

# ON WEIGHTED INEQUALITIES FOR CERTAIN FRACTIONAL INTEGRAL OPERATORS

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This paper considers the modified fractional integral operators involving the Gauss hypergeometric function and obtains weighted inequalities for these operators. Multidimensional fractional integral operators involving the H-function are also introduced.

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## 1. Introduction and preliminaries

Tuan and Saigo [7] introduced the multidimensional modified fractional integrals of order  $\alpha$  ( $\text{Re}(\alpha) > 0$ ) by

$$\begin{aligned} X_{+;n}^{\alpha} f(x) &= \frac{1}{\Gamma(\alpha+1)} \mathbf{D}^n \int_{\mathbb{R}_+^n} \left[ \min \left\{ \frac{x_1}{t_1}, \dots, \frac{x_n}{t_n} \right\} - 1 \right]_+^{\alpha} f(t) dt, \\ X_{-;n}^{\alpha} f(x) &= \frac{(-1)^n}{\Gamma(\alpha+1)} \mathbf{D}^n \int_{\mathbb{R}_+^n} \left[ 1 - \max \left\{ \frac{x_1}{t_1}, \dots, \frac{x_n}{t_n} \right\} \right]_+^{\alpha} f(t) dt, \end{aligned} \quad (1.1)$$

where  $\mathbb{R}_+^n = \{(t_1, \dots, t_n) \mid t_i > 0 (i = 1, \dots, n)\}$ ,  $\varphi_+(x)$  is a real-valued function defined in terms of the function  $\varphi(x)$  by

$$\varphi_+(x) = \begin{cases} \varphi(x), & \varphi(x) > 0, \\ 0, & \varphi(x) \leq 0, \end{cases} \quad (1.2)$$

and  $\mathbf{D}^n$  denotes the derivative operator  $\partial^n / \partial x_1, \dots, \partial x_n$ .

The operators in (1.1) provide multidimensional generalizations to the well-known one-dimensional Riemann-Liouville and Weyl fractional integral operators defined in [5] (see also [1]). The paper [7] considers several formulas and interesting properties of (1.1). By invoking the Gauss hypergeometric function  ${}_2F_1(\alpha, \beta; \gamma; x)$ , the following generalizations of the multidimensional modified integral operators (1.1) of order  $\alpha$  ( $\text{Re}(\alpha) > 0$ )

## 2 Inequalities for certain integral operators

were studied in [6]:

$$S_{+;n}^{\alpha,\beta,\gamma} f(x) = \frac{1}{\Gamma(\alpha+1)} \mathbf{D}^n \int_{\mathbb{R}_+^n} \left[ \min \left\{ \frac{x_1}{t_1}, \dots, \frac{x_n}{t_n} \right\} - 1 \right]_+^\alpha \cdot {}_2F_1 \left( \alpha + \beta, \alpha + \eta; 1 + \alpha; 1 - \min \left\{ \frac{x_1}{t_1}, \dots, \frac{x_n}{t_n} \right\} \right) f(t) dt, \quad (1.3)$$

$$S_{-;n}^{\alpha,\beta,\gamma} f(x) = \frac{(-1)^n}{\Gamma(\alpha+1)} \mathbf{D}^n \int_{\mathbb{R}_+^n} \left[ 1 - \max \left\{ \frac{x_1}{t_1}, \dots, \frac{x_n}{t_n} \right\} \right]_+^\alpha \cdot {}_2F_1 \left( \alpha + \beta, -\eta; 1 + \alpha; 1 - \max \left\{ \frac{x_1}{t_1}, \dots, \frac{x_n}{t_n} \right\} \right) f(t) dt. \quad (1.4)$$

