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ASYMMETRIC DEFORMATION OF ANISOTROPIC POROUS BODIES

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ABSTRACT. A study is made of the deformation of a transversely isotropic poro-elastic half-space under various types of loads applied on its boundary. The axis of symmetry of the medium is not taken perpendicular (as is usually done) but parallel to the boundary plane of the semi-space.

KEY WORDS AND PHRASES. *Poro-elastic bodies, Laplace transform.*
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1. INTRODUCTION.

The deformation of poro-elastic bodies has tremendous applications in various new branches of science including soil mechanics and bioscience. An extensive investigation on behavior and mechanical properties of human bone has opened a new field of an unprecedented development of medical physics and biomedical engineering in general.

Several authors including Biot [1-6], Jana [7], Evans [8], Fung [9] and Paria [10] have made investigations of the problems of deformation of poro-elastic bodies. However, relatively less attention has been given to the problems of anisotropic poro-elastic materials. Paria [10] has developed a method for solving axisymmetric deformation of poro-elastic bodies. In this paper, Paria's method has been extended for finding solution of the axisymmetric deformation of anisotropic poro-elastic bodies.

2. FUNDAMENTAL EQUATIONS.

Let us consider the deformation of a poro-elastic body containing fluid.

The Stress tensor is

$$\begin{pmatrix} \sigma_x + \sigma & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y + \sigma & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z + \sigma \end{pmatrix} \tag{2.1}$$

where σ is related to the porosity f and the fluid pressure p by

$$\sigma = -fp \, . \tag{2.2}$$

In the absence of the body forces, the equations of equilibrium are

$$\frac{\partial}{\partial x}(\sigma_x + \sigma) + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0 \, , \tag{2.3}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial}{\partial y}(\sigma_y + \sigma) + \frac{\partial \tau_{yz}}{\partial z} = 0 \, , \tag{2.4}$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial}{\partial z}(\sigma_z + \sigma) = 0 \, . \tag{2.5}$$

If the medium possesses anisotropy of the transversely isotropic type the stress-strain relations are

$$\sigma_x = 2Ne_{xx} + A(e_{xx} + e_{yy}) + Fe_{zz} + M\varepsilon \, , \tag{2.6}$$

$$\sigma_y = 2Ne_{yy} + A(e_{xx} + e_{yy}) + Fe_{zz} + M\varepsilon \, , \tag{2.7}$$

$$\sigma_z = Ce_{zz} + F(e_{xx} + e_{yy}) + Q\varepsilon \, , \tag{2.8}$$

$$\sigma = M(e_{xx} + e_{yy}) + Qe_{zz} + R\varepsilon \, , \tag{2.9}$$

$$\tau_{yz} = Le_{yz} \; ; \; \tau_{zx} = Le_{zx} \; ; \; \tau_{xy} = Ne_{xy} \, , \tag{2.10}$$

where N, A, F, M, C, Q, R and L are eight elastic constants. Also

$$e_{xx} = \frac{\partial u}{\partial x} \; ; \; e_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \, , \tag{2.11}$$

$$\varepsilon = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \, , \tag{2.12}$$

where (u, v, w) and (U, V, W) are the components of displacements of the solid and fluid respectively. According to Darcy's Law, the equations of flow for the fluid in the porous solid are -

$$\frac{\partial \sigma}{\partial x} = b_1 \frac{\partial}{\partial t}(U - u) \; ; \; \frac{\partial \sigma}{\partial y} = b_1 \frac{\partial}{\partial t}(V - v) \qquad (2.13),(2.14)$$

and

$$\frac{\partial \sigma}{\partial z} = b_2 \frac{\partial}{\partial t}(W - w) \, , \tag{2.15}$$

where t denotes time. The constant b_1 is related to the coefficient of viscosity of the fluid μ, the porosity f and Darcy's coefficient of permeability μ_1, in the direction of x and y as

$$b_1 = \mu f^2/\mu_1 \tag{2.16}$$

and similarly if μ_2 denotes the coefficient of permeability in the z-direction

$$b_2 = \mu f^2/\mu_2 \, . \tag{2.17}$$

3. STRESS FUNCTIONS.

Suppose the stress functions $\phi(x, y, z, t)$, $\Psi(x, y, z, t)$ and $\chi(x, y, z, t)$ satisfy the following relations

$$U = -\frac{\partial^2\phi}{\partial x\partial z} + \frac{\partial^2\chi}{\partial y\partial z}, \tag{3.1}$$

$$V = -\frac{\partial^2\phi}{\partial y\partial z} - \frac{\partial^2\chi}{\partial x\partial z}, \tag{3.2}$$

$$W = k_1 \nabla_1^2\phi + k_2 \frac{\partial^2\phi}{\partial z^2} + \Psi, \tag{3.3}$$

where $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $K_1 = (P + M)(F + Q + L)$, $K_2 = L/(F + L + Q)$ and $P = A + 2N$.

