Internat. J. Math. & Math. Sci.
Vol. 6 No. 3 (1983) 467-475

ON A PAIR OF RANDOM GENERALIZED NON-LINEAR CONTRACTIONS

J. ACHARI
Science College
Nanded (Maharashta), INDIA

(Received March 9, 1981 and in revised form March 8, 1982)

ABSTRACT. A fixed point theorem for a pair of random generalized non-linear contraction mappings involving four points of the space under consideration is proven. It is shown that this result includes the result of Lee and Padgett [1]. Also an application of the result is given.

KEY WORDS AND PHRASES. Complete probability measure space, Banach spaces, σ-algebras, Borel subsets, random variable, random operator, separable Banach space, random generalized nonlinear contraction, upper semicontinuous functions, Bochner integral, unique fixed point.

1980 MATHEMATICS SUBJECT CLASSIFICATION CODES. Primary 60H99, 47H10; Secondary 60H20.

1. INTRODUCTION.

The idea of fixed points plays a very important role in solving deterministic operator equations. Recently the idea of random fixed point theorems which are the stochastic generalization of the classical fixed point theorems has become a very important part of the theory of some operator equations which can be regarded as random operator equations. Many interesting results have been established by various authors (see for example Bharucha-Reid [2], Hans [3], Padgett [4], Tsokos [5], Tsokos and Padgett [6], Lee and Padgett [7]) in this area.

Recently, a fixed point theorem for a pair of generalized non-linear contraction mappings involving four points of the space under consideration, which includes many well known results as special cases has been established by Achari [8] (see also Achari [9], Pittanuer [10]).

The object of this paper is to study a stochastic version of a pair of generalized non-linear contraction mappings of Achari [8]. Also it has been shown that this result

generalizes the result of Lee and Padgett [1]. It is interesting to note that with suitable modification of the conditions of the theorem, we can easily obtain stochastic generalizations of the results of different classical fixed points. Finally, we apply Theorem 2 to prove the existence of a solution in a Banach space of a random nonlinear integral equation of the form

$$x(t;\omega) = h(t;\omega) + \int_S k(t,s;\omega) f(s,x(s;\omega)) d\mu(s) \tag{1.1}$$

where S is a locally compact metric space with metric d defined on $S \times S$, μ is a complete σ-finite measure defined on the collection of Borel subsets of S and the integral is a Bochner integral.

2. PRELIMINARIES.

In this section, we state some definitions as used by Lee and Padgett [1]. Let (Ω,S,P) be a complete probability measure space, and let $(X,\mathcal{B})$ and (Y,C) be two measurable spaces, where X and Y are Banach spaces and B and C are σ-algebras of Borel subsets of X and Y, respectively. First, we state the usual definitions of a Banach space-valued random variable and of a random operator.

DEFINITION 1. A function $V: \Omega \to X$ is said to be an X-valued random variable (Random element in X, or generalized random variable) if $\{\omega \in \Omega: V(\omega) \in B\} \in S$ for each $B \in \beta$.

DEFINITION 2. A mapping $T(\omega): \Omega \times X \to Y$ is said to be a random operator if $y(\omega) = T(\omega)x$ is a Y-valued random variable for every $x \in X$.

DEFINITION 3. Any X-valued random variable $x(\omega)$ which satisfies the condition

$$P(\{\omega: T(\omega)x(\omega) = y(\omega)\}) = 1$$

is said to be a random solution of the random operator equation $T(\omega)x = y(\omega)$.

DEFINITION 4. Let $y_j(\omega)$, $j = 1,2,\ldots,n$ be second order real valued random variables on a probability space (Ω,S,P), that is $y_j(\omega) \in L_2(\Omega,S,P)$. The collection of all n-component random vectors $y'(\omega) = (y_1(\omega), \ldots, y_n(\omega))$ constitutes a linear vector space if all equivalent random vectors are identified. Define the norm of y by

$$||y||_{L_2^n} = \max_{1\le j\le n} ||y_j||_{L_2} = \max_{1\le j\le n} \left(\int_\Omega |y_j|^2 dP\right)^{1/2}.$$

The space of all n-component random vectors y with second-order components and norm given by $||\cdot||_{L_2^n}$ above is separable Banach space and will be denoted by $L_2^n(\Omega,S,P)$ or

simply L_2^n. Let S be locally compact metric space with metric d defined on $S \times S$ and let μ be a complete σ-finite measure defined on the Borel subsets of S.

