WEAK AND STRONG FORMS OF IRRESOLUTE MAPS

MIGUEL CALDAS CUEVA

(Received 23 March 1998 and in revised form 29 July 1998)

ABSTRACT. We consider new weak and stronger forms of irresolute and semi-closure via the concept sg-closed sets which we call ap-irresolute maps, ap-semi-closed maps and contra-irresolute and use it to obtain a characterization of semi- $T_{1/2}$ spaces.

Keywords and phrases. Topological spaces, sg-closed sets, semi-open sets, semi-closed maps, irresolute maps.

2000 Mathematics Subject Classification. Primary 54C10, 54D10.

1. Introduction. The concept of a semi-generalized closed set (written in short as sg-closed set) of a topological space was introduced by Bhattacharyya and Lahiri [2]. These sets were also considered by various authors (e.g., Sundaram, Maki and Balachandran [15], Caldas [4] and Dontchev and Maki [9]).

In this paper, we introduce the concept of irresoluteness called ap-irresolute maps and ap-semi-closed maps by using sg-closed sets and study some of their basic properties. This definition enables us to obtain conditions under which maps and inverse maps preserve sg-closed sets. Also, in this paper, we present a new generalization of irresoluteness called contra-irresolute. We define this last class of map by the requirement that the inverse image of each semi-open set in the codomain is semi-closed in the domain. This notion is a stronger form of ap-irresoluteness. Finally, we also characterize the class of semi- $T_{1/2}$ spaces in terms of ap-irresolute and ap-semi-closed maps.

Throughout this paper, (X, τ) , (Y, σ) , and (Z, γ) represent nonempty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset *A* of a space (X, τ) , Cl(A), and Int(A) denote the closure of *A* and the interior of *A*, respectively.

2. Preliminaries. Since we require the following known definitions, notations, and some properties, we recall them in this section.

DEFINITION 2.1. A subset *A* of a space (X, τ) is said to be semi-open [11] if there exists $O \in \tau$ such that $O \subseteq A \subseteq Cl(O)$. The semi-interior [6] of *A* denoted by SInt(A), is defined by the union of all semi-open sets of (X, τ) contained in *A*.

REMARK 2.2. (i) A subset *A* is semi-open [6] if and only if sInt(A) = A. (ii) $sInt(A) = A \cap Cl(Int(A))$ [10].

By SO(X, τ) we mean the collection of all semi-open sets in (X, τ).

DEFINITION 2.3. A subset *B* of (X, τ) is said to be semi-closed [3] if its complement B^c is semi-open in (X, τ) . The semi-closure [3] of a set *B* of (X, τ) denoted by

 $sCl_X(B)$, briefly sCl(B), is defined to be the intersection of all semi-closed sets of (X, τ) containing *B*.

REMARK 2.4. (i) A subset *B* is semi-closed [13] if and only if sCl(B) = B. (ii) $sCl(B) = B \cup Int(Cl(B))$ [10].

DEFINITION 2.5. A map $f : (X, \tau) \to (Y, \sigma)$ is called irresolute [7] if $f^{-1}(O)$ is semiopen in (X, τ) for every $O \in SO(Y, \sigma)$.

DEFINITION 2.6. A map $f : (X, \tau) \to (Y, \sigma)$ is called pre-semi-closed (resp., presemi-open) [7] if for every semi-closed (resp., semi-open) set *B* of (X, τ) , f(B) is semiclosed (resp., semi-open) in (Y, σ) .

DEFINITION 2.7. A subset *F* of (X, τ) is said to be semi-generalized closed (written in short as sg-closed) in (X, τ) [2] if sCl(*F*) \subseteq *O* whenever *F* \subseteq *O* and *O* is semi-open in (X, τ) . A subset *B* is said to be semi-generalized open (written as sg-open) in (X, τ) [2] if its complement $B^c = X - B$ is sg-closed in (X, τ) .

3. Ap-irresolute, ap-semi-closed and contra-irresolute maps. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map from a topological space (X, τ) into a topological space (Y, σ) .

DEFINITION 3.1. A map $f : (X, \tau) \to (Y, \sigma)$ is said to be approximately irresolute (or ap-irresolute) if $sCl(F) \subseteq f^{-1}(O)$ whenever O is a semi-open subset of (Y, σ) , F is a sg-closed subset of (X, τ) , and $F \subseteq f^{-1}(O)$.