With the help of (2.6) - (2.12) and (3.1) - (3.3) the relations (2.3) - (2.5) reduce to

$$\frac{\partial}{\partial x}\{(F + Q + L)\frac{\partial\Psi}{\partial z} + (M + R)\varepsilon\} + \frac{\partial^2}{\partial y\partial z}\{L\frac{\partial^2\chi}{\partial z^2} + N\nabla_1^2\chi\} = 0, \tag{3.4}$$

$$\frac{\partial}{\partial y}\{F + Q + L)\frac{\partial\Psi}{\partial z} + (M + R)\varepsilon\} + \frac{\partial^2}{\partial x\partial z}\{L\frac{\partial^2\chi}{\partial z^2} + N(\frac{\partial^2\chi}{\partial x^2} + \frac{\partial^2\chi}{\partial y^2})\} = 0. \tag{3.5}$$

Both these relations are satisfied if

$$\varepsilon = -K_3\frac{\partial\Psi}{\partial z} \tag{3.6}$$

where $K_3 = (F + L + Q)/(M + R)$. (3.7)

Then relation (2.5) gives

$$LK_1 \nabla_1^4\phi + \{K_1(C + Q) + K_2L - (F + M + L)\}\nabla_1^2\frac{\partial^2\phi}{\partial z^2} + K_2(C + Q)\frac{\partial^2\phi}{\partial z^4} + \nabla_1^2\Psi L$$
$$+ \{C + Q - K_3(Q + R)\}\frac{\partial^2\Psi}{\partial z^2} = 0. \tag{3.8}$$

The equation (3.8) may be written in the form

$$K_1(\nabla_1^2 + \frac{1}{r_1^2}\frac{\partial^2}{\partial z^2})(\nabla_1^2 + \frac{1}{r_2^2}\frac{\partial^2}{\partial z^2})\phi + (\nabla_1^2 + \frac{1}{r_3^2}\frac{\partial^2}{\partial z^2})\Psi = 0 \tag{3.9}$$

where

$$K_1(\frac{1}{r_1^2} + \frac{1}{r_2^2}) = \{K_1(C + Q) + K_2L - (F + M + L)\}/L, \tag{3.10}$$

$$K_1L = K_2(C + Q)\, r_1^2\, r_2^2, \tag{3.11}$$

$$\{C + Q - K_3(Q + R)\}\, r_3^2 = L. \tag{3.12}$$

We can write the equations (2.13) - (2.15) in the combined form as

$$\frac{1}{b_1}\nabla_1^2\sigma + \frac{1}{b_2}\frac{\partial^2\sigma}{\partial z^2} = \frac{\partial}{\partial t}(\varepsilon - e), \tag{3.13}$$

where

$$e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}. \tag{3.14}$$

Since the σ can be expressed in terms of ϕ, Ψ and χ using (2.6) - (2.12) and (3.1) - (3.5) as

$$\sigma = \frac{\partial}{\partial z}\{(QK_1 - M)\nabla_1^2\phi + QK_2\frac{\partial^2\phi}{\partial z^2} + (Q - RK_3)\Psi\} . \quad (3.15)$$

Equation (3.13) can also be expressed with the help of (2.6) - (2.10), (3.1) - (3.3) and (3.6) - (3.7) and it is satisfied if

$$(\nabla_1^2 + \frac{b_1}{b_2}\frac{\partial^2}{\partial z^2})\{(QK_1 - M)\nabla_1^2\phi + QK_2\frac{\partial^2\phi}{\partial z^2} + (Q - RK_3)\Psi\}$$

$$= b_1\frac{\partial}{\partial t}\{(1 - K_1)\nabla_1^2\phi - K_2\frac{\partial^2\phi}{\partial z^2} - (1 + K_3)\psi\} . \quad (3.16)$$

