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Research Article

On b-I-Open Sets and b-I-Continuous Functions

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We studied some more properties of b-I-open sets and obtained several characterizations of b-I-continuous functions which are introduced by Caksu Guler and Aslim (2005). We also investigated their relationship with other types of functions.

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1. Introduction

One of the important and basic topics in the theory of classical point set topology and several branches of mathematics, which have been researched by many authors, is continuity of functions. This concept has been extended to the setting of I-continuity of functions. Janković and Hamlett [1, 2] introduced the notion of I-open sets in topological spaces. Abd El-Monsef et al. [3] further investigated I-open sets and I-continuous functions. Dontchev [4] introduced the notion of pre-I-open sets and obtained a decomposition of I-continuity. The notion of semi-I-open sets to obtain decomposition of continuity was introduced by Hatir and Noiri [5, 6]. In addition to this, Caksu Guler and Aslim [7] have introduced the notion of b-I-sets and b-I-continuous functions. In the light of the above results, the purpose of this paper is to study b-I-open sets and b-I-continuous functions and to obtain several characterizations and properties of these concepts.

2. Preliminaries

Throughout this paper, $\operatorname{int}(A)$ and $\operatorname{Cl}(A)$ denote the interior and closure of A, respectively. An ideal is defined as a nonempty collection I of subsets of X satisfying the

following two conditions. (1) If $A \in I$ and $B \subset A$, then $B \in I$. (2) If $A \in I$ and $B \in I$, then $A \cup B \in I$. An ideal topological space is a topological space (X,τ) with an ideal I on X, and it is denoted by (X,τ,I). For a subset $A \subset X$, $A^*(I) = \{x \in X : U \cap A \notin I$ for each neighborhood U of $x\}$ is called the local function of A with respect to I and τ [8]. We simply write A^* instead of $A^*(I)$ to be brief. X^* is often a proper subset of X. The hypothesis $X = X^*$ [9] is equivalent to the hypothesis $\tau \cap I = \varnothing$ [10]. For every ideal topological space (X,τ,I), there exists a topology $\tau^*(I)$, finer than τ, generated by $\beta(I,\tau) = \{U - \mathrm{I} : U \in \tau \text{ and } \mathrm{I} \in I\}$, but in general $\beta(I,\tau)$ is not always a topology [1]. Additionally, $\mathrm{Cl}^*(A) = A \cup (A)^*$ defines a Kuratowski closure operator for $\tau^*(I)$. For a subset $A \subset X$, A is called $*$-dense in itself [9] (resp., τ^*-closed [1], $*$-perfect [9]) if $A \subset A^*$ (resp., $A^* \subset A$, $A = A^*$). Given a space (X,τ,I) and $A \subset X$, A is called I-open if $A \subset \mathrm{int}(A^*)$ and a subset K is called I-closed if its complement is I-open [3].

3. b-I-open sets

First we will recall some definitions used in sequel.

Definition 3.1. A subset S of a topological space X is said to be

(a) α-open set [11] if $S \subset \mathrm{int}(\mathrm{cl}(\mathrm{int}(S)))$,
(b) semiopen set [12] if $S \subset \mathrm{cl}(\mathrm{int}(S))$,
(c) preopen set [13] if $S \subset \mathrm{int}(\mathrm{cl}(S))$,
(d) β-open set [14] if $S \subset \mathrm{cl}(\mathrm{int}(\mathrm{cl}(S)))$,
(e) b-open set (or γ-open [15]) [16] if $S \subset \mathrm{cl}(\mathrm{int}(S)) \cup \mathrm{int}(\mathrm{cl}(S))$.

The class of all semiopen (preopen, α-open) sets in X will be denoted by $\mathrm{SO}(X,\tau)$ $(\mathrm{PO}(X,\tau), \alpha\mathrm{O}(X,\tau))$.

Definition 3.2. A subset S of an ideal topological space X is said to be

(a) α-I-open set [5] if $S \subset \mathrm{int}(\mathrm{cl}^*(\mathrm{int}(S)))$,
(b) semi-I-open set [5] if $S \subset \mathrm{cl}^*(\mathrm{int}(S))$,
(c) pre-I-open set [3] if $S \subset \mathrm{int}(\mathrm{cl}^*(S))$,
(d) β-I-open set [5] if $S \subset \mathrm{cl}^*(\mathrm{int}(\mathrm{cl}^*(S)))$,
(e) I-open set [1] if $S \subset \mathrm{int}(S^*)$,
(f) b-I-open set [7] if $S \subset \mathrm{cl}^*(\mathrm{int}(S)) \cup \mathrm{int}(\mathrm{cl}^*(S))$.

The class of all semi-I-open (pre-I-open, α-open, and b-I-open) sets in X will be denoted by $\mathrm{SIO}(X,\tau)$ $(\mathrm{PIO}(X,\tau)$, $\alpha\mathrm{IO}(X,\tau)$, and $\mathrm{BIO}(X,\tau))$.

Proposition 3.3. *For a subset of an ideal topological space, the following conditions hold:*

(a) *every b-I-open set is b-open;*
(b) *every pre-I-open set is b-I-open [7];*
(c) *every semi-I-open set is b-I-open [7];*
(d) $\mathrm{SIO}(X,\tau) \cup \mathrm{PIO}(X,\tau) \subset \mathrm{BIO}(X,\tau)$.

