

SOME OPERATIONS ON LATTICE IMPLICATION ALGEBRAS

E. H. ROH, S. Y. KIM, Y. XU, and Y. B. JUN

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ABSTRACT. We introduce the concept of a \otimes -closed set and a \otimes -homomorphism in lattice implication algebras, and we discuss some of their properties. Next, we introduce the fuzzy implicative filter and obtain equivalent conditions. Finally, we discuss the operation \otimes , fuzzy filters, and fuzzy implicative filters.

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1. Introduction. In order to research the logical system whose propositional value is given in a lattice from the semantic viewpoint, Xu [3] proposed the concept of lattice implication algebras, and discussed some of their properties in [3, 4]. Xu and Qin [5] introduced the notion of filters in a lattice implication algebra, and investigated their properties. In [8], Xu and Qin defined the fuzzy filter in a lattice implication algebra, and they discussed some of their properties. Recently, Xu et al. [9] defined a congruence relation on lattice implication algebras induced by fuzzy filters and they proved the fuzzy homomorphism fundamental theorem, and Liu and Xu [1] introduced the notion of the new binary operation \otimes on lattice implication algebras and they investigated their properties.

2. Preliminaries. We recall a few definitions and properties.

DEFINITION 2.1 (see [4]). By a *lattice implication algebra* we mean a bounded lattice $(L, \vee, \wedge, 0, 1)$ with order-reversing involution “ \prime ” and a binary operation “ \rightarrow ” satisfying the following axioms:

- (I1) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$,
- (I2) $x \rightarrow x = 1$,
- (I3) $x \rightarrow y = y' \rightarrow x'$,
- (I4) $x \rightarrow y = y \rightarrow x = 1 \Rightarrow x = y$,
- (I5) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$,
- (L1) $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$,
- (L2) $(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$,

for all $x, y, z \in L$. If $(L, \vee, \wedge, 0, 1)$ satisfies the conditions (I1), (I2), (I3), (I4), and (I5), is called a *quasi lattice implication algebra*.

We can define a partial ordering \leq on a lattice implication algebra L by $x \leq y$ if and only if $x \rightarrow y = 1$.

In a lattice implication algebra L , the following hold: for all $x, y, z \in L$,

- (1) $0 \rightarrow x = 1$, $1 \rightarrow x = x$, and $x \rightarrow 1 = 1$,
- (2) $x \leq y$ implies $z \rightarrow x \leq z \rightarrow y$ and $x \rightarrow z \geq y \rightarrow z$,

- (3) $(x \rightarrow y) \rightarrow ((y \rightarrow z) \rightarrow (x \rightarrow z)) = 1$,
 (4) $x \rightarrow ((x \rightarrow y) \rightarrow y) = 1$,
 (5) $((x \rightarrow y) \rightarrow y) \rightarrow y = x \rightarrow y$.

DEFINITION 2.2 (see [7]). Let $(L, \vee, \wedge, \prime, \rightarrow)$ be a lattice implication algebra. A subset F of L is called a *filter* of L if it satisfies for all $x, y \in L$

- (F1) $1 \in F$,
 (F2) $x \in F$ and $x \rightarrow y \in F$ imply $y \in F$.

A subset F of L is called an *implicative filter* of L , if it satisfies (F1) and

- (F3) $x \rightarrow (y \rightarrow z) \in F$ and $x \rightarrow y \in F$ imply $x \rightarrow z \in F$

for all $x, y, z \in L$.

The following propositions are clear.

PROPOSITION 2.3. *In a lattice implication algebra, every implicative filter is a filter.*

PROPOSITION 2.4. *Every filter F of a lattice implication algebra L has the following property:*

$$x \leq y, \quad x \in F \text{ imply } y \in F. \quad (2.1)$$

DEFINITION 2.5 (see [6]). Let $(L, \vee, \wedge, \prime, \rightarrow)$ be a lattice implication algebra and $S \subseteq L$. If S satisfies the following condition: S is called a *subalgebra* of L if

- (i) $0 \in S$,
 (ii) for all $x, y \in S$, $x \rightarrow y \in S$.