The above relation (3.16) may be written in the form

$$(\nabla_1^2 + \frac{1}{r_o^2}\frac{\partial^2}{\partial z^2})(\nabla_1^2 + \frac{1}{r_4^2}\frac{\partial^2}{\partial z^2})\phi + \nu_1(\nabla_1^2 + \frac{1}{r_5^2}\frac{\partial^2}{\partial z^2})\frac{\partial\phi}{\partial t} + \nu_2(\nabla_1^2 + \frac{1}{r_o^2}\frac{\partial^2}{\partial z^2})\Psi + \nu_3\frac{\partial\Psi}{\partial t} = 0$$

$$(3.17)$$

where

$$\frac{1}{r_o^2} = \frac{b_1}{b_2} ; \frac{1}{\gamma_4^2} = \frac{QK_2}{QK_1 - M} ; \frac{1}{r_5^2} = K_2/K_1 - 1 ; \quad (3.18)$$

$$\nu_1 = \frac{b_1(K_1 - 1)}{QK_1 - M} ; \nu_2 = \frac{Q - RK_3}{QK_1 - M} ; \nu_3 = \frac{b_1(1 + K_3)}{QK_1 - M} . \quad (3.19)$$

Now eliminating Ψ between equation (3.9) and (3.17) we obtain the differential equation for ϕ :

$$(\nabla_1^2 + \frac{1}{r_o^2}\frac{\partial^2}{\partial z^2})\{(\nabla_1^2 + \frac{1}{r_3^2}\frac{\partial^2}{\partial z^2})(\nabla_1^2 + \frac{1}{r_4^2}\frac{\partial^2}{\partial z^2}) - K_1\nu_2(\nabla_1^2 + \frac{1}{r_1^2}\frac{\partial^2}{\partial z^2})(\nabla_1^2 + \frac{1}{r_2^2}\frac{\partial^2}{\partial z^2})\}\phi$$

$$= \{K_1\nu_3(\nabla_1^2 + \frac{1}{r_1^2}\frac{\partial^2}{\partial z^2})(\nabla_1^2 + \frac{1}{r_2^2}\frac{\partial^2}{\partial z^2}) - \nu_1(\nabla_1^2 + \frac{1}{r_3^2}\frac{\partial^2}{\partial z^2})(\nabla_1^2 + \frac{1}{r_5^2}\frac{\partial^2}{\partial z^2})\}\frac{\partial\phi}{\partial t} . \quad (3.20)$$

This equation is written in the form

$$\nabla_{10}^2\nabla_{11}^2\nabla_{12}^2\phi = \alpha\nabla_{13}^2\nabla_{14}^2\frac{\partial\phi}{\partial t} , \quad (3.21)$$

where

$$\nabla_{1i}^2 \equiv \nabla_1^2 + \frac{1}{\delta_i^2}\frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{1}{\delta_i^2}\frac{\partial^2}{\partial z^2} \quad (i = 0, 1, 2, 3, 4). \quad (3.22)$$

With $\delta_o = r_o$,

$$(1 - K_1\nu_2)(\frac{1}{\delta_1^2} + \frac{1}{\delta_2^2}) = \frac{1}{r_3^2} + \frac{1}{r_4^2} - K_1\nu_2(\frac{1}{r_1^2} + \frac{1}{r_2^2}) , \quad (3.23)$$

$$\frac{1 - K_1 \nu_2}{\delta_1^2 \delta_2^2} = \frac{1}{r_3^2 r_4^2} - \frac{K_1 \nu_2}{r_1^2 r_2^2} \; ; \; \alpha = \frac{K_1 \nu_3 - \nu_1}{1 - K_1 \nu_2} \, , \tag{3.24}$$

$$(K_1 \nu_3 - \nu_1)(\frac{1}{\delta_3^2} + \frac{1}{\delta_4^2}) = K_1 \nu_3 (\frac{1}{r_1^2} + \frac{1}{r_2^2}) - \nu (\frac{1}{r_3^2} + \frac{1}{r_5^2}) \, , \tag{3.25}$$

$$\frac{K_1 \nu_3 - \nu_1}{\delta_3^2 \delta_4^2} = \frac{K_1 \nu_3}{r_1^2 r_2^2} - \frac{\nu_1}{r_3^2 \cdot r_5^2} \, . \tag{3.26}$$

