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A FUNDAMENTAL SPECIAL-RELATIVISTIC THEORY VALID FOR ALL REAL-VALUED SPEEDS

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ABSTRACT. This paper constitutes a fundamental rederivation of special relativity based on the c-invariance postulate but independent of the assumption $ds'^2 = \pm\, ds^2$ (Einstein [1], Kittel et al [2], Recami [3]), the equivalence principle, homogeneity of space-time, isotropy of space, group properties and linearity of space-time transformations or the coincidence of the origins of inertial space-time frames. The mathematical formalism is simpler than Einstein's [4] and Recami's [3]. Whilst Einstein's subluminal and Recami's superluminal theories are rederived in this paper by further assuming the equivalence principle and "mathematical inverses" [4,3], this paper derives (independent of these assumptions) with physico-mathematical motivation an alternate singularity-free special-relativistic theory which replaces Einstein's factor $[1/(1-V^2/c^2)]^{\frac{1}{2}}$ and Recami's extended-relativistic factor $[1/(V^2/c^2-1)]^{\frac{1}{2}}$ by $[(1-(V^2/c^2)^n)/(1-V^2/c^2)]^{\frac{1}{2}}$, where n equals the value of $(m(V)/m_o)^2$ as $|V| \to c$. In this theory both Newton's and Einstein's subluminal theories are experimentally valid on account of negligible terms. This theory implies that non-zero rest mass luxons will not be detected as ordinary non-zero rest mass bradyons because of spatial collapse, and non-zero rest mass tachyons are undetectable because they exist in another cosmos, resulting in a supercosmos of matter, with the possibility of infinitely many such supercosmoses, all moving forward in time. Furthermore this theory is not based on any assumption giving rise to the twin paradox controversy. The paper concludes with a discussion of the implications of this theory for general relativity.

KEY WORDS AND PHRASES. *Singularity-free special-relativistic theory, supercosmos of matter, possibility of infinitely many such supercosmoses all moving forward in time.*
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1. INTRODUCTION.

Based on the c-invariance postulate, this theory is independent of

(a) the assumption $ds'^2 = \pm\, ds^2$ (Einstein [1], Kittel et al [2], Recami [3])

(b) the equivalence principle (Einstein [4], Recami [3], Recami, Mignani [5], Lugiato, Gorini [6], Gorini, Zecca [7], Berzi, Gorini [8], Gorini [9])

(c) homogeneity of space-time and isotropy of space (Recami, Mignani [5], Lugiato, Gorini [6], Gorini, Zecca [7], Berzi, Gorini [8], Gorini [9])

(d) the tacit assumption that physical inverses of mappings from one space-time reference frame to another are mathematical inverses (Einstein [4], Recami [3], Recami, Mignani [5])

(e) linearity of these mappings (Gorini [9], Einstein [1], Kittel et al [2]).

This theory also does not require the retarded causality postulate and RIP (Recami, [3]) because the outcome of negative energy particles moving backward in time is avoided nor does it require the coincidence of origins of the inertial space-time frames of observers (Recami [3], Recami, Mignani [5], Gorini [9], Einstein [1]). Recami [3] considers only subluminal and superluminal frames disregarding luminal frames as "unphysical"; this theory also considers luminal frames.

Compared to the group of generalised Lorentz transformations (Recami [3], Recami, Mignani [5]) to account for all space-time rotations, this theory is more simple and fundamental in its mathematical content; in particular it does not assume that the V-dependent transformations form a group (Gorini [9], Lugiato, Gorini [6], Berzi, Gorini [8]) nor does it presume a covariant form of laws (Recami [3]). Using a coordinate-increment approach the special-relativistic speed transformation equations are rederived, these being of fundamental importance to the theory. An analysis of these equations leads to a function G(V) in terms of which the equations relating the space-time metrics of inertial observers and determining the mapping are given. From these equations are rederived equations, valid for all real-valued V, relating

(a) time intervals, lengths, masses and signal frequencies of moving and stationary clocks, rods, particles and transmitters respectively

(b) the force exerted on a particle and its rate of change of linear momentum

(c) the energy of a particle and its rest mass

(d) the energy and 3-momentum of a particle.

According to these equations

(a) the infinity at $V:|V| = c$ does not follow

(b) the mathematical unreality for $V:|V| > c$ in Einstein's theory is not valid

(c) Lorentz invariance in the sense of (Einstein [1,4,10], Kittel et al [2]) is not valid for $V:|V| \geq c$

(d) the assumption of a continuous G(V) implies that Lorentz invariance is, in addition to (c), not valid for at least values of $V:|V|$ is crucially close to c

(e) the assumption of a continuous G(V) also implies that, for $V:|V| \geq c$ and for at least values of $V:|V|$ is crucially close to c, ds^2 and η_{ij} are not an invariant and a tensor respectively, η_{ij} being defined by

$$ds^2 = \eta_{ij}dx^i dx^j.$$

The theory offers an explanation in terms of spatial collapse and the inadequacy of the concept of space as a real manifold why experimental apparatus would fail to detect non-zero rest mass luxons or tachyons respectively. In the latter part of the paper, using further the equivalence principle and the tacit assumption, the results of Einstein's subluminal and Recami's superluminal special-relativistic theories are rederived. The theory also shows that the unconditional acceptance of the aforementioned tacit assumption leads to physical contradictions. Finally, however, an alternate theory, with physico-mathematical motivation, for a continuous $G(V)$ removing the singularity at $V:|V| = c$ is derived incorporating the experimentally verified results of Einstein's theory but differing from Recami's superluminal theory. Just as Newton's theory is experimentally valid in Einstein's theory only on account of negligible terms, in this theory both Newton's and Einstein's theories are experimentally valid only on account of negligible terms. However, the assumptions in the derivation of this theory do not give rise to the twin paradox controversy. The paper is concluded with a discussion of the possible consequences of this theory for the applicability of general relativity to black holes.

2. THE DERIVATION OF THE MAPS $(dx,dt) \to (dx',dt')$ AND $(dx',dt') \to (dx,dt)$ AND THE RELATIONS BETWEEN ds^2 AND ds'^2 FROM THE SPECIAL-RELATIVISTIC SPEED TRANSFORMATION EQUATIONS

Let the axes of the frames Oxyzt and O'x'y'z't' of the inertial observers O and O' respectively be aligned in the conventional manner. Let O' moving parallel to the x axis have speed V relative to O. The light postulate implies that for light ray propagation and particle motion parallel to the x axis

$$dx/dt = c \Rightarrow dx'/dt' = c \tag{2.1}$$

$$dx/dt = -c \Rightarrow dx'/dt' = -c \tag{2.2}$$

$$dx/dt = 0 \Rightarrow dx'/dt' = -V \tag{2.3}$$

for all $V \in R$.

From (2.1) and (2.2) and the physical concepts therein it appears that

$$\left.\begin{array}{l} dx' - cdt' = k(dx - cdt) \\ \text{and} \\ dx' + cdt' = l(dx + cdt), \end{array}\right\} \tag{2.4}$$

where k and l are arbitrary functions, is the most appropriate mathematical assumption. Eq. (2.4)$\Rightarrow$

$$\left.\begin{array}{l} dx' = Xdx + cYdt \\ dt' = Ydx/c + Xdt, \end{array}\right\} \tag{2.5}$$

where $X = (1 + k)/2$ and $Y = (1 - k)/2$, defines the map $(dx,dt) \overset{\alpha}{\to} (dx',dt')$.

Eq. (2.3) and (2.5) $\Rightarrow$

$$\left.\begin{array}{l} dx' = X(dx - Vdt) \\ dt' = X(-Vdx/c^2 + dt) \end{array}\right\} \tag{2.6}$$

and

$$v' = (v - V)/(1 - Vv/c^2). \tag{2.7}$$

The analogous argument for $(dx',dt') \overset{\Omega}{\rightarrow} (dx,dt)$ yields

$$\left.\begin{aligned} dx &= X'(dx' + Vdt') \\ dt &= X'(Vdx'/c^2 + dt') \end{aligned}\right\} \tag{2.8}$$

and

$$v = (v' + V)/(1 + Vv'/c^2), \tag{2.9}$$

where $X' = (1' + k')/2$.

In deriving Ω as independently as α, I do not in particular assume that $\Omega = \alpha^{-1}$. In this respect and in respect of the foregoing derivation, this theory differs from (Einstein [1,4,10], Kittel et al [2], Recami [3]); in particular while Recami [3] accepts the validity of the light postulate for $V:|V| > c$ and $V:|V| < c$, in this theory the validity for $V \in R$ is accepted.

