Internat. J. Math. & Math. Sci.
VOL. 12 NO. 1 (1989) 39-46

A BOUNDED CONSISTENCY THEOREM FOR STRONG SUMMABILITIES

C.S. CHUN and A.R. FREEDMAN

Department of Mathematics and Statistics
Simon Fraser University
Burnaby, British Columbia Canada V5A 1S6

(Received September 23, 1987)

ABSTRACT. The study of R-type summability methods is continued in this paper by showing that two such methods are identical on the bounded portion of the strong summability field associated with the methods. It is shown that this "bounded consistency" applies for many non-matrix methods as well as for regular matrix methods.

KEY WORDS AND PHRASES. Sequences Spaces, Regular Summability Methods, Zeroclass.
1980 AMS SUBJECT CLASSIFICATION CODES. 40D20, 40F05, 40H05.

1. INTRODUCTION.

In [1] a relation between densities and strong convergence fields was studied for R-type summability methods (RSM). On the space m, the space of all bounded real sequences, RSMs are equivalent to the bounded generalized limits (see [2]). In the main Theorem of [3] Freedman actually proved a consistency theorem for the strong convergence fields of generalized limits. This statement will be explained at the end of Section 2. In this paper we extend the results of [1] and [3] to obtain a Bounded Consistency Theorem for strong summability fields of RSMs. We will require a characterization of RSMs in terms of zeroclasses. This paper is a continuation of [2], therefore we will accept notation and definitions of [1], [2] and [3]. In particular a class X of subsets of I, the set of positive integers, is called a **zeroclass** if the following conditions holds:

(a) A is finite $\Rightarrow A \in X$
(b) $A, B \in X \Rightarrow A \cup B \in X$
(c) $A \subset B \in X \Rightarrow A \in X$
(d) $I \notin X$.

Further, if $x \in \omega$, $r \in R$ and $A \subset I$ with $I - A$ infinite, then by $x \underset{(A)}{\to} r$ we shall mean that for any $\varepsilon > 0$ there exists $N > 0$ such that $|x_n - r| < \varepsilon$ whenever $n \geq N$ and $n \notin A$. If X is a zeroclass, then $\omega_X = \{x \in \omega\colon x \underset{(A)}{\to} r$ for some $A \in X$ and real $r\}$ is called the space of all X-nearly convergent sequences. The sequence space ω_X contains c, the space of all convergent sequences. By a summability method we will simply mean a real valued linear functional S defined on some spaces $C_S \subset \omega$. We shall call S regular if $c \subset C_S$ and $S(x) = \lim x$ for each $x \in c$. We let

$$C_S{}^o = \{x \in C_S: S(x) = 0\},$$

$$|C_S|^o = \{\chi \in \omega: |x| \in C_S{}^o\}$$

and

$$|C_S| = \{x \in \omega: x - r \in |C_S|^o \text{ for some real } r\}.$$

The set $|C_S|$ is called the strong summability field associated with the method S. A method S will be called an RSM when S is regular and $m|C_S|^o = |C_S|^o$. (i.e., $|C_S|^o$ is solid). If S is an RSM, then $|C_S|$, $|C_S|^o$ are subspaces of C_S ([1] Proposition 4.9). If $S: C_S \to R$ is an RSM, then $X_S = \{A \subset I: S(\chi_A) = 0\}$ is a zeroclass where χ_A is the characteristic function of A. We shall say that X_S is the zeroclass related to S. For any $x \in \omega$ and $\varepsilon > 0$, let

$$N_\varepsilon(x) = \{y \in \omega: \sup\{|x_i - y_i|: i = 1,2,3,\ldots\} < \varepsilon\}$$

Then the class $\{N_\varepsilon(x): x \in \omega, \varepsilon > 0\}$ forms a base for the topology T_∞ on ω. For any RSM $S:(C_S,T_\infty) \to R$, S is continuous ([2]).