DEFINITION 2.6 (see [3]). Let $(L, \vee, \wedge, \prime, \rightarrow, \otimes, 0, 1)$, and $(L^*, \vee, \wedge, \prime, \rightarrow, \otimes, 0, 1)$ be lattice implication algebras and $f: L \rightarrow L^*$ be a mapping. Then f is called the *implication homomorphism* if for all $x, y \in L$,

$$f(x \rightarrow y) = f(x) \rightarrow f(y), \quad (2.2)$$

and f is called the *lattice implication homomorphism* if for all $x, y \in L$,

$$f(x \vee y) = f(x) \vee f(y), \quad f(x \wedge y) = f(x) \wedge f(y), \quad f(x') = f(x)'. \quad (2.3)$$

LEMMA 2.7 (see [3]). *Let $(L, \vee, \wedge, \prime, \rightarrow, 0, 1)$ and $(L^*, \vee, \wedge, \prime, \rightarrow, 0, 1)$ be lattice implication algebras. If $f: L \rightarrow L^*$ is a function. Then f is a lattice implication homomorphism if and only if f is an implication homomorphism and $f(a') = f(a)'$ for all $a \in L$.*

DEFINITION 2.8 (see [1]). Let $(L, \vee, \wedge, \prime, \rightarrow, 0, 1)$ be a quasi-lattice implication algebra and given elements a, b of L , we define

$$A(a, b) := \{x \in L \mid a \leq b \rightarrow x\}. \quad (2.4)$$

If for all $x, y \in L$, $A(x, y)$ has a least element, written $x \otimes y$, then the quasi-lattice implication algebra is said to be *with property (P)*.

LEMMA 2.9 (see [1]). *Any lattice implication algebra is with property (P), in fact $a \otimes b = (a \rightarrow b)'$.*

LEMMA 2.10 (see [1]). *Let $(L, \vee, \wedge, ', \rightarrow, 0, 1)$ be a lattice implication algebra. Then the following hold: for all $a, b, c \in L$,*

- (6) $a \otimes b \leq a \wedge b \leq a$,
- (7) $a \leq b$ if and only if $a \otimes b' = 0$,
- (8) $a \otimes b = b \otimes a$,
- (9) $(a \rightarrow b) \otimes a \leq b$,
- (10) $(a \otimes b) \rightarrow c = b \rightarrow (a \rightarrow c)$.

We now review some fuzzy logic concepts. Let X be a set. A function $\mu : X \rightarrow [0, 1]$ is called a *fuzzy subset* on X .

In the following, it will be convenient to write simply L for $(L, \vee, \wedge, ', \rightarrow, \otimes, 0, 1)$, and L^* for $(L^*, \vee, \wedge, ', \rightarrow, \otimes, 0, 1)$.

3. \otimes -closed sets and \otimes -homomorphisms. In this section, we introduce the concept of a \otimes -closed set and a \otimes -homomorphism in lattice implication algebras, and we discuss their properties. If L is a lattice implication algebra, then by Lemma 2.9, we know that $a \otimes b \in L$ for all $a, b \in L$. Now we define the \otimes -closed sets as follows.

DEFINITION 3.1. Let L be a lattice implication algebra and A a nonempty subset of L . Then A is said to be *\otimes -closed* of L if $a \otimes b \in A$ whenever $a, b \in A$.

THEOREM 3.2. *Let F be a filter of a lattice implication algebra L . Then F is a \otimes -closed set in L .*

PROOF. Let F be a filter of L and $a, b \in F$. Then by Proposition 2.4, we have $b \rightarrow (a \otimes b) \in F$, and so we get $a \otimes b \in F$. Therefore F is \otimes -closed in L . \square

THEOREM 3.3. *In a lattice implication algebra, every subalgebra is \otimes -closed.*

PROOF. Let S be a subalgebra of a lattice implication algebra L and $a, b \in S$. Then, since $x' = x \rightarrow 0 \in S$ for all $x \in S$, we have

$$a \otimes b = (a \rightarrow b')' \in S, \quad (3.1)$$

and so S is \otimes -closed in L . \square

REMARK 3.4. The converse of Theorems 3.2 and 3.3 are not true, and so we know that \otimes -closed sets are the generalization of subalgebras and filters in a lattice implication algebra.

