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WEAK CONTINUITY AND ALMOST CONTINUITY

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ABSTRACT. Two relationships considered by Weston [1] for a pair of topologies on a set X are translated to a function setting. An attempt to characterize the two resulting types of functions leads to new characterizations of weak continuity and almost continuity. After showing that weak continuity and almost continuity are independent, interrelationships are sought. This leads to the definition of subweak continuity and a new characterization for almost openness. Finally, several published results are strengthened or slightly extended.

KEY WORDS AND PHRASES. *Weakly continuous function, almost continuous function, subweakly continuous function.*
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1. INTRODUCTION.

Two of the most studied generalizations of continuity are the weak continuity of Levine [2] and the almost continuity of Husain [3]. Both have been used to characterize certain classes of topological spaces ([4], [5]). Almost continuity has also played an important role in obtaining topological versions of the classical closed graph theorem [6]. We will establish new characterizations for weak continuity and almost continuity, determine conditions under which each will imply the other, and then use these results to extend certain known results in the literature.

2. DEFINITIONS AND PRELIMINARIES.

Unless otherwise specified, $f : X \to Y$ will denote an arbitrary function between arbitrary topological spaces X and Y. Further, for any subset A of a topological space, Cl A and Int A will denote the closure and the interior respectively of A.

DEFINITION 1. (Levine [2]). A function $f : X \to Y$ is weakly continuous if for each neighborhood V of f(x) there exists a neighborhood U of x such that $f(U) \subseteq \mathrm{Cl}\ V$.

Some straightforward consequences of Definition 1 follow.

THEOREM 1. (Levine [2]) A function $f : X \to Y$ is weakly continuous if and only if $f^{-1}(V) \subseteq \mathrm{Int}\, f^{-1}(\mathrm{Cl}\, V)$ for each open subset V of Y.

THEOREM 2. (Levine [2]) If $f : X \to Y$ is weakly continuous and Y is regular, then f is continuous.

THEOREM 3. (Noiri [7]) If $f : X \to Y$ is weakly continuous and Y is Hausdorff, then f has a closed graph $G(f) \subseteq X \times Y$.

DEFINITION 2. (Husain [3]) A function $f : X \to Y$ is almost continuous if, for each $x \in X$ and each neighborhood V of $f(x)$, $\mathrm{Cl}\, f^{-1}(V)$ is a neighborhood of x. (Almost continuity was studied for real-valued functions on Euclidean space by Blumberg [8] in 1922.)

THEOREM 4. A function $f : X \to Y$ is almost continuous if and only if $f^{-1}(V) \subseteq \mathrm{Int}\, \mathrm{Cl}\, f^{-1}(V)$ for each open subset V of Y.

Clearly continuity for a function $f : X \to Y$ implies weak continuity and almost continuity. The independence of weak continuity and almost continuity is shown by the following examples.

EXAMPLE 1. Let Z be any set with at least two elements. Let X = Z have the indiscrete topology, Y = Z have the discrete topology, and $f : X \to Y$ be the identity function on Z. Then f is discontinuous and Y is regular so that, by Theorem 2, f is not weakly continuous. However, f is almost continuous since every nonempty subset of X is dense in X.

EXAMPLE 2. Let R be the set of real numbers and let S be the smallest extension of the usual topology containing Q, the set of rational numbers. Let T be the smallest extension of the usual topology containing R − Q, the set of irrational numbers. If X = (R,S) and Y = (R,T) and $f : X \to Y$ is the identity function on R, then f is weakly continuous but not almost continuous.

By the above examples, weak continuity and almost continuity are strictly weaker than continuity and Example 3 of [9] gives a discontinuous function which is both weakly continuous and almost continuous.

The following useful result says that to insure weak continuity (almost continuity) by Theorem 1 (Theorem 4) above, it is sufficient to consider only basic open subsets of the range.

THEOREM 5. A function $f : X \to Y$ is weakly (almost) continuous if and only if there is an open basis B for the topology on Y such that $f^{-1}(V) \subseteq \mathrm{Int}\, f^{-1}(\mathrm{Cl}\, V)$ $[f^{-1}(V) \subseteq \mathrm{Int}\, \mathrm{Cl}\, f^{-1}(V)]$ for each $V \in B$.

Singal and Singal [10] introduced a different almost continuity which herein is referred to as almost continuity (S and S).