Proof. The proof is obvious. □

Remark 3.4. For several sets defined above, we have the following implications:

$$\begin{tikzcd}
 & \alpha\text{-open} & & \\
\text{open} \arrow[r, Rightarrow] & \alpha\text{-}I\text{-open} \arrow[u, Leftrightarrow] \arrow[r, Rightarrow] \arrow[d, Rightarrow] & \text{semi-}I\text{-open} \arrow[r, Rightarrow] & \text{semiopen} \arrow[d, Rightarrow] \\
I\text{-open} \arrow[r, Rightarrow] & \text{pre-}I\text{-open} \arrow[r, Rightarrow] \arrow[d, Rightarrow] & b\text{-}I\text{-open} \arrow[r, Rightarrow] & b\text{-open} \\
 & \text{preopen} \arrow[rr, Rightarrow] & & b\text{-open} \arrow[u, Leftrightarrow]
\end{tikzcd} \tag{3.1}$$

Example 3.5. Consider the set $\mathbb{R}$ of real numbers with the usual topology with ideal $I = \{\varnothing\}$ and let $S = [0,1] \cup ((1,2) \cap \mathbb{Q})$, where $\mathbb{Q}$ stands for the set of rational numbers. Then S is b-I-open set but neither semi-I-open nor pre-I-open. On the other hand, let $T = [0,1) \cap \mathbb{Q}$. Then T is not b-I-open.

Example 3.6. Let $(\mathbb{R},\tau)$ be the real numbers with the usual topology and I the ideal of all finite sets of R. Let $\mathbb{Q}$ be the set of all rationals. Since $\mathbb{Q}^*(I) = \mathbb{R}$, then $\mathbb{Q}$ is b-I-open. Since $\mathrm{cl}^*(\mathrm{int}\,\mathbb{Q}) = \varnothing$, $\mathbb{Q}$ is not semi-I-open.

Example 3.7. Let $X = \{a,b,c,d\}$ be the topological space by setting

$$\tau = \{X,\varnothing,\{b\},\{c,d\},\{b,c,d\}\}, \qquad I = \{\{c\},\{d\},\{c,d\},\varnothing\}. \tag{3.2}$$

Then $A = \{a,b\}$ is not pre-I-open but it is b-I-open.

Proposition 3.8. *Let S be a b-I-open set such that* $\mathrm{int}\,S = \varnothing$. *Then S is pre-I-open set.*

Proof. Since $S \subset \mathrm{cl}^*(\mathrm{int}\,S) \cup \mathrm{int}(\mathrm{cl}^*(S)) = \mathrm{cl}^*(\varnothing) \cup \mathrm{int}(\mathrm{cl}^*(S)) = \mathrm{int}(\mathrm{cl}^*(S))$, then S is pre-I-open. □

Lemma 3.9. *Let A and B be subsets of a space (X,τ,I) [1]. Then*
(1) *if $A \subset B$, then $A^* \subset B^*$;*
(2) *if $U \in \tau$, then $U \cap A^* \subset (U \cap A)^*$.*

Proposition 3.10. *Let (X,τ,I) be an ideal topological space and A, B subsets of X.*
(a) *If $U_\alpha \in \mathrm{BIO}(X,\tau)$ for each $\alpha \in \Delta$, then $\cup\{U_\alpha : \alpha \in \Delta\} \in \mathrm{BIO}(X,\tau)$.*
(b) *If $A \in \mathrm{BIO}(X,\tau)$ and $B \in \tau$, then $A \cap B \in \mathrm{BIO}(X,\tau)$ [7].*

Proof. (a) Since $U_\alpha \in \mathrm{BOI}(X,\tau)$, we have $U_\alpha \subset \mathrm{cl}^*(\mathrm{int}(U_\alpha)) \cup \mathrm{int}(\mathrm{cl}^*(U_\alpha))$ for each $\alpha \in \Delta$. Then by using Lemma 3.9, we have

$$\begin{aligned}
\bigcup_{\alpha\in\Delta} U_\alpha &\subset \bigcup_{\alpha\in\Delta} [\mathrm{cl}^*(\mathrm{int}(U_\alpha)) \cup \mathrm{int}(\mathrm{cl}^*(U_\alpha))] \\
&\subset \bigcup_{\alpha\in\Delta} \{[(\mathrm{int}(U_\alpha) \cup (\mathrm{int}(U_\alpha))^*] \cup [\mathrm{int}(U_\alpha \cup (U_\alpha)^*)]\} \\
&\subset \left[\mathrm{int}\left(\bigcup_{\alpha\in\Delta} U_\alpha\right) \cup \left(\bigcup_{\alpha\in\Delta} (\mathrm{int}(U_\alpha))^*\right)\right] \cup \left[\mathrm{int}\left(\left(\bigcup_{\alpha\in\Delta} U_\alpha\right) \cup \left(\bigcup_{\alpha\in\Delta} (U_\alpha)^*\right)\right)\right] \\
&\subset \left[\mathrm{int}\left(\bigcup_{\alpha\in\Delta} U_\alpha\right) \cup \left(\mathrm{int}\left(\bigcup_{\alpha\in\Delta} U_\alpha\right)\right)^*\right] \cup \left[\mathrm{int}\left(\left(\bigcup_{\alpha\in\Delta} U_\alpha\right) \cup \left(\bigcup_{\alpha\in\Delta} (U_\alpha)\right)^*\right)\right] \\
&\subset \mathrm{cl}^*\left[\mathrm{int}\left(\bigcup_{\alpha\in\Delta} U_\alpha\right)\right] \cup \left[\mathrm{int}\,\mathrm{cl}^*\left(\bigcup_{\alpha\in\Delta} U_\alpha\right)\right].
\end{aligned} \tag{3.3}$$

Hence $\bigcup_{\alpha\in\Delta} U_\alpha$ is b-I-open.