2. BOUNDED CONSISTENCY ON STRONG CONVERGENCE FIELDS.

In this section we first develop a theory of strong convergence fields with the help of the zeroclass concept.

DEFINITION 2.1. For any zeroclass X we denote

$$V_x{}^o = \{x \in \omega: \text{For any } \alpha > 0, \{i: \alpha < |x_i|\} \in X\}$$

$$V_x = \{x \in \omega: x-r \in V_x{}^o \text{ for some } r \in R\}.$$

PROPOSITION 2.1. For any zeroclass X,

V_x is a linear space of sequences. (2.1)

$V_x{}^o$ is a subspace of V_x. (2.2)

PROOF. Suppose that $x,y \in V_x, r_1, r_2 \in R$ with $x-r_1, y-r_2 \in V_x{}^o$. For each i and for any $\alpha > 0$,

$$|x_i-r_1| \leq \alpha/2 \text{ and } |y_i-r_2| \leq \alpha/2 \implies |x_i+y_i - (r_1+r_2)| \leq \alpha.$$

Thus

$$\{i: |x_i+y_i - (r_1+r_2)| > \alpha\} \subset \{i: |x_i-r_1| > \alpha/2\} \cup \{i: |y_i-r_2| > \alpha/2\}.$$

By the definition of $V_x{}^o$ and the properties of zeroclasses $\{i: |x_i-r_1| > \alpha/2\} \cup \{i: |y_i-r_2| > \alpha/2\} \in X$ and $\{i: |x_i+y_i - (r_1+r_2)| > \alpha\} \in X$. Consequently we get that $x+y \in V_x$. If $k \in R$, then for any $\alpha > 0$,

$$\{i: \alpha<|kx_i-kr_1|\} = \begin{cases} \phi, & \text{if } k = 0, \\ \{i: \alpha/|k| < |x_i-r_1|\}, & \text{if } k \neq 0, \end{cases}$$

Therefore for any $\alpha > 0$, $\{i: \alpha<|kx_i-kr_1|\} \varepsilon X$, which implies $kx \varepsilon V_x$. Hence V_x is a linear space of sequences.

(2.2) is obvious.

PROPOSITION 2.2. For any zeroclass X, let $T_x: V_x \to R$ be the function from V_x to R defined by $T_x(x) = r$ when $x-r \varepsilon V_x^o$. Then T_x is an RSM with domain V_x, $|V_x| = V_x$ and $|V_x|^o = V_x^o$. Further, X is related to T_x.

PROOF. In this proof we will write T_x as T for convenience. First, we show T is well defined. If $x \varepsilon V_x$, $T(x)=r_1$ and $T(x)=r_2$, so that $x-r_1$, $x-r_2 \varepsilon V_x^o$, then, since V_x^o is a linear space, $(x-r_1) - (x-r_2) = (r_2-r_1)e \varepsilon V_x^o$ where $e = (1,1,1,\ldots)$. From the fact that for any $\alpha > 0$

$$\{i: \alpha< |(r_2-r_1)e_i|\} = \begin{cases} \phi & \text{if } \alpha \geq |r_2-r_1|, \\ I, & \text{if } \alpha< |r_2-r_1| \end{cases}$$

and $\{i: \alpha < |(r_2-r_1)e_i|\} \varepsilon X$, it follows that $r_1=r_2$.

Suppose that $x, y \varepsilon V_x$ and $T(x)=r_1$ and $T(y)=r_2$. Then $x-r_1$, $y-r_2 \varepsilon V_x^o$. Since V_x^o is a linear space, $(x+y)-(r_1+r_2) \varepsilon V_x^o$ and $kx-kr_1 \varepsilon V_x^o$ for any $k \varepsilon R$. Therefore $T(x+y)=T(x)+T(y)$ and $T(kx)=kT(x)$. Hence T is a linear functional.