DEFINITION 3.2. A map $f : (X, \tau) \to (Y, \sigma)$ is said to be approximately semi-closed (or ap-semi-closed) if $f(B) \subseteq \text{sInt}(A)$ whenever A is a sg-open subset of (Y, σ) , B is a semi-closed subset of (X, τ) , and $f(B) \subseteq A$.

Clearly irresolute maps are ap-irresolute and pre-semi-closed maps are ap-semiclosed, but not conversely.

The proof follows from Definition 3.1 and [2, Def. 1] (resp., Definition 3.2 and [2, Thm. 6]).

The following example shows the converse implications do not hold.

EXAMPLE 3.3. Let $X = \{a, b\}$ be the Sierpinski space with the topology, $\tau = \{\emptyset, \{a\}, X\}$. Let $f : X \to X$ be defined by f(a) = b and f(b) = a. Since the image of every semi-closed set is semi-open, then f is ap-semi-closed (similarly, since the inverse image of every semi-open set is semi-closed, then f is ap-irresolute). However $\{b\}$ is semi-closed in (X, τ) (resp., $\{a\}$ is semi-open) but $f(\{b\})$ is not semi-closed (resp., $f^{-1}(\{a\})$ is not semi-open in (X, τ)). Therefore f is not pre-semi-closed (resp., f is not irresolute).

THEOREM 3.4. (i) $f: (X, \tau) \to (Y, \sigma)$ is ap-irresolute if $f^{-1}(O)$ is semi-closed in (X, τ) for every $O \in SO(Y, \sigma)$.

(ii) $f: (X,\tau) \to (Y,\sigma)$ is ap-semi-closed if $f(B) \in SO(Y,\sigma)$ for every semi-closed subset B of (X,τ) .

PROOF. (i) Let $F \subseteq f^{-1}(O)$, where $O \in SO(Y, \sigma)$ and F is a sg-closed subset of (X, τ) . Therefore $SCl(F) \subseteq SCl(f^{-1}(O)) = f^{-1}(O)$. Thus f is ap-irresolute.

(ii) Let $f(B) \subseteq A$, where *B* is a semi-closed subset of (X, τ) and *A* is a sg-open subset of (Y, σ) . Therefore $\operatorname{sInt}(f(B)) \subseteq \operatorname{sInt}(A)$. Then $f(B) \subseteq \operatorname{sInt}(A)$. Thus *f* is apsemi-closed.

This theorem was used in Example 3.3.

REMARK 3.5. Let (X, τ) denote the topological space defined in Example 3.3. Then the identity map on (X, τ) is both ap-irresolute and ap-semi-closed, it is clear that the converses of Theorem 3.4 do not hold.

In the following theorem, we get under certain conditions that the converse of Theorem 3.4 is true.

THEOREM 3.6. Let $f : (X, \tau) \to (Y, \sigma)$ be a map from a topological space (X, τ) in a topological space (Y, σ) .

(i) If the semi-open and semi-closed sets of (X,τ) coincide, then f is ap-irresolute if and only if $f^{-1}(O)$ is semi-closed in (X,τ) for every $O \in SO(Y,\sigma)$.

(ii) If the semi-open and semi-closed sets of (Y, σ) coincide, then f is ap-semi-closed if and only if $f(B) \in SO(Y, \sigma)$ for every semi-closed subset B of (X, τ) .

PROOF. (i) Assume *f* is ap-irresolute. Let *A* be an arbitrary subset of (X, τ) such that $A \subseteq Q$, where $Q \in SO(X, \tau)$. Then by hypothesis $SCl(A) \subseteq SCl(Q) = Q$. Therefore all subsets of (X, τ) are sg-closed (and hence all are sg-open). So, for any $O \in SO(Y, \sigma)$, $f^{-1}(O)$ is sg-closed in (X, τ) . Since *f* is ap-irresolute $SCl(f^{-1}(O)) \subseteq f^{-1}(O)$. Therefore $SCl(f^{-1}(O)) = f^{-1}(O)$, i.e., $f^{-1}(O)$ is semi-closed in (X, τ) .

The converse is clear by Theorem 3.4.