DEFINITION 5. We define the space $C(S,L_2^n(\Omega,S,P))$ to be the space of all continuous functions from S into $L_2^n(\Omega,S,P)$ with the topology of uniform convergence on compacta. It may be noted that $C(S,L_2^n(\Omega,S,P))$ is locally convex space whose topology is defined by a countable family of seminorms given by

$$||x(t;\omega)||_j = \sup_{t\in C_j} ||x(t;\omega)||_{L_2^n}, \quad j = 1,2,\ldots .$$

DEFINITION 6. Let B and D be Banach spaces. The pair (B,D) is said to be admissible with respect to a random operator $U(\omega)$ if $U(\omega)(B) \subset D$.

DEFINITION 7. A random operator $T(\omega)$ on a Banach space X with domain $D(T(\omega))$ is said to be a random generalized nonlinear contraction if there exists non-negative real-valued upper semicontinuous functions $\phi_i(\omega)$, $i = 1,2,\ldots,5$ satisfying $\phi_i(\omega)(r) < \frac{r}{5}$ for $r > 0$, $\phi_i(\omega)(0) = 0$ and such that

$$||T(\omega)x_1 - T(\omega)x_2|| \le \phi_1(||x_1 - x_2||) + \phi_2(||x_1 - T(\omega)x_1||) + \phi_3(||x_2 - T(\omega)x_2||)$$
$$+ \phi_4(||x_1 - T(\omega)x_2||) + \phi_5(||x_2 - T(\omega)x_1||)$$

for all $x_1,x_2 \in D(T(\omega))$.

3. A FIXED POINT THEOREM FOR A PAIR OF RANDOM GENERALIZED NONLINEAR CONTRACTIONS.

THEOREM 1. Suppose $A_1(\omega)$ and $A_2(\omega)$ are a pair of random operators from a separable Banach space X into itself such that

$$||A_1(\omega)x_1 - A_2(\omega)x_2|| \le \phi_1(||x_1 - x_2||) + \phi_2(||x_1 - A_1(\omega)x_3||) + \phi_3(||x_2 - A_2(\omega)x_4||)$$
$$+ \phi_4(||x_1 - A_2(\omega)x_4||) + \phi_5(||x_2 - A_1(\omega)x_3||) \tag{3.1}$$

where $\phi_i(\omega)$, $i = 1,2,\ldots,5$, are non-negative real-valued upper semicontinuous functions satisfying $\phi_i(\omega)(r) < \frac{r}{5}$ for $r > 0$, $\phi_{i=0}(\omega)(0)$ and for all $x_1,x_2,x_3,x_4 \in X$. Then there exists an X-valued random variable $\eta(\omega)$ which is the unique common fixed point of $A_1(\omega)$ and $A_2(\omega)$.

PROOF. Let x,y X and we define

$$x_1 = A_2(\omega)y,\ x_2 = A_1(\omega)x,\ x_3 = x,\ x_4 = y,$$

Then (3.1) takes the form

$$||A_1(\omega)A_2(\omega)y - A_2(\omega)A_1(\omega)x|| \le \phi_1(||A_1(\omega)x - A_2(\omega)y||) + \phi_2(||A_1(\omega)x - A_2(\omega)y||) + \phi_3(||A_1(\omega)x - A_2(\omega)y||). \tag{3.2}$$

Let $x_o \in X$ be arbitrary and construct a sequence $\{x_n\}$ defined by

$$A_1(\omega)x_{n-1} = x_n,\ A_2(\omega)x_n = x_{n+1},\ A_1(\omega)x_{n+1} = x_{n+2},\quad n = 1,2,\ldots .$$