EXAMPLE 3.5. Let $L := \{0, a, b, c, 1\}$. Define the partially ordered relation on L as $0 < a < b < c < 1$, and define $x \wedge y := \min\{x, y\}$, $x \vee y := \max\{x, y\}$ for all $x, y \in L$, and “ $'$ ” and “ \rightarrow ” as follows:

x	x'
0	1
a	c
b	b
c	a
1	0

\rightarrow	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	b	c	1	1	1
c	a	b	c	1	1
1	0	a	b	c	1

Then $(L, \vee, \wedge, \prime, \rightarrow)$ is a lattice implication algebra and we can find the following \otimes -table:

\otimes	0	a	b	c	1
0	0	0	0	0	0
a	0	0	0	0	a
b	0	0	0	a	b
c	0	0	a	b	c
1	0	a	b	c	1

Now we consider the \otimes -closed set $A := \{0, a, b\}$, which is neither a filter nor a sub-algebra of L since $0 \in A$ and $b \rightarrow a = c \notin A$.

DEFINITION 3.6. Let L and L^* be lattice implication algebras and let $f : L \rightarrow L^*$ be a function. Then F is called a \otimes -homomorphism if for all $a, b \in L$,

$$f(a \otimes b) = f(a) \otimes f(b). \quad (3.2)$$

THEOREM 3.7. Let $f : L \rightarrow L^*$ be a \otimes -homomorphism of lattice implication algebras. If $f^{-1}(0) := \{a \in L \mid f(a) = 0\} \neq \emptyset$, then $f^{-1}(0)$ is \otimes -closed in L .

PROOF. Let $a, b \in f^{-1}(0)$. Then we have $f(a \otimes b) = f(a) \otimes f(b) = 0 \otimes 0 = 0$, and so $a \otimes b \in f^{-1}(0)$, that is, $f^{-1}(0)$ is a \otimes -closed set in L . \square

We discuss the connection between lattice implication homomorphisms and \otimes -homomorphisms.

THEOREM 3.8. Let $f : L \rightarrow L^*$ be a function of lattice implication algebras. If f is a lattice implication homomorphism, then f is a \otimes -homomorphism.

PROOF. For any $a, b \in L$, we have

$$f(a \otimes b) = f((a \rightarrow b)') = f(a \rightarrow b)' = (f(a) \rightarrow f(b))' = f(a) \otimes f(b), \quad (3.3)$$

and hence f is a \otimes -homomorphism. \square

We consider the converse of [Theorem 3.8](#), so we have the following theorem.

THEOREM 3.9. Let $f : L \rightarrow L^*$ be a function of lattice implication algebras. If f is a \otimes -homomorphism and $f(a') = f(a)'$ for all $a \in L$, then f is a lattice implication homomorphism.

PROOF. For any $a, b \in L$, we have

$$f(a \rightarrow b) = f((a \otimes b)') = f(a) \rightarrow f(b). \quad (3.4)$$

Thus by [Lemma 2.7](#), we obtain that f is a lattice implication homomorphism. \square

By [Theorems 3.8](#) and [3.9](#), we have the following corollary.

COROLLARY 3.10. *Let $f : L \rightarrow L^*$ be a function of lattice implication algebras. Then f is a lattice implication homomorphism if and only if f is a \otimes -homomorphism and $f(a') = f(a)'$ for all $a \in L$.*

THEOREM 3.11. *Let L and L^* be lattice implication algebras, and f and g be functions from L into L^* . If $f(a \otimes b) = g(a \otimes b)$ for all $a, b \in L$, then $f = g$.*

PROOF. For any $x \in L$, we have $f(x) = f(x \otimes 1) = g(x \otimes 1) = g(x)$, and so $f = g$. \square

