SIX-DIMENSIONAL CONSIDERATIONS OF EINSTEIN'S CONNECTION FOR THE FIRST TWO CLASSES II. THE EINSTEIN'S CONNECTION IN 6- g -UFT

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ABSTRACT. Lower dimensional cases of Einstein's connection were already investigated by many authors for $n = 2, 3, 4, 5$. In the following series of two papers, we present a surveyable tensorial representation of 6-dimensional Einstein's connection in terms of the unified field tensor:

- I. The recurrence relations in 6- g -UFT.
- II. The Einstein's connection in 6- g -UFT.

In our previous paper [2], we investigated some algebraic structure in Einstein's 6-dimensional unified field theory (i.e., 6- g -UFT), with emphasis on the derivation of the recurrence relations of the third kind which hold in 6- g -UFT. This paper is a direct continuation of [2]. The purpose of the present paper is to prove a necessary and sufficient condition for a unique Einstein's connection to exist in 6- g -UFT and to display a surveyable tensorial representation of 6-dimensional Einstein's connection in terms of the unified field tensor, employing the powerful recurrence relations of the third kind obtained in the first paper [2].

All considerations in this paper are restricted to the first and second classes of the 6-dimensional generalized Riemannian manifold X_6 , since the case of the third class, the simplest case, was already studied by many authors.

Keywords and phrases. Generalized Riemannian manifold, 6-dimensional Einstein's unified field theory, Einstein's connection in 6- g -UFT.

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This paper is a direct continuation of our previous paper [2], which will be denoted by I in the present paper. All considerations in this paper are based on the results and symbolism of I. Whenever necessary, they will be quoted in the present paper.

In the following theorem, we prove a necessary and sufficient condition for a unique Einstein's connection to exist in 6- g -UFT.

THEOREM 1. *A necessary and sufficient condition for the existence and uniqueness of the solution of I-(2.4) or I-(2.20) in 6- g -UFT is*

FOR THE FIRST CLASS.

$$g_{AB}(C^2 - 4CD + E) \neq 0, \quad (1)$$

where

$$A = 1 - 2K_4 + (K_4)^2 - 4K_2K_6 - 8K_6, \quad (2a)$$

$$B = 1 - 3K_2 + 9K_4 - 27K_6, \quad (2b)$$

$$C = (1 - K_2 + K_4 - K_6)^2 - 4[K_4 - 6K_6 - (K_4)^2 + 16(K_6)^2 + 10K_4K_6 + 2K_2K_6 - 2K_6(k_2)^2], \quad (2c)$$

$$D = (2 - K_2 + 3K_6)(3K_6 - K_2K_4), \quad (2d)$$

$$E = 16[(2 - K_2 + 4K_6)^2 + (K_6)^2][9(K_6)^2 - 6K_2K_4K_6 + (K_4)^3 + (K_2)^3K_6] - 16K_6(2 - K_2 + 4K_6) \times [-2(K_2)^3K_6 + (K_2)^2(K_4)^2 + 6K_2K_4K_6 - 2(K_4)^3 - 9(K_6)^2]. \quad (2e)$$

FOR THE SECOND CLASS WITH THE SECOND CATEGORY.

$$(1 + K_2 + K_4)(1 - K_2 + K_4)(1 - K_4)(1 - 3K_2 + 9K_4) \times [(1 - K_2 - 3K_4)^2 - 4K_4((K_2)^2 - 4K_4)] \neq 0. \quad (3)$$

FOR THE SECOND CLASS WITH THE FIRST CATEGORY.

$$1 - (K_2)^2 \neq 0. \quad (4)$$

PROOF OF THE FIRST CLASS. The symmetric scalars M_{xyz} defined by I-(2.23) take values as in the following table, in virtue of I-(4.3a) and I-(4.4a):