DEFINITION 3. (Singal and Singal [10]). A function $f : X \to Y$ is almost continuous (S and S) if, for each $x \in X$ and for each neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(U) \subseteq \mathrm{Int}\, \mathrm{Cl}\, V$.

Long and Herrington [11] have noted that a function $f : X \to (Y,T)$ is almost continuous (S and S) if and only if $f : X \to (Y,T_s)$ is continuous, where T_s is the semiregular subtopology of T generated by the regular open sets in (Y,T). Continuity implies almost continuity (S and S) which in turn implies weak continuity and neither implication is reversible. Singal and Singal [10] gave an example of a function which is weakly continuous but not almost continuous (S and S) and the function of Example 2 above is almost continuous (S and S) but not almost continuous and hence not continuous. The function of Example 1 is almost continuous but not almost continuous (S and S). Further, the discontinuous function of Example 3 of [9] is both almost continuous (S and S) and almost continuous.

3. NEW CHARACTERIZATIONS.

Before weak continuity and almost continuity had been formally introduced into the literature, Weston [1] considered the following two conditions for a set X endowed with two topologies T and T*. Let Cl A and Cl* A denote the T and T* closures of A respectively for each subset A of X.

$$\mathrm{Cl}\ U \subseteq \mathrm{Cl}^*\ U \text{ for each } U \in T^* \tag{3.1}$$

$$\mathrm{Cl}\ U \subseteq \mathrm{Cl}^*\ U \text{ for each } U \in T \tag{3.2}$$

To express (3.1) and (3.2) in function language, let us agree that, for any function $f : (X,T) \to (Y, T^{**})$, we will let $T^* = f^{-1}(T^{**}) = \{f^{-1}(V) : V \in T^{**}\}$. Define $f : X \to Y$ to be a $W_1(W_2)$ function if (3.1) ((3.2)) is satisfied.

Attempting to characterize the W_1 (W_2) functions in terms of the domain and range topologies only led to the following related conditions for a function $f : X \to Y$.

$$\mathrm{Cl}\ f^{-1}(V) \subseteq f^{-1}(\mathrm{Cl}\ V) \text{ for each open subset } V \text{ of } Y \tag{3.3}$$

$$f(\mathrm{Cl}\ U) \subseteq \mathrm{Cl}\ f(U) \text{ for each open subset } U \text{ of } X \tag{3.4}$$

These conditions relate to weak continuity and almost continuity as follows.

THEOREM 6. For any function $f : X \to Y$ the following are equivalent.

1) f is almost continuous
2) f is a W_2 function
3) $f(\mathrm{Cl}\ U) \subseteq \mathrm{Cl}\ f(U)$ for each open subset U of X

PROOF: The equivalence of 1) and 2) was proven by Weston [1]. It remains to show that 2) implies 3) and 3) implies 1). If f is a W_2 function and U is any open subset of X, then $f(\mathrm{Cl}\ U) \subseteq f(\mathrm{Cl}^*\ U)$. Since $f : (X,T^*) \to Y$ is continuous, $f(\mathrm{Cl}^*\ U) \subseteq \mathrm{Cl}\ f(U)$ so that $f(\mathrm{Cl}\ U) \subseteq \mathrm{Cl}\ f(U)$. To show that 3) implies 1), suppose that $f(\mathrm{Cl}\ U) \subseteq \mathrm{Cl}\ f(U)$ for each open subset U of X. Let V be an open subset of Y and let $x \in f^{-1}(V)$. If $x \notin \mathrm{Int}\ \mathrm{Cl}\ f^{-1}(V)$, then $x \in \mathrm{Cl}\ U$ where $U = X - \mathrm{Cl}\ f^{-1}(V)$ so that $f(x) \in \mathrm{Cl}\ f(U) \cap V$ and hence $f(U) \cap V \neq \emptyset$. But this contradicts the fact that $U \subseteq X - f^{-1}(V)$. Thus $x \in \mathrm{Int}\ \mathrm{Cl}\ f^{-1}(V)$ and f is almost continuous.

From Theorem 6 we have the following interesting comparison of almost continuity with continuity.

COROLLARY. A function $f : X \to Y$ is (almost) continuous if and only if $f(\mathrm{Cl}\ U) \subseteq \mathrm{Cl}\ f(U)$ for each (open) subset U of X.

Turning now to weak continuity we have the following characterization.