For  $\beta = -\alpha$ , the operators (1.3) and (1.4) reduce to the modified integral operators defined in (1.1), respectively. In [8], the integral operators  $X_{+;n}^\alpha f(x)$  and  $X_{-;n}^\alpha f(x)$  defined on the space  $\mathcal{M}_\gamma(\mathbb{R}_+^n)$  are shown to satisfy some  $L_p - L_q$  weighted inequalities. The space  $\mathcal{M}_\gamma(\mathbb{R}_+^n)$  represents the space of functions  $f$  which are defined on  $\mathbb{R}_+^n$ , and are entire functions of exponential type (see [7]). The present paper is devoted to finding inequalities for the generalized multidimensional modified integral operators (1.3) and (1.4) by making use of the inequality stated in [8] (which was established with the aid of Pitt's inequality). Multidimensional operators have also been studied in [3, 4].

### 2. Inequalities for operators (1.3) and (1.4)

If  $(\mathcal{H}f)(x)$  denotes the integral operator

$$(\mathcal{H}f)(x) = \int_{\mathbb{R}_+^n} k(xy) f(y) dy, \quad (2.1)$$

then following [8], we have

$$\int_{\mathbb{R}_+^n} k(xy) f(y) dy = \frac{1}{(2\pi i)^n} \int_{(1/2)} k^*(s) f^*(1-s) x^{-s} ds, \quad (2.2)$$

where the integral over  $(1/2)$  stands for the multiple integral

$$\int_{(1/2)} = \int_{1/2-i\infty}^{1/2+i\infty} \cdots \int_{1/2-i\infty}^{1/2+i\infty}, \quad (2.3)$$

and  $k^*(s)$  and  $f^*(1-s)$  are the Mellin transforms of the functions  $k(y)$  and  $f(y)$ , respectively. It is proved in [8] that if

$$|k^*(s)| \leq C |s|^{-\alpha} \left( s \in \left( \frac{1}{2} \right), \alpha \geq 0 \right), \quad (2.4)$$

then there holds the inequality

$$\| |\log y - t|^{-b} y^{1/2-1/r} (\mathcal{H}f)(y) \|_{L_r(\mathbb{R}_+^n)} \leq C \| |\log y - t|^d y^{1/2-1/q} f(y) \|_{L_q(\mathbb{R}_+^n)}, \quad (2.5)$$

or equivalently,

$$\| |\log y - t|^{-b} y^{1/2-1/r} (\mathcal{H}f)(y) \|_{L_r(\mathbb{R}_+^n)} \leq C \| |\log y - t|^{a-b+n(1/r-1/q)} y^{1/2-1/q} f(y) \|_{L_q(\mathbb{R}_+^n)}, \quad (2.6)$$

for all  $t \in \mathbb{R}_+^n$ , provided that

$$\left[ \begin{array}{l} \max \left\{ \frac{2}{r}, \frac{\alpha}{n} + \frac{1}{r} \right\} - 1 \leq \frac{b}{n} \leq \min \left\{ 0, \frac{\alpha}{n} - \frac{1}{q}, \frac{\alpha}{n} + \frac{1}{q} - 1 \right\} + \frac{1}{r}, \\ \max \left\{ 0, n \left( \frac{1}{r} + \frac{1}{q} - 1 \right) \right\} \leq \alpha \leq n, \quad 1 < q \leq r < \infty. \end{array} \right]. \quad (2.7)$$

In the paper [6], it was established that if

$$\operatorname{Re}(\alpha) > 0, \quad \operatorname{Re}(h_j) < \frac{1}{2} \quad (j = 1, \dots, n), \quad \sum_{j=1}^n \operatorname{Re}(h_j) < \frac{n}{2} + \min \{ \operatorname{Re}(\beta), \operatorname{Re}(\eta) \}, \quad (2.8)$$

then the operator  $x^h \mathcal{S}_{+;n}^{\alpha,\beta,\gamma} x^{-h} f(x)$  is a homeomorphism of the space  $\mathcal{M}_{1/2}(\mathbb{R}_+^n)$  onto itself, and

$$\begin{aligned} & x^h \mathcal{S}_{+;n}^{\alpha,\beta,\gamma} x^{-h} f(x) \\ &= \frac{1}{(2\pi i)^n} \int_{(1/2)} \frac{\Gamma(\beta + n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j) \Gamma(\eta + n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j)}{\Gamma(n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j) \Gamma(\alpha + \beta + \eta + n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j)} f^*(s) x^{-s} ds. \end{aligned} \quad (2.9)$$

