ON THE DISSIPATION LAYER OF RADIAL BEARINGS

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ABSTRACT. The dissipation boundary layer of certain radial bearings is identified. It is shown that, under certain conditions, the temperature outside this layer is constant.

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1. INTRODUCTION.

In paper [1], we discussed the solution to the coupled momentum and energy equations

$$\frac{\partial \phi}{\partial t} = \text{Pr} \frac{\partial}{\partial y} \left(\exp\left(\frac{\lambda_1}{\theta + \lambda_2}\right) \frac{\partial \phi}{\partial y} \right), \quad (1.1)$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - \left(\frac{\partial \theta}{\partial y}\right)^2 + \Delta \exp\left(\theta + \frac{\lambda_1}{\theta + \lambda_2}\right) \left(\frac{\partial \phi}{\partial y}\right)^2 \quad (1.2)$$

where λ_1 , λ_2 and Δ are constants.

The boundary and initial conditions are

$$\frac{\partial \phi}{\partial y}(0, t) = -\frac{B}{\sqrt{t}} \cdot \phi(0, t) \quad \text{given, } \phi(\infty, t) = 0 \quad (1.3)$$

$$\phi(y, 0) = 0 \quad (1.4)$$

$$\theta(0, t) = \theta(\infty, t) = 0 \quad (1.5)$$

$$\theta(y, 0) = 0 \quad (1.6)$$

The above equations arose from investigations on the flow of thin lubricating oils.

Earlier in [2], we discussed the thermal runaway of variable viscosity flows between concentric cylinders - there is a wide gap between the cylinders and we show that, under certain conditions which affect the Peclet number, the reduced Reynolds number and the Nahme-Griffith number, the width of the thermal boundary layer is $O(R)$ where R is the radius of the inner cylinder. Under the same assumptions, there exists a dissipation boundary layer of width $O(R/G^2)$ where G is the Nahme-Griffith number.

In the case of radial bearings, the width, h , of the lubricant is small and thus if R_1, R_2 are the radii of the shaft and the bearing then

$$R_1 \gg h. \quad i = 1, 2 \quad (1.7)$$

(1.7) implies that as a first approximation, we may regard the shaft and the bearing as flat surfaces, that is $R_1 \rightarrow \infty$ when h is the standard measure. We therefore need to investigate the boundary layer of the lubrication problem although $O(R/G^2)$ is adequate for [2].

2. DISSIPATION BOUNDARY LAYER.

Shampine [3] investigated the problem

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x} \left(D(c) \frac{\partial c}{\partial x} \right), \quad (2.1)$$

$$c(0, t) = 1, \quad \frac{\partial c}{\partial x}(0, t) = 0, \quad \text{given} \quad (2.2)$$

He showed that if

$$-D(1) \frac{\partial c}{\partial t}(0, t) = \frac{1}{2} + \max D(c),$$

then there is an $\eta_1 < 1$ such that $c(\eta_1) = 0$, where $\eta = x/\sqrt{t}$. For the proof of this, see [3].

In this paper as in [1], we have

$$\Pr \exp\left(\frac{\lambda_1}{\theta + \lambda_2}\right)$$

in the plane of $D(c)$ above, but a careful study of the proof advanced by Shampine showed that his result is true for (1.1). That is, there exists an $\eta_1 < 1$ such that if

$$B = 1 + \frac{1}{2} e^{-\frac{\lambda_1}{\lambda_2}} \quad (2.3)$$

then there exists an $\eta_1 < 1$ such that $\phi(\eta_1) = 0$ and $\phi(\eta) \equiv 0$ for $\eta \geq \eta_1$. This leads to

THEOREM. Let (1.1), (1.2) (1.3) - (1.6), and (2.3) hold. Then there exists an $\eta_2 \leq 1$ such that $\theta(\eta_2) = 0$ and $\theta(\eta) \equiv 0$ and $\eta \geq \eta_2$.

PROOF. $\phi(\eta_1) \equiv 0$ for $\eta \geq \eta_1$

implies $\frac{\partial \phi}{\partial y} \equiv 0$ for $\eta > \eta_1$.

That is, $\frac{\partial \phi}{\partial y} \equiv 0$ for $\eta \geq \eta_2 = \eta_1^+$

Hence $\theta \equiv 0$ for $\eta \geq \eta_2$.

3. PHYSICAL INTERPRETATION.

The above theorem shows that at time t the heat generated has only penetrated to $y = \eta_2 \sqrt{t}$. This identifies the dissipation layer. The viscosity of the lubricant outside this layer is not affected by heat under the conditions assumed in this problem.

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