(b) Let $A \in \mathrm{BIO}(X,\tau)$ and $B \in \tau$. Then $A \subset \mathrm{cl}^*(\mathrm{int}A) \cup \mathrm{int}(\mathrm{cl}^*(A))$ and

$$\begin{aligned}
A \cap B &\subset [\mathrm{cl}^*(\mathrm{int}A) \cup \mathrm{int}(\mathrm{cl}^*(A))] \cap B \\
&= [\mathrm{cl}^*(\mathrm{int}A) \cap B] \cup [\mathrm{int}(\mathrm{cl}^*(A)) \cap B] \\
&= [[(\mathrm{int}A) \cup (\mathrm{int}A)^*] \cap B] \cup [\mathrm{int}(A \cup (A)^*) \cap B] \\
&\subset [((\mathrm{int}A) \cap B) \cup ((\mathrm{int}A) \cap B)^*] \cup [\mathrm{int}[(A \cap B) \cup (A^* \cap B)]] \\
&\subset [[(\mathrm{int}(A \cap B))^*] \cup (\mathrm{int}(A \cap B))] \cup [\mathrm{int}[(A \cap B)^* \cup (A \cap B)]] \\
&= \mathrm{cl}^*(\mathrm{int}(A \cap B)) \cup \mathrm{int}(\mathrm{cl}^*(A \cap B)).
\end{aligned} \tag{3.4}$$

This shows that $A \cap B \in \mathrm{BIO}(X,\tau)$. □

Definition 3.11. A subset A of a space (X,τ,I) is said to be b-I-closed if its complement is b-I-open.

Theorem 3.12. *If a subset A of a space (X,τ,I) is b-I-closed, then* $\mathrm{int}(\mathrm{cl}^*(A)) \cap \mathrm{cl}^*(\mathrm{int}A) \subset A$.

Proof. Since A is b-I-closed, $X - A \in \mathrm{BIO}(X,\tau)$ and since $\tau^*(I)$ is finer than τ, we have

$$\begin{aligned}
X - A &\subset \mathrm{cl}^*(\mathrm{int}(X - A)) \cup \mathrm{int}(\mathrm{cl}^*(X - A)) \subset \mathrm{cl}(\mathrm{int}(X - A)) \cup \mathrm{int}(\mathrm{cl}(X - A)) \\
&= [X - [\mathrm{int}(\mathrm{cl}(A))]] \cup [X - [\mathrm{cl}(\mathrm{int}A)]] \\
&\subset [X - [\mathrm{int}(\mathrm{cl}^*(A))]] \cup [X - [\mathrm{cl}^*(\mathrm{int}A)]] \\
&= X - [[\mathrm{int}(\mathrm{cl}^*(A))] \cap [\mathrm{cl}^*(\mathrm{int}A)]].
\end{aligned} \tag{3.5}$$

Therefore, we obtain

$$[\operatorname{int}(\operatorname{cl}^*(A))] \cap [\operatorname{cl}^*(\operatorname{int} A)] \subset A. \tag{3.6}$$

□

Corollary 3.13. *Let A be a subset of (X,τ,I) such that $X - [\operatorname{int}(\operatorname{cl}^*(A))] = \operatorname{cl}^*(\operatorname{int}(X-A))$ and $X - [\operatorname{cl}^*(\operatorname{int} A)] = \operatorname{int}(\operatorname{cl}^*(X-A))$. Then A is b-I-closed if and only if $\operatorname{int}(\operatorname{cl}^*(A)) \cap \operatorname{cl}^*(\operatorname{int} A) \subset A$.*

Proof. Necessity. This is an immediate consequence of Theorem 3.12.

Sufficiency. Let $\operatorname{int}(\operatorname{cl}^*(A)) \cap \operatorname{cl}^*(\operatorname{int} A) \subset A$. Then

$$\begin{aligned} X - A &\subset X - [\operatorname{int}(\operatorname{cl}^*(A)) \cap \operatorname{cl}^*(\operatorname{int} A)] \\ &\subset [X - [\operatorname{int}(\operatorname{cl}^*(A))]] \cup [X - [\operatorname{cl}^*(\operatorname{int} A)]] \\ &= \operatorname{cl}^*(\operatorname{int}(X-A)) \cup \operatorname{int}(\operatorname{cl}^*(X-A)). \end{aligned} \tag{3.7}$$

Thus $X - A$ is b-I-open and so A is b-I-closed. □

If (X,τ,I) is an ideal topological space and A is a subset of X, we denote by $\tau|_A$. the relative topology on A and $I|_A = \{A \cap I : I \in I\}$ is obviously an ideal on A.

Lemma 3.14 (see [1]). *Let (X,τ,I) be an ideal topological space and A, B subsets of X such that $B \subset A$. Then $B^*(\tau|_A, I|_A) = B^*(\tau,I) \cap A$.*

Theorem 3.15. *Let (X,τ,I) be an ideal topological space. If $U \in \tau$ and $W \in \operatorname{BIO}(X,\tau)$, then $U \cap W \in \operatorname{BIO}(U,\tau|_U,I|_U)$.*

Proof. Since U is open, we have $\operatorname{int}_U A = \operatorname{int} A$ for any subset A of U. By using this fact and Lemma 3.14, we have

$$\begin{aligned} U \cap W &\subset U \cap [\operatorname{cl}^*(\operatorname{int}(W)) \cup \operatorname{int}(\operatorname{cl}^*(W))] \\ &= [U \cap [(\operatorname{int} W) \cup (\operatorname{int} W)^*]] \cup [U \cap [\operatorname{int}(W \cup W^*)]] \\ &\subset \{U \cap [U \cap (\operatorname{int} W) \cup U \cap (\operatorname{int} W)^*]\} \cup \{U \cap [U \cap [\operatorname{int}(W \cup W^*)]]\} \\ &\subset \{U \cap [U \cap (\operatorname{int} W) \cup (U \cap \operatorname{int} W)^*]\} \\ &\cup \{U \cap [U \cap [\operatorname{int}(W \cup W^*)]]\} \\ &\subset \{[U \cap (\operatorname{int}_U(U \cap W))] \cup (U \cap \operatorname{int}_U(U \cap W))^*\} \\ &\cup \{U \cap [\operatorname{int}((U \cap W) \cup (U \cap W)^*)]\} \\ &= \{[\operatorname{int}_U(U \cap W)] \cup [\operatorname{int}_U(U \cap W)]^*_{(\tau|_U, I|_U)}\} \\ &\cup \{U \cap [\operatorname{int}((U \cap W) \cup (U \cap W)^*)]\} \\ &= \operatorname{cl}^*_U[\operatorname{int}_U(U \cap W)] \cup [\operatorname{int}_U(\operatorname{cl}^*_U(U \cap W))]. \end{aligned} \tag{3.8}$$