(ii) Assume *f* is ap-semi-closed. Reasoning as in (i), we obtain that all subsets of (Y, σ) are sg-open. Therefore for any semi-closed subset of *B* of (X, τ) , f(B) is sg-open in *Y*. Since *f* is ap-semi-closed $f(B) \subseteq \operatorname{sInt}(f(B))$. Therefore $f(B) = \operatorname{sInt}(f(B))$, i.e, f(B) is semi-open. The converse is clear by Theorem 3.4.

As immediate consequence of Theorem 3.6, we have the following.

COROLLARY 3.7. Let $f : (X, \tau) \to (Y, \sigma)$ be a map from a topological space (X, τ) in a topological space (Y, σ) .

(i) If the semi-open and semi-closed sets of (X,τ) coincide, then f is ap-irresolute if and only if f is irresolute.

(ii) If the semi-open and semi-closed sets of (Y,σ) coincide, then f is ap-semi-closed if and only if f is pre-semi-closed.

A map $f : (X, \tau) \to (Y, \sigma)$ is called contra-irresolute if $f^{-1}(O)$ is semi-closed in (X, τ) for each $O \in SO(Y, \sigma)$, and contra-pre-semi-closed if $f(B) \in SO(Y, \sigma)$ for each semi-closed set *B* of (X, τ) .

REMARK 3.8. In fact, contra-irresoluteness and irresoluteness are independent notions. Example 3.3 shows that contra-irresoluteness does not imply irresoluteness while the reverse is shown in the following example.

EXAMPLE 3.9. An irresolute map need not be contra-irresolute. The identity map on the topological space (X, τ) where $\tau = \{\emptyset, \{a\}, X\}$ is an example of an irresolute map which is not contra-irresolute.

In the same manner, we can prove that contra-pre-semi-closed maps and pre-semiclosed are independent notions.

The following result can be easily verified. Its proof is straightforward.

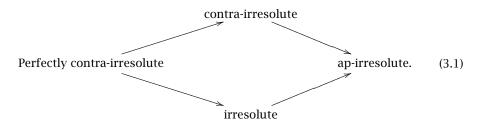
THEOREM 3.10. Let $f : (X, \tau) \to (Y, \sigma)$ be a map. Then the following conditions are equivalent:

- (i) f is contra-irresolute.
- (ii) The inverse image of each semi-closed set in Y is semi-open in X.

REMARK 3.11. By Theorem 3.4, we have that every contra-irresolute map is apirresolute and every contra-pre-semi-closed is ap-semi-closed, the converse implication do not hold.

A map $f: (X, \tau) \to (Y, \sigma)$ is called perfectly contra-irresolute if the inverse of every semi-open set in *Y* is semi-clopen in *X*. Hence, every perfectly contra-irresolute map is contra-irresolute and irresolute.

Clearly, the following diagram holds and none of its implications is reversible:



The next two theorems establish conditions under which maps and inverse maps preserve sg-closed sets.

Sundaram, Maki and Balachandran in [15, Thm. 3.7] showed that the irresolute presemi-closed inverse image of a sg-closed set is sg-closed. We strengthen this result slightly by replacing the pre-semi-closed requirement with ap-semi-closed.

THEOREM 3.12. If a map $f : (X, \tau) \to (Y, \sigma)$ is irresolute and ap-semi-closed, then $f^{-1}(A)$ is sg-closed (resp., sg-open) whenever A is sg-closed (resp., sg-open) subset of (Y, σ) .

PROOF. Let *A* be a sg-closed subset of (Y, σ) . Suppose that $f^{-1}(A) \subseteq O$ where $O \in SO(X, \tau)$. Taking complements we obtain $O^c \subseteq f^{-1}(A^c)$ or $f(O^c) \subseteq A^c$. Since *f* is an ap-semi-closed and $SInt(A) = A \cap Cl(Int(A))$ and $SCl(A) = A \cup Int(Cl(A))$, then $f(O^c) \subseteq SInt(A^c) = (SCl(A))^c$. It follows that $O^c \subseteq (f^{-1}(SCl(A)))^c$ and hence $f^{-1}(SCl(A)) \subseteq O$. Since *f* is irresolute $f^{-1}(SCl(A))$ is semi-closed. Thus we have $SCl(f^{-1}(A)) \subseteq SCl(f^{-1}(SCl(A))) = f^{-1}(SCl(A)) \subseteq O$. This implies that $f^{-1}(A)$ is sg-closed in (X, τ) . A similar argument shows that inverse images of sg-open are sg-open.