Again elimination of ϕ between (3.9) and (3.22) gives the differential equation for Ψ

$$\nabla_{10}^2 \nabla_{11}^2 \nabla_{12}^2 \Psi \equiv \alpha \nabla_{13}^2 \nabla_{14}^2 \frac{\partial \Psi}{\partial t} \, . \tag{3.27}$$

The reduced relations of (2.3) - (2.5) with the help of (2.6) - (2.12) and (3.1) - (3.3) give the differential equation for χ

$$\frac{\partial^2 \chi}{\partial z^2} + \frac{N}{L} \nabla_1^2 \chi = 0 \, . \tag{3.28}$$

4. STRESS COMPONENTS.

The stress components can now be expressed in terms of the stress functions ϕ, Ψ and χ. Thus, from (2.6) - (2.10) with the help of (2.11) - (2.15) and (3.6) we get

$$\sigma_x = \frac{\partial}{\partial z} \{(FK_1 - P) \frac{\partial^2 \phi}{\partial x^2} + (FK_1 - A) \frac{\partial^2 \phi}{\partial y^2} + FK \frac{\partial^2 \phi}{\partial z^2} + (F - MK_3)\Psi + 2N \frac{\partial^2 \chi}{\partial x \partial y}\} \, , \tag{4.1}$$

$$\sigma_y = \frac{\partial}{\partial z} \{(FK_1 - A) \frac{\partial^2 \phi}{\partial x^2} + (FK_1 - P) \frac{\partial^2 \phi}{\partial y^2} + FK_2 \frac{\partial^2 \phi}{\partial z^2} + (F - MK_3)\Psi - 2N \frac{\partial^2 \chi}{\partial x \partial y}\} \, , \tag{4.2}$$

$$\sigma_z = \frac{\partial}{\partial z} \{(CK_1 - F) \nabla_1^2 \phi + CK_2 \frac{\partial^2 \phi}{\partial z^2} + (C - QK_3)\Psi\} \, , \tag{4.3}$$

$$\tau_{xy} = -2N \frac{\partial^3 \phi}{\partial x \partial y \partial z} + N \frac{\partial}{\partial z} (\frac{\partial^2 \chi}{\partial y^2} - \frac{\partial^2 \chi}{\partial x^2}) \, , \tag{4.4}$$

$$\tau_{yz} = L \frac{\partial}{\partial y} \{K_1 \nabla_1^2 \phi + (K_2 - 1) \frac{\partial^2 \phi}{\partial z^2} + \Psi\} - L \frac{\partial^3 \chi}{\partial x \partial z^2} \, , \tag{4.5}$$

$$\tau_{xz} = L \frac{\partial}{\partial x} \{K_1 \nabla_1^2 \phi + (K_2 - 1) \frac{\partial^2 \phi}{\partial z^2} + \Psi\} + L \frac{\partial}{\partial y} (\frac{\partial^2 \chi}{\partial y^2}) \, . \tag{4.6}$$

The component σ is given by the relation (3.15).

5. HALF-SPACE UNDER GIVEN NORMAL LOADS.

Let the half-space be bounded by the plane $y = 0$ and let the positive direction of the y-axis be taken into the half-space considered. Let a distributed load of constant magnitude q per unit area be applied on the surface through a porous smooth rectangular slab whose sides are given by $x = \pm a$, $z = \pm b$. It will be assumed that the system is at rest and unstrained initially and the load is applied instantaneously. The boundary and initial conditions are then as follows:

$$\sigma_y = \begin{cases} 0 - q\delta(t) & \text{on } y = 0, \; |x| < a, \; |z| < b \, , \; t > 0 \\ 0 & \text{elsewhere on } y = 0. \end{cases} \tag{5.1}$$

$$\tau_{yz} = \tau_{xy} = 0 \text{ on } y = 0,\ |x| \geq 0,\ |z| \geq 0,\ t > 0 \tag{5.2}$$

$$\sigma = 0 \text{ on } y = 0,\ |x| \geq 0,\ |z| > 0,\ r > 0\ . \tag{5.3}$$

6. TRANSFORM SOLUTION.

We introduce the Laplace transform $\bar{f}$ (x, y, z, p) of the function f (x, y, z, t) defined by