This shows that $U \cap W \in \operatorname{BIO}(U,\tau|_U \cdot, I|_U \cdot)$. □

Proposition 3.16 (see [7]). *For an ideal topological space (X,τ,I) and $A \subset X$, we have the following.*

(1) *If $I = \varnothing$, then A is b-I-open if and only if A is b-open.*

(2) *If $I = P(x)$, then A is b-I-open if and only if $A \in \tau$.*

(3) *If $I = N$, then A is b-I-open if and only if A is b-open, where N is the ideal of all nowhere dense sets.*

Lemma 3.17 (see [1]). *Let (X,τ,I) be an ideal topological space and let $A \subset X$. Then if $U \in \tau$, $U \cap A^* = U \cap (U \cap A)^* \subset (U \cap A)^*$.*

Proposition 3.18. *Let (X,τ,I) be an ideal topological space with Δ being an arbitrary index set. Then*

(1) *if $A \in \mathrm{BIO}(X,\tau)$ and $B \in \tau^{\alpha}$, then $A \cap B \in \mathrm{BO}(X,\tau)$;*

(2) *if $A \in \mathrm{PIO}(X,\tau)$ and $B \in \mathrm{SIO}(X,\tau)$, then $A \cap B \in \mathrm{SO}(A)$;*

(3) *if $A \in \mathrm{PIO}(X,\tau)$ and $B \in \mathrm{SIO}(X,\tau)$, then $A \cap B \in \mathrm{PO}(B)$.*

Proof. (1) Since intersection of b-open and α-set is always a b-open set [16, Proposition 2.4], then the claim is clear due to Proposition 3.3.

(2)-(3) It was proved in [17] that the intersection of a preopen and a semiopen set is a preopen subset of the semiopen set and a semiopen subset of the preopen set. Thus the claim follows from [5, Proposition 2.5] and [6]. □

Proposition 3.19. *Each b-I-open subset which is τ^*-closed is semi-I-closed.*

Proof. Let A be b-I-open and τ^*-closed set. Then

$$\begin{aligned} A &\subset \operatorname{int}(\operatorname{cl}^*(A)) \cup \operatorname{cl}^*(\operatorname{int}A) = \operatorname{int}A \cup [\operatorname{int}A \cup (\operatorname{int}A)^*] \\ &= \operatorname{int}A \cup (\operatorname{int}A)^* = \operatorname{cl}^*(\operatorname{int}A). \end{aligned} \tag{3.9}$$

□

Definition 3.20. If S is a subset of a space (X,τ,I), then

(a) the b-I-closure of S, denoted by $\operatorname{cl}_b^*(S)$, is the smallest b-I-closed set containing S;

(b) the b-I-interior of S, denoted by $\operatorname{int}_b I(S)$, is the largest b-I-open set contained in S.

Lemma 3.21. (1) *Let A be a subset of a space (X,τ,I). Then A is b-I-closed if and only if $\operatorname{cl}_b^*(A) = A$.*

(2) *Let B be a subset of a space (X,τ,I). Then A is b-I-open if and only if $\operatorname{int}_b I(B) = B$.*

Proposition 3.22. *Let A, B be subsets of a space (X,τ,I) such that A is b-I-open and B is b-I-closed in X. Then there exist a b-I-open set H and a b-I-closed set K such that $A \cap B \subset K$ and $H \subset A \cup B$.*

Proof. Let $K = \operatorname{cl}_b^*(A) \cap B$ and $H = A \cup \operatorname{int}_b I(B)$. Then, K is b-I-closed and H is b-I-open. $A \subset \operatorname{cl}_b^*(A)$ implies $A \cap B \subset \operatorname{cl}_b^*(A) \cap B = K$ and $\operatorname{int}_b I(B) \subset B$ implies $A \cup \operatorname{int}_b I(B) = H \subset A \cup B$. □

Definition 3.23. (1) A subset S of a space (X,τ,I) is called b-dense if $\operatorname{cl}_b(S) = X$, where $\operatorname{cl}_b(S)$ is the smallest b-closed set containing S [15].

(2) A subset S of a space (X,τ,I) is called b-I-dense if $\operatorname{cl}_b^*(S) = X$.

Remark 3.24. Every b-I-dense subset in a space (X,τ,I) is b-dense.

4. b-I-continuous mappings

Definition 4.1. (a) A function $f:(X,\tau)\to(Y,\sigma)$ is called b-continuous (or γ-continuous) if the inverse image of each open set in Y is b-open set in X [15].

(b) A function $f:(X,\tau)\to(Y,\sigma)$ is called precontinuous if the inverse image of each open set in Y is preopen set in X [13].

(c) A function $f:(X,\tau,I)\to(Y,\sigma)$ is called pre-I-continuous if the inverse image of each open set in Y is pre-I-open set in X [18].

(d) A function $f:(X,\tau)\to(Y,\sigma)$ is called semicontinuous if the inverse image of each open set in Y is semiopen set in X [12].

(e) A function $f:(X,\tau,I)\to(Y,\sigma)$ is called semi-I-continuous if the inverse image of each open set in Y is semi-I-open set in X [5].

(f) A function $f:(X,\tau)\to(Y,\sigma)$ is called α-continuous (or γ-continuous) if the inverse image of each open set in Y is α-open set in X [14].

(g) A function $f:(X,\tau,I)\to(Y,\sigma)$ is called α-I-continuous if the inverse image of each open set in Y is α-I-open set in X [5].

(h) A function $f:(X,\tau,I)\to(Y,\sigma)$ is called b-I-continuous if the inverse image of each open set in Y is b-I-open set in X [7].

Remark 4.2 (see [7, Propositions 6 and 7]). (1) b-I-continuity implies b-continuity.