From the graphs of $v \rightarrow v'$ for all $V \in R$ defined by (2.7) it can be deduced that

(a) when $|V| > c$ then

$$\left.\begin{aligned} |v| < c &\Rightarrow |v'| > c \\ |v| > c &\Rightarrow |v'| < c \\ |v| = c &\Rightarrow |v'| = c \end{aligned}\right\} \tag{2.10}$$

(b) when $|V| < c$ then

$$\left.\begin{aligned} |v| < c &\Rightarrow |v'| < c \\ |v| > c &\Rightarrow |v'| > c \\ |v| = c &\Rightarrow |v'| = c \end{aligned}\right\} \tag{2.11}$$

(c) when $|V| = c$ then

$$|v| \leq c \text{ or } |v| > c \Rightarrow |v'| = c. \tag{2.12}$$

Similarly from the graphs of $v' \rightarrow v$ for all $V \in R$ defined by (2.9) it can be deduced that

(a) when $|V| > c$ then

$$\left.\begin{aligned} |v'| < c &\Rightarrow |v| > c \\ |v'| > c &\Rightarrow |v| < c \\ |v'| = c &\Rightarrow |v| = c \end{aligned}\right\} \tag{2.13}$$

(b) when $|V| < c$ then

$$\left.\begin{aligned} |v'| < c &\Rightarrow |v| < c \\ |v'| > c &\Rightarrow |v| > c \\ |v'| = c &\Rightarrow |v| = c \end{aligned}\right\} \tag{2.14}$$

(c) when $|V| = c$ then

$$|v'| \leq c \text{ or } |v'| > c \Rightarrow |v| = c. \tag{2.15}$$

Eq. (2.10)-(2.12) imply respectively:

(a) when $|V| > c$, then

$$\left.\begin{aligned} dx^2 - c^2dt^2 < 0 &\Rightarrow dx'^2 - c^2dt'^2 > 0 \\ dx^2 - c^2dt^2 > 0 &\Rightarrow dx'^2 - c^2dt'^2 < 0 \\ dx^2 - c^2dt^2 = 0 &\Rightarrow dx'^2 - c^2dt'^2 = 0 \end{aligned}\right\} \qquad (2.16)$$

(b) when $|V| < c$, then

$$\left.\begin{aligned} dx^2 - c^2dt^2 < 0 &\Rightarrow dx'^2 - c^2dt'^2 < 0 \\ dx^2 - c^2dt^2 > 0 &\Rightarrow dx'^2 - c^2dt'^2 > 0 \\ dx^2 - c^2dt^2 = 0 &\Rightarrow dx'^2 - c^2dt'^2 = 0 \end{aligned}\right\} \qquad (2.17)$$

(c) when $|V| = c$, then

$$dx^2 - c^2dt^2 \in R \Rightarrow dx'^2 - c^2dt'^2 = 0. \qquad (2.18)$$

Since (2.16)-(2.18) show that the relation between $dx'^2 - c^2dt'^2$ and $dx^2 - c^2dt^2$ is a function of V, (2.4) $\Rightarrow$

$$dx'^2 - c^2dt'^2 = kl(dx^2 - c^2dt^2) = G(V)(dx^2 - c^2dt^2). \qquad (2.19)$$

Eq. (2.16)-(2.18) show that

$$G(V) < 0 \;\forall\; V : |V| > c \qquad (2.20)$$

$$G(V) > 0 \;\forall\; V : |V| < c \qquad (2.21)$$

$$G(V) = 0 \;\forall\; V : |V| = c. \qquad (2.22)$$

Now because for $V = 0$, α is defined by

$$\left.\begin{aligned} dx' &= dx \\ dt' &= dt \end{aligned}\right\} \qquad (2.23)$$

$$(2.19) \Rightarrow G(0) = 1 \qquad (2.24)$$

Eq. (2.6) and (2.19) $\Rightarrow$

$$X^2(1 - V^2/c^2) = G(V) \qquad (2.25)$$

Eq. (2.20)-(2.22) $\Rightarrow \forall\; V \in R \;\exists\; X \in R$ satisfying (2.25); in particular for $V:|V| = c$, (2.25) becomes an equation of the form

$$x.0 = 0 \Rightarrow x \in R$$

from which it can be seen that the solution $X = \infty$ does not arise in (2.25) for $V:|V| = c$, a result differing from that of (Einstein [1,4,10], Kittel et al [2] Recami [3]). With the real-valued solutions X written as

$$X = \pm[G(V)/(1 - V^2/c^2)]^{\frac{1}{2}} \qquad (2.26)$$

(2.6), in view of (2.23) and (2.24), reduces to

$$\left.\begin{aligned} dx' &= [G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(dx - Vdt) \\ dt' &= [G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(-Vdx/c^2 + dt) \end{aligned}\right\} \qquad (2.27)$$

Accepting Newton's assertion $dt' = dt$ for $V:|V| << c$, (2.27) $\Rightarrow$

$$G(V) = 1 \ \forall \ V:|V| << c \tag{2.28}$$

From the physical significance of (2.27), it can be expected that

(a) the real value/s of X for $V:|V| = c$, in view of the removable discontinuity of X in (2.26) at $V:|V| = c$, equals

$$\lim_{V \to \pm c} \pm[G(V)/(1 - V^2/c^2)]^{\frac{1}{2}} \tag{2.29a}$$

this limit of the "0/0" indeterminate form being determined by L'Hospital's theorem.

(b) G(V) is a differentiable function. (2.29b)

In section II such a G(V) is determined without assuming (2.29b).

The analogous argument for Ω using (2.13)-(2.15) yields

$$dx^2 - c^2dt^2 = O(V)(dx'^2 - c^2dt'^2) \tag{2.30a}$$

and

$$\left.\begin{aligned} dx &= [O(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(dx' + Vdt') \\ dt &= [O(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(Vdx'/c^2 + dt') \end{aligned}\right\} \tag{2.30b}$$

O(V) having the properties (2.20)-(2.22), (2.24) and (2.28).

Cartesian-coordinate transformations yielding α and Ω, denoted by $\alpha^c(V)$ and $\Omega^c(V)$ respectively, are respectively

$$\left.\begin{aligned} x' &= [G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(x - Vt) + K_1 \\ t' &= [G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(-Vx/c^2 + t) + K_4 \end{aligned}\right\} \tag{2.31}$$

and

$$\left.\begin{aligned} x &= [O(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(x' + Vt') + L_1 \\ t &= [O(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(Vx'/c^2 + t') + L_4 \end{aligned}\right\} \tag{2.32}$$

The convention for positive and negative speeds used with respect to motion parallel to the x axis in (2.1)-(2.3) can be generalised for arbitrary motion relative to the frames of O and O'. In terms of a generalisation (available for examination) based on a subdivision of the surface of a unit sphere U I show that for arbitrary motion

$$v = \pm c \Rightarrow v' = \pm c \ \forall \ V \in R$$

(no correspondence in signs implied). Therefore

$$|v| = c \Rightarrow |v'| = c$$

or $\quad dx'^2 + dy'^2 + dz'^2 - c^2dt'^2 = K(dx^2 + dy^2 + dz^2 - c^2dt^2).$

Eq. (2.19) $\Rightarrow$

$$dx'^2 + dy'^2 + dz'^2 - c^2dt'^2 = G(V)(dx^2 + dy^2 + dz^2 - c^2dt^2) \tag{2.33}$$

The analogous argument for Ω yields

$$dx^2 + dy^2 + dz^2 - c^2dt^2 = O(V)(dx'^2 + dy'^2 + dz'^2 - c^2dt'^2) \qquad (2.34)$$

Eq. (2.33) and (2.34) give the relations between ds^2 and ds'^2. I avoid the use of the mathematical conclusion that $O(V) = 1/G(V)$ because

(a) it implies a contradiction of the result $G(V) = O(V) = 0$ for $V:|V| = c$ that arose from physical considerations

(b) it is shown in section 9 to be a necessary condition for $\Omega = \alpha^{-1}$.