Values of x, y, z	M_{xyz}
Two of the indices x, y, z are 1, 2 or 3, 4 or 5, 6	$1 + M_1^2, \quad 1 + M_3^2, \quad 1 + M_5^2.$
All indices are equal	$1 + 3M_1^2, \quad 1 + 3M_3^2, \quad 1 + 3M_5^2.$
No two are equal and no two take the values 1,2, 3,4 nor 5,6	$1 + \frac{MM}{1\ 3} + \frac{MM}{3\ 5} + \frac{MM}{5\ 1},$ $1 + \frac{MM}{1\ 3} - \frac{MM}{3\ 5} - \frac{MM}{5\ 1},$ $1 - \frac{MM}{1\ 3} - \frac{MM}{3\ 5} + \frac{MM}{5\ 1},$ $1 - \frac{MM}{1\ 3} + \frac{MM}{3\ 5} - \frac{MM}{5\ 1}.$
The remaining cases	$1 + M_1^2 \pm 2\frac{MM}{1\ 3}, \quad 1 + M_3^2 \pm 2\frac{MM}{3\ 5},$ $1 + M_5^2 \pm 2\frac{MM}{1\ 5}, \quad 1 + M_1^2 \pm 2\frac{MM}{1\ 5},$ $1 + M_3^2 \pm 2\frac{MM}{1\ 3}, \quad 1 + M_5^2 \pm 2\frac{MM}{3\ 5}.$

TABLE 1.

It may be easily verified that the product of 3 factors in the first row of Table 1 is g given by I-(4.1), that of 3 factors in the second row is B given by (2b), and that of 4 factors in the third row is A given by (2a). After a lengthy calculation, we find that the product of 12 factors in the fourth row of (1) is $C^2 - 4CD + E$. Consequently, our assertion follows. $\square$

PROOF OF THE SECOND CLASS. The proof of the second class may be obtained easily from (1) and (2) by simply substituting the corresponding conditions of each case. $\square$

In the following three theorems, we establish a linear system containing the torsion tensor $S = S_{\omega\lambda\mu}$ of the Einstein's connection, employing the powerful recurrence relations of the third kind obtained in I-Theorem 4.5.

THEOREM 2a (For the first class of 6- g -UFT). *The system of equations I-(2.20) or I-(2.4) is reduced to the following linear system of 43 equations:*