(2) semi-I-continuity implies b-I-continuity.

(3) pre-I-continuity implies b-I-continuity.

Definition 4.3 (see [19]). Let A be a subset of a space (X,τ,I).

Then the set $\cap\{U\in\tau : A\subset U\}$ is called the kernel of A and denoted by $\mathrm{Ker}(A)$.

Lemma 4.4 (see [20]). *Let A be a subset of a space (X,τ), then*

(a) $x\in\mathrm{Ker}(A)$ *if and only if* $A\cap F\neq\varnothing$ *for any closed subset F of X with $x\in F$;*

(b) $A\subset\mathrm{Ker}(A)$ *and* $A=\mathrm{Ker}(A)$ *if A is open in X;*

(c) *if* $A\subset B$, *then* $\mathrm{Ker}(A)\subset\mathrm{Ker}(B)$.

Definition 4.5. Let N be a subset of a space (X,τ,I) and let $x\in X$. Then N is called b-I-neighborhood of x, if there exists a b-I-open set U containing x such that $U\subset N$.

Theorem 4.6. *The following statements are equivalent for a function $f:(X,\tau,I)\to(Y,\sigma)$:*

(a) *f is b-I-continuous;*

(b) *for each $x\in X$ and each open set V in Y with $f(x)\in V$, there exists a b-I-open set U containing x such that $f(U)\subset V$;*

(c) *for each $x\in X$ and each open set V in Y with $f(x)\in V$, $f^{-1}(V)$ is a b-I-neighborhood of x;*

(d) *the inverse image of each closed set in (Y,σ) is b-I-closed;*

(e) *for every subset A of X, $f(\mathrm{int}_b I(A))\subset\mathrm{Ker}(f(A))$;*

(f) *for every subset B of Y, $\mathrm{int}_b I(f^{-1}(B))\subset f^{-1}(\mathrm{Ker}(B))$.*

Proof. (a)⇒(b). Let $x \in X$ and let V be an open set in Y such that $f(x) \in V$. Since f is b-I-continuous, $f^{-1}(V)$ is b-I-open. By putting $U = f^{-1}(V)$ which is containing x, we have $f(U) \subset V$.

(b)⇒(c). Let V be an open set in Y and let $f(x) \in V$. Then by (b), there exists a b-I-open set U containing x such that $f(U) \subset V$. So $x \in U \subset f^{-1}(V)$. Hence $f^{-1}(V)$ is a b-I-neighborhood of x.

(c)⇒(a). Let V be an open set in Y and let $f(x) \in V$. Then by (c), $f^{-1}(V)$ is a b-I-neighborhood of x. Thus for each $x \in f^{-1}(V)$, there exists a b-I-open set U_x containing x such that $x \in U_x \subset f^{-1}(V)$. Hence $f^{-1}(V) \subset \bigcup_{x \in f^{-1}(V)} U_x$ and so $f^{-1}(V) \in \mathrm{BOI}(X,\tau)$.

(a)⇔(d). It is obvious.

(a)⇒(e). Let A be any subset of X. Suppose that $y \notin \mathrm{Ker}(f(A))$. Then, by Lemma 4.4, there exists a closed subset F of Y such that $y \in F$ and $f(A) \cap F = \varnothing$. Thus we have $A \cap f^{-1}(F) = \varnothing$ and $(\mathrm{int}_b I(A)) \cap f^{-1}(F) = \varnothing$. Therefore, we obtain $f(\mathrm{int}_b I(A)) \cap F = \varnothing$ and $y \notin f(\mathrm{int}_b I(A))$. This implies that $f(\mathrm{int}_b I(A)) \subset \mathrm{Ker}(A)$.

(e)⇒(f). Let B be any subset of Y. By (e) and Lemma 4.4, we have $f(\mathrm{int}_b I(f^{-1}(B))) \subset \mathrm{Ker}(f(f^{-1}(B))) \subset \mathrm{Ker}(B)$ and $\mathrm{int}_b I(f^{-1}(B)) \subset f^{-1}(\mathrm{Ker}(B))$.

(f)⇒(a). Let V be an open set of Y. Then by Lemma 4.4 and (f), we have $\mathrm{int}_b I(f^{-1}(V)) \subset f^{-1}(\mathrm{Ker}(V)) = f^{-1}(V)$ and $\mathrm{int}_b I(f^{-1}(V)) = f^{-1}(V)$. This shows that $f^{-1}(V)$ is b-I-open. □

The following examples show that b-I-continuous functions do not need to be semi-I-continuous and pre-I-continuous, and b-continuous function does not need to be b-I-continuous.

Example 4.7. Let $X = Y = \{a,b,c,d\}$ be the topological space by setting $\tau = \sigma = \{\{a\}, \{d\}, \{a,d\}, X, \varnothing\}$, and $I = \{\varnothing, \{c\}\}$ on X.

Define a function $f : (X,\tau,I) \to (Y,\sigma)$ as follows: $f(a) = f(c) = d$ and $f(b) = f(d) = b$. Then f is b-I-continuous but it is not pre-I-continuous.

Example 4.8. Let (X,τ) be the real line with the indiscrete topology and (Y,σ) the real line with the usual topology. Then the identity function $f : (X,\tau,P(X)) \to (Y,\sigma)$ is b-continuous but not b-I-continuous.

Example 4.9. Let $X = Y = \{a,b,c\}$ be the topological space by setting $\tau = \sigma = \{X, \varnothing, \{a, b\}\}$ and $I = \{\{c\}, \varnothing\}$. Define a function $f : (X,\tau,I) \to (Y,\sigma)$ as follows: $f(a) = a$, $f(b) = c$, and $f(c) = b$. Then f is b-I-continuous but not semi-I-continuous.