Next we have

$$V_x^o = \{x\varepsilon\omega: \text{ for any } \alpha > 0, \{i: \alpha <|x_i|\} \varepsilon X\}$$
$$= \{x\varepsilon\omega: |x| \varepsilon V_x^o\} = |V_x|^o,$$
$$V_x = V_x^o \oplus \langle e\rangle = |V_x|^o \oplus \langle e\rangle = |V_x|.$$

Suppose that $x \varepsilon |V_x|^o$ and $|y| \leq |x|$ (i.e., $|y_i| \leq |x_i|$ for any i). Thus for any $\alpha > 0$, $\{i: \alpha<|y_i|\} \subset \{i: \alpha < |x_i|\} \varepsilon X$ and so $\{i: \alpha <|y_i|\} \varepsilon X$. Thus we have $y \varepsilon |V_x|^o$ ([1] Proposition 1). Hence T is an RSM.

For any $A \subset I$,

$$\{i: \alpha < \chi_A(i)\} = \begin{cases} A, & \text{if } 0 < \alpha < 1, \\ \phi, & \text{if } 1 \leq \alpha. \end{cases}$$

Hence $\chi_A \varepsilon V_x^o = |C_T|^o$ if an only if $A \varepsilon X$. Thus T and X are related.

Note that these results can be written in the following notation: $V_x = C_T$, $|V_x| = |C_T|$, $|V_x|^o = |C_T|^o$.

PROPOSITION 2.3. For any zeroclass X, V_x is closed with respect to the topological space (ω, T_∞).

PROOF. Supposed that $x \in \bar{V}_x$ and choose $\{x^n\} \subset V_x$ such that $\|x^n - x\|_\infty = \sup_i |x_i^n - x_i| < 1/n$ $(n \geq 1)$. Suppose that $T_x(x^n) = r_n \in R$. Since x^n converges to x as $n \to \infty$, we have, for any $\varepsilon > 0$, there exists $N \in I$ such that $n,m \geq N \Rightarrow \|x^n - x^m\|_\infty < \varepsilon$. We show that $\lim_n r_n$ exists. Suppose that $n,m \geq N$. For each $i \in I$,

$$|r_n - r_m| \leq |r_n - x_i^n| + |x_i^n - x_i^m| + |x_i^m - r_m|$$

$$< |r_n - x_i^n| + \varepsilon + |x_i^m - r_m|.$$

Clearly

$$I = \{i: |r_n - r_m| - \varepsilon < |r_n - x_i^n| + |x_i^m - r_m|\}$$

$$\subset \{i: (|r_n - r_m| - \varepsilon)/2 < |r_n - x_i^n|\} \cup \{i: i: (|r_n - r_m| - \varepsilon)/2 < |x_i^m - r_m|\}.$$

If $|r_n - r_m| > \varepsilon$, then $\{i: (|r_n - r_m| - \varepsilon)/2 < |r_n - x_i|\} \in X$ and $\{i: i: (|r_n - r_m| - \varepsilon)/2 < |x_i - r_m|\} \in X$ so that $I \in X$, a contradiction. Hence $|r_n - r_m| \leq \varepsilon$ and $\{r_n\}$ is a Cauchy sequence of real numbers. Let $\lim r_n = r \in R$.

Now we show that $x \in V_x$. For any $\alpha > 0$, we choose $N \in I$ such that $n > N \Longrightarrow \|x - x^n\|_\infty < \alpha/3$ and $|r_n - r| < \alpha/3$. For any $i \in I$,

$$|x_i - r| \leq |x_i - x_i^n| + |x_i{}^n - r_n| + |r_n - r| < 2\alpha/3 + |x_i^n - r_n|.$$

Therefore

$$\{i; \alpha < |x_i - r|\} \subset \{i: \alpha/3 < |x_i^n - r_n|\}.$$

Since $T(x^n) = r_n$, we have $\{i: \alpha/3 < |x_i - r_n|\} \in X$. Hence $x \in V_x$, and so V_x is closed.