$$\begin{aligned}
 B &= S + \overset{110}{S} + 2 \overset{(10)1}{S}, & B &= \overset{(10)1}{S} + \overset{(21)1}{S} + \overset{(20)2}{S} + \overset{112}{S}, \\
 B &= \overset{(12)1}{S} + \overset{(12)1}{S} + \overset{(23)1}{S} + \overset{222}{S} + \overset{(13)2}{S}, & B &= \overset{(20)2}{S} + \overset{(20)2}{S} + \overset{(31)2}{S} + \overset{(30)3}{S} + \overset{(21)3}{S}, \\
 B &= \overset{(23)1}{S} + \overset{(23)1}{S} + \overset{(34)1}{S} + \overset{332}{S} + \overset{(24)2}{S}, & B &= \overset{(13)2}{S} + \overset{(13)2}{S} + \overset{(24)2}{S} + \overset{(23)3}{S} + \overset{(14)3}{S}, \\
 S &= \overset{(30)3}{S} + \overset{(30)3}{S} + \overset{(41)3}{S} + \overset{(40)4}{S} + \overset{(31)4}{S}, & B &= \overset{(12)3}{S} + \overset{(12)3}{S} + \overset{(23)3}{S} + \overset{224}{S} + \overset{(13)4}{S}, \\
 2 B &= 2 \overset{(34)1}{S} + 2 \overset{(34)1}{S} + 2K_4 \overset{(32)1}{S} + 2K_6 \overset{(21)1}{S} + 2K_6 \overset{(30)1}{S} + K_4 \overset{222}{S} + \overset{442}{S} + 2K_6 \overset{(20)2}{S} - K_2 \overset{332}{S} + K_6 \overset{112}{S}, \\
 2 B &= 2 \overset{(24)2}{S} - \overset{(24)2}{S} - \overset{442}{S} - K_2 \overset{332}{S} + 2K_6 \overset{(20)2}{S} + K_4 \overset{222}{S} + K_6 \overset{112}{S} - 2K_2 \overset{(23)3}{S} + 2K_6 \overset{(10)3}{S}, \\
 B &= \overset{(23)3}{S} + \overset{(23)3}{S} + \overset{(43)3}{S} + \overset{334}{S} + \overset{(24)4}{S}, \\
 2 B &= 2 \overset{(14)3}{S} - 2 \overset{(14)3}{S} - 2K_2 \overset{(32)3}{S} + 2K_6 \overset{(10)3}{S} - \overset{334}{S} - 2K_2 \overset{(13)4}{S} + 2 \overset{(24)4}{S} - K_4 \overset{114}{S}, \\
 2 B &= 2 \overset{(40)4}{S} - 2K_2 \overset{(40)4}{S} - \overset{(31)4}{S} - \overset{334}{S} - K_4 \overset{114}{S} - 2 \overset{(32)5}{S} - 2K_2 \overset{(30)5}{S} - 2K_2 \overset{(21)5}{S} - 2K_4 \overset{(10)5}{S}, \\
 B &= \overset{(31)4}{S} + \overset{(31)4}{S} + \overset{(42)4}{S} + \overset{(41)5}{S} + \overset{(32)5}{S}, & B &= \overset{(30)1}{S} + \overset{(30)1}{S} + \overset{(41)1}{S} + \overset{(40)2}{S} + \overset{(31)2}{S}, \\
 B &= \overset{(10)3}{S} + \overset{(10)3}{S} + \overset{(21)3}{S} + \overset{(20)4}{S} + \overset{114}{S}, \\
 2 B &= 2 \overset{(43)3}{S} + 2K_4 \overset{(43)3}{S} + 2K_6 \overset{(32)3}{S} + 2K_6 \overset{(21)3}{S} + \overset{(30)3}{S} + \overset{444}{S} + \overset{224}{S} + 2K_6 \overset{(20)4}{S} + K_6 \overset{114}{S} - K_2 \overset{334}{S}, \\
 2 B &= 2 \overset{(24)4}{S} - \overset{(24)4}{S} - \overset{444}{S} - K_2 \overset{334}{S} + 2K_6 \overset{(20)4}{S} + K_4 \overset{224}{S} + K_6 \overset{114}{S} - 2K_2 \overset{(23)5}{S} + 2K_6 \overset{(10)5}{S}, \\
 B &= 2 \overset{(32)5}{S} + \overset{(32)5}{S} - K_2 \overset{(43)5}{S} - K_4 \overset{(42)4}{S} - K_6 \overset{(42)2}{S} - K_2 \overset{334}{S} - K_4 \overset{332}{S} - K_6 \overset{330}{S}, \\
 B &= \overset{(30)5}{S} + \overset{(30)5}{S} - K_2 \overset{(14)5}{S} - K_4 \overset{(40)4}{S} - K_6 \overset{(40)2}{S} - K_2 \overset{(40)0}{S} - K_4 \overset{(31)4}{S} - K_6 \overset{(31)2}{S} - K_6 \overset{(13)0}{S}, \\
 B &= \overset{(21)5}{S} + \overset{(21)5}{S} - K_2 \overset{(32)5}{S} - K_4 \overset{(31)4}{S} - K_6 \overset{(31)2}{S} - K_2 \overset{(13)0}{S} - \overset{224}{S} - K_4 \overset{222}{S} - K_6 \overset{220}{S}, \\
 B &= \overset{(10)5}{S} + \overset{(10)5}{S} - K_2 \overset{(21)5}{S} - K_4 \overset{(20)4}{S} - K_6 \overset{(20)2}{S} - K_2 \overset{(20)0}{S} - \overset{114}{S} - K_4 \overset{112}{S} - K_6 \overset{110}{S}, \\
 2 B &= 2 \overset{(14)5}{S} - 2 \overset{(14)5}{S} - 2K_2 \overset{(34)5}{S} + 2K_6 \overset{(23)5}{S} - 2K_2 \overset{(10)5}{S} - 2K_4 \overset{(24)4}{S} - 2K_6 \overset{(24)2}{S} - 2K_6 \overset{(24)0}{S} + 2(K_2)^2 \overset{(31)4}{S} \\
 &\quad + (K_4)^2 \overset{112}{S} + 2K_2 K_6 \overset{(31)0}{S} + K_6 \overset{330}{S} + K_4 K_6 \overset{110}{S} + K_2 \overset{334}{S} + K_2 K_4 \overset{114}{S} + 2K_2 K_4 \overset{(31)2}{S} + K_4 \overset{332}{S}, \\
 2 B &= 2 \overset{(41)1}{S} - 2 \overset{(41)1}{S} - 2K_2 \overset{(43)1}{S} + 2K_6 \overset{(32)1}{S} - 2K_2 \overset{(10)1}{S} - \overset{(31)2}{S} - \overset{332}{S} - K_4 \overset{112}{S} + 2 \overset{(42)2}{S},
 \end{aligned} \tag{5}$$