Proposition 4.10. *Let $f : (X,\tau,I) \to (Y,\sigma,J)$ and $g : (Y,\sigma,J) \to (Z,\nu)$ be two functions, where I and J are ideals on X and Y, respectively. Then $g \circ f$ is b-I-continuous if f is b-I-continuous and g is continuous.*

Proof. The proof is clear. □

Theorem 4.11. *Let $f : (X,\tau,I) \to (Y,\sigma)$ be b-I-continuous and $U \in \tau$. Then the restriction $f_{|U} : (U,\tau_{|U},I_{|U}) \to (Y,\sigma)$ is b-I-continuous.*

Proof. Let V be any open set of (Y,σ). Since f is b-I-continuous, $f^{-1}(V) \in \mathrm{BIO}(X,\tau)$ and by Theorem 3.15, $(f_{|U})^{-1}(V) = f^{-1}(V) \cap U \in \mathrm{BIO}(U,I_{|U})$. This shows that $f_{|U} : (U,\tau_{|U},I_{|U}) \to (Y,\sigma)$ is b-I-continuous. □

Theorem 4.12. *Let $f : (X,\tau,I) \to (Y,\sigma,J)$ be a function and let $\{U_\alpha : \alpha \in \Delta\}$ be an open cover of X. If the restriction function $f|_{U\alpha}$ is b-I-continuous for each $\alpha \in \Delta$, then f is b-I-continuous.*

Proof. The proof is similar to that of Theorem 4.11. □

Theorem 4.13. *A function $f : (X,\tau,I) \to (Y,\sigma)$ is b-I-continuous if and only if the graph function $g : X \to X \times Y$ defined by $g(x) = (x, f(x))$ for each $x \in X$ is b-I-continuous.*

Proof. Necessity. Let f be b-I-continuous. Now let $x \in X$ and let W be any open set in $X \times Y$ containing $g(x) = (x, f(x))$. Then there exists a basic open set $U \times V$ such that $g(x) \subset U \times V \subset W$. Since f is b-I-continuous, there exists a b-I-open set U_1 in X such that $x \in U_1 \subset X$ and $f(U_1) \subset V$. By Proposition 3.10, $U_1 \cap U \in \mathrm{BOI}(X,\tau)$ and $U_1 \cap U \subset U$, then $g(U_1 \cap U) \subset U \times V \subset W$. This shows that g is b-I-continuous.

Sufficiency. Suppose that g is b-I-continuous and let V be open set in Y containing $f(x)$. Then $X \times V$ is open set in $X \times Y$ and by the b-I-continuity of g, there exists a b-I-open set U containing x such that $g(U) \subset X \times V$. Therefore, we obtain $f(U) \subset V$. This shows that f is b-I-continuous. □

Theorem 4.14. *Let $\{X_\alpha : \alpha \in \Delta\}$ be any family of ideal topological spaces. If $f : (X,\tau,I) \to (\prod_{\alpha\in\Delta} X_\alpha, \sigma)$ is a b-I-continuous function, then $P_\alpha \circ f : X \to X_\alpha$ is b-I-continuous for each $\alpha \in \Delta$, where P_α is the projection of $\prod X_\alpha$ onto X_α.*

Proof. We will consider a fixed $\alpha_\circ \in \Delta$. Let G_{α_0} be an open set of X_{α_0}. Then $(P_{\alpha_0})^{-1}(G_{\alpha_0})$ is open set in $\prod_{\alpha\neq\alpha_0} X_\alpha$. Since f is b-I-continuous, $f^{-1}((P_{\alpha_0})^{-1}(G_{\alpha_0})) = (P_{\alpha_0} \circ f)^{-1}(G_{\alpha_0})$ is b-I-open in X. Thus $P_{\alpha_0} \circ f$ is b-I-continuous. □

Lemma 4.15 (see [21]). *For any function $f : (X,\tau,I) \to (Y,\sigma)$, $f(I)$ is an ideal on Y.*

Definition 4.16 (see [21]). An ideal topological space (X,τ,I) is said to be I-compact if for every I-open cover $\{W_\alpha : \alpha \in \Delta\}$ of X, there exists a finite subset $\Delta_\circ$ of Δ such that $(X - \cup\{W_\alpha : \alpha \in \Delta_\circ\}) \in I$.

Definition 4.17. An ideal topological space (X,τ,I) is said to be b-I-compact if for every b-I-open cover $\{W_\alpha : \alpha \in \Delta\}$ of X, there exists a finite subset $\Delta_\circ$ of Δ such that $(X - \cup\{W_\alpha : \alpha \in \Delta_\circ\}) \in I$.

Theorem 4.18. *The image of a b-I-compact space under a b-I-continuous surjective function is $f(I)$-compact.*

Proof. Let $f : (X,\tau,I) \to (Y,\sigma)$ be a b-I-continuous surjection and $\{V_\alpha : \alpha \in \Delta\}$ be an open cover of Y. Then $\{f^{-1}(V_\alpha) : \alpha \in \Delta\}$ is a b-I-open cover of X due to our assumption on f. Since X is b-I-compact, then there exists a finite subset $\Delta_\circ$ of Δ such that $(X - \cup\{f^{-1}(V_\alpha) : \alpha \in \Delta_\circ\}) \in I$. Therefore $(Y - \cup\{V_\alpha : \alpha \in \Delta_\circ\}) \in f(I)$ which shows that $(Y,\sigma,f(I))$ is $f(I)$-compact. □

Definition 4.19. An ideal topological space (X,τ,I) is said to be b-I-normal if for each pair of nonempty disjoint closed sets of X, it can be separated by disjoint b-I-open sets.

Definition 4.20. An ideal topological space (X,τ,I) is said to be b-I-connected if X is not the union of two disjoint b-I-open subsets of X.

Definition 4.21 (see [22]). A topological space (X,τ) is said to be ultra normal if for each pair of nonempty disjoint closed sets of X, it can be separated by disjoint clopen sets.