Let us put $x = x_{n-1}$ and $y = x_n$ in (3.1), then we have

$$||A_1(\omega)A_2(\omega)x_n - A_2(\omega)A_1(\omega)x_{n-1}|| \le \phi_1(||A_1(\omega)x_{n-1} - A_2(\omega)x_n||)\quad \phi_2(||A_1(\omega)x_{n-1} - A_2(\omega)x_n||) + \phi_3(||A_1(\omega)x_{n-1} - A_2(\omega)x_n||)$$

or

$$||x_{n+2} - x_{n+1}|| \le \phi_1(||x_n - x_{n+1}||) + \phi_2(||x_n - x_{n+1}||) + \phi_3(||x_n - x_{n+1}||) \tag{3.3}$$

We take n to be even and set $\alpha_n = ||x_{n-1} - x_n||$. Then

$$\alpha_{n+2} = ||x_{n+1} - x_{n+2}|| \le \phi_1(||x_n - x_{n+1}||) + \phi_2(||x_n - x_{n+1}||) + \phi_3(||x_n - x_{n+1}||)$$
$$\le \phi_1(\alpha_{n+1}) + \phi_2(\alpha_{n+1}) + \phi_3(\alpha_{n+1}). \tag{3.4}$$

From (3.4) it is clear that α_n decreases with n and hence $\alpha_n \to \alpha$ as $n \to \infty$. Let $\alpha > 0$. Then since ϕ_i is upper semicontinuous, we obtain in the limit as $n \to \infty$

$$\alpha \le \phi_1(\alpha) + \phi_2(\alpha) + \phi_3(\alpha) < \frac{3}{5}\alpha$$

which is impossible unless $\alpha = 0$.

Now, we shall show that $\{x_n\}$ is a Cauchy sequence. If not, then there is an $\varepsilon > 0$ and for all positive integers k, there exist $\{m(k)\}$ and $\{n(k)\}$ with $m(k) > n(k) \ge k$, such that

$$d_k = ||x_{m(k)} - x_{n(k)}|| \ge \varepsilon. \tag{3.5}$$

We may assume that

$$||x_{m(k)-1} - x_{n(k)}|| < \varepsilon,$$

by choosing m(k) to be the smallest number exceeding n(k) for which (3.5) holds.

Then we have

$$d_k \le ||x_{m(k)} - x_{m(k)-1}|| + ||x_{m(k)-1} - x_{n(k)}||$$
$$\le \alpha_{m(k)} + \varepsilon < \alpha_k + \varepsilon$$

which implies that $d_k \to \varepsilon$ as $k \to \infty$. Now the following cases are to be considered.

(i) m is even and n is odd,

(ii) m and n are both odd,

(iii) m is odd and n is even,

(iv) m and n are both even.

(e (i):

$$d_k = ||x_m - x_n|| \le ||x_m - x_{m+1}|| + ||x_{m+1} - x_{n+1}|| + ||x_n - x_{n+1}||$$

$$\le \alpha_{m+1} + \alpha_{n+1} + ||A_1(\omega)x_m - A_2(\omega)x_n||$$

By putting $x_1 = x_n$, $x_2 = x_m$, $x_3 = x_{n-1}$, $x_4 = x_{m-1}$ in (3.1), we have

$$\le \alpha_{m+1} + \alpha_{n+1} + \phi_1(||x_m - x_n||) + \phi_2(||x_n - A_1(\omega)x_{n-1}||) + \phi_3(||x_m - A_2(\omega)x_{m-1}||)$$

$$+ \phi_4(||x_n - A_2(\omega)x_{m-1}||) + \phi_5(||x_m - A_1(\omega)x_{n-1}||)$$

$$\le \alpha_{m+1} + \alpha_{n+1} + \phi_1(d_k) + \phi_4(d_k) + \phi_5(d_k).$$

Letting $k \to \infty$ we have

$$\varepsilon \le \frac{3}{5}\varepsilon.$$

This is a contradiction if $\varepsilon > 0$. In the Case (ii), we have

$$d_k = ||x_m - x_n|| \le ||x_m - x_{m+1}|| + ||x_{m+2} - x_{m+1}|| + ||x_{m+2} - x_{n+1}|| + ||x_n - x_{n+1}||$$

$$\le \alpha_{m+2} + \alpha_{m+1} + \alpha_{n+1} + ||A_2(\omega)x_{m+1} - A_1(\omega)x_n||.$$