We note that

$$\frac{\Gamma(\beta + n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j) \Gamma(\eta + n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j)}{\Gamma(n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j) \Gamma(\alpha + \beta + \eta + n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j)} = O(|s|^{-\alpha}), \quad (2.10)$$

so we can apply the inequality (2.6) to the multidimensional operator defined by (2.9), which leads to

$$\begin{aligned} & \| |\log y - t|^{-b} y^{1/2-1/r+h} \mathcal{S}_{+;n}^{\alpha,\beta,\gamma} y^{-h} f(y) \|_{L_r(\mathbb{R}_+^n)} \\ & \leq C \| |\log y - t|^{a-b+n(1/r-1/q)} y^{1/2-1/q} f(y) \|_{L_q(\mathbb{R}_+^n)}, \end{aligned} \quad (2.11)$$

valid for all  $t \in \mathbb{R}_+^n$ , provided that the constraints (2.7) and (2.8) are satisfied. On the other hand, (see [6]) if

$$\operatorname{Re}(\alpha) > 0, \quad \operatorname{Re}(h_j) > \frac{1}{2} \quad (j = 1, \dots, n), \quad \sum_{j=1}^n \operatorname{Re}(h_j) > \frac{n}{2} + \operatorname{Re}(\beta - \eta) - 1, \quad (2.12)$$

#### 4 Inequalities for certain integral operators

then the operator  $x^h \mathcal{S}_{-;n}^{\alpha,\beta,\gamma} x^{-h} f(x)$  is a homeomorphism of the space  $\mathcal{M}_{1/2}(\mathbb{R}_+^n)$  onto itself, and we obtain

$$\begin{aligned} & x^h \mathcal{S}_{-;n}^{\alpha,\beta,\gamma} x^{-h} f(x) \\ &= \frac{1}{(2\pi i)^n} \int_{(1/2)} \frac{\Gamma(1-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j) \Gamma(1-\beta+\eta-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j)}{\Gamma(1-\beta-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j) \Gamma(1+\alpha+\eta-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j)} \\ & \quad \times f^*(s) x^{-s} ds. \end{aligned} \tag{2.13}$$

By noting the estimate that

$$\frac{\Gamma(1-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j) \Gamma(1-\beta+\eta-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j)}{\Gamma(1-\beta-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j) \Gamma(1+\alpha+\eta-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j)} = O(|s|^{-\alpha}), \tag{2.14}$$

we again apply the inequality (2.6) to the multidimensional operator defined by (2.13) to get

$$\begin{aligned} & \| |\log y - t|^{-b} y^{1/2-1/r+h} \mathcal{S}_{-;n}^{\alpha,\beta,\gamma} y^{-h} f(y) \|_{L_r(\mathbb{R}_+^n)} \\ & \leq C \| |t \log y - t|^{a-b+n(1/r-1/q)} y^{1/2-1/q} f(y) \|_{L_q(\mathbb{R}_+^n)}, \end{aligned} \tag{2.15}$$

valid for all  $t \in \mathbb{R}_+^n$ , provided that the constraints (2.7) and (2.12) are satisfied.