$$\begin{aligned}
2 \overset{(40)2}{B} &= 2 \overset{(40)2}{S} - 2K_2 \overset{(31)2}{S} - \overset{332}{S} - K_4 \overset{112}{S} - 2 \overset{(32)3}{S} - 2K_2 \overset{(30)3}{S} - 2K_2 \overset{(21)3}{S} - 2K_4 \overset{(10)3}{S}, \\
\overset{(20)4}{B} &= \overset{(20)4}{S} + \overset{(31)4}{S} + \overset{(30)5}{S} + \overset{(21)5}{S}, \\
2 \overset{(43)5}{B} &= 2 \overset{(43)5}{S} + 2K_4 \overset{(32)5}{S} + 2K_6 \overset{(21)5}{S} + 2K_6 \overset{(30)5}{S} + (K_2)^2 \overset{334}{S} - K_2 K_4 \overset{224}{S} \\
&\quad - K_2 K_6 \overset{114}{S} - 2K_2 K_6 \overset{(20)4}{S} + K_2 K_4 \overset{332}{S} - (K_4)^2 \overset{222}{S} - 2K_4 K_6 \overset{(20)2}{S} - K_4 K_6 \overset{112}{S} \\
&\quad + K_2 K_6 \overset{330}{S} - K_4 K_6 \overset{220}{S} - 2(K_6)^2 \overset{(20)0}{S} - (K_6)^2 \overset{110}{S} - K_2 \overset{444}{S} - K_4 \overset{442}{S} - K_6 \overset{440}{S}, \\
2 \overset{(24)0}{B} &= 2 \overset{(24)0}{S} - \overset{440}{S} - K_2 \overset{330}{S} + 2K_6 \overset{(20)0}{S} + K_4 \overset{220}{S} + K_6 \overset{110}{S} - 2K_2 \overset{(23)1}{S} + 2K_6 \overset{(10)1}{S}, \\
2 \overset{(40)0}{B} &= 2 \overset{(40)0}{S} - 2K_2 \overset{(31)0}{S} - \overset{330}{S} - K_4 \overset{110}{S} - 2 \overset{(32)1}{S} - 2K_2 \overset{(30)1}{S} - 2K_2 \overset{(21)1}{S} - 2K_4 \overset{(10)1}{S}, \\
\overset{(13)0}{B} &= \overset{(13)0}{S} + \overset{(24)0}{S} + \overset{(23)1}{S} + \overset{(14)1}{S}, \quad \overset{(20)0}{B} = \overset{(20)0}{S} + \overset{(31)0}{S} + \overset{(30)1}{S} + \overset{(21)1}{S}, \\
\overset{110}{B} &= \overset{110}{S} + \overset{220}{S} + 2 \overset{(12)1}{S}, \quad \overset{112}{B} = \overset{112}{S} + \overset{222}{S} + 2 \overset{(12)3}{S}, \quad \overset{222}{B} = \overset{222}{S} + \overset{332}{S} + 2 \overset{(32)3}{S}, \\
\overset{332}{B} &= \overset{332}{S} + \overset{442}{S} + 2 \overset{(43)3}{S}, \quad \overset{224}{B} = \overset{224}{S} + \overset{334}{S} + 2 \overset{(32)5}{S}, \\
\overset{442}{B} &= (1 + K_2) \overset{442}{S} + 2K_4 \overset{(42)2}{S} + 2K_6 \overset{(13)2}{S} + K_4 \overset{332}{S} + K_6 \overset{222}{S} + 2K_6 \overset{(40)2}{S} + 2K_4 \overset{(23)3}{S} \\
&\quad + 2K_6 \overset{(21)3}{S} + 2K_6 \overset{(30)3}{S}, \\
\overset{334}{B} &= \overset{334}{S} + \overset{444}{S} + 2 \overset{(43)5}{S}, \quad \overset{114}{B} = \overset{114}{S} + \overset{224}{S} + 2 \overset{(12)5}{S}, \\
\overset{444}{B} &= (1 + K_2) \overset{444}{S} + 2K_4 \overset{(42)4}{S} + 2K_6 \overset{(13)4}{S} + K_4 \overset{334}{S} + K_6 \overset{224}{S} + 2K_6 \overset{(40)4}{S} + 2K_4 \overset{(23)5}{S} \\
&\quad + 2K_6 \overset{(21)5}{S} + 2K_6 \overset{(30)5}{S}, \\
\overset{330}{B} &= \overset{330}{S} + \overset{440}{S} + 2 \overset{(43)1}{S}, \quad \overset{220}{B} = \overset{220}{S} + \overset{330}{S} + 2 \overset{(23)1}{S}, \\
\overset{440}{B} &= (1 + K_2) \overset{440}{S} + 2K_4 \overset{(42)0}{S} + 2K_6 \overset{(13)0}{S} + K_4 \overset{330}{S} + K_6 \overset{220}{S} + 2K_6 \overset{(40)0}{S} + 2K_4 \overset{(23)1}{S} \\
&\quad + 2K_6 \overset{(21)1}{S} + 2K_6 \overset{(30)1}{S}.
\end{aligned}$$