Theorem 4.22. *If $f:(X,\tau,I)\to(Y,\sigma)$ is a b-I-continuous, closed injection and Y is normal, then X is b-I-normal.*

Proof. Let F_1 and F_2 be disjoint closed subsets of X. Since f is closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of Y. Since Y is normal, $f(F_1)$ and $f(F_2)$ are separated by disjoint open sets V_1 and V_2, respectively. Hence $F_1\subset f^{-1}(V_1)$, $F_2\subset f^{-1}(V_2)$, $f^{-1}(V_1)\in \mathrm{BIO}(X,\tau)$, $f^{-1}(V_2)\in \mathrm{BIO}(X,\tau)$, and $f^{-1}(V_1)\cap f^{-1}(V_2)=\varnothing$. Thus X is b-I-normal. □

Corollary 4.23. *If $f:(X,\tau,I)\to(Y,\sigma)$ is a b-I-continuous, closed injection and Y is ultra normal, then X is b-I-normal.*

Theorem 4.24. *A b-I-continuous image of a b-I-connected space is connected.*

Proof. Let $f:(X,\tau,I)\to(Y,\sigma)$ be a b-I-continuous function of a b-I-connected space X onto a topological space Y. If possible, let Y be disconnected. Let A and B form a disconnected set of Y. Then A and B are clopen and $Y=A\cup B$, where $A\cap B=\varnothing$. Since f is b-I-continuous, $X=f^{-1}(Y)=f^{-1}(A\cup B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are nonempty b-I-open sets in X. Also $f^{-1}(V_1)\cap f^{-1}(V_2)=\varnothing$. Hence X is non-b-I-connected, which is a contradiction. Therefore, Y is connected. □

Definition 4.25. A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is called b-I-open (resp., b-I-closed) if for each $U\in\tau$ (resp., closed set F), $f(U)$ (resp., $f(F)$) is b-J-open (resp., b-J-closed).

Remark 4.26. Every b-I-open (resp., b-I-closed) function is b-open (resp., b-closed) and the converses are false in general.

Example 4.27. Let $X=\{a,b,c\}$, $\tau_1=\{X,\varnothing,\{b,c\}\}$, $\tau_2=\{X,\varnothing,\{a,b\},\{b\},\{a\}\}$, and $I=\{\{a\},\varnothing\}$. Then the identity function $f:(X,\tau_1)\to(X,\tau_2,I)$ is b-open but not b-I-open.

Example 4.28. Let $X=\{a,b,c\}$, $\tau_1=\{X,\varnothing,\{a\}\}$, $\tau_2=\{X,\varnothing,\{b,c\},\{b\},\{c\}\}$, and $I=\{\{c\},\varnothing\}$. Define a function $f:(X,\tau_1)\to(X,\tau_2,I)$ as follows: $f(a)=a, f(b)=f(c)=b$. Then, f is b-closed but not b-I-closed.

Definition 4.29. (a) A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is called semi-I-open (resp., semi-I-closed) if for each $U\in\tau$ (resp., closed set F), $f(U)$ (resp., $f(F)$) is semi-J-open (resp., semi-J-closed) [6] .

(b) A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is called pre-I-open (resp., pre-I-closed) if for each $U\in\tau$ (resp., closed set F), $f(U)$ (resp., $f(F)$) is pre-J-open (resp., pre-J-closed).

(c) A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is called α-I-open (resp., α-I-closed) if for each $U\in\tau$ (resp., U is closed), $f(U)$ is α-I-open (resp., α-I-closed).

Remark 4.30. (1) Every semi-I-open (resp., semi-I-closed) function is b-I-open (resp., b-I-closed); (2) every pre-I-open (resp., pre-I-closed) function is b-I-open (resp., b-I-closed).

Theorem 4.31. *A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is b-I-open if and only if for each $x\in X$ and each neighborhood U of x, there exists $V\in \mathrm{BJO}(Y,\sigma)$ containing $f(x)$ such that $V\subset f(U)$.*

Proof. Suppose that f is a b-I-open function. For each $x\in X$ and each neighborhood U of x, there exists $U_\circ\in\tau$ such that $x\in U_\circ\subset U$. Since f is b-I-open, $V=f(U_\circ)\in \mathrm{BJO}(Y,\sigma)$ and $f(x)\in V\subset f(U)$. Conversely, let U be an open set of (X,τ). For each $x\in U$, there exists $V_x\in \mathrm{BIO}(X,\tau)$ such that $f(x)\in V_x\subset f(U)$. Therefore we obtain $f(U)=\cup\{V_x : x\in U\}$ and hence by Proposition 3.10, $f(U)\in \mathrm{BJO}(Y,\sigma)$. This shows that f is b-I-open. □

Theorem 4.32. *Let $f:(X,\tau,I)\to(Y,\sigma,J)$ be b-I-open (resp., b-I-closed). If W is any subset of Y and F is a closed (resp., open) set of X containing $f^{-1}(W)$, then there exists a b-I-closed (resp., b-I-open) subset H of Y containing W such that $f^{-1}(H)\subset F$.*

Proof. Suppose that f is a b-I-open function. Let W be any subset of Y and F a closed subset of X containing $f^{-1}(W)$. Then $X-F$ is open and since f is b-I-open, $f(X-F)$ is b-I-open. Hence $H=Y-f(X-F)$ is b-I-closed. It follows from $f^{-1}(W)\subset F$ that $W\subset H$. Moreover, we obtain $f^{-1}(H)\subset F$. For a b-I-closed function, we can prove Theorem 4.32 similarly. □

Theorem 4.33. *For any bijective function $f:(X,\tau)\to(Y,\sigma,J)$, the following are equivalent:*

(i) *$f^{-1}:(Y,\sigma,J)\to(X,\tau)$ is b-I-continuous;*

(ii) *f is b-I-open;*

(iii) *f is b-I-closed.*

Proof. It is straightforward. □

Definition 4.34 ([4]). A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is called $*$-I-continuous if the preimage of every open set in (Y,σ) is $*$-dense in itself.