### 3. Classes of multidimensional operators

We introduce the following classes of multidimensional modified fractional integral operators involving the well-known H-function [2, Section 8.3] (see also [1, page 343]) defined by

$$\begin{aligned} & \left( \mathbf{H}_{\mathbf{P},\mathbf{Q},+;n}^{\mathbf{M},\mathbf{N}} \left| \begin{matrix} (\mathbf{a}_p, \alpha_p) \\ (\mathbf{b}_q, \beta_q) \end{matrix} \right. f \right) (x) = \left( \mathbf{H}_{\mathbf{P},\mathbf{Q},+;n}^{\mathbf{M},\mathbf{N}} \left| \begin{matrix} (\mathbf{a}_1, \alpha_1), \dots, (\mathbf{a}_p, \alpha_p) \\ (\mathbf{b}_1, \beta_1), \dots, (\mathbf{b}_q, \beta_q) \end{matrix} \right. \mathbf{f} \right) (x) \\ & = \mathbf{D}^n \int_{\mathbb{R}_+^n} H_{P,Q}^{\mathbf{M},\mathbf{N}} \left[ \min \left\{ \frac{x_1}{t_1}, \dots, \frac{x_n}{t_n} \right\} \left| \begin{matrix} (a_p, \alpha_p) \\ (b_q, \beta_q) \end{matrix} \right. \right] f(t) dt, \end{aligned} \tag{3.1}$$

$$\begin{aligned} & \left( \mathbf{H}_{\mathbf{P},\mathbf{Q},-;n}^{\mathbf{M},\mathbf{N}} \left| \begin{matrix} (\mathbf{a}_p, \alpha_p) \\ (\mathbf{b}_q, \beta_q) \end{matrix} \right. f \right) (x) = \left( \mathbf{H}_{\mathbf{P},\mathbf{Q},-;n}^{\mathbf{M},\mathbf{N}} \left| \begin{matrix} (a_1, \alpha_1), \dots, (a_p, \alpha_p) \\ (b_1, \beta_1), \dots, (b_q, \beta_q) \end{matrix} \right. \mathbf{f} \right) (x) \\ & = (-1)^n \mathbf{D}^n \int_{\mathbb{R}_+^n} H_{P,Q}^{\mathbf{M},\mathbf{N}} \left[ \max \left\{ \frac{x_1}{t_1}, \dots, \frac{x_n}{t_n} \right\} \left| \begin{matrix} (a_p, \alpha_p) \\ (b_q, \beta_q) \end{matrix} \right. \right] f(t) dt, \end{aligned} \tag{3.2}$$

where we assume that the parameters of the H-function involved in (3.1) and (3.2) satisfy the existence conditions as given in [2].

The special cases of the operators of interest in this paper are the operators which emerge from (3.1) and (3.2) in the case when  $N = 0$ ,  $P = M$ ,  $Q = M$ , and the parameters  $\alpha_1 = \alpha_2 = \dots = \alpha_m = 1$ , and  $\beta_1 = \beta_2 = \dots = \beta_m = 1$ . Thus, we have the following multidimensional fractional integral operators (defined in terms of Meijer's G-function)

(see [6]):

$$\begin{aligned}
 (\mathbf{G}_{+;n}^{(\mathbf{a}_m);(\mathbf{b}_m)} \mathbf{f})(x) &= (\mathbf{G}_{+;n}^{(\mathbf{a}_1, \dots, \mathbf{a}_m);(\mathbf{b}_1, \dots, \mathbf{b}_m)} \mathbf{f})(x) \\
 &= \mathbf{D}^n \int_{\mathbb{R}_+^n} G_{m,m}^{m,0} \left[ \min \left\{ \frac{x_1}{t_1}, \dots, \frac{x_n}{t_n} \right\} \right]_{(b_m)}^{(a_m)} f(t) dt, \\
 (\mathbf{G}_{-;n}^{(\mathbf{a}_m);(\mathbf{b}_m)} \mathbf{f})(x) &= (\mathbf{G}_{+;n}^{(\mathbf{a}_1, \dots, \mathbf{a}_m);(\mathbf{b}_1, \dots, \mathbf{b}_m)} \mathbf{f})(x) \\
 &= (-1)^n \mathbf{D}^n \int_{\mathbb{R}_+^n} G_{m,m}^{m,0} \left[ \max \left\{ \frac{x_1}{t_1}, \dots, \frac{x_n}{t_n} \right\} \right]_{(b_m)}^{(a_m)} f(t) dt.
 \end{aligned} \tag{3.3}$$