THEOREM 3.12. *Let $f : L \rightarrow L^*$ be a surjective \otimes -homomorphism of lattice implication algebras. If A is \otimes -closed in L , then $f(A)$ is \otimes -closed in L^* .*

PROOF. For any $a, b \in f(A)$, there exist x, y in A such that $f(x) = a$ and $f(y) = b$. Thus we have $a \otimes b = f(x) \otimes f(y) = f(x \otimes y)$. Since A is \otimes -closed in L , we get $a \otimes b \in f(A)$, and hence $f(A)$ is \otimes -closed in L^* . \square

THEOREM 3.13. *Let $f : L \rightarrow L^*$ be a \otimes -homomorphism of lattice implication algebras. If M^* is \otimes -closed in L^* , then $f^{-1}(M^*)$ is \otimes -closed in L .*

PROOF. Suppose that M^* is \otimes -closed in L^* and let $a, b \in f^{-1}(M^*)$. Then we have $f(a) \in M^*$ and $f(b) \in M^*$. Since M^* is \otimes -closed in L^* , we get $f(a \otimes b) = f(a) \otimes f(b) \in M^*$, and so $a \otimes b \in f^{-1}(M^*)$, that is, $f^{-1}(M^*)$ is \otimes -closed in L . \square

4. The operation \otimes and the fuzzy (implicative) filters. In this section, we introduce the fuzzy implicative filter and obtain equivalent conditions of implicative filters. Moreover, we discuss the operation \otimes , fuzzy filters and fuzzy implicative filters.

DEFINITION 4.1 (see [8]). A fuzzy subset μ of a lattice implication algebra L is called a *fuzzy filter* if it satisfies for any $x, y \in L$,

$$(FF1) \quad \mu(1) \geq \mu(x),$$

$$(FF2) \quad \mu(y) \geq \min\{\mu(x \rightarrow y), \mu(x)\}.$$

PROPOSITION 4.2 (see [8]). *Let μ be a fuzzy filter of a lattice implication algebra L and $x, y \in L$. Then $x \leq y$ implies $\mu(x) \leq \mu(y)$.*

THEOREM 4.3. *If μ is a fuzzy filter of a lattice implication algebra L , then for any $x, y \in L$,*

$$(FF3) \quad \mu(x \otimes y) \geq \min\{\mu(x), \mu(y)\}.$$

PROOF. Let μ be a fuzzy filter of L . Then for all $x, y \in L$, we have

$$\mu(x \otimes y) \geq \min\{\mu(y \rightarrow (x \otimes y)), \mu(y)\} \geq \min\{\mu(x), \mu(y)\}. \quad (4.1)$$

\square

Let μ be a fuzzy filter of a lattice implication algebra L and $x, y \in L$. Then, using [Proposition 4.2](#) and (FF3), we have $\mu(1) \geq \mu(x)$ and

$$\mu(y) \geq \mu(x \wedge y) = \mu(x \otimes (x \rightarrow y)) \geq \min\{\mu(x), \mu(x \rightarrow y)\}. \quad (4.2)$$

Thus by [Theorem 4.3](#), we have the following theorem.

THEOREM 4.4. *Let L be a lattice implication algebra. A fuzzy set μ in L is a fuzzy filter if and only if μ is an order preserving set and (FF3) holds.*

We provide an equivalent condition for a fuzzy set to be a fuzzy filter.

THEOREM 4.5. *Let L be a lattice implication algebra. A fuzzy set μ in L is a fuzzy filter if and only if it satisfies for all $x, y, z \in L$,*

$$(FF4) \quad x \geq y \otimes z \text{ implies } \mu(x) \geq \min\{\mu(y), \mu(z)\}.$$

PROOF. Suppose that μ is a fuzzy filter of L and let $x, y, z \in L$ be such that $x \geq y \otimes z$. Then by [Theorem 4.3](#), we have

$$\begin{aligned} \mu(x) &\geq \min\{\mu((y \otimes z) \rightarrow x), \mu(y \otimes z)\} \\ &= \min\{\mu(1), \mu(y \otimes z)\} \\ &\geq \min\{\mu(y), \mu(z)\}, \end{aligned} \tag{4.3}$$

and so (FF4) holds.