PROOF. These equations may be obtained from I-(3.10), using I-(4.2a) and I-(4.16). $\square$

THEOREM 2b (For the second class of the second category of 6- g -UFT). *The system of equations I-(2.20) or I-(2.4) is reduced to the following linear system of 12 equations:*

$$\begin{aligned}
B &= \overset{110}{S} + \overset{(10)1}{S} + 2 \overset{(10)1}{S}, \quad \overset{(10)1}{B} = \overset{(10)1}{S} + \overset{(21)1}{S} + \overset{(20)2}{S} + \overset{112}{S}, \\
2 \overset{(12)1}{B} &= 2 \overset{(12)1}{S} + 2K_4 \overset{(10)1}{S} + \overset{222}{S} - K_2 \overset{112}{S} + K_4 \overset{002}{S}, \\
2 \overset{(02)2}{B} &= 2 \overset{(02)2}{S} - \overset{222}{S} - K_2 \overset{112}{S} + K_4 \overset{002}{S} - 2K_2 \overset{(10)3}{S}, \\
\overset{(01)3}{B} &= \overset{(01)3}{S} + \overset{(12)3}{S} - K_2 \overset{112}{S} - K_4 \overset{110}{S} - K_2 \overset{(02)2}{S} - K_4 \overset{(02)0}{S},
\end{aligned}$$

$$\begin{aligned}
2 \stackrel{(12)3}{B} &= 2 \stackrel{(12)3}{S} + 2K_4 \stackrel{(10)3}{S} - K_2 \stackrel{222}{S} - K_4 \stackrel{220}{S} + (K_2)^2 \stackrel{112}{S} + K_2 K_4 \stackrel{110}{S} - K_2 K_4 \stackrel{002}{S} - (K_4)^2 S, \quad (6) \\
2 \stackrel{(02)0}{B} &= 2 \stackrel{(02)0}{S} - K_2 \stackrel{110}{S} - 2K_2 \stackrel{(01)1}{S} - \stackrel{220}{S} + K_4 S, \quad \stackrel{110}{B} = \stackrel{110}{S} + \stackrel{220}{S} + 2 \stackrel{(12)1}{S}, \\
\stackrel{112}{B} &= \stackrel{112}{S} + \stackrel{222}{S} + 2 \stackrel{(21)3}{S}, \quad \stackrel{222}{B} = (1+K_2) \stackrel{222}{S} + K_4 \stackrel{112}{S} + 2K_4 \stackrel{(10)3}{S} + 2K_4 \stackrel{(20)2}{S}, \\
\stackrel{002}{B} &= \stackrel{002}{S} + \stackrel{112}{S} + 2 \stackrel{(10)3}{S}, \quad \stackrel{220}{B} = (1+K_2) \stackrel{220}{S} + 2K_4 \stackrel{(10)1}{S} + K_4 \stackrel{110}{S} + 2K_4 \stackrel{(20)0}{S}.
\end{aligned}$$