By setting the parameters

$$m = 2, \quad a_1 = 1 - \beta, \quad a_2 = 1 - \eta, \quad b_1 = 1 - \alpha - \beta - \eta, \quad b_2 = 0, \tag{3.4}$$

in (3.1), and

$$m = 2, \quad a_1 = 1 - \beta, \quad a_2 = 1 + \alpha + \eta, \quad b_1 = 1 - \beta + \eta, \quad b_2 = 0, \tag{3.5}$$

in (3.2), and noting the relation (see [1, equation (1.1.18), page 18] )

$$\begin{aligned}
 G_{2,2}^{2,0} \left[ \sigma \middle| \begin{matrix} a_1, a_2 \\ b_1, b_2 \end{matrix} \right] &= \frac{\sigma^{b_2} (1 - \sigma)^{a_1 + a_2 - b_1 - b_2 - 1}}{\Gamma(a_1 + a_2 - b_1 - b_2)} \\
 &\cdot {}_2F_1(a_2 - b_1, a_1 - b_1; a_1 + a_2 - b_1 - b_2; 1 - \sigma) \quad (\sigma < 1),
 \end{aligned} \tag{3.6}$$

we observe the following relationships:

$$\begin{aligned}
 (\mathbf{G}_{+;n}^{(1-\beta, 1-\eta);(1-\alpha-\beta-\eta, 0)} \mathbf{f})(x) &= (-1)^\alpha S_{+;n}^{\alpha, \beta, \gamma} f(x), \\
 (\mathbf{G}_{-;n}^{(1-\beta, 1+\alpha+\eta);(1-\beta+\eta, 0)} \mathbf{f})(x) &= S_{-;n}^{\alpha, \beta, \gamma} f(x),
 \end{aligned} \tag{3.7}$$

in terms of the multidimensional modified fractional integral operators (1.3) and (1.4).

We state below two useful lemmas concerning the multidimensional Mellin transform of the functions  $f(\max[x_1, \dots, x_n])$  and  $f(\min[x_1, \dots, x_n])$  (see [3, 6]).

LEMMA 3.1. *Let  $\text{Re}(s_j) > 0$  ( $j = 1, \dots, n$ ) and let  $\tau^{s-1} f(\tau) \in L_1(\mathbb{R}_+)$ , then*

$$\int_{\mathbb{R}_+^n} x^{s-1} f(\max[x_1, \dots, x_n]) dx = \frac{|s|}{s^1} f^*(|s|), \tag{3.8}$$

where  $s^1$  denotes the product  $s_1, \dots, s_n$ , and  $|s| = s_1 + \dots + s_n$ .

LEMMA 3.2. *Let  $\text{Re}(s_j) < 0$  ( $j = 1, \dots, n$ ) and let  $\tau^{s-1} f(\tau) \in L_1(\mathbb{R}_+)$ , then*

$$\int_{\mathbb{R}_+^n} x^{s-1} f(\min[x_1, \dots, x_n]) dx = (-1)^{n-1} \frac{|s|}{s^1} f^*(|s|). \tag{3.9}$$