PROPOSITION 2.4. For any zeroclass X, $V_x{}^o$ is a closed subset of (ω, T_∞).

PROOF. $T_x: (V_x, T_\infty) \to R$ is an RSM and so it is continuous. Thus $T_x{}^{-1}(0) = V_x{}^o$ is closed subset of (V_x, T_∞). Since V_x is also closed in (ω, T_∞), $V_x{}^o$ is closed in (ω, T_∞).

PROPOSITION 2.5. For any zeroclass X, $\bar{\omega}_x = V_x$ (where $\bar{\omega}_x$ denotes the closure of ω_x with respect to the topology T_∞).

PROOF. Suppose that $x \in \omega_x$, $r \in R$ and $A \in X$ with $x \underset{(A)}{\to} r$. Then by the definition of $x \underset{(A)}{\to} r$, we have, for any $\alpha > 0$, there exist $N \in I$ such that $\{i: \alpha < |x_i - r|\} \subset A \cup \{1,2,3,\dots,N\}$. Since A and $\{1,2,3,\dots,N\} \in X$, we have $\{i: \alpha < |x_i - r|\} \in X$. Hence

$x \in V_x$. Therefore $\omega_x \subset V_x$. Since V_x is closed, we have $\bar{\omega}_x \subset V_x$.

Suppose that $x \in V_x$ and $T(x)=r$. For each n, let $\{i: 1/n<|x_i-r|\}=A_n$. Then $A_n \varepsilon X$.

Let us define $x^n \varepsilon \omega$ by

$$x_i^n = \begin{cases} r & \text{if} \quad i \in I-A_n \\ x_i & \text{if} \quad i \in A_n. \end{cases}$$

Obviously, $x^n \underset{(A_n)}{\to} r$ and $A_n \varepsilon X$, thus $x^n \varepsilon \omega_x$.

Since

$$|x_i^n - x_i| = \begin{cases} |r-x_i| & \text{if} \quad i \in I-A_n \\ 0 & \text{if} \quad i \in A_n, \end{cases}$$

we get $\|x^n - x\|_\infty \leq 1/n$. It follows that $x \in \overline{\omega_x}$. Hence $\overline{\omega_x}=V_x$.

Replacing ω_x by ω_x^o, V_x by V_x^o and r by 0 we obtain

PROPOSITION 2.6. For any zeroclass X, $\overline{\omega_x^o}=V_x^o$ where $\omega_x^o= \{x\varepsilon\omega: \exists\, A\varepsilon X \;\; x \underset{(A)}{\to} 0\}$.

PROPOSITION 2.7. (see [1] Proposition 4.10). If the zeroclass X is related to the RSM S, then

$$\omega_x^o \cap m \subset |C_S|^o \subset V_x^o, \tag{2.3}$$

$$\omega_x \cap m \subset |C_S| \subset V_x, \tag{2.4}$$

$$S \text{ and } T_x \text{ have same value on } |C_S|. \tag{2.5}$$

PROOF. (2.3) Let $x \in \omega_x^o \cap m$. Then there exists a set $A \subset I$ such that $A \in X$ and and $x \underset{(A)}{\to} 0$. Since $A \in X$, we have $\chi_A \varepsilon |C_S|^o$. Since $x \in m$ and S is an RSM, $x \cdot \chi_A \varepsilon |C_S|^o$. Further $x \cdot \chi_{I-A} \varepsilon c_o \subset |C_S|^o$ ([1] Proposition 4.9). Thus $x = x \cdot \chi_A + x \cdot \chi_{I-A} \varepsilon |C_S|^o$. Next, for any $x \varepsilon |C_S|^o$ and for any $\alpha > 0$, $\alpha \chi_{\{i:\, \alpha < |x_i|\}} \leq |x| \varepsilon |C_S|^o$. Thus $\alpha \chi_{\{i:\, \alpha < |x_i|\}} \varepsilon |C_S|^o$ and so $\chi_{\{i:\, \alpha < |x_i|\}} \varepsilon |C_S|^o$ equivalently $\{i: \alpha < |x_i|\} \varepsilon X$.

