepsilon\to+0} f(x + i(y_1+\varepsilon))$ and $f_2 = \lim_{\varepsilon\to+0} f(x+i(y_2-\varepsilon))$ in the $\mathcal{D}'$ topology. Then for $y_1 < \mathrm{Im}(z) < y_2$

$$f(z) = \frac{1}{2\pi i}\langle f_1, \frac{1}{t+iy_1-z}\rangle - \frac{1}{2\pi i}\langle f_2, \frac{1}{t+iy_2-z}\rangle , \tag{4.1}$$

where the Cauchy representation of f_1 is analytic in the upper half-plane $\mathrm{Im}(z) > y_1$, and the Cauchy representation of f_2 is analytic in the lower half-plane $\mathrm{Im}(z) < y_2$.

PROOF. Let $y_1 < a < b < y_2$. Since $f(z)$ tends uniformly to zero as $|z| \to \infty$ in Δ, an application of Cauchy's integral formula [7, Lemma 1, p.293] leads to the decomposition $f(z) = f^+(z) + f^-(z)$, where

$$f^+(z) = \frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia}\frac{f(\zeta)}{\zeta-z}\,d\zeta ,$$

$$f^-(z) = -\frac{1}{2\pi i}\int_{-\infty+ib}^{\infty+ib}\frac{f(\zeta)}{\zeta-z}\,d\zeta .$$

We recall that the function $f^+(z)$ is analytic in the upper half-plane $\mathrm{Im}(z) > a$, and $f^-(z)$ is analytic in the lower half-plane $\mathrm{Im}(z) < b$. By virtue of the arbitrarily closeness of the points a and b to the points y_1 and y_2 respectively, the strip Δ is the common domain of analyticity for $f^+(z)$ and $f^-(z)$. In order to investigate the behavior of these functions at the point at infinity consider the equality

$$z\, f^+(z) = \frac{1}{2\pi i} \int_{-\infty+ia}^{\infty+ia} \frac{z\, f(\zeta)}{\zeta - z}\, d\zeta$$

$$= \frac{1}{2\pi i} \int_{-\infty+ia}^{\infty+ia} \frac{\zeta\, f(\zeta)}{\zeta - z}\, d\zeta - \frac{1}{2\pi i} \int_{-\infty+ia}^{\infty+ia} f(\zeta)\, d\zeta\ . \tag{4.2}$$

The integral of the Cauchy type in (4.2) vanishes as $|z| \to \infty$ in the upper half-plane $\mathrm{Im}(z) > y_1$, while other one converges since $f(\zeta) = 0\left(\frac{1}{|\zeta|^{1+\lambda}}\right)$, $\lambda > 0$. From this we conclude that $f^+(z) = 0\left(\frac{1}{|z|}\right)$ as $|z| \to \infty$. Also, from a similar integral representation for $z\, f^-(z)$ we infer $f^-(z) = 0\left(\frac{1}{|z|}\right)$ as $|z| \to \infty$ in the lower half-plane $\mathrm{Im}(z) < y_2$.

Further, we must verify that the functions $f^+(z)$ and $f^-(z)$ really converge in the $\mathcal{D}'$ topology to certain boundary values on $\mathrm{Im}(z) = y_1$ and $\mathrm{Im}(z) = y_2$ respectively (from the interior of Δ). Let $z = x + i(a + \varepsilon)$ be a point in the half-plane $\mathrm{Im}(z) > a$. Then in the distributional setting

$$f^+(x + i(a + \varepsilon)) = \frac{1}{2\pi i} \left\langle f(t + ia), \frac{1}{t - (x + i\varepsilon)} \right\rangle,$$

$$f^+(x + i(y_1 + \varepsilon)) = \lim_{a \to y_1} f^+(x + i(a + \varepsilon))$$

$$\lim_{a \to y_1} \frac{1}{2\pi i} \left\langle f(t + ia), \frac{1}{t - (x + i\varepsilon)} \right\rangle$$

By Lemma 3.1 the analyticity of $f(z) = 0\left(\frac{1}{|z|}\right)$ in $\Delta(|z| \to \infty)$ and the convergence of $f(t + ia)$ to f_1 as $a \to y_1$ together imply $f_1 \in \mathfrak{G}'_\alpha$ $(-1 \le \alpha < 0)$. On the other hand, according to Lemma 3.2 we have

$$f^+(x + i(y_1 + \varepsilon)) = \frac{1}{2\pi i} \left\langle f_1, \frac{1}{t - (x + i\varepsilon)} \right\rangle .$$

Now, in view of the distributional Plemelj formulas [5, Theorem 2] we get

$$f_x^+ = \lim_{\varepsilon \to +0} f^+(x + i(y_1 + \varepsilon)) = \frac{1}{2} f_1 - \frac{1}{2\pi i} \langle f_1 * \mathrm{vp} \frac{1}{x} \rangle$$

in the $\mathcal{D}'$ topology.