$$\bar{f}\,(x, y, z, p) = \int_0^\infty f(x, y, z, t)\, e^{-pt} dt\ , \tag{6.1}$$

the inverse of Laplace transforms $\bar{\phi}$, $\bar{\Psi}$, $\bar{x}$ being $\phi = \frac{1}{2\pi i}\int_{\nu-i\alpha}^{\nu+i\alpha} \bar{\phi} e^{pt} dp$, etc. The functions $\bar{\bar{\Phi}}$, $\bar{\bar{\Psi}}$ and $\bar{\bar{\chi}}$ are assumed in the form

$$\bar{\bar{\Phi}}(\xi, y, \eta, p) = {}_0\!\int^\infty {}_0\!\int^\infty \bar{\phi}(x, y, z, p) \cos(\xi x)\sin(\eta z) dx dz\ , \tag{6.2}$$

$$\bar{\bar{\Psi}}(\xi, y, \eta, p) = {}_0\!\int^\infty {}_0\!\int^\infty \bar{\Psi}(x, y, z, p) \cos(\xi x)\sin(\eta z) dx dz\ , \tag{6.3}$$

$$\bar{\bar{\chi}}(\xi, y, \eta, p) = {}_0\!\int^\infty {}_0\!\int^\infty \bar{\chi}(x, y, z, p) \sin(\xi x)\sin(\eta z) dx dz\ . \tag{6.4}$$

Then using the conditions (5.1) - (5.3) and the definitions (6.1) - (6.4), the equations (3.22), (3.27) and (3.28) can be written as

$$\nabla_{10}^2\, \nabla_{11}^2\, \nabla_{12}^2 \bar{\bar{\phi}} = \alpha p\, \nabla_{13}^2\, \nabla_{14}^2 \bar{\bar{\phi}} \tag{6.5}$$

$$\nabla_{10}^2\, \nabla_{11}^2\, \nabla_{12}^2 \bar{\bar{\Psi}} = \alpha p\, \nabla_{13}^2\, \nabla_{14}^2 \bar{\bar{\Psi}} \tag{6.6}$$

and

$$\frac{\partial^2 \bar{\bar{\chi}}}{\partial z^2} + \frac{N}{L} \nabla_1^2 \bar{\bar{\chi}} = 0\ . \tag{6.7}$$

This last relation with the help of $\bar{\bar{\phi}}$ given by (6.2) is satisfied if

$$\Big[\{D^2 - (\xi^2 + \frac{\eta^2}{\delta_0^2})\}\{D^2 - (\xi^2 + \frac{\eta^2}{\delta_1^2})\}\{D^2 - (\xi^2 + \frac{\eta^2}{\delta_2^2})\}$$

$$- \alpha p\, \{D^2 - (\xi^2 + \frac{\eta^2}{\delta_3^2})\}\{D^2 - (\xi^2 + \frac{\eta^2}{\delta_4^2})\}\Big] \bar{\bar{\phi}} = 0, \quad \text{where } D = \frac{\partial}{\partial y}\ . \tag{6.8}$$

This will give $\bar{\bar{\Phi}}$ in the form

$$\bar{\bar{\Phi}} = \sum_{i=1}^{3} B_i\, e^{-m_i y}\ , \tag{6.9}$$

where m_1, m_2, m_3 are roots, with positive real parts, of

$$\{m^2 - (\xi^2 + \frac{\eta^2}{\delta_0^2})\}\{m^2 - (\xi^2 + \frac{\eta^2}{\delta_1^2})\}\{m^2 - (\xi^2 + \frac{\eta^2}{\delta_2^2})\}$$

$$-\alpha p\, \{m^2 - (\xi^2 + \frac{\eta^2}{\delta_3^2})\}\{m^2 - (\varepsilon^2 + \frac{\eta^2}{\delta_4^2})\} = 0\ . \tag{6.10}$$

Similarly we can obtain

$$\bar{\bar{\Psi}} = \sum_{i=1}^{3} C_i e^{-m_i y} . \tag{6.11}$$

From (3.9) we get

$$K_1 B_i \{(\xi^2 + \frac{\eta^2}{r_1^2} - m_i^2)(\xi^2 + \frac{\eta^2}{r_2^2} - m_i^2)\} + C_i \{\xi^2 + \frac{\eta^2}{r_3^2} - m_i^2\} = 0 \text{ for } i = 1, 2, 3. \tag{6.12}$$