Proposition 4.35. *For a subset $A\subset(X,\tau,I)$, if the condition $(\mathrm{int}(A^*))^*\subset\mathrm{int}(A^*)$ holds, then the following are equivalent:*

(1) *A is I-open;*

(2) *A is b-I-open and $*$-dense in itself.*

Proof. (1)⇒(2) Let A be an I-open subset of (X,τ,I). Then $A\subset\mathrm{int}(A^*)\subset A^*$, which shows that A is $*$-dense in itself. Since A is I-open, then A is pre-I-open and so $A\subset\mathrm{int}(\mathrm{cl}^*(A))\subset\mathrm{int}(\mathrm{cl}^*(A))\cup\mathrm{cl}^*(\mathrm{int}A)$. Thus A is b-I-open.

(2)⇒(1) Let A be a b-I-open and $*$-dense in itself.

Then since $(\mathrm{int}(A^*))^*\subset\mathrm{int}(A^*)$, $A\subset\mathrm{int}(\mathrm{cl}^*(A))\cup\mathrm{cl}^*(\mathrm{int}A)=\mathrm{int}(A\cup A^*)\cup(\mathrm{int}A\cup(\mathrm{int}A)^*)\subset\mathrm{int}(A^*)\cup\mathrm{int}(A)\cup(\mathrm{int}(A))^*=\mathrm{int}(A^*)\cup(\mathrm{int}(A))^*=\mathrm{int}(A^*)$. □

Proposition 4.36. *If a function $f:(X,\tau,I)\to(Y,\sigma)$ is I-continuous and if $(\mathrm{int}(A^*))^*\subset\mathrm{int}(A^*)$ for each subset A of X, then f is b-I-continuous and $*$-I-continuous.*

Proof. From Proposition 4.35, the proof is clear. □

Definition 4.37. A space (X,τ) is called

(1) b-space if every b-open set of X is open in X [23],

(2) submaximal if every dense set of X is open in X, and equivalently, if every pre-open set is open,

(3) extremally disconnected if the closure of every open set of X is open in X.

Corollary 4.38. *If a function $f:(X,\tau,I)\to(Y,\sigma)$ is continuous, then f is b-I-continuous.*

Corollary 4.39. *If (X,τ) is b-space, then for any ideal I on X,* $\mathrm{BIO}(X,\tau)=\mathrm{BO}(X,\tau)=\tau$.

Corollary 4.40 ([18]). *If (X,τ) is submaximal, then for any ideal I on X,* $\mathrm{PIO}(X,\tau)=\mathrm{PO}(X,\tau)=\tau$.

Corollary 4.41. *If (X,τ) is b-space, then for any ideal I on X,* $\mathrm{BIO}(X,\tau)=\mathrm{BO}(X,\tau)=\mathrm{PIO}(X,\tau)=\mathrm{PO}(X,\tau)=\tau$.

Corollary 4.42. *If (X,τ) is extremally disconnected and submaximal, then for any ideal I on X,* $\mathrm{PIO}(X,\tau)=\mathrm{SIO}(X,\tau)=\mathrm{SO}(X,\tau)=\mathrm{PO}(X,\tau)=\alpha\mathrm{O}(X,\tau)=\alpha\mathrm{IO}(X,\tau)=\tau$.

Corollary 4.43. *If (X,τ) is b-space, then for any ideal I on X,* $\mathrm{BIO}(X,\tau)=\mathrm{BO}(X,\tau)=\mathrm{SIO}(X,\tau)=\mathrm{SO}(X,\tau)=\tau$.

Corollary 4.44. *If (X,τ) is b-space, then for any ideal I on X,* $\mathrm{BIO}(X,\tau)=\mathrm{BO}(X,\tau)=\mathrm{PIO}(X,\tau)=\mathrm{SIO}(X,\tau)=\mathrm{SO}(X,\tau)=\mathrm{PO}(X,\tau)=\alpha\mathrm{O}(X,\tau)=\alpha\mathrm{IO}(X,\tau)=\tau$.

Corollary 4.45. *Let $f:(X,\tau,I)\to(Y,\sigma)$ be a function and let (X,τ) be b-space, then the following are equivalent:*

(1) *f is b-I-continuous,*

(2) *f is b-continuous,*

(3) *f is pre-I-continuous,*

(4) *f is precontinuous,*

(5) *f is semi-I-continuous,*

(6) *f is semicontinuous,*

(7) *f is α-I-continuous,*

(8) *f is α-continuous,*

(9) *f is continuous.*

Remark 4.46. For b-I-open, b-open, semi-I-open, semiopen, pre-I-open, pre-open, α-I-open, and open functions, we have similar corollary if (X,τ) is b-space.

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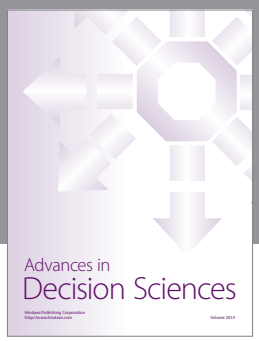

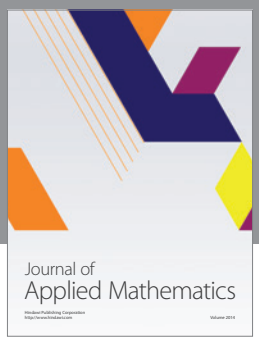

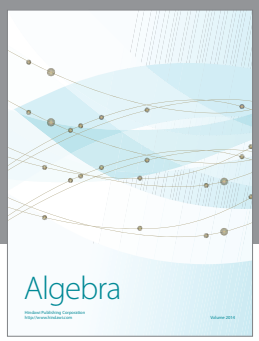

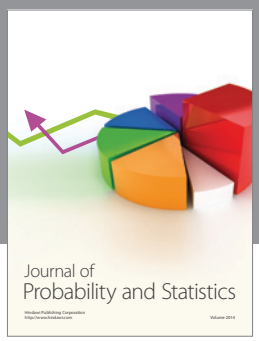

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