By putting $x_1 = x_n$, $x_2 = x_{m+1}$, $x_3 = x_{n-1}$, $x_4 = x_m$ in (3.1), we get

$$\le \alpha_{m+2} + \alpha_{m+1} + \alpha_{n+1} + \phi_1(||x_{m+1} - x_n||) + \phi_2(||x_n - A_1(\omega)x_{n-1}||)$$

$$+ \phi_3(||x_{m+1} - A_2(\omega)x_m||) + \phi_4(||x_n - A_2(\omega)x_m||) + \phi_5(||x_m - A_1(\omega)x_{n-1}||)$$

$$\le \alpha_{m+2} + \alpha_{m+1} + \alpha_{n+1} + \phi_1(d_k + \alpha_{m+1}) + \phi_4(d_k + \alpha_{m+1}) + \phi_5(d_k + \alpha_{m+1}).$$

Letting $k \to \infty$ in the above inequality we obtain $\varepsilon < \frac{3}{5}\varepsilon$, which is a contradiction if $\varepsilon > 0$. Similarly, the cases (iii) and (iv) may be disposed of. This leads us to conclude that $\{x_n\}$ is a Cauchy sequence. Let $\eta(\omega)$ be the limit of the sequence. We shall now show that $A_1(\omega)\eta(\omega) = \eta(\omega) = A_2(\omega)\eta(\omega)$. Putting $x_1 = x_{n-1}$, $x_2 = \eta(\omega)$, $x_3 = x_{n+1}$, $x_4 = x_n$, in (3.1), we get

$$||A_1(\omega)x_{n-1} - A_2(\omega)\eta(\omega)|| \le \phi_1(||x_{n-1} - \eta(\omega)||) + \phi_2(||x_{n-1} - A_1(\omega)x_{n+1}||)$$

$$+ \phi_3(||\eta(\omega) - A_2(\omega)x_n||) + \phi_4(||x_{n-1} - A_2(\omega)x_n||) + \phi_5(||\eta(\omega) - A_1(\omega)x_{n+1}||).$$

Letting $n \to \infty$, we get $||\eta(\omega) - A_2(\omega)\eta(\omega)|| \le 0$ which is a contradiction and hence $\eta(\omega) = A_2(\omega)\eta(\omega)$. In the same way, it is possible to show that $\eta(\omega) = A_1(\omega)\eta(\omega)$. Thus $\eta(\omega)$ is a common fixed point of $A_1(\omega)$ and $A_2(\omega)$. Suppose there is another fixed

point $\xi(\omega) \neq \eta(\omega)$ of $A_1(\omega)$ and $A_2(\omega)$. Then putting $x_1 = x_4 = \eta(\omega)$ and $x_2 = x_4 = \xi(\omega)$ in (3.1), we have

$$||\eta(\omega) - \xi(\omega)|| \leq \phi_1(||\eta(\omega) - \xi(\omega)||) + \phi_2(||\eta(\omega) - \xi(\omega)||) + \phi_3(||\eta(\omega) - \xi(\omega)||)$$
$$< \frac{3}{5}\, ||\eta(\omega) - \xi(\omega)||$$

which is a contradiction. Hence $\eta(\omega) = \xi(\omega)$. This completes the proof. If in Theorem 1, we put $A_1(\omega) = A_2(\omega) = A(\omega)$ and $x_1 = x_3 = x$, $x_2 = x_4 = y$ then we have the following theorem which we only state without proof.