The results

$$O(V) = G(V) = 1 \ \forall \ V:|V| < c \qquad (2.36)$$

and

$$O(V) = G(V) = -1 \ \forall \ V:|V| > c \qquad (2.37)$$

of (Einstein [1,4,10], Kittel et al [2]) and (Recami [3]) respectively could be regarded as special cases of this theory. Eq. (2.33) and (2.34) show

(a) if $|V| < c$, then a bradyon relative to O(O') is a bradyon relative to O'(O); a luxon relative to O(O') is a luxon relative to O'(O), and a tachyon relative to O(O') is a tachyon relative to O'(O) (2.38)

(b) if $|V| > c$, then a bradyon relative to O(O') is a tachyon relative to O'(O); a luxon relative to O(O') is a luxon relative to O'(O), and a tachyon relative to O(O') is a bradyon relative to O'(O) (2.39)

(c) if $|V| = c$, then bradyons, luxons and tachyons relative to O are luxons relative to O'; and bradyons, luxons and tachyons relative to O' are luxons relative to O. (2.40)

Eq. (2.33) and (2.34) also show that ds^2 is not Lorentz invariant in the sense of (Einstein [1,4,10], Kittel et al [2]) for values of $V:|V| \geq c$, and (2.29) in addition imply ds^2 is not Lorentz invariant for at least the values of $V:|V|$ is crucially close to c. Similarly the extended Lorentz invariance concept of Recami [3] for $V:|V| > c$ is acceptable except for at least the values of $V:|V|$ is crucially close to c.

3. THE KINETICS AND ENERGY EQUATIONS, THE RELATIVITY OF TIME, MASS AND SPATIAL MEASUREMENTS, THE EQUATION RELATING 3-LINEAR MOMENTUM AND ENERGY, AND THE FREQUENCY-SHIFT EQUATION. PHYSICAL IMPLICATIONS.

In an independent rederivation (available for examination) I show that

(a) $$t = [G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}\tau \qquad (3.1)$$

$$t = [O(V)/(1 - V^2/c^2)]^{\frac{1}{2}}\tau \qquad (3.2)$$

are the relations between proper time τ and moving observer's time t from the viewpoints of O and O' respectively

(b) $$l = |1 - V^2/c^2|[G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}l_o \qquad (3.3)$$

$$l = |1 - V^2/c^2|[O(V)/(1 - V^2/c^2)]^{\frac{1}{2}}l_o \qquad (3.4)$$

are the relations between proper lengths l_o and moving observer's lengths l from the viewpoints of O and O' respectively

$$\text{(c)}\quad \underline{f} = (d/dt)(m_o[G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}\underline{V}) \tag{3.5}$$

$$m(V) = [G(V)/(1 - V^2/c^2)]^{\frac{1}{2}} m_o \tag{3.6}$$

$$E = m_o c^2 \{ \sqrt{\frac{G(V)}{1 - V^2/c^2}} - \frac{1}{2} \int (\sqrt{\frac{1 - V^2/c^2}{G(V)}} [(d/dt)G(V)])dt\} + d, \tag{3.7}$$

d an integration constant

$$\underline{p}\cdot\underline{p} = [(E - d)/c + (m_o c/2) \int \{ \sqrt{\frac{1 - V^2/c^2}{G(V)}} [(d/dt)G(V)]\} dt]^2 - m_o^2 c^2 G(V) \tag{3.8}$$

$$\nu_r = \nu_e / \{(1 + V/c)[G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}\} \tag{3.9}$$

are from the viewpoint of O respectively the kinetics equation, the equation relating rest mass m_o and moving mass m(V), the energy equation, the equation relating 3-linear momentum and energy, and the equation relating frequencies of emission and receipt.

For $V:|V| << c$, (2.28) leads to

(a) the usual Newtonian results in (3.3)-(3.6) and (3.9)

(b) $E = m_o c^2 + m_o V^2/2 + d$ + negligible V-dependent terms in (3.7)

(b) $\underline{p}\cdot\underline{p} = (E - d)^2/c^2 - m_o^2 c^2$ in (3.8).

Experimental physics appears to yield d = 0 or negligible.

With velocity $\underline{V} = V\mathbf{i}$, (3.5) and (3.6) ⇒

$$\underline{f} = (Vdm/dV + m)(dV/dt)\mathbf{i} + mVd\mathbf{i}/dt.$$

Via a simple argument (available for examination) I show that this does not violate

$$\underline{f} = \underline{0} \Rightarrow \underline{\ddot{r}} = \underline{0} \tag{3.10}$$

a fundamental postulate of physics. The argument leads to the necessary condition $Vdm/dV + m > 0$ wherein an immediately obvious sufficient condition is that

m(V) increases as $|V|$ increases. (3.11)

Since experimental physics shows that the increase in mass is the same irrespective of direction of motion, (3.6) ⇒ G(V) = G(-V). (3.12)

The discussion of solutions X of (2.25) and the physical significance of X in (3.6), (3.5), (3.7), (3.8), (3.1) and (3.3) show that, unlike results of (Einstein [1,4,10], Kittel et al [2], Recami [3]), m(V), E, t and l in particular

(a) do not have infinite values for non-zero rest mass luxons hence making it physically possible for

  (i) O to accelerate a non-zero rest mass bradyon to the speed ±c relative to O or for a non-zero rest mass bradyon, e.g., a spaceship in empty space, to accelerate to the speed ±c relative to O, the increase in m(V), implied by (3.11), however, making the objective of the former exercise more difficult to realise.

  (ii) a luxon to have non-zero rest mass.

(b) have positive real values for positive real rest mass tachyons, making the acceleration to superluminal speeds physically possible also.

While (Einstein [1,4,10], Kittel et al [2], Asimov [11]) allow tachyons with imaginary m(V), E, t and l, Recami [3] allows tachyons with negative m(V), E, t and l.

Eq. (3.11) via (3.6), (3.1) and (3.2) implies time dilation. Eq. (3.3) and (3.4) imply length contraction for at least values of $V:|V|$ is crucially close to c. Eq. (2.27) and (2.30b) imply relativity of simultaneity. Einstein's assumption (Einstein [1,10], Kittel et al [2]) of G(V) = 1 yields standard special-relativistic results in (3.1)-(3.9).

Since non-zero rest mass bradyons relative to O physically exist and are detectable, and for such particles X in (2.25) are positive real-valued on account of (2.27), it follows that the corresponding real-valuedness for non-zero rest mass luxons and tachyons relative to O implies that these particles also exist and are detectable. However, the physical existence of particles with speeds $V:V \leq -c$ relative to O imply causality complications in view of the signal-transmission speed c and (3.9). These complications are avoided in this theory by an explanation offered via the next section for the *physical non-existence and undetectability*, relative to O, of non-zero rest mass luxons and tachyons *as ordinary bradyons* relative to O.

4. THREE-DIMENSIONAL EXTENSIONS OF α(V) AND Ω(V) AND THE SPECIAL-RELATIVISTIC SPEED TRANSFORMATION

On account of the identical orientation of the sets of axes {y',y} and {z',z} w.r.t. each other and the x and x' axes, and the form of (2.5), it can be assumed that

$$\left.\begin{aligned} dy' &= a_1 dx + a_2 dy + a_3 dz + a_4 dt \\ dz' &= a_1 dx + a_3 dy + a_2 dz + a_4 dt \end{aligned}\right\} \qquad \text{(a)}$$

with the a_i such that the three-dimensional extension of α(V) defined by (2.27) and (a) reduces to

$$\alpha(0): \begin{cases} dx' = dx \\ dy' = dy \\ dz' = dz \\ dt' = dt \end{cases} \qquad (4.1)$$

the extension of (2.23). It can be proved that (2.33), (2.19), (2.27) and (a) ➡

$$a_1 = 0 \text{ and } a_4 = 0 \qquad \text{(b)}$$

$$a_3{}^2 + a_2{}^2 = G(V) \qquad \text{(c)}$$

$$a_2 a_3 = 0 \,. \qquad \text{(d)}$$

In view of (4.1) and the non-zero valuedness of G(V) for $V:|V| \neq c$, it can be proved that of the cases implied by (d), the case $a_3 = 0, a_2 \neq 0$ which leads to

$$\alpha(V): \begin{cases} dx' = [G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(dx - Vdt) \\ dy' = G(V)^{\frac{1}{2}}dy \\ dz' = G(V)^{\frac{1}{2}}dz \\ dt' = [G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(-Vdx/c^2 + dt) \end{cases} \tag{4.2}$$

is physically acceptable. It is shown in the next section that, from a physical point of view, the implications of the acceptance of the possible applicability of the remaining cases

$$a_3 = a_2 = 0 \tag{4.3}$$

and

$$a_2 = 0, \; a_3 \neq 0 \tag{4.4}$$

to $V:|V| = c$ and $V:|V| > c$ respectively do not contradict the implications of the acceptance of (4.2) for $V \varepsilon R$, the latter being preferred for reasons similar to those for the preference of (2.27). Eq. (4.2) $\Rightarrow$

$$\left. \begin{aligned} dx'/dt' &= (dx/dt - V)/[1 - (dx/dt)(V/c^2)] \\ dy'/dt' &= (1 - V^2/c^2)^{\frac{1}{2}}(dy/dt)/[1 - (dx/dt)(V/c^2)] \\ dz'/dt' &= (1 - V^2/c^2)^{\frac{1}{2}}(dz/dt)/[1 - (dx/dt)(V/c^2)] \end{aligned} \right\} \tag{4.5}$$

and cartesian coordinate transformations yielding (4.2) are

$$\left. \begin{aligned} x' &= [G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(x - Vt) + K_1 \\ y' &= G(V)^{\frac{1}{2}}y + K_2 \\ z' &= G(V)^{\frac{1}{2}}z + K_3 \\ t' &= [G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(-Vx/c^2 + t) + K_4 \end{aligned} \right\} \tag{4.6}$$

Eq. (4.5) and (4.6) are the three-dimensional extensions of (2.7) and (2.31) respectively.