Conversely, suppose that μ satisfies (FF4). Since $1 \geq x \otimes x$ for all $x \in L$, it follows from (FF4) that

$$\mu(1) \geq \min\{\mu(x), \mu(x)\} = \mu(x) \quad \forall x \in L. \tag{4.4}$$

Note that $(y \rightarrow x) \otimes y \leq x$ for all $x, y \in L$. Hence, by (FF4), we have $\mu(x) \geq \min\{\mu(y \rightarrow x), \mu(y)\}$, which proves (FF2). Therefore μ is a fuzzy filter of L . \square

Now, we give an equivalent condition of implicative filters as follows.

LEMMA 4.6 (see [2]). *Let F be an implicative filter of L . Then for any $x, y \in L$,*

$$(x \rightarrow y) \rightarrow x \in F \text{ implies } x \in F. \tag{4.5}$$

THEOREM 4.7. *A nonempty subset F of L is an implicative filter if and only if it satisfies (F1) and*

$$(F4) \quad z \rightarrow ((x \rightarrow y) \rightarrow x) \in F \text{ and } z \in F \text{ imply } x \in F \text{ for all } x, y, z \in L.$$

PROOF. Suppose that F satisfies (F1) and (F4). Let $x, z \in L$ be such that $z \rightarrow x \in F$ and $z \in F$. In (F4), we take $y = x$, then

$$z \rightarrow ((x \rightarrow x) \rightarrow x) = z \rightarrow (1 \rightarrow x) = z \rightarrow x \in F. \tag{4.6}$$

Thus by (F4), we have $x \in F$. This says that F is a filter of L . Let $x, y, z \in L$ be such that $z \rightarrow (y \rightarrow x) \in F$ and $z \rightarrow y \in F$. Since $z \rightarrow (y \rightarrow x) = y \rightarrow (z \rightarrow x) \leq (z \rightarrow y) \rightarrow (z \rightarrow (z \rightarrow x))$, we have $z \rightarrow (z \rightarrow x) \in F$. As $((z \rightarrow x) \rightarrow x) \rightarrow (z \rightarrow x) = z \rightarrow (((z \rightarrow x) \rightarrow x) \rightarrow x) = z \rightarrow (z \rightarrow x) \in F$, it follows that $1 \rightarrow (((z \rightarrow x) \rightarrow x) \rightarrow (z \rightarrow x)) \in F$. Combining (F1) and (F4) we obtain $z \rightarrow x \in F$. This means that F is an implicative filter of L .

Conversely, suppose that F is an implicative filter of L . Let $x, y, z \in L$ be such that $z \rightarrow ((x \rightarrow y) \rightarrow x) \in F$ and $z \in F$. Then by [Proposition 2.4](#), we have $(x \rightarrow y) \rightarrow x \in F$. By [Lemma 4.6](#), we get $x \in F$. \square

We state the fuzzification of implicative filters.

DEFINITION 4.8. A fuzzy subset μ of a lattice implication algebra L is called a *fuzzy implicative filter* if it satisfies (FF1) and

$$(FF5) \quad \mu(x) \geq \min\{\mu(z \rightarrow ((x \rightarrow y) \rightarrow x)), \mu(z)\} \text{ for all } x, y, z \in L.$$

THEOREM 4.9. *In a lattice implication algebra, every fuzzy implicative filter is a fuzzy filter.*

PROOF. In (FF5), we take $y = x$, then we have

$$\begin{aligned} \mu(x) &\geq \min\{\mu(z \rightarrow ((x \rightarrow x) \rightarrow x)), \mu(z)\} \\ &= \min\{\mu(z \rightarrow (1 \rightarrow x)), \mu(z)\} \\ &= \min\{\mu(z \rightarrow x), \mu(z)\}. \end{aligned} \tag{4.7}$$

This means that F is a filter of L . \square

We state an equivalent condition for a fuzzy set to be a fuzzy implicative filter.