## 6 Inequalities for certain integral operators

Making use of (3.1), we have

$$\begin{aligned} \left( \mathbf{H}_{P,Q,+;n}^{M,N} \left| \begin{array}{l} (a_p, \alpha_p) \\ (b_q, \beta_q) \end{array} \right. \mathbf{x}^{-s} \right) (x) &= \mathbf{D}^n \int_{\mathbb{R}_+^n} H_{P,Q}^{M,N} \left[ \min \left\{ \frac{x_1}{t_1}, \dots, \frac{x_n}{t_n} \right\} \left| \begin{array}{l} (a_p, \alpha_p) \\ (b_q, \beta_q) \end{array} \right. \right] t^{-s} dt \\ &= \mathbf{D}^n x^{1-s} \int_{\mathbb{R}_+^n} t^{s-2} H_{P,Q}^{M,N} \left[ \min \left\{ t_1, \dots, t_n \right\} \left| \begin{array}{l} (a_p, \alpha_p) \\ (b_q, \beta_q) \end{array} \right. \right] dt. \end{aligned} \quad (3.10)$$

Applying now (3.9) of Lemma 3.2, and the following result giving the Mellin transform of the H-function [1, equation (E.20), page 348], namely,

$$\begin{aligned} \left\{ H_{P,Q}^{M,N} \left[ x \left| \begin{array}{l} (a_p, \alpha_p) \\ (b_q, \beta_q) \end{array} \right. \right] \right\}^* (s) &= \frac{\prod_{j=1}^M \Gamma(b_j + \beta_j s) \prod_{i=1}^N \Gamma(1 - a_i - \alpha_i s)}{\prod_{i=N+1}^P \Gamma(a_i + \alpha_i s) \prod_{j=M+1}^Q \Gamma(1 - b_j - \beta_j s)} \\ &\quad \times \left( - \min_{1 \leq j \leq M} \left[ \frac{\operatorname{Re}(b_j)}{\beta_j} \right] < \operatorname{Re}(s) < \min_{1 \leq i \leq N} \left[ \frac{1 - \operatorname{Re}(a_i)}{\alpha_i} \right] \right), \end{aligned} \quad (3.11)$$

we obtain

$$\begin{aligned} \left( \mathbf{H}_{P,Q,+;n}^{M,N} \left| \begin{array}{l} (a_p, \alpha_p) \\ (b_q, \beta_q) \end{array} \right. \mathbf{x}^{-s} \right) (x) &= \frac{\Gamma(1+n-|s|) \prod_{j=1}^M \Gamma(b_j + \beta_j(|s|-n)) \prod_{i=1}^N \Gamma(1-a_i-\alpha_i(|s|-n))}{\Gamma(n-|s|) \prod_{i=N+1}^P \Gamma(a_i + \alpha_i(|s|-n)) \prod_{j=M+1}^Q \Gamma(1-b_j-\beta_j(|s|-n))} x^{-s}. \end{aligned} \quad (3.12)$$

Similarly, by using the multidimensional operator (3.2), we obtain

$$\begin{aligned} \left( \mathbf{H}_{P,Q,-;n}^{M,N} \left| \begin{array}{l} (a_p, \alpha_p) \\ (b_q, \beta_q) \end{array} \right. \mathbf{x}^{-s} \right) (x) &= \frac{(-1)^n \Gamma(1-n+|s|) \prod_{j=1}^M \Gamma(b_j + \beta_j(|s|-n)) \prod_{i=1}^N \Gamma(1-a_i-\alpha_i(|s|-n))}{\Gamma(-n+|s|) \prod_{i=N+1}^P \Gamma(a_i + \alpha_i(|s|-n)) \prod_{j=M+1}^Q \Gamma(1-b_j-\beta_j(|s|-n))} x^{-s}. \end{aligned} \quad (3.13)$$

The result (3.13) on specializing the parameters in accordance with (3.5) yields the formula [7, equation (3.6), page 148] involving the multidimensional modified integral operator (1.4). Similarly, we can deduce a result from (3.12) which involves the multidimensional modified integral operator (1.3).

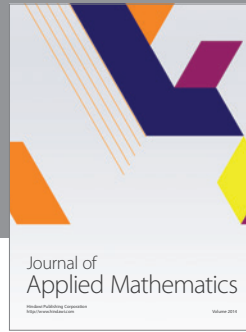
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