Hindawi Publishing Corporation
International Journal of Mathematics and Mathematical Sciences
Volume 2010, Article ID 317107, 10 pages
doi:10.1155/2010/317107

Research Article

Two Fixed-Point Theorems for Mappings Satisfying a General Contractive Condition of Integral Type in the Modular Space

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Received 16 February 2010; Revised 22 July 2010; Accepted 28 October 2010

Academic Editor: Oscar Blasco

First we prove existence of a fixed point for mappings defined on a complete modular space satisfying a general contractive inequality of integral type. Then we generalize fixed-point theorem for a quasicontraction mapping given by Khamsi (2008) and Ciric (1974) .

1. Introduction

In [1], Branciari established that a function f defined on a complete metric space satisfying a contraction condition of the form

$$\int_0^{d(fx,fy)} \varphi(t)dt \le c \int_0^{d(x,y)} \varphi(t)dt \tag{1.1}$$

has a unique attractive fixed point where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesgue-integrable mapping and $c \in [0,1)$.

In [2], Rhoades extended this result to a quasicontraction function f. The purpose of this paper is to extend these theorems in modular space.

First, we introduce the notion of modular space.

Definition 1.1. Let X be an arbitrary vector space over $K(= \mathbb{R}$ or $\mathbb{C})$. A functional $\rho : X \to [0,+\infty)$ is called modular if

(1) $\rho(x) = 0$ if and only if $x = 0$;

(2) $\rho(\alpha x) = \rho(x)$ for $\alpha \in K$ with $|\alpha| = 1$, for all $x, y \in X$;

(3) $\rho(\alpha x + \beta y) \leq \rho(x) + \rho(y)$ if $\alpha, \beta \geq 0$, $\alpha + \beta = 1$, for all $x, y \in X$.

If (2.14) in Definition 1.1 is replaced by

$$\rho(\alpha x + \beta y) \leq \alpha^s \rho(x) + \beta^s \rho(y) \tag{1.2}$$

for $\alpha, \beta \geq 0$, $\alpha^s + \beta^s = 1$ with an $s \in (0,1]$, then the modular ρ is called an s-convex modular; and if $s = 1$, ρ is called a convex modular.

Definition 1.2. A modular ρ defines a corresponding modular space, that is, the space X_ρ is given by

$$X_\rho = \{x \in X \mid \rho(\lambda x) \longrightarrow 0 \text{ as } \lambda \longrightarrow 0\}. \tag{1.3}$$

Definition 1.3. Let X_ρ be a modular space.

(1) A sequence $\{x_n\}_n$ in X_ρ is said to be

(a) ρ-convergent to x if $\rho(x_n - x) \rightarrow 0$ as $n \rightarrow +\infty$,

(b) ρ-Cauchy if $\rho(x_n - x_m) \rightarrow 0$ as $n, m \rightarrow +\infty$.

(2) X_ρ is ρ-complete if any ρ-Cauchy sequence is ρ-convergent.

(3) A subset $B \subset X_\rho$ is said to be ρ-closed if for any sequence $\{x_n\}_n \subset B$ with $x_n \rightarrow x$ then $x \in B$. $\overline{B}^\rho$ denotes the closure of B in the sense of ρ.

(4) A subset $B \subset X_\rho$ is called ρ-bounded if

$$\delta_\rho(B) = \sup_{x,y \in B} \rho(x - y) < +\infty, \tag{1.4}$$

where $\delta_\rho(B)$ is called the ρ-diameter of B.

(5) We say that ρ has Fatou property if

$$\rho(x - y) \leq \liminf \rho(x_n - y_n) \tag{1.5}$$

whenever

$$x_n \overset{\rho}{\longrightarrow} x, \qquad y_n \overset{\rho}{\longrightarrow} y. \tag{1.6}$$

(6) ρ is said to satisfy the Δ_2-condition if: $\rho(2x_n) \rightarrow 0$ as $n \rightarrow +\infty$ whenever $\rho(x_n) \rightarrow 0$ as $n \rightarrow +\infty$.

Remark 1.4. Note that since ρ does not satisfy a priori the triangle inequality, we cannot expect that if $\{x_n\}$ and $\{y_n\}$ are ρ-convergent, respectively, to x and y then $\{x_n + y_n\}$ is ρ-convergent to $x + y$, neither that a ρ-convergent sequence is ρ-Cauchy.