(2.4) Obviously, $\omega_x \cap m = (\omega_x^o \oplus \langle e \rangle) \cap m \subset (|C_S|^o \oplus \langle e \rangle) \cap m = |C_S| \cap m$. also, $|C_S| = |C_S|^o \oplus \langle e \rangle \subset V_x^o \oplus \langle e \rangle = V_x$.

(2.5) Let $x \varepsilon |C_S|$. Then there exists $r \varepsilon R$ such that $x-r \varepsilon |C_S|^o \subset C_S^o$ so that $S(x-r)=0$ or $S(x)=r$. By (2.3), $x-r \varepsilon V_x^o$. Therefore $T_x(x)=r$.

PROPOSITION 2.8. If X_1 and X_2 are zeroclasses with $X_1 \subset X_2$, then we have

$$V_{x_1}{}^{o} \subset V_{x_2}{}^{o}, \tag{2.6}$$

$$V_{x_1} \subset V_{x_2}, \tag{2.7}$$

$$T_{x_2}\big|V_{x_1} = T_{x_1}. \tag{2.8}$$

PROOF. (2.6) Suppose that $x \varepsilon V_{x_1}{}^{o}$. Then for any $\alpha>0$, $\{i: \alpha<|x_i|\} \varepsilon X_1 \subset X_2$. Therefore, for any $\alpha>0$, $\{i: \alpha<|x_i|\} \varepsilon X_2$ or $x \varepsilon V_{x_2}{}^{o}$. For (2.7) and (2.8) let $x \varepsilon V_{x_1}$ and $T_{x_1}(x) = r$. Then we have $x-r \varepsilon V_{x_1}{}^{o} \subset V_{x_2}{}^{o}$. Thus $x-r \varepsilon V_{x_2}{}^{o}$ and so $x \varepsilon V_{x_2}$ and $T_{x_1}(x) = r = T_{x_2}(x)$.

PROPOSITION 2.9. (Bounded Consistency Theorem on Strong Convergence Fields). Let $S_1:C_{S_1} \to R$ be an RSM related with the zeroclass X_1 and $S_2:C_{S_2} \to R$ be an RSM related with the zeroclass X_2. Suppose that $X_1 \subset X_2$ and $C_{s_1} \cap m \subset C_{s_2}$. Then we have:

$$|C_{S_1}|^{o} \cap m \subset |C_{S_2}|^{o} \cap m, \tag{2.9}$$

$$|C_{S_1}| \cap m \subset |C_{S_2}| \cap m, \tag{2.10}$$

$$S_1(|C_{S_1}| \cap m) = S_2|(|C_{S_1}| \cap m). \tag{2.11}$$

PROOF. (2.9) If $x \varepsilon |C_{S_1}|^{o} \cap m$, then $|x| \varepsilon C_{S_1} \cap m$, $S_1(|x|) = 0$, $|x| \varepsilon V_{X_1}{}^{o}$, (Proposition 2.7) and $T_{X_1}(|x|) = 0$. Since $|x| \varepsilon C_{S_1} \cap m \subset C_{S_2}$, $S_2(|x|)$ is defined. By the previous proposition $|x| \varepsilon V_{X_1}^{o} \cap m \subset V_{X_2}{}^{o} \cap m = \bar{\omega}_{x_2} \cap m$ and by Proposition 2.7 $\omega_{x_2} \cap m \subset |C_{S_2}| \cap m \subset \bar{\omega}_{x_2} \cap m$. Thus we can find a sequence $\{x^n\}$ in $|C_{S_2}| \cap m$ such that $x^n \to |x|$ in (ω, T_∞). Since S_2 is an RSM, S_2 is continuous. Thus $S_2(x^n) \to S_2(|x|)$. Since $x^n \varepsilon |C_{S_2}| \cap m \subset V_{x_2}$, $T_{x_2}(x^n) = S_2(x^n)$. On the other hand $|x| \varepsilon V_{x_1}{}^{o} \subset V_{x_2}$, thus $0 = T_{x_1}(x) = T_{x_2}(x)$. Hence we have $0 = T_{x_2}(x) = \lim_n T_{x_2}(x^n) = \lim_n S_2(x^n) = S_2(x)$. Therefore $x \varepsilon |C_{S_2}|^{o}$.