THEOREM 7. A function $f : X \to Y$ is weakly continuous if and only if $\mathrm{Cl}\ f^{-1}(V) \subseteq f^{-1}(\mathrm{Cl}\ V)$ for each open subset V of Y.

PROOF: T. Noiri [12] proved the necessity. For the sufficiency let V be an open subset of Y and let $W = Y - \mathrm{Cl}\, V$. Then $\mathrm{Cl}\, f^{-1}(W) \subseteq f^{-1}(\mathrm{Cl}\, W)$ implies $X - \mathrm{Int}\, f^{-1}(\mathrm{Cl}\, V) \supseteq X - f^{-1}(\mathrm{Int}\, \mathrm{Cl}\, V) \subseteq X - f^{-1}(V)$. Thus $f^{-1}(V) \subseteq \mathrm{Int}\, f^{-1}(\mathrm{Cl}\, V)$ so that f is weakly continuous.

THEOREM 8. (1) Every continuous function is a W_1 function and (2) every W_1 function is weakly continuous.

PROOF: If $f : X \to Y$ is continuous, then $T^* \subseteq T$ and $\mathrm{Cl}\, U \subseteq \mathrm{Cl}^* U$ for all subsets U of X so that f is a W_1 function. (2) If f is a W_1 function and V is an open subset of Y, then $f^{-1}(V) \in T^*$ and $\mathrm{Cl}\, f^{-1}(V) \subseteq \mathrm{Cl}^* f^{-1}(V) \subseteq \mathrm{Cl}^* f^{-1}(\mathrm{Cl}\, V) = f^{-1}(\mathrm{Cl}\, V)$ since $f : (X, T^*) \to Y$ is continuous. By Theorem 7, f is weakly continuous.

It can be shown that the class of W_1functions properly contains the class of continuous functions and is properly contained in the class of weakly continuous functions. Furthermore this class neither contains nor is contained in the class of almost continuous (S and S) functions. Example 2.3 of Singal and Singal [10] gives a W_1 function which is not almost continuous (S and S) and hence not continuous. The following example presents an almost continuous (S and S) function, and hence weakly continuous, which is not a W_1 function.

EXAMPLE 3. Let $X = \{a, b\}$ have the indiscrete topology $T = \{\emptyset, X\}$. Let $Y = \{a,b,c\}$ have the topology $T^{**} = \{\emptyset, Y, \{a, c\}, \{b, c\} \{c\}\}$ and define $f : X \to Y$ to be the inclusion function. Then $T^* = f^{-1}(T^{**}) = \{\emptyset, X, \{a\}, \{b\}\}$ is the discrete topology on X. For $\{a\} \in T^*$, $\mathrm{Cl}\, \{a\} = X$ whereas $\mathrm{Cl}^* \{a\} = \{a\}$. Thus f is not a W_1 function. But for each nonempty $V \in T^{**}$, $\mathrm{Int}\, \mathrm{Cl}\, V = Y$ so that f is almost continuous (S and S) and hence also weakly continuous.

A partial converse to Theorem 8 (2) is now found.

THEOREM 9. If $f : X \to Y$ is weakly continuous and $f(X)$ is an open subset of Y, then f is a W_1 function.

PROOF: Let V be any open subset of Y and suppose that $x \in \mathrm{Cl}\, f^{-1}(V)$. To show that $x \in \mathrm{Cl}^* f^{-1}(V)$, let W be an open subset of Y with $x \in f^{-1}(W)$. By Theorem 7, $\mathrm{Cl}\, f^{-1}(V) \subseteq f^{-1}(\mathrm{Cl}\, V)$ so that $f(x) \in (\mathrm{Cl}\, V) \cap (W \cap f(X))$. Since $W \cap f(X)$ is open, $V \cap (W \cap f(X)) = (V \cap W) \cap f(X) \neq \emptyset$ so that $f^{-1}(V \cap W) = f^{-1}(V) \cap f^{-1}(W) \neq \emptyset$ and $x \in \mathrm{Cl}^* f^{-1}(V)$.

COROLLARY. If $f : X \to Y$ is a surjection, then f is weakly continuous if and only if f is a W_1 function.

PROOF: For a surjection $f : X \to Y$, $f(X) = Y$ so that the conclusion follows from Theorem 9 and Theorem 8 (2).

The W_1 function $f : X \to Y$ given in Example 2.3 of Singal and Singal [10] is such that $f(X)$ is not an open subset of Y. Thus the hypothesis of Theorem 9 is sufficient but not necessary for a W_1 function.