This is known (see [15]) that the semi-continuous pre-semi-closed image of a sgclosed set is sg-closed. The following theorem test this result replacing the semicontinuous requirement with ap-irresolute. **THEOREM 3.13.** If a map $f : (X,\tau) \to (Y,\sigma)$ is ap-semi-irresolute and pre-semiclosed, then for every sg-closed F of (X,τ) , f(F) is sg-closed set of (Y,σ) .

PROOF. Let *F* be a sg-closed subset of (X, τ) . Let $f(F) \subseteq O$ where $O \in SO(Y, \sigma)$. Then $F \subseteq f^{-1}(O)$ holds. Since *f* is ap-irresolute $sCl(F) \subseteq f^{-1}(O)$ and hence $f(sCl(F)) \subseteq O$. Therefore, we have $sCl(f(F)) \subseteq sCl(f(sCl(F))) = f(sCl(F)) \subseteq O$. Hence f(F) is sg-closed in (Y, σ) .

Now, reasoning as in [9], we obtain that the composition of two contra-irresolute maps need not be contra-irresolute. Really, Let $X = \{a, b\}$ be the Sierpinski space and set $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{b\}, X\}$. The identity maps $f : (X, \tau) \to (X, \sigma)$ and $g : (X, \sigma) \to (X, \tau)$ are both contra-irresolute but their composition $g \circ f : (X, \tau) \to (X, \tau)$ is not contra-irresolute.

However the following theorem holds. The proof is easy and hence omitted.

THEOREM 3.14. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \gamma)$ be two maps such that $g \circ f : (X, \tau) \to (Z, \gamma)$. Then,

(i) $g \circ f$ is contra-irresolute, if g is irresolute and f is contra-irresolute.

(ii) $g \circ f$ is contra-irresolute, if g is contra-irresolute and f is irresolute.

In an analogous way, we have the following.

THEOREM 3.15. Let $f : (X, \tau) \to (Y, \sigma)$, $g : (Y, \sigma) \to (Z, \gamma)$ be two maps such that $g \circ f : (X, \tau) \to (Z, \gamma)$. Then,

(i) $g \circ f$ is ap-semi-closed, if f is pre-semi-closed and g is ap-semi-closed.

(ii) $g \circ f$ is ap-semi-closed, if f is ap-semi-closed and g is pre-semi-open and g^{-1} preserves sg-open sets.

(iii) $g \circ f$ is ap-irresolute, if f is ap-irresolute and g is irresolute.

PROOF. To prove statement (i), suppose *B* is an arbitrary semi-closed subset in (X, τ) and *A* is a sg-open subset of (Z, γ) for which $g \circ f(B) \subseteq A$. Then f(B) is semi-closed in (Y, σ) because *f* is pre-semi-closed. Since *g* is ap-semi-closed, $g(f(B)) \subseteq$ sInt(*A*). This implies that $g \circ f$ is ap-semi-closed.

To prove statement (ii), suppose *B* is an arbitrary semi-closed subset of (X, τ) and *A* is a sg-open subset of (Z, γ) for which $g \circ f(B) \subseteq A$. Hence $f(B) \subseteq g^{-1}(A)$. Then $f(B) \subseteq \text{sInt}(g^{-1}(A))$ because $g^{-1}(A)$ is sg-open and *f* is ap-semi-closed. Thus,

$$(g \circ f)(B) = g(f(B)) \subseteq g(\operatorname{sInt}(g^{-1}(A))) \subseteq \operatorname{sInt}(gg^{-1}(A)) \subseteq \operatorname{sInt}(A).$$
(3.2)

This implies that $g \circ f$ is ap-semi-closed.

To prove statement (iii), suppose *F* is an arbitrary sg-closed subset of (X, τ) and $O \in$ SO(*Z*, γ) for which $F \subseteq (g \circ f)^{-1}(O)$. Then $g^{-1}(O) \in$ SO(*Y*, σ) because *g* is irresolute. Since *f* is ap-irresolute, sCl(*F*) $\subseteq f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$. This proves that $g \circ f$ is ap-irresolute.

As a consequence of Theorem 3.15, we have the following.

COROLLARY 3.16. Let $f_{\alpha} : X \to Y_{\alpha}$ be a map for each $\alpha \in \Omega$ and $f : X \to \prod Y_{\alpha}$ the product map given by $f(x) = (f_{\alpha}(x))$. If f is ap-irresolute, then f_{α} is ap-irresolute for each α .