2. Main Result

Theorem 2.1. *Let X_ρ be a complete modular space, where ρ satisfies the Δ_2-condition. Assume that $\psi : \mathbb{R}^+ \to [0,\infty)$ is an increasing and upper semicontinuous function satisfying*

$$\psi(t) < t, \quad \forall t > 0. \tag{2.1}$$

Let $\varphi : [0,+\infty) \to [0,+\infty]$ be a nonnegative Lebesgue-integrable mapping which is summable on each compact subset of $[0,+\infty[$ and such that for $\varepsilon > 0$, $\int_0^\varepsilon \varphi(t)dt > 0$ and let $f : X_\rho \to X_\rho$ be a mapping such that there are $c, l \in \mathbb{R}^+$ where $l < c$,

$$\int_0^{\rho(c(fx-fy))} \varphi(t)dt \le \psi\left(\int_0^{\rho(l(x-y))} \varphi(t)dt\right), \tag{2.2}$$

for each $x, y \in X_\rho$. Then f has a unique fixed point in X_ρ.

Proof. First, we show that for $x \in X_\rho$, the sequence $\{\rho(c(f^n x - f^{n-1}x))\}$ converges to 0. For $n \in \mathbb{N}$, we have

$$\begin{aligned}
\int_0^{\rho(c(f^n x-f^{n-1}x))} \varphi(t)dt &\le \psi\left(\int_0^{\rho(l(f^{n-1}x-f^{n-2}x))} \varphi(t)dt\right) \\
&< \int_0^{\rho(l(f^{n-1}x-f^{n-2}x))} \varphi(t)dt \\
&< \int_0^{\rho(c(f^{n-1}x-f^{n-2}x))} \varphi(t)dt.
\end{aligned} \tag{2.3}$$

Consequently, $\{\int_0^{\rho(c(f^n x-f^{n-1}x))} \varphi(t)dt\}$ is decreasing and bounded from below. Therefore $\{\int_0^{\rho(c(f^n x-f^{n-1}x))} \varphi(t)dt\}$ converges to a nonnegative point a.

Now, if $a \neq 0$,

$$\begin{aligned}
a &= \lim_{n\to\infty} \int_0^{\rho(c(f^n x-f^{n-1}x))} \varphi(t)dt \\
&\le \lim_{n\to\infty} \psi\left(\int_0^{\rho(l(f^{n-1}x-f^{n-2}x))} \varphi(t)dt\right) \\
&\le \lim_{n\to\infty} \psi\left(\int_0^{\rho(c(f^{n-1}x-f^{n-2}x))} \varphi(t)dt\right),
\end{aligned} \tag{2.4}$$

then

$$a \le \psi(a), \tag{2.5}$$

which is a contradiction, so $a = 0$ and

$$\int_0^{\rho(c(f^n x - f^{n+1}x))} \varphi(t)dt \longrightarrow 0^+ \quad \text{as } n \longrightarrow +\infty. \tag{2.6}$$

This concludes $\rho(c(f^n x - f^{n+1}x)) \to 0$. Suppose that

$$\lim_{n\to\infty} \sup \rho\Big(c\Big(f^n x - f^{n+1}x\Big)\Big) = \varepsilon > 0 \tag{2.7}$$

then there exist a $\nu_\varepsilon \in \mathbb{N}$ and a sequence $(f^{n_\nu}x)_{\nu \geq \nu_\varepsilon}$ such that

$$\begin{gathered} \rho\Big(c\Big(f^{n_\nu} x - f^{n_\nu+1}x\Big)\Big) \longrightarrow \varepsilon > 0, \quad \nu \longrightarrow \infty, \\ \rho\Big(c\Big(f^{n_\nu} x - f^{n_\nu+1}x\Big)\Big) \geq \frac{\varepsilon}{2}, \quad \forall \nu \geq \nu_\varepsilon, \end{gathered} \tag{2.8}$$

then we get the following contradiction:

$$0 = \lim_{\nu\to\infty} \int_0^{\rho(c(f^{n_\nu} x - f^{n_\nu+1}x))} \varphi(t)dt \geq \int_0^{\varepsilon/2} \varphi(t)dt > 0. \tag{2.9}$$

Now, we prove for each $x \in X_\rho$ the sequence $\{f^n x\}_{n\in\mathbb{N}}$ is a ρ-Cauchy sequence.