Let $z = x + i(b - \varepsilon)$ be a point in the half-plane $\mathrm{Im}(z) < y_2$. Starting from

$$f^-(x + i(b - \varepsilon)) = - \frac{1}{2\pi i} \langle f(t + ib), \frac{1}{t - (x - i\varepsilon)} \rangle$$

and proceeding along the same lines as before, we find

$$f^-(x + i(y_2 - \varepsilon)) = - \frac{1}{2\pi i} \langle f_2, \frac{1}{t - (x - i\varepsilon)} \rangle ,$$

$$f_x^- = \lim_{\varepsilon \to +0} f^-(x + i(y_2 - \varepsilon)) = \frac{1}{2} f_2 + \frac{1}{2\pi i} \langle f_2 * \mathrm{vp} \frac{1}{x} \rangle$$

in the $\mathcal{D}'$ topology.

So we have proved that the function $f^+(z)$ [resp. $f^-(z)$] is analytic in the half-plane $\mathrm{Im}(z) > y_1$ [resp. $\mathrm{Im}(z) < y_2$] with the order relation $O(\frac{1}{|z|})$ as $|z| \to \infty$, and that it converges in the $\mathcal{D}'$ topology to f_x^+ on $\mathrm{Im}(z) = y_1$ [resp. f_x^- on $\mathrm{Im}(z) = y_2$]. In view of Lemma 3.3, it follows that

$$\frac{1}{2\pi i} \langle f_t^+ , \frac{1}{t + iy_1 - z} \rangle = f^+(z) \quad \text{for} \quad \mathrm{Im}(z) > y_1 ,$$
$$= 0 \quad \text{for} \quad \mathrm{Im}(z) < y_1 .$$

Analogously,

$$- \frac{1}{2\pi i} \langle f_t^- , \frac{1}{t + iy_2 - z} \rangle = f^-(z) \quad \text{for} \quad \mathrm{Im}(z) < y_2 ,$$
$$= 0 \quad \text{for} \quad \mathrm{Im}(z) > y_2 .$$

Now we shall compute the value of the integral

$$\frac{1}{2\pi i} \langle f^+(t + iy_2), \frac{1}{t + iy_2 - z} \rangle \tag{4.3}$$

for $\mathrm{Im}(z) < y_2$. For such z the function

$$\zeta \mapsto \frac{f^+(\zeta)}{\zeta - z}$$

is analytic inside the closed path which consists of the segment $[-r + iy_2, r + iy_2]$ and the semicircle L_r of radius r lying in $\mathrm{Im}(z) > y_2$. According to Cauchy integral theorem, we may write

$$\frac{1}{2\pi i} \int_{-r}^{r} \frac{f^+(t + iy_2)}{t + iy_2 - z} \, dt + \frac{1}{2\pi i} \int_{L_r} \frac{f^+(\zeta)}{\zeta - z} \, d\zeta = 0 .$$

The integral along L_r tends to zero as $r \to \infty$. Thus the integral (4.3) is equal to zero for $\mathrm{Im}(z) < y_2$. Also, as an immediate consequence of the derivation above,

$$\frac{1}{2\pi i} \langle f^-(t + iy_1), \frac{1}{t + iy_1 - z} \rangle = 0 \tag{4.4}$$

for $\mathrm{Im}(z) > y_1$. Combining the Cauchy representation of f_t^+ and f_t^- with (4.4) and (4.3) respectively, we have

$$f^+(z) = \frac{1}{2\pi i} \langle f_t^+ + f^-(t + iy_1), \frac{1}{t + iy_1 - z} \rangle \quad \text{for} \quad \mathrm{Im}(z) > y_1 ,$$

$$f^-(z) = -\frac{1}{2\pi i} \langle f_t^- + f^+(t + iy_2), \frac{1}{t + iy_2 - z} \rangle \quad \text{for} \quad \mathrm{Im}(z) < y_2 .$$

From the decomposition $f(z) = f^+(z) + f^-(z)$ we see that $f_1 = f_t^+ + f^-(t + iy_1)$ is the boundary value of $f(z)$ on $\mathrm{Im}(z) = y_1$ in the $\mathcal{D}'$ topology and $f_2 = f_t^- + f^+(t + iy_2)$ is the boundary value of $f(z)$ on $\mathrm{Im}(z) = y_2$ in the same topology. Consequently,

$$f^+(z) = \frac{1}{2\pi i} \langle f_1, \frac{1}{t + iy_1 - z} \rangle \quad \text{for} \quad \mathrm{Im}(z) > y_1 ,$$

$$f^-(z) = -\frac{1}{2\pi i} \langle f_2, \frac{1}{t + iy_2 - z} \rangle \quad \text{for} \quad \mathrm{Im}(z) < y_2 .$$

Again returning to the decomposition of the function $f(z)$, the representation (4.1) follows at once.

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