THEOREM 4.10. *Let μ be a fuzzy filter of a lattice implication algebra L . Then the following are equivalent:*

- (i) μ is a fuzzy implicative filter.
- (ii) $\mu(x) \geq \mu((x \rightarrow y) \rightarrow x)$ for all $x, y \in L$.
- (iii) $\mu(x) = \mu((x \rightarrow y) \rightarrow x)$ for all $x, y \in L$.

PROOF. (i) \Rightarrow (ii). Let μ be a fuzzy implicative filter of L . Then by (FF5), we have

$$\mu(x) \geq \min\{\mu(1 \rightarrow ((x \rightarrow y) \rightarrow x)), \mu(1)\} = \mu((x \rightarrow y) \rightarrow x) \tag{4.8}$$

for all $x, y \in L$. Hence the condition (ii) holds.

(ii) \Rightarrow (iii). Observe that $x \leq (x \rightarrow y) \rightarrow x$ for all $x, y \in L$. Then, by [Proposition 2.4](#), we have $\mu(x) \leq \mu((x \rightarrow y) \rightarrow x)$. It follows from (ii) that $\mu(x) = \mu((x \rightarrow y) \rightarrow x)$. Hence the condition (iii) holds.

(iii) \Rightarrow (i). Suppose that the condition (iii) holds. Since μ is a fuzzy filter, by (FF2) we have

$$\mu((x \rightarrow y) \rightarrow x) \geq \min\{\mu(z \rightarrow ((x \rightarrow y) \rightarrow x)), \mu(z)\}. \tag{4.9}$$

Combining (iii) we obtain

$$\mu(x) \geq \min\{\mu(z \rightarrow ((x \rightarrow y) \rightarrow x)), \mu(z)\}, \tag{4.10}$$

thus μ satisfies (FF5). Therefore μ is a fuzzy implicative filter of L . \square

THEOREM 4.11. *Let L be a lattice implication algebra. A fuzzy set μ in L is a fuzzy implicative filter if and only if it satisfies*

$$(FF6) \quad z \otimes u \leq (x \rightarrow y) \rightarrow x \text{ in } L \text{ implies } \mu(x) \geq \min\{\mu(z), \mu(u)\} \text{ for all } x, y, z, u \in L,$$

PROOF. Suppose that μ is a fuzzy implicative filter of L and let $x, y, z, u \in L$ be such that $z \otimes u \leq (x \rightarrow y) \rightarrow x$. Since μ is a fuzzy filter of L by [Theorem 4.9](#), it follows from [Theorems 4.5](#) and [4.10](#) that

$$\mu(x) = \mu((x \rightarrow y) \rightarrow x) \geq \min\{\mu(z), \mu(u)\}. \quad (4.11)$$

Conversely, suppose that μ satisfies (FF6). Obviously, μ satisfies (FF1). Since $(z \rightarrow ((x \rightarrow y) \rightarrow x)) \otimes z \leq (x \rightarrow y) \rightarrow x$, (FF6) implies that

$$\mu(x) \geq \min\{\mu(z \rightarrow ((x \rightarrow y) \rightarrow x)), \mu(z)\}, \quad (4.12)$$

which proves (FF5). Therefore μ is a fuzzy implicative filter of L . \square

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E. H. ROH: DEPARTMENT OF MATHEMATICS EDUCATION, CHINJU NATIONAL UNIVERSITY OF EDUCATION, CHINJU 660-756, KOREA

E-mail address: ehroh@cue.ac.kr

S. Y. KIM: DEPARTMENT OF MATHEMATICS EDUCATION, CHINJU NATIONAL UNIVERSITY OF EDUCATION, CHINJU 660-756, KOREA

Y. XU: DEPARTMENT OF APPLIED MATHEMATICS, SOUTHWEST JIAOTONG UNIVERSITY, CHENGDU, SICHUAN 610031, CHINA

Y. B. JUN: DEPARTMENT OF MATHEMATICS EDUCATION, GYEONGSANG NATIONAL UNIVERSITY, CHINJU 660-701, KOREA

E-mail address: ybjun@nongae.gsnu.ac.kr



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