Assume that there is an $\varepsilon > 0$ such that for each $\nu \in \mathbb{N}$ there exist $m_\nu, n_\nu \in \mathbb{N}$ that $m_\nu > n_\nu > \nu$,

$$\rho(l(f^{m_\nu} x - f^{n_\nu} x)) \geq \varepsilon. \tag{2.10}$$

Then we choose the sequence $(m_\nu)_{\nu\in\mathbb{N}}$ and $(n_\nu)_{\nu\in\mathbb{N}}$ such that for each $\nu \in \mathbb{N}$, m_ν is minimal in the sense that

$$\rho(l(f^{m_\nu} x - f^{n_\nu} x)) \geq \varepsilon. \tag{2.11}$$

But

$$\rho\Big(l\Big(f^h x - f^{n_\nu} x\Big)\Big) < \varepsilon, \tag{2.12}$$

for each $h \in \{n_\nu + 1, \ldots, m_\nu - 1\}$.

Now, let $\alpha \in \mathbb{R}^+$ be such that $l/c + 1/\alpha = 1$, then we have

$$\begin{aligned}
\int_0^{\varepsilon} \varphi(t)dt &\le \int_0^{\rho(l(f^{m_\nu}x - f^{n_\nu}x))} \varphi(t)dt \\
&\le \int_0^{\rho(c(f^{m_\nu}x - f^{n_\nu+1}x))} \varphi(t)dt + \int_0^{\rho(\alpha l(f^{n_\nu+1}x - f^{n_\nu}x))} \varphi(t)dt \\
&\le \varphi\left(\int_0^{\rho(l(f^{m_\nu-1}x - f^{n_\nu}x))} \varphi(t)dt\right) + \int_0^{\rho(\alpha l(f^{n_\nu+1}x - f^{n_\nu}x))} \varphi(t)dt \\
&\le \int_0^{\rho(l(f^{m_\nu-1}x - f^{n_\nu}x))} \varphi(t)dt + \int_0^{\rho(\alpha l(f^{n_\nu+1}x - f^{n_\nu}x))} \varphi(t)dt \\
&\le \int_0^{\varepsilon} \varphi(t)dt + \int_0^{\rho(\alpha l(f^{n_\nu+1}x - f^{n_\nu}x))} \varphi(t)dt.
\end{aligned} \tag{2.13}$$

Thus, as $\nu \to \infty$, by Δ_2-condition, $\int_0^{\rho(\alpha l(f^{n_\nu+1}x - f^{n_\nu}x))} \varphi(t)dt \to 0$. Therefore

$$\int_0^{\rho(l(f^{m_\nu}x - f^{n_\nu}x))} \varphi(t)dt \longrightarrow \varepsilon^+, \qquad \nu \longrightarrow \infty. \tag{2.14}$$

Now,

$$\begin{aligned}
\int_0^{\rho(l(f^{m_\nu}x - f^{n_\nu}x))} \varphi(t)dt &\le \int_0^{\rho(c(f^{m_\nu+1}x - f^{n_\nu+1}x))} \varphi(t)dt + \int_0^{\rho(2\alpha l(f^{m_\nu}x - f^{m_\nu+1}x))} \varphi(t)dt \\
&\quad + \int_0^{\rho(2\alpha l(f^{n_\nu+1}x - f^{n_\nu}x))} \varphi(t)dt \\
&\le \varphi\left(\int_0^{\rho(l(f^{m_\nu}x - f^{n_\nu}x))} \varphi(t)dt\right) + \int_0^{\rho(2\alpha l(f^{m_\nu}x - f^{m_\nu+1}x))} \varphi(t)dt \\
&\quad + \int_0^{\rho(2\alpha l(f^{n_\nu+1}x - f^{n_\nu}x))} \varphi(t)dt.
\end{aligned} \tag{2.15}$$

If $\nu \to \infty$ we get

$$\int_0^{\varepsilon} \varphi(t)dt \le \varphi\left(\int_0^{\varepsilon} \varphi(t)dt\right), \tag{2.16}$$

which is a contradiction for $\varepsilon > 0$. Therefore $\{lf^n x\}$ is a ρ-Cauchy sequence and by Δ_2-condition $\{f^n x\}$ is ρ-Cauchy. By the fact that X_ρ is ρ-complete, there is a $z \in X_\rho$ such that $\rho(f^n z - z) \to 0$ as $n \to \infty$. Furthermore, z is the fixed point for f. In fact

$$\rho\left(\frac{c}{2}(z - fz)\right) \le \rho\left(c\left(z - f^n z\right)\right) + \rho\left(c\left(f^n z - fz\right)\right) \longrightarrow 0, \qquad n \longrightarrow \infty \tag{2.17}$$

then $\rho((c/2)(z - fz)) = 0$ and $fz = z$.