4. SUBWEAK CONTINUITY AND INTERRELATIONSHIPS.

For general spaces X and Y, what property for a function $f : X \to Y$ would cause almost continuity to imply weak continuity for f? The only published answer (known to the author) is given by Noiri [12], who found that any almost continuous function satisfying (3.3) is weakly continuous. Since (3.3) is equivalent to weak continuity by Theorem 8, the almost continuity in Noiri's result is superfluous and the above question is still open. To answer this question, we define subweak continuity as follows.

DEFINITION 4. A function $f : X \to Y$ is subweakly continuous if there is an open basis B for the topology on Y such that $\mathrm{Cl}\, f^{-1}(V) \subseteq f^{-1}(\mathrm{Cl}\, V)$ for each $V \in B$.

Before showing that every subweakly continuous almost continuous function is weakly continuous, the following example demonstrates that subweak continuity by itself does not imply weak continuity. However, by Theorem 8, every weakly continuous function is subweakly continuous.

EXAMPLE 4. Let X be any set with a non-discrete T_1 topology (singleton sets are closed), T. Let Y = X have the discrete topology T** and let $f : X \to Y$ be the identity function. Let B be the open basis of singleton subsets of Y. Then $\mathrm{Cl}\, f^{-1}(y) = f^{-1}(\mathrm{Cl}\, y)$ for each $y \in Y$ so that f is subweakly continuous. But since f is discontinuous and Y is regular, by Theorem 2, f is not weakly continuous.

THEOREM 10. If $f : X \to Y$ is an almost continuous subweakly continuous function, then f is weakly continuous.

PROOF: Let B be an open basis for the topology on Y relative to which f is subweakly continuous. If $V \in B$, $f^{-1}(V) \subseteq \mathrm{Int}\,\mathrm{Cl}\, f^{-1}(V) \subseteq \mathrm{Int}\, f^{-1}(\mathrm{Cl}\, V)$ so that by Theorem 5, the conclusion follows.

Examples 1 and 2 show the independence of subweak continuity and almost continuity. Also, the function of Example 4 cannot be almost continuous by Theorem 10.

We now turn to the question "Under what condition for a function $f : X \to Y$ will weak continuity imply almost continuity?" Singal and Singal [10] proved that every open weakly continuous function is almost continuous (S and S). Long and Carnahan [13] showed that every open almost continuous (S and S) function is almost continuous. Thus "openness" is an answer. We will show that "almost openness" as defined below is a better answer being weaker than "openness".

DEFINITION 5. A function $f : X \to Y$ is almost open if $f(U) \subseteq \mathrm{Int}\,\mathrm{Cl}\, f(U)$ for each open subset U of X.

Clearly every open function is almost open. Also, note that for a bijection $f : X \to Y$, f is (almost) continuous if and only if f^{-1} is (almost) open. Thus Example 1 shows that almost openness does not imply openness.

REMARK. The almost openness defined above is the almost openness as applied to homomorphisms between topological groups by Brown [14] and others to obtain topological versions of the open mapping theorem.

Next we will observe that, just as (3.4) is equivalent to almost continuity by Theorem 6, by replacing f with f^{-1} and U with any open subset V of Y in (3.4), a characterization for almost openness is obtained.

THEOREM 11. A function $f : X \to Y$ is almost open if and only if $f^{-1}(\mathrm{Cl}\, V) \subseteq \mathrm{Cl}\, f^{-1}(V)$ for each open subset V of Y.

PROOF: Assume first that f is almost open. If V is an open subset of Y, $x \in f^{-1}(\mathrm{Cl}\, V)$, and U is an open subset of X containing x, then $f(x) \in f(U) \cap \mathrm{Cl}\, V \subseteq [\mathrm{Int}\,\mathrm{Cl}\, f(U)] \cap \mathrm{Cl}\, V$ so that $V \cap \mathrm{Int}\,\mathrm{Cl}\, f(U) \neq \emptyset$. Thus $V \cap f(U) \neq \emptyset$ so that $U \cap f^{-1}(V) \neq \emptyset$. Hence $x \in \mathrm{Cl}\, f^{-1}(V)$. Conversely, suppose that $f^{-1}(\mathrm{Cl}\, V) \subseteq \mathrm{Cl}\, f^{-1}(V)$ for each open subset V of Y. If f is not almost open then for some open subset U of

X, $f(U) \not\subseteq \operatorname{Int} \operatorname{Cl} f(U)$. Let $V = Y - \operatorname{Cl} f(U)$. Then $f(U) \cap V = \emptyset$ but $f(U) \cap \operatorname{Cl} V \neq \emptyset$. Thus $U \cap f^{-1}(\operatorname{Cl} V) \neq \emptyset$ and, by hypothesis, $U \cap \operatorname{Cl} f^{-1}(V) \neq \emptyset$. Hence, $U \cap f^{-1}(V) \neq \emptyset$, in contradiction to the fact that $f(U) \cap V = \emptyset$.