The analogous argument for $\Omega(V)$ yields

$$\Omega(V): \begin{cases} dx = [O(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(dx' + Vdt') \\ dy = O(V)^{\frac{1}{2}}dy' \\ dz = O(V)^{\frac{1}{2}}dz' \\ dt = [O(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(Vdx'/c^2 + dt'] \end{cases} \tag{4.7}$$

$$\left. \begin{aligned} dx/dt &= (dx'/dt' + V)/[1 + (dx'/dt')(V/c^2)] \\ dy/dt &= (1 - V^2/c^2)^{\frac{1}{2}}(dy'/dt')/[1+ (dx'/dt')(V/c^2)] \\ dz/dt &= (1 - V^2/c^2)^{\frac{1}{2}}(dz'/dt')/[1 + (dx'/dt')(V/c^2)] \end{aligned} \right\} \tag{4.8}$$

and

$$\left.\begin{aligned} x &= [\Omega(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(x' + Vt') + L_1 \\ y &= \Omega(V)^{\frac{1}{2}}y' + L_2 \\ z &= \Omega(V)^{\frac{1}{2}}z' + L_3 \\ t &= [\Omega(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(Vx'/c^2 + t') + L_4 \end{aligned}\right\} \tag{4.9}$$

the three-dimensional extensions of (2.30b), (2.9) and (2.32) respectively.

5. PHYSICAL IMPLICATIONS OF THE THREE-DIMENSIONAL EXTENSIONS OF $\alpha(V)$ AND $\Omega(V)$.

Let P_i be a material body relative to an observer O. Define

$$s^0_{P_i} = \{\text{spatial points, relative to O, occupied by } P_i\}.$$

Then, since space relative to an observer O is assumed to be

$$S_0 = R \times R \times R \tag{5.1}$$

P_i physically exists relative to O as a material body only if

$$s^0_{P_i} \subset S_0 \tag{5.2}$$

and

$$\dim s^0_{P_i} = 3. \tag{5.3}$$

Let P_a be a rectangular prism at rest relative to Oxyzt and with one side parallel to the xy plane. Then, in the context of (4.6) the volume of $s^0_{P_a}$ is

$$\Delta v_a = |\Delta x_a|.|\Delta y_a|.|\Delta z_a|,$$

this volume being measured by an extension, implying four events relative to O at suitable corners of P_a, of the technique that yielded (3.3). For every material body P_i, there exists such a rectangular prism P_a with

$$s^0_{P_i} \subset s^0_{P_a}. \tag{5.4}$$

Relative to O', the volume of $s^{0'}_{P_a}$ is

$$\Delta v'_a = |\Delta x'_a|.|\Delta y'_a|.|\Delta z'_a|$$

which, if $V \neq 0$, is the spatial volume occupied by a rectangular prism with the aforementioned orientation but in motion relative to an observer or, in view of (5.4), a spatial volume in which a material body P_i moving relative to an observer exists.

When $|V| << c$, (2.28), (4.6), (3.3), (5.2) and (5.3) imply that P_a physically exists as a material body relative to O and O', and the shape and volume of P_a relative to O' are the same as those relative to O; in view of (5.4) the same results follow for any P_i. When $|V| < c$, but $|V| << c$ does not apply, $\Delta v'_a \neq \Delta v_a$ and, for at least the values of $V{:}|V|$ is crucially close to c,

$$\Delta v'_a < \Delta v_a \text{ with } \Delta v'_a \to 0 \text{ as } |V| \to c \text{ irrespective of how large } \Delta v_a \text{ is.} \tag{5.5}$$

However, since for $V{:}|V| < c$, (4.6) $\Rightarrow$

$$(s^0_{P_a} \subset S_0) \Rightarrow (s^{0'}_{P_a} \subset S_{0'}) \tag{5.6}$$

and

$$(\dim s^{0}_{P_a} = 3) \Rightarrow (\dim s^{0'}_{P_a} = 3) \tag{5.7}$$

P_a physically exists as a material body relative to 0 and 0' but the shape and volume of P_a relative to 0' are not the same as those relative to 0 with the same results for P_i. Eq. (5.5) further implies that 0' will experience greater difficulties in directly detecting P_a or P_i by laboratory apparatus the closer $|V|$ is to c. Eq. (5.6) and (5.7) imply that P_a (having non-zero rest mass relative to 0) bradyons relative to 0' will physically exist relative to 0' as ordinary non-zero rest mass material bodies or particles.

When $|V| = c$, (4.6) or the coordinate transformation corresponding to (4.3) imply that, irrespective of how large Δv_a is,

$$\Delta v'_a = 0 \tag{5.8}$$

$$(s^{0}_{P_a} \subset S_0) \Rightarrow (s^{0'}_{P_a} \subset S_{0'}) \tag{5.9}$$

and

$$(\dim s^{0}_{P_a} = 3) \Rightarrow (\dim s^{0'}_{P_a} = 0) \tag{5.10}$$

Eq. (5.8) and (5.10) show that not only $s^{0}_{P_a}$ but also S_0, both having three-dimensional existence relative to 0, have collapsed relative to 0'. Thus P_a, which physically exists relative to 0 as an ordinary non-zero rest mass material body, exists relative to 0' on account of (5.9), (5.2) and (5.1) but, in view of (5.10) and (5.3), *not as an ordinary material body*. Relative to 0, P_a has shape and volume of ordinary material bodies but relative to 0' P_a has zero volume and the "shape" of a mathematical point. In view of (5.4) these results follow for any P_i. Thus P_a or P_i, having non-zero rest mass relative to 0 but luxons relative to 0', will not be detected by 0' in laboratory apparatus as he directly detects ordinary bradyons. In addition to its invisibility, a non-zero rest mass luxon will, on account of (5.8) and (5.10), manifest no track of its path in, for example, a bubble chamber.

When $|V| > c$, (4.6) or the coordinate transformation corresponding to (4.4) imply that, irrespective of how large Δv_a is,

$\Delta v'_a$ has no physical meaning because $|a|$ is defined for real-valued a (5.11)

$$(s^{0}_{P_a} \subset S_0) \Rightarrow (s^{0'}_{P_a} \not\subset S_{0'}) \tag{5.12}$$

$$(\dim s^{0}_{P_a} = 3) \Rightarrow \text{an undefined } \dim s^{0'}_{P_a} \tag{5.13}$$

$$(p = (x,0,0) \in s^{0}_{P_a} \subset S_0) \Rightarrow (p' = (x',0,0) \in s^{0'}_{P_a} \subset S_{0'}) \tag{5.14}$$

Eq. (5.11)-(5.14) show that P_a which physically exists relative to 0 as an ordinary non-zero rest mass material body does not physically exist relative to 0', in view of (5.2), (5.3) and (5.6)-(5.10), as either a bradyon (directly observable) or luxon (not directly observable) exists. Hence while a non-zero rest mass luxon P_a is itself of a physically complex form, a non-zero rest mass tachyon P_a is of an even more complex form. Further, on account of the real-valuedness in (3.6), (3.7), (3.1) and (3.3), and the imaginary y_a' and z_a' but real x_a' corresponding to real x_a, y_a and z_a, the existence of non-zero rest mass tachyons, in my view, *transcends our conventional classical consciousness and model pertaining to space and the physical existence of matter therein, even more than the existence of non-zero rest mass luxons.* Eq. (3.1) implies that non-zero rest mass tachyons exist in *time* exactly as non-zero rest mass bradyons and luxons do but, in view of the fundamental property of imaginary y_a' and z_a' but real x_a', their existence and location in *space* do not accord with our conventional concepts of spatial existence and location accepted in (5.1)-(5.3). Eq. (5.11)-(5.14) show that 0' will not detect non-zero rest mass tachyons by laboratory apparatus.