Also

$$\bar{\chi} = D_1 e^{-my} , \tag{6.13}$$

where m is the square root with positive real part such that

$$\xi^2 + \eta^2 \frac{L}{N} = m^2 . \tag{6.14}$$

Now the Laplace transform of (5.1) gives

$$\bar{\sigma}_y = -q \text{ on } y = 0 \text{ for } |x| < a, |z| < b , \tag{6.15}$$

and

$$\bar{\tau}_{yz} = \bar{\tau}_{xy} = 0 \text{ and } \bar{\sigma} = 0 \text{ on } y = 0 . \tag{6.16}$$

These conditions can be expressed as

$$\bar{\bar{\sigma}}_y (x, z) = - \frac{4q}{\pi^2} \int_0^\infty \int_0^\infty \frac{\sin(\xi a)\sin(\eta b)}{\xi\eta} \cos(\xi x)\cos(\eta z) d\xi d\eta \tag{6.17}$$

and

$$\bar{\bar{\tau}}_{yz} = \bar{\bar{\tau}}_{xy} = 0 \text{ and } \bar{\bar{\sigma}} = 0 \text{ on } y = 0 . \tag{6.18}$$

Equations (3.15), (4.4) give

$$\bar{\bar{\sigma}}_y = \frac{\partial}{\partial z} \{(FK_1 - A) \frac{\partial^2 \bar{\phi}}{\partial x^2} + (FK_1 - P) \frac{\partial^2 \bar{\phi}}{\partial y} + FK_2 \frac{\partial^2 \phi}{\partial z^2} + (F - MK_3)\bar{\Psi} - 2N \frac{\partial^2 \bar{\chi}}{\partial x \partial y}\} , \tag{6.19}$$

$$\bar{\tau}_{xy} = -2N \frac{\partial^3 \bar{\phi}}{\partial x \partial y \partial z} + N \frac{\partial}{\partial z} (\frac{\partial^2 \bar{\chi}}{\partial x^2} - \frac{\partial^2 \bar{\chi}}{\partial x^2}) , \tag{6.20}$$

$$\bar{\tau}_{yz} = L \frac{\partial}{\partial y} \{K_1 \nabla_1^2 \bar{\phi} + (K_2 - 1) \frac{\partial^2 \bar{\phi}}{\partial z^2} + \bar{\Psi}\} - L \frac{\partial^3 \bar{\chi}}{\partial x \partial z^2} . \tag{6.21}$$

From the boundary conditions (6.17) - (6.18) on y = 0, we obtain (from (6.2) - (6.4) and (6.14))

$$\bar{\sigma}_y = -(FK_1 - A)\xi^2\eta \sum_{i=0}^{3} B_i + (FK_1 - P) \sum_{i=1}^{3} m_i^2 B_i - FK_2\eta^3 \sum_{i=1}^{3} B_i$$

$$+ (F - MK_3)\eta \sum_{i=1}^{3} C_i + 2N\xi\eta D_1 m = - \frac{4q}{\pi^2} \int_0^\infty \int_0^\infty \frac{\sin(\xi a)\sin(\eta b)}{\xi\eta} d\xi d\eta \tag{6.22}$$

$$\bar{\bar{\tau}}_{yz} = -LM\left[\{K_1 (m_i^2 - \xi^2) + (K_2 - 1)\eta^2\} B_i + \sum_{i=1}^{3} C_i\right] + L\xi\eta^2 D_1 = 0 , \tag{6.23}$$

$$\bar{\bar{\tau}}_{xy} = -2N\xi\eta \sum_{i=1}^{3} B_i m_i + N\eta D_1 (m^2 + \xi^2) = 0 , \tag{6.24}$$

$$\bar{\bar{\sigma}} = (QK_1 - M)\eta \sum_{i=1}^{3} B_i (m_i^2 - \xi^2) - QK_2\eta^3 \sum_{i=1}^{3} B_i + (Q - RK_3)\eta \sum_{i=1}^{3} C_i = 0 . \tag{6.25}$$

From (6.12) and (6.22)-(6.25) we can formally calculate B_i; C_i (i = 1, 2, 3) and D_1.

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