(2.10) By (2.9), $|C_{S_1}| \cap m = (|C_{S_1}|^{o} \oplus \langle e \rangle)\, m \subset (|C_{S_2}|^{o} \oplus \langle e \rangle) \cap m = |C_{S_2}| \cap m$.

(2.11) By Proposition 2.7 (2.5), $S_1 \big| |C_{S_1}| \cap m = T_{x_1} \big| |C_{s_1}| \cap m$ and $S_2 \big| |C_{S_2}| \cap m = T_{x_2} \big| |C_{S_2}| \cap m$. By Proposition 2.8, we have $T_{x_1}|V_{x_1} = T_{x_2}|V_{x_1}$. By (2.10) and the fact that $|C_{S_1}| \cap m \subset V_{x_1}$, we have the result.

COROLLARY 1. Let $S_1 : C_{S_1} \to R$ and $S_2 : C_{S_2} \to R$ be RSMs defined on the same domain $C_S = C_{S_1} = C_{S_2}$ and with same related zeroclass X. Then we have $|C_{S_1}| \cap m = |C_{S_2}| \cap m$ and $S_1|(|C_{S_1}| \cap m) = S_2|(|C_{S_1}| \cap m)$.

REMARK. Let F be the collection of all RSMs which are related to a fixed zeroclass X. Then T_x is a member of F and for any RSM $S:C_s \to R$ in F, S and T_x have the same values on the bounded strong convergence field associated with S.

Finally we look at RSMs on m.

PROPOSITION 2.10. Let $S:C_s \to R$ be an RSM and let $X=X_s$. Suppose that $C_s = m$. Then $|C_s| = V_x \cap m$ and $S(x)=T_x(x)$ for any $x \varepsilon |C_s|$.

PROOF. By Proposition 2.7, we have $\omega_x \cap m \subset |C_s| \subset V_x \cap m$. Let $x \varepsilon V_x \cap m$. Since V_x is the closure of ω_x in (ω, T_∞), we can find a sequence $\{x^n\} \subset |C_s|$ which converges to x in (ω, T_∞). Suppose that $x^n - r_n \varepsilon |C_s|^o$ and $S(x)=r$. Since $S(x^n)=r_n$ and S is continuous, we have $r_n \to r$ in R. Thus $|x^n - r_n| \to |x-r|$ in (ω, T_∞). Note that $x \varepsilon m$. Thus $|x-r|$ is also in m, which is the domain of S. Since S is continuous and $S(|x^n - r_n|)=0$, $S(|x-r|)=0$, which means $x \varepsilon |C_s|$.

In the main Theorem of [3], Freedman proved (in the terminology of this paper) the following:

THEOREM. If Y is a zeroclass, then $x \varepsilon V_Y \cap m$ if and only if for any two RSMs S_1, S_2 on m with $Y \subset X_{s_1}$, $Y \subset X_{s_2}$, $S_1(x) = S_2(x)$.

Suppose that $S_i : m \to R$, $X_i (i=1,2)$ satisfy the hypothesis of Proposition 2.9, we show that the above Theorem implies that the conclusion of Proposition 2.9 also holds for S_1, S_2. If $x \varepsilon |C_{s_1}|$ then $x \varepsilon V_{x_1} \cap m$. It is clear that $S_1(x)=S_2(x)$ since $X_1 \subset X_2$.