PROOF. For each β let P_{β} : $\prod Y_{\alpha} \to Y_{\beta}$ be the projection map. Then $f_{\beta} = P_{\beta} \circ f$, where P_{β} is irresolute. By Theorem 3.15(iii), f_{β} is ap-irresolute.

Regarding the restriction f_A of a map $f: (X, \tau) \to (Y, \sigma)$ to a subset A of X, we have the following.

THEOREM 3.17. (i) If $f: (X, \tau) \to (Y, \sigma)$ is ap-semi-closed and A is a semi-closed set of (X, τ) , then its restriction $f_A: (A, \tau_A) \to (Y, \sigma)$ is ap-semi-closed.

(ii) If $f: (X, \tau) \to (Y, \sigma)$ is ap-irresolute and A is an open, sg-closed subset of (X, τ) , then $f_A: (A, \tau_A) \to (Y, \sigma)$ is ap-irresolute.

PROOF. (i) Suppose *B* is an arbitrary semi-closed subset of (A, τ_A) and *O* a sg-open subset of (Y, σ) for which $f_A(B) \subseteq O$. By [12, Thm. 2.6] *B* is semi-closed of (X, τ) because *A* is semi-closed of (X, τ) . Then $f_A(B) = f(B) \subseteq O$. Using Definition 3.2, we have $f_A(B) \subseteq \operatorname{sInt}(O)$. Thus f_A is an ap-semi-closed map.

(ii) Assume that *F* is a sg-closed subset relative to *A*, i.e., sg-closed in (A, τ_A) , and *G* is a semi-open subset of (Y, σ) for which $F \subseteq (f_A)^{-1}(G)$. Then $F \subseteq f^{-1}(G) \cap A$. By [2, Thm. 3] *F* is sg-closed in *X*. Since *f* is ap-irresolute $sC(F) \subseteq f^{-1}(G)$. Then $sCl(F) \cap A \subseteq f^{-1}(G) \cap A$. Using the fact that $sCl(F) \cap A = sCl_A(F)$ for every pre-open subset [14, Thm. 2.4], we have $sCl_A(F) \subseteq (f_A)^{-1}(G)$. Thus $f_A : (A, \tau_A) \to (Y, \sigma)$ is ap-irresolute.

Observe that restrictions of ap-semi-closed maps can fail to be ap-semi-closed.

Really, as in [1], let *X* be an indiscrete space. Then *X* and \emptyset are the only semi-open subsets of *X*. Hence the semi-closed subsets of *X* are also *X* and \emptyset . Let *A* a nonempty proper subset of *X*. The identity map $f : X \to X$ is ap-semi-closed, but $f_A : A \to X$ fails to be ap-semi-closed. In fact, f(A) is sg-open (every subset of *X* is sg-open) and *A* is closed in *A*. Therefore semi-closed in (A, τ_A) , but $f(A) \subseteq \text{sInt}(f(A))$.

4. A characterization of semi- $T_{1/2}$ spaces. In the following theorem, we give a characterization of a class of topological space called semi- $T_{1/2}$ space by using the concepts of ap-irresolute maps and ap-semi-closed maps.

We recall that a topological space (X, τ) is said to be semi- $T_{1/2}$ space [2], if every sg-closed set is semi-closed.

THEOREM 4.1. Let (X, τ) be a topological space. Then the following statements are equivalent:

(i) (X, τ) is a semi- $T_{1/2}$ space.

(ii) For every space (Y, σ) and every map $f: (X, \tau) \rightarrow (Y, \sigma)$, f is ap-irresolute.

PROOF. (i) \Rightarrow (ii): Let *F* be a sg-closed subset of (X, τ) and suppose that $F \subseteq f^{-1}(O)$, where $O \in SO(Y, \sigma)$. Since (X, τ) is a semi- $T_{1/2}$ space, *F* is semi-closed (i.e., F = sCl(F)). Therefore $sCl(F) \subseteq f^{-1}(O)$. Then *f* is ap-irresolute.