Eq. (4.6) and (4.9) show that

(i) when 0 observes 0' as a non-zero rest mass bradyon then 0' will also observe 0 as a non-zero rest mass bradyon

(ii) when 0 observes the collapse of $S_{0'}$, then 0' in turn observes the collapse of S_0

(iii) when 0 observes that 0' does not exist as a non-zero rest mass bradyon or luxon then 0' will observe the same in respect of 0.

Thus the physical non-existence and undetectability, relative to 0, of non-zero rest mass luxons and tachyons as ordinary bradyons relative to 0, avoids not only the causality complications mentioned in section 3 but also the complications in (4.5) and (4.8) for $V: |V| \geq c$. Finally it is essential to stress that the above physico-spatial results pertaining to the existence of matter are consequences of (2.33) and (2.34) which in turn mathematically follow from the fundamental light postulate.

6. A SUPERCOSMOS

Relative to any observer 0, the union C^0 of the sets $C^0_<$, $C^0_=$, $C^0_>$ of bradyons, luxons and tachyons respectively represents, in the context of section 5, a supercosmos of matter. Whilst in respect of any two observers 0 and 0',

$$0' \in C^0_< \Leftrightarrow 0 \in C^{0'}_< ;\ 0' \in C^0_= \Leftrightarrow 0 \in C^{0'}_= ;\ 0' \in C^0_> \Leftrightarrow 0 \in C^{0'}_> \tag{6.1}$$

it can be proved using (2.38)-(2.40) and the light postulate that

$$0' \in C^0_< \Rightarrow C^0_< = C^{0'}_< ;\ C^0_> = C^{0'}_> ;\ C^0_= = C^{0'}_= \tag{6.2}$$

$$0' \in C^0_> \Rightarrow C^0_< = C^{0'}_> ;\ C^0_> = C^{0'}_< ;\ C^0_= = C^{0'}_= \tag{6.3}$$

$$0' \in C^0_= \Rightarrow C^0_< \cup C^0_> \cup C^0_= \subset C^{0'}_= \text{ and } C^{0'}_< \cup C^{0'}_> \cup C^{0'}_= \subset C^0_= \tag{6.4}$$

Eq. (6.2) and (6.3) imply the following: an observer 0 and all bradyons relative to 0 belong to one and the same subcosmos $C^0_<$; relative to any member of $C^0_<$ there exists one and the same subcosmos $C^0_>$; all members of $C^0_>$ exist for each other as bradyons, and relative to them exists one and the same tachyon subcosmos this in turn being $C^0_<$; relative to any member of $C^0_< \cup C^0_>$ there exists one and the same luxon subcosmos $C^0_=$.

Eq. (6.4) imply the following: relative to any given member 0' of $C^0_=$, not only is every other member of $C^0_=$ a luxon (as required by the light postulate) but the light postulate also implies that $C^0_< \cup C^0_>$ is luxon, the latter also being expected from the spatial collapse discussed in section 5; furthermore, in view of an equivalence of 0 and 0' apparent in (6.1), the supercosmos of 0' differs from that of the members of $C^0_< \cup C^0_>$ and is in turn for the same reasons luxon relative to the members of $C^0_< \cup C^0_>$. Hence relative to each member of $C^0_=$ exists a unique supercosmos of matter in the luxon subcosmos of which exist every other member of $C^0_=$ and the supercosmos of every other member of $C^0_=$. Thus $C^0_= \subset C^0$ implies that C^0 incorporates, in addition to $C^0_<$ and $C^0_>$ respectively detectable and undetectable by 0, many supercosmoses, possibly infinite, not detectable by 0 on account of spatial collapse.

It follows that when an observer $0' \varepsilon C^0_<$ accelerates relative to 0 or any element of $C^0_<$ to a superluminal speed then 0' will assume a physical existence as a bradyon relative to members of $C^0_>$ only and vice versa. However, when $0' \varepsilon C^0_<$ accelerates to the speed V exactly equal to $\pm c$ (which is physically extremely improbable because the nature of physical measurements implies invariably, if not always, a margin of error) then 0' will cease to physically exist as a bradyon relative to members of $C^0_< \cup C^0_>$ but will exist in a supercosmos $C^{0'}$ spatially collapsed relative to members of $C^0_< \cup C^0_>$; if 0' now wishes to accelerate again, 0' can do so only relative to $C^{0'}$ and hence despite any acceleration he will forever remain existing in supercosmoses spatially collapsed relative to members of $C^0_< \cup C^0_>$. Thus in particular non-zero rest mass luxons relative to 0 will remain luxons relative to 0.

For the mathematical description of the supercosmos C^0 (and motion therein), (5.1) (and hence the speed convention generalisation related to U of section 2), which are adequate to describe $C^0_< \cup C^0_=$, are inadequate and have to be extended to incorporate the description of $C^0_>$. Eq. (4.6) and (4.9) imply:

(i) (5.1) has to be extended to $\mathcal{S}_0 = S_0 \cup {*S_0}$ where

$*S_0 = R \times \{ki + K_2 : k \varepsilon R\} \times \{li + K_3 : l \varepsilon R\}$

(ii) the convention has to be extended to a convention based on a subdivision of the surfaces of a union of U and *U. Such an extension is always possible so that (2.1) - (2.3) are still valid.

Since an extension to higher dimensions, as conventionally accepted, implies an addition of real axes orthogonal to the real axes being used, this case of the addition of imaginary axes I_y and I_z does not constitute such an extension. Whilst the association of any relationships between sets of real axes with sets containing imaginary axes would thus not necessarily be correct, the separate existence of S_0 and $*S_0$ spanned by the sets of axes $\{x, y, z\}$ and $\{x, I_y, I_z\}$ respectively may be considered analogous to the separate existence of the spaces spanned by $\{x, y\}$ and $\{x, z\}$ respectively, the latter being visualisable on account of our three-dimensional consciousness in the context of (5.1). All matter in C^0 exist relative to each other in a forward surge of time since (3.1) and (3.2) $\Rightarrow$

$$\Delta t > 0 \Leftrightarrow \Delta t' > 0 \tag{6.5}$$

In accord with the transcendence of consciousness concluded in section 5

(i) a material body moves in $S_0 \subset \mathcal{S}_0$ while it is a bradyon, spatially collapses into $(S_0 \cap {*S_0}) \subset \mathcal{S}_0$ when it becomes a luxon, and then emerges in $*S_0 \subset \mathcal{S}_0$ when it becomes a tachyon

(ii) a figure, like the usual figure for the world $W_0 = S_0 X\ R$, for the extended world $\mathcal{W}_0 = \mathcal{S}_0\ X\ R$ showing the common t axis of $C^0_<$, $C^0_=$ and $C^0_>$ is possible if S_0 and $*S_0$ are shown "suppressed" as two straight lines perpendicular to the t axis provided that the space (of our three-dimensional awareness) between the world "planes" $S_0\ X\ R$ and $*S_0\ X\ R$ is conceived as non-existent. Whilst for a given non-zero rest mass luxon, a null world line exists in both world planes, the world line segment of an accelerating non-zero rest mass bradyon breaks off in one plane at the instant it becomes a tachyon and continues as a timelike world line segment in the other plane, with the world point associated with the instant it becomes a luxon appearing in both planes.

If photons satisfy (5.8)-(5.10), conditions for non-zero rest mass luxons, then photons whose source exists in the space line of only one of the world planes will be visible in the space lines of both world planes only if these photons travel through coinciding empty spaces in both spaces. In view of the incessant motion of matter in $C^0_<$ and $C^0_>$, and the need for an observer suitably positioned relative to these empty spaces, seeing such photons is highly improbable.

7. ACCORD BETWEEN EXPERIMENTAL OBSERVATION AND PHYSICAL PREDICTIONS OF THE THEORY

Whilst it is well known that (3.11) (and hence time dilation), length contraction, the expression of (3.7) for $V:|V| << c$ and for, in particular, $V = 0$, the incorrectness of the Newtonian kinetic energy expression for $V:|V|$ is large compared to c, and the non-detection of non-zero rest mass luxons and tachyons by laboratory apparatus have experimental support, the following observations (Calder [12]) regarding a neutrino, a luxon whose physical existence is only indirectly inferred, are in accord with the outcomes of (5.8) and (5.10):

(i) Neutrinos are "invisible" and have "no track" in a bubble chamber

(ii) "Neutrinos colliding with the earth would pass straight through and come out on the other side. Indeed a neutrino could travel through many millions of miles of rock without feeling anything"

The experimental finding (Lubimov et al [13]) that a neutrino has non-zero rest mass is also in accord with section 3.