THEOREM 12. If $f : X \to Y$ is an almost open and weakly continuous function, then f is almost continuous.

PROOF: By Theorems 1 and 11, $f^{-1}(V) \subseteq \operatorname{Int} f^{-1}(\operatorname{Cl} V) \subseteq \operatorname{Int} \operatorname{Cl} f^{-1}(V)$ for each open subset V of Y so that f is almost continuous by Theorem 4.

In a letter of correspondence Takashi Noiri improved the aforementioned result of Singal and Singal [10] by replacing "openness" with "almost openness". The improved result with Noiri's proof is now given.

THEOREM 13. (Noiri) If $f : X \to Y$ is almost open and weakly continuous, then f is almost continuous (S and S).

PROOF: Let F be a regular closed subset of Y. By Theorems 7 and 11, $f^{-1}(\operatorname{Cl} V) = \operatorname{Cl} f^{-1}(V)$ for each open subset V of Y. Thus $f^{-1}(F) = f^{-1}(\operatorname{Cl} \operatorname{Int} F) = \operatorname{Cl} f^{-1}(\operatorname{Int} F)$ is a closed subset of X.

5. APPLICATIONS. Some results in the literature can now be strengthened. The following theorem improves the result of Long and Carnahan [13] that, for every open almost continuous (S and S) function $f : X \to Y$, $\operatorname{Cl} f^{-1}(V) = f^{-1}(\operatorname{Cl} V)$ for each open subset V of Y.

THEOREM 14. A function $f : X \to Y$ is almost open and almost continuous (S and S) if and only if $\operatorname{Cl} f^{-1}(V) = f^{-1}(\operatorname{Cl} V)$ for each open subset V of Y.

PROOF: By Theorem 13, a function $f : X \to Y$ is almost open and almost continuous (S and S) if and only if f is almost open and weakly continuous which by Theorems 7 and 11 is equivalent to $\operatorname{Cl} f^{-1}(V) = f^{-1}(\operatorname{Cl} V)$ for each open subset V of Y.

Replacing "almost continuous (S and S)" by "weakly continuous" in Theorem 14 would be an apparent, but not an actual, strengthening of the theorem since the two are equivalent for almost open functions. Theorem 14 implies the following.

COROLLARY. If $f : X \to Y$ is an almost open function, then f is almost continuous (S and S) if and only if $\operatorname{Cl} f^{-1}(V) = f^{-1}(\operatorname{Cl} V)$ for each open subset V of Y.

This corollary improves another result of Long and Carnahan [13] which insured the same conclusion for every open and almost continuous function; i.e., the openness could be weakened and almost continuity dropped.

Long and McGehee [15] proved that every almost continuous function $f : X \to Y$ into a locally connected regular space Y is continuous if $\operatorname{Cl} f^{-1}(C) \subseteq f^{-1}(\operatorname{Cl} C)$ for each connected subset C of Y. This follows as a corollary to the following theorem.

THEOREM 15. If $f : X \to Y$ is an almost continuous function such that $\operatorname{Cl} f^{-1}(C) \subseteq f^{-1}(\operatorname{Cl} C)$ for each connected subset C of Y and if Y is locally connected then f is weakly continuous.

PROOF: If V is a connected basic open subset of Y, then $\operatorname{Cl} f^{-1}(V) \subseteq f^{-1}(\operatorname{Cl} V)$ and hence f is subweakly continuous. The conclusion now follows from Theorem 10. Further, if Y is regular, f is continuous by Theorem 2.

Long and McGehee [15] also proved that, an almost continuous function $f : X \to Y$ from a Hausdorff space X to a regular Hausdorff locally connected space Y is continuous, if the function and its inverse, preserve connected subsets. Without requiring X and Y to be Hausdorff, we have the following.