(ii) \Longrightarrow (i): Let *B* be a sg-closed subset of (X, τ) and let *Y* be the set *X* with the topology $\sigma = \{\emptyset, B, Y\}$. Finally let $f : (X, \tau) \to (Y, \sigma)$ be the identity map. By assumption *f* is ap-irresolute. Since *B* is sg-closed in (X, τ) and semi-open in (Y, σ) and $B \subseteq f^{-1}(B)$, it follows that sCl $(B) \subseteq f^{-1}(B) = B$. Hence *B* is semi-closed in (X, τ) and therefore is semi- $T_{1/2}$.

258

THEOREM 4.2. Let (Y, σ) be a topological space. Then the following statements are equivalent:

- (i) (Y, σ) is a semi- $T_{1/2}$ space.
- (ii) For every space (X,τ) and every map $f:(X,\tau) \to (Y,\sigma)$, f is ap-semi-closed.

PROOF. Analogous to Theorem 4.1 making the obvious changes.

We refer the reader to [2, 4, 5, 15] for other results on semi- $T_{1/2}$ spaces.

References

- C. W. Baker, On preserving g-closed sets, Kyungpook Math. J. 36 (1996), no. 1, 195-199. MR 97d:54026. Zbl 908.54011.
- P. Bhattacharyya and B. K. Lahiri, Semi-generalized closed sets in topology, Indian J. Math. 29 (1987), no. 3, 375-382. MR 90a:54004. Zbl 687.54002.
- [3] N. Biswas, On characterizations of semi-continuous functions, Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8) 48 (1970), 399–402. MR 44#998. Zbl 199.25602.
- [4] M. Caldas, *Semi-T*_{1/2} *spaces*, Pro-Math. **VIII** (1994), 115–121.
- [5] _____, A separation axiom between semi-T₀ and semi-T₁, Mem. Fac. Sci. Kôchi Univ. Ser. A Math. 18 (1997), 37-42. MR 98c:54015. Zbl 893.54014.
- [6] S. G. Crossley and S. K. Hildebrand, Semi-closure, Texas J. Sci. 22 (1971), 99-112.
- [7] _____, Semi-topological properties, Fund. Math. 74 (1972), no. 3, 233–254. MR 46 846.
 Zbl 206.51501.
- [8] J. Dontchev, Contra-continuous functions and strongly S-closed spaces, Internat. J. Math. Math. Sci. 19 (1996), no. 2, 303–310. MR 96m:54021. Zbl 840.54015.
- J. Dontchev and H. Maki, On sg-closed sets and semi-λ-closed sets, Questions Answers Gen. Topology 15 (1997), no. 2, 259–266. MR 98h:54002. Zbl 990.12413.
- [10] D. S. Jankovic and I. L. Reilly, On semiseparation properties, Indian J. Pure Appl. Math. 16 (1985), no. 9, 957–964. MR 87a:54029. Zbl 572.54010.
- [11] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly 70 (1963), 36-41. MR 29#4025. Zbl 113.16304.
- S. N. Maheshwari and R. Prasad, On s-regular spaces, Glas. Mat. Ser. III 10 (30) (1975), no. 2, 347–350. MR 53 1518. Zbl 316.54017.
- T. Noiri, A note on semi-continuous mappings, Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8) 55 (1974), 400-403. MR 51 4139. Zbl 336.54012.
- [14] T. Noiri and B. Ahmad, A note on semi-open functions, Math. Sem. Notes Kobe Univ. 10 (1982), no. 2, 437-441. MR 84g:54015. Zbl 515.54010.
- [15] P. Sundaram, H. Maki, and K. Balachandran, Semi-generalized continuous maps and semi- $T_{1/2}$ spaces, Bull. Fukuoka Univ. Ed. Part III **40** (1991), 33-40. MR 92j:54032c. Zbl 790.54010.

CALDAS: DEPARTAMENTO OF MATEMÁTICA, APLICADA-IMUFF, UNIVERSIDADE FEDERAL FLUMI-NENSE, RUA MÁRIO SANTOS BRAGA s/n^o , CEP: 24020-140, NITERÓI RJ, BRASIL *E-mail address*: gmamccs@vm.uff.br

259



Advances in **Operations Research**



The Scientific World Journal







Hindawi

Submit your manuscripts at http://www.hindawi.com



Algebra



Journal of Probability and Statistics



International Journal of Differential Equations





Complex Analysis

International Journal of

Mathematics and Mathematical Sciences





Mathematical Problems in Engineering



Abstract and Applied Analysis

Discrete Dynamics in Nature and Society





Function Spaces



International Journal of Stochastic Analysis