THEOREM 2. If $A(\omega)$ is a random generalized nonlinear contraction from a separable Banach space X into itself, then there exists an X-valued random variable which is the unique fixed point of $A(\omega)$.

We now have the following corollary of Theorem 2.

COROLLARY 1. If $A^b(\omega)$ is a random generalized contraction from X into itself for some positive integer b, then $A(\omega)$ has a unique fixed point $\eta(\omega)$ which is an X-valued random variable.

PROOF. Since $A^b(\omega)$ is a random generalized nonlinear contraction operator on X, by Theorem 2, there exists a unique X-valued random variable $\eta(\omega)$ such that

$$A^b(\omega)\eta(\omega) = \eta(\omega).$$

We claim that $A(\omega)\eta(\omega) = \eta(\omega)$. If not, consider

$$||A^{b+1}(\omega)\eta(\omega) - A^b(\omega)\eta(\omega)|| \leq \phi_1(||A(\omega)\eta(\omega) - \eta(\omega)||) + \phi_2(||A(\omega)\eta(\omega)-A^{b+1}(\omega)\eta(\omega)||)$$
$$+\phi_3(||\eta(\omega)-A^b(\omega)\eta(\omega)||)+\phi_4(||A(\omega)\eta(\omega)-A^b(\omega)\eta(\omega)||)+\phi_5(||\eta(\omega)-A^{b+1}(\omega)\eta(\omega)||), \quad (3.6)$$

Moreover, the left hand side of (3.6) is

$$||A^{b+1}(\omega)\eta(\omega) - A^b(\omega)\eta(\omega)|| = ||A(\omega)\eta(\omega) - \eta(\omega)||. \quad (3.7)$$

From (3.6) and (3.7), we have

$$||A(\omega)\eta(\omega) - \eta(\omega)|| \leq \phi_1(||A(\omega)\eta(\omega)-\eta(\omega)||)+\phi_3(||A(\omega)\eta(\omega)-\eta(\omega)||)+\phi_5(||A(\omega)\eta(\omega)-\eta(\omega)||)$$

which is a contradiction and hence $A(\omega)\eta(\omega) = \eta(\omega)$.

We remark that under the conditions $A_1(\omega) = A_2(\omega) = A(\omega)$ and $x_1 = x_3 = x$, $x_2 = x_4 = y$, $\phi_1 = \phi$, $\phi_j(\omega)(r) = 0$, $j = 2,3,4,5$, the Theorem 1 reduces to the following corollary.

COROLLARY 2. (Lee and Padgett [1]). If $A(\omega)$ is a random nonlinear contraction operator from a separable Banach space X into itself, then there exists an X-valued random variable $\eta(\omega)$ which is the unique random fixed point of $A(\omega)$.

4. APPLICATION TO A RANDOM NONLINEAR INTEGRAL EQUATION.

In this section we give an application of Theorem 2 to a random nonlinear integral equation. To do so we have followed the steps of Lee and Padgett [1] with necessary modifications as required for the more general settings. We shall assume the following conditions concerning the random kernel $k(t,s;\omega)$. The function $k(.,.;.)$: $S \times S \times \Omega \to R$ is such that

(i) $k(t,s;\omega)$: $S \times S \to L_\infty(\Omega,S,P)$ such that $|||k(t,s;\omega)||| \cdot ||x(s;\omega)||_{L_2^n}$ is μ-integrable with respect to $s \in S$ for each $t \in S$ and $x \in C(S,L_2^n(\Omega,S,P))$ where for each $(t,s) \in S \times S$

$$|||k(t,s;\omega)||| = ||k(t,s;\omega)||_{L_\infty(\Omega,S,P)}$$

is the norm in $L_\infty(\Omega,S,P)$;

(ii) for each $s \in S$, $k(t,s;\omega)$ is continuous in $t \in S$ from S into $L_\infty(\Omega,S,P)$; for each $t \in S$, $k(t,s;\omega)$ is continuous in $s \in S$ from S into $L_\infty(\Omega,S,P)$; and