Now, assume that we have more than one fixed point for f. Let z and u be two distinct fixed points, then

$$\begin{aligned}\int_0^{\rho(c(z-u))}\varphi(t)dt &= \int_0^{\rho(c(fz-fu))}\varphi(t)dt \le \psi\left(\int_0^{\rho(l(z-u))}\varphi(t)dt\right)\\ &< \int_0^{\rho(l(z-u))}\varphi(t)dt \le \int_0^{\rho(c(z-u))}\varphi(t)dt,\end{aligned} \tag{2.18}$$

which is a contradiction. So $z = u$ and the proof is complete. □

Corollary 2.2 (see [1]). *Let X_ρ be a complete modular space, where ρ satisfies the Δ_2-condition. Let $f : X_\rho \to X_\rho$ be a mapping such that there exists an $\lambda \in (0,1)$ and $c, l \in \mathbb{R}^+$ where $l < c$ and for each $x, y \in X_\rho$,*

$$\int_0^{\rho(c(fx-fy))}\varphi(t)dt \le \lambda\left(\int_0^{\rho(l(x-y))}\varphi(t)dt\right), \tag{2.19}$$

then f has a unique fixed point.

Corollary 2.3 (see [3]). *Let X_ρ be a complete modular space, where ρ satisfies the Δ_2-condition. Assume that $\psi : \mathbb{R}^+ \to [0,\infty)$ is an increasing and upper semicontinuous function satisfying*

$$\psi(t) < t, \quad \forall t > 0. \tag{2.20}$$

Let B be a ρ-closed subset of X_ρ and $T : B \to B$ be a mapping such that there exist $c, l \in \mathbb{R}^+$ with $c > l$,

$$\rho(c(Tx - Ty)) \le \psi(\rho(l(x-y))) \tag{2.21}$$

for all $x, y \in B$. Then T has a fixed point.

In the next theorem we use the following notation:

$$m(x,y) = \max\left\{\rho(x-y), \rho(x-Tx), \rho(y-Ty), \frac{\rho(1/2(x-Ty)) + \rho(1/2(y-Tx))}{2}\right\}. \tag{2.22}$$

Theorem 2.4. *Let (X_ρ, ρ) be a ρ-complete modular space that ρ satisfies the Δ_2-condition and let $T : X_\rho \to X_\rho$ be a mapping such that for each $x, y \in X_\rho$,*

$$\int_0^{\rho(Tx-Ty)} \phi(t)dt \le \psi\left(\int_0^{m(x,y)} \phi(t)dt\right), \tag{2.23}$$

where $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ and $\psi : \mathbb{R}^+ \to [0,\infty)$ are as in Theorem 2.1. Then T has a unique fixed point.

Proof. Let $x \in X_\rho$, we will show that $\{T^n x\}$ is a Cauchy sequence. First, we prove that $\{\rho(T^n x - T^{n-1}x)\}$ converges to 0. From (2.23),

$$\int_0^{\rho(T^n x - T^{n-1}x)} \phi(t)dt \le \psi\left(\int_0^{m(T^{n-1}x, T^{n-2}x)} \phi(t)dt\right). \tag{2.24}$$

By the definition of $m(x, y)$,

$$\begin{aligned} m\left(T^{n-1}x, T^{n-2}x\right) &= \max\left\{\rho\left(T^n x - T^{n-1}x\right), \rho\left(T^{n-1} - T^{n-2}x\right), \frac{\rho(1/2(T^n x - T^{n-2}x))}{2}\right\}, \\ \frac{\rho(1/2(T^n x - T^{n-2}x))}{2} &\le \frac{\rho(T^n x - T^{n-1}x) + \rho(T^{n-1} - T^{n-2}x)}{2} \\ &\le \max\left\{\rho\left(T^n x - T^{n-1}x\right), \rho\left(T^{n-1} - T^{n-2}x\right)\right\}. \end{aligned} \tag{2.25}$$