THEOREM 16. If $f : X \to Y$ is almost continuous and Y is locally connected, then f is weakly continuous if f and f^{-1} both preserve connected sets.

PROOF: Let V be a connected basic open subset of Y. If $x \in \mathrm{Cl}\, f^{-1}(V) - f^{-1}(\mathrm{Cl}\, V)$, then there is a connected open set W containing $f(x)$ such that $W \cap V = \emptyset$. Since $f^{-1}(V) \neq \emptyset$ and $f^{-1}(W) \neq \emptyset$, $(W \cup V) \cap f(X)$ is disconnected. But $f^{-1}(V)$ and $f^{-1}(W)$ are connected and $[\mathrm{Cl}\, f^{-1}(V)] \cap f^{-1}(W) \neq \emptyset$ so that $f^{-1}(V) \cup f^{-1}(W) = f^{-1}(V \cup W)$ is connected. Thus $f(f^{-1}(V \cup W)) = (V \cup W) \cap f(X)$ is connected. This is a contradiction and therefore $\mathrm{Cl}\, f^{-1}(V) \subseteq f^{-1}(\mathrm{Cl}\, V)$ so that f is subweakly continuous. By Theorem 10, f is weakly continuous and, if Y is regular, f is continuous by Theorem 2.

Long and McGehee [15] also proved that , if in addition to the hypotheses of Theorem 16, Y is Hausdorff, then the graph of f, $G(f)$, is closed. This follows from Theorem 16 above and Theorem 3.

Jones [16] defined a function $f : X \to Y$ to be semi-connected if $f^{-1}(C)$ is closed and connected whenever C is a closed and connected subset of Y.

Evidently, for a semi-connected function $f : X \to Y$, $\mathrm{Cl}\, f^{-1}(C) \subseteq f^{-1}(\mathrm{Cl}\, C)$ for each connected subset C of Y, since $\mathrm{Cl}\, C$ is connected if C is connected. Therefore a semi-connected function into a locally connected space must be subweakly continuous. This proves the following theorem.

THEOREM 17. If $f : X \to Y$ is an almost continuous semi-connected function and Y is locally connected, then f is weakly continuous.

A corollary is the result of Long and McGehee [15] where, in addition to the hypotheses of Theorem 17, Y is assumed regular, and in conclusion f is continuous.

The following theorem implies results of Singal and Singal [10] and Long and Carnahan [13] by replacing "Urysohn" by "Hausdorff".

THEOREM 18. If $f : X \to Y$ is a weakly continuous bijection from a compact space X to a Hausdorff space Y, then f is both open and almost continuous (and almost continuous (S and S)).

PROOF: Let K be a closed subset of X. Then K is compact. By Theorem 3, the graph of f, $G(f) \subseteq X \times Y$, is closed so that $f(K)$ is a closed subset of Y. It follows that f is open. By Theorems 12 and 13, f is almost continuous and almost continuous (S and S).

The notion of C-continuity for a function $f : X \to Y$ was introduced by Gentry and Hoyle [17]. Long and Hendrix [18] proved that a function $f : X \to Y$ is C-continuous if and only if $f^{-1}(C)$ is closed for each closed and compact subset C of Y, and, as a consequence, proved that every closed graph function is C-continuous. A space is strongly locally compact if each point has a closed compace neighborhood. The following theorem and corollaries are immediate consequences.

THEOREM 19. If $f : X \to Y$ is a C-continuous function into a strongly locally compact space Y, then f is subweakly continuous.

PROOF: If V is a basic open set with compact closure, then $\mathrm{Cl}\, f^{-1}(V) \subseteq f^{-1}(\mathrm{Cl}\, V)$.

COROLLARY. If $f : X \to Y$ is an almost continuous and C-continuous function into a strongly locally compact space Y, then f is weakly continuous.

COROLLARY. If $f : X \to Y$ is an almost continuous function with a closed graph into a strongly locally compact space Y, then f is weakly continuous.

Every locally compact Hausdorff or locally compact regular space is strongly locally compact and regular. Therefore, the last corollary above implies a closed graph theorem of Long and McGehee [15], since weak continuity is equivalent to continuity for functions into a regular space. Actually, in [9], it is shown that, under the hypotheses of the final corollary above, f is continuous without requiring Y to be regular. This follows from the fact that strongly locally compact spaces are rim-compact and closed graph weakly continuous functions into rim-compact spaces are continuous.

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