(iii) there exists a positive real-valued function H on S such that $H(s)||x(s;\omega)||_{L_2^n}$ is μ-integrable for $x \in C(S,L_2^n(\Omega,S,P))$ and such that for each $t,s \in S$

$$|||k(t,u;\omega) - k(s,u;\omega)||| \cdot ||x(u;\omega)||_{L_2^n} \le H(u)||x(u;\omega)||_{L_2^n} .$$

Thus, for each $(t,s) \in S \times S$, we have $k(t,s;\omega)x(s;\omega) \in L_2^n(\Omega,S,P)$. We now define the random integral operator $T(\omega)$ on $C(S,L_2^n(\Omega,S,P))$ by

$$[T(\omega)x](t;\omega) = \int_S k(t,s;\omega)x(s;\omega)d\mu(s) \tag{4.1}$$

where the integral is a Bochner integral. Moreover, we have that for each $t \in S$, $[T(\omega)x](t;\omega) \in L_2^n(\Omega,S,P)$ and that is a continuous linear operator from $C(S,L_2^n(\Omega,S,P))$ into itself. We now have the following theorem.

THEOREM 3. We consider the stochastic integral equation (1.1) subject to the following conditions:

(a) B and D are Banach spaces stronger (cf. [1]) then $C(S,L_2^n(\Omega,S,P))$ such that (B,D) is admissible with respect to the integral operator defined by (4.1);

(b) $x(t;\omega) \to f(t,x(t;\omega))$ is an operator from the set

$$Q(\rho) = \{x(t;\omega):\ x(t;\omega) \in D,\ ||x(t;\omega)||_D \le \rho\}$$

into the space B satisfying

$$||f(t,x(t;\omega)) - f(t,y(t;\omega))||_B \le \phi_1(\omega)(||x(t;\omega) - y(t;\omega)||_D) + \phi_2(\omega)(||x(t;\omega) - f(t,x(t;\omega))||_D) + \phi_3(\omega)(||y(t;\omega) - f(t,y(t;\omega))||_D) + \phi_1(\omega)(||x(t;\omega) - f(t,y(t;\omega))||_D) + \phi_5(\omega)(||y(t;\omega) - f(t,x(t;\omega))||_D)$$

for $x(t;\omega)$, $y(t;\omega) \in Q(\rho)$, where $\phi_i(\omega)$, $i = 1,2,\ldots,5$ are non-negative real-valued upper semicontinuous functions satisfying $\phi_i(\omega)(r) < \frac{r}{5}$ for $r > 0$ and $\phi_i(\omega)(0) = 0$;

(c) $h(t;\omega) \in D$.

Then there exists a unique random solution of (1.1) in $Q(\rho)$, provided $c(\omega) < 1$ and $||h(t;\omega)||_D + 2c(\omega)||f(t;0)||_B \le \rho(1 - c(\omega))$ where $c(\omega)$ is the norm of $T(\omega)$.

PROOF. Define the operator $U(\omega)$ from $Q(\rho)$ into D by

$$[U(\omega)x](t;\omega) = h(t;\omega) + \int_S k(t,s;\omega)f(s;\omega))d\mu(s).$$

Now

$$||[U(\omega)x](t;\omega)||_D \le ||h(t;\omega)||_D + c(\omega)||f(t,x(t;\omega))||_B$$
$$\le ||h(t;\omega)||_D + c(\omega)||f(t;0)||_B + c(\omega)||f(t,x(t;\omega)) - f(t;0)||_B.$$

Then from the conditions of the theorem

$$c(\omega)||f(t,x(t;\omega))-f(t,0)||_B \le c(\omega)[\phi_1(\omega)(||x(t;\omega)||_D) + \phi_2(\omega)(||x(t;\omega)-f(t,x(t;\omega))||_D) + \phi_3(\omega)(||f(t;0)||_D) + \phi_4(\omega)(||x(t;\omega)||_D) + \phi_5(\omega)(||f(t,x(t;\omega))||_D),$$

i.e. $\frac{3}{5}c(\omega)||f(t,x(t;\omega) - f(t;0)||_B < \frac{3}{5}c(\omega)\rho + \frac{3}{5}c(\omega)||f(t;0)||_B$

Hence

$$||[U(\omega)x](t;\omega)||_D \le ||h(t;\omega)||_D + 2c(\omega)||f(t;0)||_B + c(\omega)\rho$$
$$< \rho(1 - c(\omega)) + c(\omega)\rho$$
$$< \rho.$$

Hence $[U(\omega)x](t;\omega) \in Q(\rho)$.