Hence,

$$m\left(T^{n-1}x, T^{n-2}x\right) = \max\left\{\rho\left(T^n x - T^{n-1}x\right), \rho\left(T^{n-1} - T^{n-2}x\right)\right\} \tag{2.26}$$

and therefore,

$$\begin{aligned} \int_0^{\rho(T^n x - T^{n-1}x)} \phi(t)dt &\le \psi\left(\int_0^{m(T^{n-1}x, T^{n-2}x)} \phi(t)dt\right) \\ &\le \int_0^{m(T^{n-1}x, T^{n-2}x)} \phi(t)dt \\ &= \int_0^{\max\{\rho(T^n x - T^{n-1}x), \rho(T^{n-1} - T^{n-2}x)\}} \phi(t)dt \\ &= \max\left\{\int_0^{\rho(T^n x - T^{n-1}x)} \phi(t)dt, \int_0^{\rho(T^{n-1} - T^{n-2}x)} \phi(t)dt\right\} \\ &= \int_0^{\rho(T^{n-1} - T^{n-2}x)} \phi(t)dt. \end{aligned} \tag{2.27}$$

This means that $\{\rho(T^n x - T^{n-1}x)\}$ is decreasing and since it is bounded from below, it is a convergent sequence. Similarly to Theorem 2.1, it is easy to show that

$$\left\{\rho\left(T^n x - T^{n-1}x\right)\right\} \longrightarrow 0. \tag{2.28}$$

Now, we show that $\{T^n x\}$ is Cauchy. If not, then there exist an $\varepsilon > 0$ and subsequences $\{m(p)\}$ and $\{n(p)\}$ such that $m(p) < n(p) < m(p+1)$ with

$$\rho\left(T^{m(p)}x - T^{n(p)}x\right) \geq \varepsilon, \qquad \rho\left(2\left(T^{m(p)}x - T^{n(p)-1}x\right)\right) < \varepsilon. \tag{2.29}$$

From (2.22),

$$\begin{aligned} m\left(T^{m(p)-1}x, T^{n(p)-1}x\right) = \max\Bigg\{ &\rho\left(T^{m(p)-1}x - T^{n(p)-1}x\right), \\ &\rho\left(T^{m(p)}x - T^{m(p)-1}x\right), \rho\left(T^{n(p)}x - T^{n(p)-1}x\right), \\ &\frac{\rho(1/2(T^{m(p)}x - T^{n(p)-1}x)) + \rho(1/2(T^{n(p)}x - T^{m(p)-1}x))}{2}\Bigg\}. \end{aligned} \tag{2.30}$$

By using (2.28), we get

$$\lim_p \int_0^{\rho(T^{m(p)}x - T^{m(p)-1}x)} \phi(t)dt = \lim_p \int_0^{\rho(T^{n(p)}x - T^{n(p)-1}x)} \phi(t)dt = 0. \tag{2.31}$$

On the other hand,

$$\begin{aligned} \rho\left(T^{m(p)-1}x - T^{n(p)-1}x\right) &\leq \rho\left(2\left(T^{m(p)-1}x - T^{m(p)}x\right)\right) + \rho\left(2\left(T^{m(p)}x - T^{n(p)-1}x\right)\right) \\ &\leq \rho\left(2\left(T^{m(p)-1}x - T^{m(p)}x\right)\right) + \varepsilon, \end{aligned} \tag{2.32}$$

thus by the Δ_2-condition,

$$\lim_p \int_0^{\rho(T^{m(p)-1}x - T^{n(p)-1}x)} \phi(t)dt \leq \int_0^{\varepsilon} \phi(t)\mathrm{d}t. \tag{2.33}$$

For the last term in $m(T^{m(p)-1}x, T^{n(p)-1}x)$ by the fact that $\rho(cx)$ is an increasing function of c we have

$$\begin{aligned}
v(m,n) :&= \frac{\rho\left(1/2\left(T^{m(p)}x - T^{n(p)-1}x\right)\right) + \rho\left(1/2\left(T^{n(p)}x - T^{m(p)-1}x\right)\right)}{2} \\
&\leq \frac{\rho\left(T^{m(p)}x - T^{m(p)-1}x\right) + \rho\left(2\left(T^{n(p)}x - T^{n(p)-1}x\right)\right)}{2} \\
&\quad + \frac{\rho\left(2\left(T^{m(p)}x - T^{n(p)-1}x\right)\right) + \rho\left(1/2\left(T^{m(p)}x - T^{n(p)-1}x\right)\right)}{2} \\
&\leq \varepsilon + \frac{\rho\left(T^{m(p)}x - T^{m(p)-1}x\right) + \rho\left(2\left(T^{n(p)}x - T^{n(p)-1}x\right)\right)}{2}.
\end{aligned} \tag{2.34}$$