3. RSMs WITH A RELATED ULTRAZEROCLASS.

In [2] we studied RSMs induced from matrices. For a regular matrix A, we define the linear functional $f_A : C_A \to R$ by $f_A(x) = \lim_n Ax$ for any $x \varepsilon C_A$. The ordinary Bounded Consistency Theorem (BCT) (see, e.g. [4]) says that for any regular matrices A,B with $C_A \cap m \subset C_B$, $f_A(x)=f_B(x)$ for any $x \varepsilon C_A \cap m$.

We can easily see that the BCT for strong convergence fields (Proposition 9) is included in the ordinary BCT for matrices when the RSMs are induced from regular matrices. Therefore we would like to find examples of summabilities such that the bounded consistency in the strong convergence fields of these summabilities is not implied by the matrix BCT.

DEFINITION 3.1. An ultrazeroclass on I is a zeroclass X such that there is no zeroclass on I which is strictly finer than X.

PROPOSITION 3.1. Let X be an ultrazeroclass on I. Then for any $A\varepsilon 2^I$, $A\varepsilon X$ or $I-A\varepsilon X$.

PROOF. Let $F = \{A\varepsilon 2^I : I-A\varepsilon X\}$. Then F is an ultrafilter. Thus for any $A\varepsilon 2^I$, $A\varepsilon F$ or $I-A\varepsilon F$.

PROPOSITION 3.2. X is an ultrazeroclass if and only if $m\subset V_x$.

PROOF. Suppose that X is an ultrazeroclass. Then for any $A\varepsilon 2^I$, $A\varepsilon X$ or $I-A\varepsilon X$, equivalently, $\chi_A \varepsilon V_x$ or $\chi_{(I-A)}\varepsilon V_x$, that is $\chi_A\varepsilon V_x$ or $1-\chi_A\varepsilon V_x$.

It follows that $\chi_A \ \varepsilon\ V_x$. Since V_x is linear space, $m_o \subset V_x$. Since V_x is closed in (ω, T_∞), $\bar{m}_o = m \subset V_x$.

Suppose that X is not an ultrazeroclass, then there exists $A\varepsilon 2^I$ such that $A\notin X$ and $I-A\notin X$. Assume that $\chi_A\varepsilon V_x$. Then there exists $r\varepsilon R$ such that $\{i:\alpha<|\chi_A(i)-r|\}\ \varepsilon\ X$ for any $\alpha>0$.

If $r=1$, then $\{i:1/2<|\chi_A(i)-r|\} = I-A\notin X$.

If $r=0$, then $\{i:1/2<|\chi_A(i)-r|\}=A\notin X$.

If $r\notin\{0,1\}$, then $\{i:0.5\min\{|r|,|1-r|\} <|\chi_A(i)-r|\}= I\notin X$.

This is a contradiction. Hence $\chi_A\ \varepsilon\ m-V_x$.

PROPOSITION 3.3. If X is an ultrazeroclass then there does not exist a regular matrix A such that f_A is an RSM and $|V_x| \cap m = C_A \cap m$.

PROOF. Since X is an ultrazeroclass, $m \subset V_x$ and thus $|V_x| \cap m = V_x \cap m = m$. On the other hand, for any regular matrix A, $m- C_A \neq \phi$

It follows from the above and Proposition 2.9 that the value of any RSM, S, on its bounded strong convergence field is determined by any ultrazeroclass containing the zeroclass related to S.

REFERENCES

1. FREEDMAN, A.R. and SEMBER, J.J. Densities and Summability, Pacific J. Math., 95 (1981), 293-305.

2. CHUN, C.S. and FREEDMAN, A.R. Theorems and Examples for R-type Summability Methods (To appear).

3. FREEDMAN, A.R. Generalized limits and sequence spaces, Bull. London Math. Soc., 13 (1981), 224-228.

4. RUCKLE, W.H. The bounded consistency theorem, Amer. Math. Monthly 86 (1979), 566-571.