With regard to accelerating a non-zero rest mass particle to a speed $V:|V| \geq c$, the following extract (Marion [14]), in my view, supports the outcomes of sections 3 and 5, viz. that (i) a finite external force is required and (ii) when a non-zero rest mass particle attains such a speed laboratory apparatus would not detect it:

"One sometimes hears questions of the following type: "I know that relativity theory says that we must always have $v < c$, but what would happen *if* an object or a particle could be accelerated from some small velocity to a velocity greater than c?" There is no answer to such a question within the realm of physical science because there is no conceivable way in which we can put the question to an experimental test. Physical science cannot address itself to any question that does not permit a *measurement* to settle the issue."

The extract does not assert that experimental findings indeed prove that an infinite force is required to realise c (a finite speed) but rather that a "*measurement*" to settle the issue is not permitted, the latter, in view of the 0.999999999c electron speed already realised in the Stanford Linear Accelerator (Marion [14]), being surely due to the failure by laboratory apparatus to detect the particle.

That a photon as a luxon does not manifest the usual properties associated with bradyons appears to be in accord with the conclusion that luxons exist as non-ordinary material bodies. Furthermore the dual particle-wave nature of a photon also appears to be in accord with the existence of luxons on the 'border' of the bradyon subcosmos $C^0_<$; this seems to suggest that matter in $C^0_>$ would be wavelike but occupying space in a manner transcending classical consciousness in the context of section 5.

8. THE EQUIVALENCE PRINCIPLE

The motions of 0' relative to 0 and 0 relative to 0' are distinguished in the context of α and Ω by, respectively, the functions G(V) and O(V). If we assume that actual and apparent motion are physically indistinguishable and hence that the spatial-time reference frames of 0 and 0' are physically equivalent in the sense that the factors $[G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}$ and $[O(V)/(1 - V^2/c^2)]^{\frac{1}{2}}$ are equal, the identicalness of equations (3.1)-(3.9) from the viewpoints of 0 and 0' being an example of the special principle of relativity, then

$$G(V) = O(V) \ \forall \ V \in R . \tag{8.1}$$

Eq. (8.1), a necessary condition for the principle in the context of this theory, does not contradict any physical prediction of sections 3, 5 and 6, nor does it in particular imply a mathematical infinity at $V:|V| = c$, a result in Einstein's and Recami's special-relativistic theories. Eq. (8.1) however gives rise to the twin paradox controversy in (3.1) and 3.2).

9. PHYSICAL AND MATHEMATICAL INVERSES

It can be proved that, with regard to the matrix of $\alpha(V)$

$$[\alpha] = \begin{bmatrix} [G(V)/(1 - V^2/c^2)]^{\frac{1}{2}} & 0 & 0 & -V[G(V)/(1 - V^2/c^2)]^{\frac{1}{2}} \\ 0 & G(V)^{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & G(V)^{\frac{1}{2}} & 0 \\ -V[G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}/c^2 & 0 & 0 & [G(V)/(1 - V^2/c^2)]^{\frac{1}{2}} \end{bmatrix}$$

and the corresponding matrix $[\Omega]$, $[\alpha].[\Omega] = [\Omega].[\alpha] =$

$$\begin{bmatrix} \dfrac{(G(V)O(V))^{\frac{1}{2}}(1 - V^2/c^2)}{|1 - V^2/c^2|} & 0 & 0 & 0 \\ 0 & G(V)^{\frac{1}{2}}O(V)^{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & G(V)^{\frac{1}{2}}O(V)^{\frac{1}{2}} & 0 \\ 0 & 0 & 0 & \dfrac{(G(V)O(V))^{\frac{1}{2}}(1 - V^2/c^2)}{|1 - V^2/c^2|} \end{bmatrix} \tag{9.1}$$

It must be re-stressed that α and Ω have each been independently derived from fundamental physical observations, and the same physical assumptions without assuming any mathematical relationship between them, and they may thus be termed *physical* inverses of each other. Eq. (9.1) shows that these physical inverses cannot possibly be *mathematical* inverses for values of $V:|V| \geq c$ even if (8.1) is assumed. (9.2)
Furthermore (2.29) imply that these physical inverses would not be mathematical inverses even if (8.1) is again assumed for in addition at least bradyon inertial frames with $V:|V|$ is crucially close to c. (9.3)

This result shows that the tacit assumption that physical inverses are mathematical inverses is not necessarily true when the physical variables are space-*time* coordinates.

10. PHYSICAL CONSEQUENCES OF AN UNCONDITIONAL ACCEPTANCE OF THE TACIT ASSUMPTION IN ADDITION TO (8.1) : EINSTEIN'S SUBLUMINAL AND RECAMI'S SUPERLUMINAL THEORIES

If the conditions (9.2) and (9.3) are disregarded and the tacit assumption is accepted unconditionally in addition to (8.1), as Einstein [4] and Recami [3] did, then it can be proved that for $V:|V| < c$, (9.1) $\Rightarrow$

$$G(V) = 1 \tag{10.1}$$

resulting in Einstein's subluminal special-relativistic theory.

For $V:|V| > c$, (9.1) implies that (4.2) or (4.7) must be suitably adjusted. From a mathematical point of view this adjustment is not unique and hence G(V) is not uniquely determined. However, it can be proved that (2.7), (2.9), (2.33) and (2.34), all of which follow from physical observations, imply only 8 possible adjustments, viz.

(a)

$$\alpha(V):\begin{cases} dx' = +[G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(dx - Vdt) \\ dy' = \pm G(V)^{\frac{1}{2}}dy \\ dz' = \pm G(V)^{\frac{1}{2}}dz \\ dt' = +[G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(-Vdx/c^2 + dt) \end{cases}$$

provided that correspondingly

$$\Omega(V):\begin{cases} dx = -[G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(dx' + Vdt') \\ dy = \mp G(V)^{\frac{1}{2}}dy' \\ dz = \mp G(V)^{\frac{1}{2}}dz' \\ dt = -[G(V)/(1 - V^2/c^2)]^{\frac{1}{2}}(Vdx'/c^2 + dt') \end{cases}$$

and

$$G(V) = -1$$

(b) $\alpha(V)$ and $\Omega(V)$ are the same as those for (a) in all respects except that the plus signs of the first and fourth equations of $\alpha(V)$ in (a) are interchanged with the minus signs of the first and fourth equations of $\Omega(V)$ in (a).

Eq. (10.1) appears in accord with high-energy experimental physics and is in accord with (3.11). However, (10.1) and (3.5) imply that nothing short of the sum total of all physical forces in the infinite supercosmos is required to accelerate a non-zero rest mass particle, even as microscopic as, say, an electron, to the speed c. In view of the finiteness of c and the 0.999999999c electron speed already realised I regard this a physically unacceptable result. Whilst this result also implies that it will be physically impossible for matter from $C^0_<$ to enter $C^0_>$, it cannot be used as a reason for concluding that $C^0_>$ does not physically exist since for instance $C^0_=$ exists despite this result. Similarly (10.1) also implies physically unacceptable results in (3.6), (3.7), (3.1) and (3.3).

In addition to the foregoing physically unacceptable results for $V:|V| = c$, (a) implies the following physically contradictory results for $V:|V| > c$ i.e., for $0' \varepsilon\ C^0_>$:

(i) Eq. (3.1) and (3.2) respectively become

$$dt' = [1/(V^2/c^2 - 1)]^{\frac{1}{2}}dt \tag{c}$$

$$dt = -[1/(V^2/c^2 - 1)]^{\frac{1}{2}}dt' \tag{d}$$

which imply that, relative to 0', 0 moves forward in time but, relative to 0, 0' moves backward in time; (6.5) is contradicted.

(ii) Using (d) in place of (c) in section 3 it can be proved that, relative to 0', 0 has positive mass (energy) but, relative to 0, 0' has negative mass (energy).

(iii) Two simultaneous events relative to 0 at (x,0,0) and $(x + \Delta x,0,0)$ imply that

$$\Delta x' = [1/(V^2/c^2 - 1)]^{\frac{1}{2}}\Delta x\ . \tag{e}$$

However, two simultaneous events relative to 0' at (x',0,0) and $(x' + \Delta x',0,0)$ imply that

$$\Delta x = -[1/(V^2/c^2 - 1)]^{\frac{1}{2}}\Delta x'\ . \tag{f}$$

Eq. (e) implies that corresponding to a positive (negative) Δx relative to 0 is a positive (negative) $\Delta x'$ relative to 0' but (f) implies that corresponding to a positive (negative) $\Delta x'$ relative to 0' is a negative (positive) Δx relative to 0, showing that relative to 0 the 'x space' of 0' has "inverted" but relative to 0' the 'x space' of 0 has not "inverted".