Hence, from (2.28) we get

$$\lim_{p} \int_0^{v(m,n)} \phi(t)dt \leq \int_0^{\varepsilon} \phi(t)dt. \tag{2.35}$$

Therefore from (2.31), (2.33), and (2.35) it can be concluded that

$$\begin{aligned}
\int_0^{\varepsilon} \phi(t)dt &\leq \int_0^{\rho(T^{m(p)}x - T^{n(p)}x)} \phi(t)dt \leq \psi\left(\int_0^{m(T^{m(p)-1}x, T^{n(p)-1}x)} \phi(t)dt\right) \\
&< \int_0^{m(T^{m(p)-1}x, T^{n(p)-1}x)} \phi(t)dt \leq \int_0^{\varepsilon} \phi(t)dt
\end{aligned} \tag{2.36}$$

which is a contradiction, when p is large enough. Therefore, $\{T^n x\}$ is Cauchy and since X_ρ is ρ-complete there is an $z \in X_\rho$ that $T^n x \to z$. Now, we should prove that z is the fixed point for T. In fact,

$$\begin{aligned}
\int_0^{\rho(1/2(Tz - z))} \phi(t)dt &\leq \int_0^{\rho(Tz - T^n z)} \phi(t)dt + \int_0^{\rho(T^n z - z)} \phi(t)dt \\
&\leq \psi\left(\int_0^{m(z, T^{n-1}z)} \phi(t)dt\right) + \int_0^{\rho(T^n z - z)} \phi(t)dt \longrightarrow 0 \quad \text{as } n \longrightarrow \infty,
\end{aligned} \tag{2.37}$$

by the definition of m. It follows that $Tz = z$.

Let $w \in X_\rho$ be another fixed point of T. Then,

$$\begin{aligned}
\int_0^{\rho(w-z)} \phi(t)dt &= \int_0^{\rho(Tw - Tz)} \phi(t)dt \leq \psi\left(\int_0^{m(w,z)} \phi(t)dt\right) \\
&< \int_0^{m(w,z)} \phi(t)dt = \int_0^{\rho(w-z)} \phi(t)dt.
\end{aligned} \tag{2.38}$$

That is because

$$
\begin{aligned}
m(w,z) &= \max\left\{\rho(z-w),\rho(z-z),\rho(w-w),\frac{\rho(1/2(z-w))+\rho(1/2(w-z))}{2}\right\} \\
&= \rho(w-z),
\end{aligned}
\tag{2.39}
$$

thus $z = w$. □

Corollary 2.5 (see [2]). *Let (X,d) be complete metric space, $k \in [0,1)$, $f : X \to X$ a mapping such that, for $x, y \in X$,*

$$
\int_0^{d(f(x),f(y))} \phi(t)dt \leq k\int_0^{m(x,y)} \phi(t)dt,
\tag{2.40}
$$

where $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesgue-integrable mapping which is summable, nonnegative, and such that

$$
\int_0^{\varepsilon} \phi(t)dt > 0 \quad \forall\varepsilon > 0,
\tag{2.41}
$$

and where

$$
m(x,y) = \max\left\{d(x,y),d(x,fx),d(y,fy),\frac{d(x,fy)+d(y,fy)}{2}\right\}.
\tag{2.42}
$$

Then f has a unique fixed point.

Corollary 2.6 (see [4]). *Let (X,ρ) be a modular space such that ρ satisfies the Fatou property. Let C be a ρ-complete nonempty subset of X_ρ and $T : C \to C$ be quasicontraction. Let $x \in C$ such that $\delta_\rho(x) < \infty$. Then $\{T^n x\}$ ρ-converges to $\omega \in C$. Here $\delta_\rho(x) = \sup\{\rho(T^n x - T^m x); n, m \in \mathbb{N}\}$.*

Acknowledgments

The authors would like to thank the anonymous referees for helpful comments to improve this paper. The first author thanks the Islamic Azad University-Kermanshah branch for supporting this research.

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