Now, for $x(t;\omega)$, $y(t;\omega) \in Q(\rho)$ we have by condition (b)

$$||[U(\omega)x](t;\omega) - [U(\omega)y](t;\omega)||_D$$
$$= ||\int_S k(t,s;\omega)[f(s,x(s;\omega)) - f(s,y(s;\omega))]d\mu(s)||_D$$
$$\le c(\omega)||f(t,x(t;\omega)) - f(t,y(t;\omega))||_B \le c(\omega)[\phi_1(\omega)(||x(t;\omega) - y(t;\omega)||_D)$$
$$+ \phi_2(\omega)(||x(t;\omega) - f(t,x(t;\omega))||_D) + \phi_3(\omega)(||y(t;\omega) - f(t,y(t;\omega))||_D)$$
$$+ \phi_4(\omega)(||x(t;\omega) - f(t,y(t;\omega))|| + \phi_5(\omega)(||y(t;\omega) - f(t,x(t;\omega))||_D)$$

$$\leq \phi_1(\omega)(||x(t;\omega) - y(t;\omega)||_D + \phi_2(\omega)(||x(t;\omega) - f(t,x(t;\omega))||_D$$

$$+ \phi_3(\omega)(||y(t;\omega) - f(t,y(t;\omega))||_D) + \phi_4(\omega)(||x(t;\omega) - f(t,y(t;\omega))||_D)$$

$$+ \phi_5(\omega)(||y(t;\omega) - f(t,x(t;\omega))||_D)$$

since $c(\omega) \leq 1$. Thus $U(\omega)$ is a random nonlinear contraction operator on $Q(\rho)$. Hence, by Theorem 2 there exists a unique X-valued random variable $x^*(t;\omega) \in Q(\rho)$ which is a fixed point of $U(\omega)$, that is $x^*(t;\omega)$ is the unique random solution of the Equation (1.1).

ACKNOWLEDGEMENT. The author is thankful to the referee for his valuable comments. This work was supported by UGC Grant No. F. 25-9 (11658)/80.

REFERENCES

1. LEE, ARTHUR C.H. and PADGETT, W.J. On random nonlinear contraction, Math. Systems Theory 11 (1977), 77-84.

2. BHARUCHA-REID, A.T. Random Integral Equations, Academic Press, New York, (1972).

3. HANS, O. Random fixed point theorems, Trans. First Prague Conf. on Information Theory, Statist. Decision Functions on Random Process, (1957), 105-125.

4. PADGETT, W.J. On a nonlinear stochastic integral equations of Hammerstein type, Proc. Amer. Math. Soc. 38 (1973), 625-631.

5. TSOKOS, C.P. On a stochastic integral equation of Voltera type, Math. Systems Theory 3 (1969), 222-231.

6. TSOKOS, C.P. and PADGETT, W.J. Random Integral Equations with Applications in Life Sciences and Engineering, Academic Press, New York, (1974).

7. LEE, A.C.H. and PADGETT, W.J. On a heavily non-linear stochastic integral equations, Utilitas Mathematica 9 (1976), 128-138.

8. ACHARI, J. Fixed point theorems in complete metric spaces, Resultate der Mathematik 3 (1980), 1-6.

9. ACHARI, J. A result on fixed points, Kyungpook Math. Journal 19 (1979), 245-248.

10. PITTNAUER, F. A fixed point theorem in complete metric spaces, to appear in Periodica Math. Hungarica.