(iv) Eq. (c), (d) and (3.9) imply that if 0' emits a signal at frequency ν_e the frequency of receipt ν_r would differ only in sign from the frequency ν_r if 0 instead emits a signal at frequency ν_e.

The retarded causality postulate and consequently the RIP (Recami [3]) giving rise to antimatter are thus necessitated by the tacit assumption. These results show that, relative to 0, 0' is antimatter but, relative to 0', 0 is not antimatter. Eq. (3.5) shows that 0 (0') has to exert an infinite 'negative' (positive) force to accelerate antimatter 0' (matter 0) to the speed c.

11. AN ALTERNATE THEORY

Sections 3,5 and 8 show that neither the fundamental light postulate nor the assumption of the equivalence principle leads to the physically unacceptable results for $V:|V| = c$ or the physically contradictory results for $V:|V| > c$. The results for $V:|V| > c$ in fact contradict the equivalence principle, showing that (8.1), a necessary condition for the principle, is not a sufficiently strong condition if at the same time the tacit assumption is unconditionally accepted.

Furthermore if (10.1) is indeed valid for all $V:|V| < c$ then, in view of the singularity at $V:|V| = c$, experimental findings in particle-accelerators must show that irrespective of any increase in the accelerating force the particle forever remains detectable because the stage at which (5.8)-(5.10) apply would never be reached. The extract quoted in section 7 does not confirm this; it instead presents evidence in terms of section 5 that the particle has attained a speed $V:|V| \geq c$. Hence, while (10.1) appears valid for high-energy experimental physics, there must exist an ε-neighbourhood I of $\pm c$ on which (10.1) is not valid, in accord with (2.26) and (2.29).

More importantly, the singularity is physically unacceptable for the further reason that it implies unaccelerated forced motion at some stage, violating not only (3.10) but also, in view of m(V), conservation of energy. The existence of such an I would permit the validity of the principle of conservation of energy in a closed system of supercosmoses.

Also experimental confirmation of the singularity is inconceivable. In particular any claim of the experimental confirmation of $G(V) = 1$ *up to* $\pm c$ via orbital particle-accelerators is false because, according to Bransden et al [15],

(a) this would, in view of the availability of only finite magnetic fields, require cyclotrons, synchrotons and synchrocyclotrons of infinite radii

(b) the rotation frequency of an accelerating particle at speed $\pm c$ in these orbital accelerators would have the physically absurd value of zero

(c) in addition at the speed $\pm c$ an accelerating particle in a synchroton would lose via electromagnetic radiation an infinite amount of energy per revolution.

Orbital accelerators are consequently inefficient for the realisation of speeds most likely to lie in I; according to Bransden et al [15] orbital accelerators are not efficient to realise electrons with energy much above 1 GeV. On the other hand, the principles used in the design of a linear accelerator, such as the aforementioned Stanford Linear Accelerator, are independent of special-relativistic effects; according to Bransden et al [15] a 2-mile long linear accelerator realises 20 GeV electrons.

From the mathematical viewpoint, in this theory (2.25) is the important *fundamental* equation which determines X appearing in particular in the basic equations (2.27). The discussion of the solutions X of (2.25) showed that the solution $X = \infty$ for $V:|V| = c$ is inadmissible, the real-valued X being subsequently written in the form (2.26). Furthermore the tacit assumption which is the additional assumption from which the singularity follows is not valid at $V:|V| = c$ on an account of (2.22). Nor is it valid for $V:|V| > c$ if physical coherence is required of the theory. Therefore from considerations of:

(a) the physical implications of the spatial collapse predicted at

$V:|V| = c$ (in particular spatial relationships between observers expected at 'customary' speeds do not apply at the speed $\pm c$)

(b) the observed *continuity* in change of value of physical variables associated with accelerating particles in accelerators

(c) the lack of absolute exactness in experimental measurement

it is reasonable to accept the invalidity of the tacit assumption in at least some such neighbourhood I.

Furthermore the tacit assumption which is nonetheless valid for transformations between spatial reference frames stationary relative to each other would in its application to space-*time* reference frames appear to disregard not only the fact of the relative motion of the frames but also the essential difference between a spatial and a time variable as physical entities. Also there is no physical motivation for the validity of the tacit assumption in respect of V-dependent spacetime transformations especially at such crucially large speeds, and, in view of the aforementioned continuity of change, there is no motivation for the essential discontinuity at $V:|V| = c$ in $G(V)$ more especially when this essentiality implies singularities in related functions. At any rate a singularity is most unwelcome in a physical theory because it invariably implies physical incongruities. Moreover, the assumption (8.1) used to determine (10.1) gives rise to the twin paradox controversy.

It would thus appear that for a physically more acceptable theory in accord with experimental evidence and the mathematical properties of $G(V)$, the tacit assumption subject to conditions (9.2) and (9.3) with a differentiable, piecewise-continuous

$$G(V) = 1 \ \forall\ V:|V| < c,\ V \notin I \tag{11.1}$$

is acceptable. However, in view of the inevitability of marginal error in experimental physics and the limitations imposed by the relevant apparatus on the largeness of $|V|$ relative to c realisable therein, the *apparent* experimental confirmation of $G(V) = 1$ for $V:|V| < c$ in the aforementioned context is confirmation of an element of a 1-parameter set of functions $G(V)$ determined in the sequel, this set being more preferable than (11.1) on account of its determination without the tacit assumption or the equivalence principle and for all real-valued V.

As a motivation for a more general $G(V)$ satisfying (2.20)-(2.22) and (2.24) but not piecewise continuity, assume a polynomial $G(V)$. Eq. (2.22) $\Rightarrow$

$$G(V) = (1 - V^2/c^2)\phi(V)$$

where, assuming for simplicity $\pm c$ are not repeated roots of $G(V) = 0$, $\phi(V) > 0 \ \forall\ V \in R$ on account of (2.20) and (2.21). Guided by (2.28) which expresses the experimental validity of Newton's theory or, if one prefers, the apparent experimental confirmation of $G(V) = 1$, an appropriate and obvious choice for $\phi(V)$ which avoids the singularity is a finite sub-series

$$\sum_{r=0}^{r=n-1} (V^2/c^2)^r$$

of the infinite Maclaurin series expansion of $(1 - V^2/c^2)^{-1}$. Then $G(V)$ becomes

$$G(n,V) = 1 - (V^2/c^2)^n,\ n \text{ a positive integer,}$$

which can be generalised without contradicting the aforementioned properties, in particular (2.28), to

$$G(n,V) = 1 - (V^2/c^2)^n, \ n \geq 1, \ n \in R. \tag{11.2}$$

Since as $n \to \infty$, $G(n,V) \to 1$ for <u>all</u> $V:|V| < c$, for finitely very large n, $G(n,V)$ from an experimental point of view does not differ significantly from 1 for $V:|V| < c$ except for V in some such aforementioned ε-neighbourhood I of $V = \pm c$ where $\varepsilon \to 0$ as $n \to \infty$. Such finite n yield $G(n,V)$ having properties of (11.1) except piecewise-continuity. Since

$$\lim_{|V| \to c} \sqrt{\frac{1 - (V^2/c^2)^n}{1 - V^2/c^2}} = n^{\frac{1}{2}}$$

n is, in the context of (3.6), section 7 and the aforementioned, the experimentally determinable finite value of $(m(V)/m_o)^2$ as $|V| \to c$, a direct measurement at $V:|V| = c$ not being permitted on account of the physical implications of the theoretically-predicted spatial collapse. The value of n is thus most probably non-integral.

However, in the following table possible integral n-values are considered and corresponding to each n a value $m > 0$ is given where for $V/c \in [-m,m]$, (11.2) will correct to ten decimal places, be experimentally determined to be 1.

n	$5x10^5$	$5x10^6$	$5x10^7$	$5x10^8$	$5x10^9$	$5x10^{10}$	$5x10^{11}$	$5x10^{12}$
m	$1-\varepsilon_1$	$1-\varepsilon_2$	$1-\varepsilon_3$	$1-\varepsilon_4$	$1-\varepsilon_5$	$1-\varepsilon_6$	$1-\varepsilon_7$	$1-\varepsilon_8$

where $\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4,\varepsilon_5,\varepsilon_6,\varepsilon_7,\varepsilon_8$ are respectively $2.3718717x10^{-5}$, $2.371897x10^{-6}$, $2.37190x10^{-7}$, $2.3719x10^{-8}$, $2.372x10^{-9}$, $2.38x10^{-10}$, $2.4x10^{-11}$ and $3x10^{-12}$. For instance if $n = 5x10^{10}$, i.e., if $m(V) \to 5^{\frac{1}{2}}x10^5 m_o$ as $|V| \to c$, then (11.2) is, correct to thirty decimal places, experimentally determined to be 1 at speeds up to 0.9999999993c, i.e., a speed $0.20985475 \pm 0000002 1 ms^{-1}$ less than c where $c = (2.997925 \pm 0.000003)x10^8 \ ms^{-1}$ has been assumed. Hence (11.2) which removes the physically unacceptable singularity is experimentally confirmed in the apparent experimental confirmation of $G(V) = 1$.