PROOF. This assertion also follows from I-(3.10), using I-(4.2b) and I-(4.17). $\square$

THEOREM 2c (For the second class of the first category of 6- g -UFT). *The system of equations I-(2.20) or I-(2.4) is reduced to the following linear system of two equations:*

$$B = (1+K_2)S + 2 \stackrel{(10)1}{S}, \quad \stackrel{(10)1}{B} = (1-K_2) \stackrel{(10)1}{S}. \quad (7)$$

PROOF. The system (7) may be obtained from I-(3.10), using I-(4.2c) and I-(4.18). $\square$

Using the Gauss-Jordan Elimination Method, each of the system (5) and (6) may be solved for $S = S_{\omega\lambda\mu}$ by operating computer. Since the solution for the case of the first class is too complicated, we just display, in the following two theorems, the solution for the case of the second class only.

THEOREM 3a (For the second class of the second category of 6- g -UFT). *The unique solution of the system I-(2.20) may be given by*

$$N(S-B) = 4B \stackrel{1}{(K_4-1)} + B \stackrel{2}{(1-K_2+5K_4)} + 2B \stackrel{3}{[1-2K_2+(K_2)^2-5K_4]}, \quad (8a)$$

where

$$\begin{aligned}
N &= (1+K_2+K_4) \left[(1-K_2+5K_4)^2 - 4K_4(2-K_2)^2 \right], \\
B \stackrel{1}{=} &= (K_4)^2 B + 2 \stackrel{(12)3}{B} + K_2 K_4 \stackrel{002}{B} + \left[2K_4 - (K_2)^2 \right] \stackrel{112}{B} - 2K_4 \stackrel{(12)1}{B} + K_4 \left[2+2K_2+(K_2)^2 \right] \stackrel{110}{B} \\
&\quad + K_2 \stackrel{222}{B} + 2K_4 \stackrel{(20)2}{B} - 2K_4(1+K_2) \stackrel{(10)3}{B} - K_4(1+K_2) \stackrel{220}{B} - 2K_4(1+K_2)^2 \stackrel{(10)1}{B}, \\
B \stackrel{2}{=} &= -(K_4)^2 B + 2 \left[(K_2)^2 - 1 + K_4 + 2K_2 K_4 \right] \stackrel{(10)1}{B} + (2+K_2) \stackrel{112}{B} - \stackrel{222}{B} - K_4 \stackrel{002}{B} + 2 \stackrel{(20)2}{B} \\
&\quad + 2(K_2+2K_4) \stackrel{(10)3}{B} + 2K_4 \stackrel{(20)0}{B} - \left[(K_2)^2 - 1 + K_4 + 2K_2 K_4 \right] \stackrel{110}{B} + (K_2-1+2K_4) \stackrel{220}{B}, \\
B \stackrel{3}{=} &= 2(K_4)^2 B + 2 \stackrel{(12)3}{B} - K_4 \stackrel{002}{B} + K_2 \stackrel{112}{B} + 2(1+K_2) \stackrel{(21)1}{B} - \stackrel{222}{B} + 2K_4 \stackrel{(10)3}{B} - (1+K_4)(1+K_2) \stackrel{110}{B} \\
&\quad + (1+K_4) \stackrel{220}{B} + 2K_4(1+K_2) \stackrel{(10)1}{B} - 2K_4 \stackrel{(20)0}{B}.
\end{aligned} \quad (8b)$$

THEOREM 3b (For the second class of the first category of 6- g -UFT). *The unique solution of the system I-(2.20) may be given by*

$$\left[1 - (K_2)^2 \right] (B-S) = K_2(1-K_2)B + 2 \stackrel{(10)1}{B}. \quad (9)$$

Now that we have obtained the tensor $S = S_{\omega\lambda\mu}$ in terms of $g_{\lambda\mu}$, it is possible to determine the tensor $U^\nu_{\lambda\mu}$ and eventually the 6-dimensional Einstein's connection $\Gamma^\nu_{\lambda\mu}$ in terms of $g_{\lambda\mu}$ by simply substituting for S into I-(2.19) and I-(2.18), respectively.

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