Eq. (11.2) and (4.6) show that

(i) Newton's theory is valid for values of V for which V^2/c^2 is negligible

(ii) Einstein's theory is valid for values of V for which $(V^2/c^2)^n$ is negligible, or that analogous to the observation that in Einstein's theory Newton's theory is the limiting case as $c \to \infty$ is the additional observation that in this theory Einstein's theory is the limiting case as $n \to \infty$.

Using the infinite series expansion

$$(1 - x)^m = \sum_{k=0}^{k=\infty} \binom{m}{k} (-x)^k \text{ for } |x| < 1$$

it can be shown that (3.7) for $V:|V| < c$ becomes

$$E_n = m_o c^2\{\sum_{k=0}^{k=\infty}\sum_{l=0}^{l=\infty}\binom{\frac{1}{2}}{k}\binom{-\frac{1}{2}}{l}(-1)^{k+l}(V^2/c^2)^{nk+l} +$$

$$\frac{n}{2}\sum_{k=0}^{k=\infty}\sum_{l=0}^{l=\infty}\frac{\binom{\frac{1}{2}}{k}\binom{-\frac{1}{2}}{l}(-1)^{k+l}(V^2/c^2)^{n(l+1)+k}}{n(l+1)+k}\} + d\ . \tag{11.3}$$

Using the result

$$\sum_{k=0}^{k=p}\binom{m}{k}\binom{-m}{p-k} = 0$$

derivable from the infinite series expansion of $(1 \pm x)^m$, it can be shown that $n = 1$ in (11.3) $\Rightarrow$

$$E_1 = m_o c^2 + m_o V^2/2 + d \ \forall\ V:|V| < c.$$

E_1,(3.3), (2.27), (3.1) and (3.6) show that the case $n = 1$ implies a modification of Newton's theory only in respect of the introduction of (a) the rest energy term and an integration constant (b) length contraction of a moving rod and (c) relativity of simultaneity, there being neither time dilation/contraction of a moving clock nor mass increase/decrease of a moving particle. This case, however, appears inconsistent with experimental observations.

It can be shown in (11.3) that for V for which V^2/c^2 is negligible $n \geq 2$, i.e., an infinite number of G(n,V), $\Rightarrow$

$$E_n = m_o c^2 + m_o V^2/2 + d + \text{negligible V-dependent terms}\ . \tag{11.4}$$

A motivation for the acceptance of Einstein's energy expression in the derivation of which d is assumed to be zero or negligible is the implication in (11.4) that Newton's kinetic energy expression is experimentally valid only for such V, this validity being due solely to the negligibility of the terms.

Furthermore it can be shown that for V for which $(V^2/c^2)^n$ is negligible (11.3) $\Rightarrow$

$$E_n = m_o c^2\sum_{l=0}^{l=[n]}\binom{-\frac{1}{2}}{l}(-1)^l(V^2/c^2)^l + d + \text{negligible V-dependent terms}$$

or

$$E_n = m_o c^2(1 - V^2/c^2)^{-\frac{1}{2}} - \text{the negligible term } m_o c^2\sum_{l=[n]+1}^{l=\infty}\binom{-\frac{1}{2}}{l}(-1)^l(V^2/c^2)^l + d$$
$$+ \text{other negligible V-dependent terms} \tag{11.5}$$

where [n] is the least integer greater than n. Eq. (11.5) in turn shows that Einstein's energy expression is valid only for such V, this validity being solely due to the negligibility of the terms. It can also be shown that

$$\lim_{n\to\infty} E_n = m_o c^2(1 - V^2/c^2)^{-\frac{1}{2}} + d.$$

Eq. (11.2) yields the physically more acceptable shift of the singularity to $V = \pm\infty$. Furthermore because (11.2) is derived using only the properties of G(V) without any dependence on the O(V) properties, this theory is not based on the reciprocity principle (Berzi, Gorini [8]) or on any assumption giving rise to the twin paradox controversy. The controversy does not arise in this theory if in (3.2)

$$O(V) = (1 - V^2/c^2)^2/G(V). \qquad (11.6)$$

Since

$$\lim_{|V| \to c} (1 - V^2/c^2)^2/(1 - (V^2/c^2)^n) = 0,$$

(11.6) does not contradict the properties of continuity of O(V) analagous to (2.20)-(2.22), (2.24) and (2.28). Eq. (11.6) implies that (9.1) yields $(1 - V^2/c^2)[\delta_{ij}]$ for $V : |V| < c$ showing the physical acceptability of the tacit assumption only for $V : |V| << c$.

Whilst I prefer (11.2) it may be argued that one may determine G(V) for $V:|V| > c$ without the contradictions of section 10 by a common mathematical argument wherein the tacit assumption is not *explicitly* made: (2.33), (2.34) and (8.1) $\Rightarrow G(V)^2 = 1$ provided $V \neq \pm c \Rightarrow$

$$G(V) = \begin{cases} 1 \ \forall \ V:|V| < c \\ -1 \ \forall \ V:|V| > c. \end{cases}$$

This result however, is based on the equivalence principle (which gives rise to the twin paradox controversy) and yields the physically unacceptable singularity.

12. IMPLICATIONS FOR GENERAL RELATIVITY

In postulating the physical equivalence of an observer A in a gravitational field and an accelerating observer B, Einstein (a) assumes on account of (10.1) that

$$ds'^2 = ds^2 \qquad (12.1)$$

and hence g_{ij} is a tensor and B cannot attain or exceed c and (b) in view of (a) does not specify the acceleration of B corresponding to the position of A in the field.

However, in view of the alternate theory

$$ds'^2 = [1 - (V^2/c^2)^n]ds^2 \qquad (12.2)$$

necessitates the specification of the acceleration. Now via a *kinematical* argument based on the experimentally supported assertion $\ddot{r} = -GM/r^2$ it can be shown that an observer falling freely from rest at $r = \infty$ attains the speed c at $r = 2GM/c^2$ in the gravitational field of a generating source (without charge or angular momentum) satisfying $R < 2GM/c^2$ where M and R are respectively the mass and radius of the source. If further this is accepted, then the experimental validity of (12.1) in (12.2) shows that Einstein's general relativity will be experimentally valid for all r values except $r \leq 2GM/c^2$ and r crucially close to $r = 2GM/c^2$ in the field of such a source. Thus, if this physical equivalence of observers is indeed true, then 'black holes' would be 'doors' to $C^0_>$. Einstein's theory would appear experimentally valid if $R < 2GM/c^2$ is not satisfied.

13. CONCLUSION

Whilst the alternate special-relativistic theory derived has been shown to be as valid within experimental limitations as Einstein's theory, its removal of the singularity from $V = \pm c$ to $V = \pm\infty$ is physically more satisfactory. Causality complications and the violation of the principle of energy conservation and of the law that the necessary and sufficient condition for particle-acceleration is the application of a non-zero force are avoided by spatial collapse and the existence of a supercosmos of matter. The theory also differs from Recami's extended special-relativistic theory in its superluminal results.

In particular when an astronaut exceeds c relative to us he would be undetectable by us as he would enter another cosmos of matter perhaps very much like our own and unlike the implications of Recami's and Einstein's theories would still be moving forward in time. Furthermore in like manner matter from the other cosmos can materialise in our cosmos. Since exits from empty spaces of our cosmos need not imply entries into empty spaces in the other cosmos an astronaut seeking to reach other stars of our cosmos will have to travel at subluminal speeds and perhaps exploit time travel. Finally also unlike Einstein's theory luxons need not have zero rest mass. If one, however, accepts for other reasons the concept of a zero rest mass particle, then one may determine the energy E of such a particle moving at *arbitrary* speeds by determining the expression of (3.8) (obtained for non-zero rest mass particles) as m_o approaches 0, viz.

$$p = (E - d)/c.$$

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