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Research Article

Identities on the Bernoulli and Genocchi Numbers and Polynomials

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We give some interesting identities on the Bernoulli numbers and polynomials, on the Genocchi numbers and polynomials by using symmetric properties of the Bernoulli and Genocchi polynomials.

1. Introduction

Let p be a fixed odd prime number. Throughout this paper $\mathbb{Z}_p, \mathbb{Q}_p$, and $\mathbb{C}_p$ will denote the ring of p-adic rational integers, the field of p-adic rational numbers, and the completion of the algebraic closure of $\mathbb{Q}_p$. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{Z}_+ = \mathbb{N} \cup \{0\}$. The p-adic norm on C_p is normalized so that $|p|_p = p^{-1}$. Let $C(\mathbb{Z}_p)$ be the space of continuous functions on $\mathbb{Z}_p$. For $f \in C(\mathbb{Z}_p)$, the fermionic p-adic integral on $\mathbb{Z}_p$ is defined by Kim as follows:

$$I_{-1}(f) = \int_{\mathbb{Z}_p} f(x) d\mu_{-1}(x) = \lim_{N \to \infty} \sum_{x=0}^{p^N-1} f(x)(-1)^x \tag{1.1}$$

(see [1–16]). From (1.1), we have

$$I_{-1}(f_1) = -I_{-1}(f) + 2f(0) \tag{1.2}$$

(see [1–16]), where $f_1(x) = f(x+1)$.

Let us take $f(x) = e^{xt}$. Then, by (1.2), we get

$$t\int_{\mathbb{Z}_p} e^{xt} d\mu_{-1}(x) = \frac{2t}{e^t+1} = \sum_{n=0}^{\infty} G_n \frac{t^n}{n!}, \tag{1.3}$$

where G_n are the nth ordinary Genocchi numbers (see [8, 15]).

From the same method of (1.3), we can also derive the following equation:

$$t\int_{\mathbb{Z}_p} e^{(x+y)t} d\mu_{-1}(y) = \frac{2t}{e^t+1} e^{xt} = \sum_{n=0}^{\infty} G_n(x) \frac{t^n}{n!}, \tag{1.4}$$

where $G_n(x)$ are called the nth Genocchi polynomials (see [14, 15]).

By (1.3), we easily see that

$$G_n(x) = \sum_{l=0}^{n} \binom{n}{l} G_l x^{n-l} \tag{1.5}$$

(see [15]). By (1.3) and (1.4), we get Witt's formula for the nth Genocchi numbers and polynomials as follows:

$$\int_{\mathbb{Z}_p} x^n d\mu_{-1}(x) = \frac{G_{n+1}}{n+1}, \quad \int_{\mathbb{Z}_p} (x+y)^n d\mu_{-1}(y) = \frac{G_{n+1}(x)}{n+1}, \quad \text{for } n \in \mathbb{Z}_+. \tag{1.6}$$

From (1.2), we have

$$\int_{\mathbb{Z}_p} (x+1)^n d\mu_{-1}(x) + \int_{\mathbb{Z}_p} x^n d\mu_{-1}(x) = 2\delta_{0,n}, \tag{1.7}$$

where the symbol $\delta_{0,n}$ is the Kronecker symbol (see [4, 5]).

Thus, by (1.5) and (1.7), we get

$$(G+1)^n + G_n = 2\delta_{1,n} \tag{1.8}$$

(see [15]). From (1.4), we can derive the following equation:

$$\int_{\mathbb{Z}_p} (1-x+y)^n d\mu_{-1}(y) = (-1)^n \int_{\mathbb{Z}_p} (x+y)^n d\mu_{-1}(y). \tag{1.9}$$

By (1.6) and (1.9), we see that

$$\frac{G_{n+1}(1-x)}{n+1} = (-1)^n \frac{G_{n+1}(x)}{n+1}. \tag{1.10}$$

Thus, by (1.10), we get $G_{n+1}(2)/(n+1) = (-1)^n (G_{n+1}(-1)/(n+1))$.

From (1.5) and (1.8), we have

$$\frac{G_{n+1}(2)}{n+1} = 2 - \frac{G_{n+1}(1)}{n+1} = 2 + \frac{G_{n+1}}{n+1} - 2\delta_{1,n+1}. \tag{1.11}$$

The Bernoulli polynomials $B_n(x)$ are defined by

$$\frac{t}{e^t - 1}e^{xt} = e^{B(x)t} = \sum_{n=0}^{\infty} B_n(x)\frac{t^n}{n!} \tag{1.12}$$

(see [6, 9, 12]) with the usual convention about replacing $B^n(x)$ by $B_n(x)$.

In the special case, $x = 0$, $B_n(0) = B_n$ is called the n-th Bernoulli number. By (1.12), we easily see that

$$B_n(x) = \sum_{l=0}^{n}\binom{n}{l}x^{n-l}B_l = (B + x)^n \tag{1.13}$$

(see [6]). Thus, by (1.12) and (1.13), we get reflection symmetric formula for the Bernoulli polynomials as follows:

$$B_n(1 - x) = (-1)^n B_n(x), \tag{1.14}$$

$$B_0 = 1, \quad (B + 1)^n - B_n = \delta_{1,n} \tag{1.15}$$

(see [6, 9, 12]). From (1.14) and (1.15), we can also derive the following identity:

$$(-1)^n B_n(-1) = B_n(2) = n + B_n(1) = n + B_n + \delta_{1,n}. \tag{1.16}$$

In this paper, we investigate some properties of the fermionic p-adic integrals on $\mathbb{Z}_p$. By using these properties, we give some new identities on the Bernoulli and the Euler numbers which are useful in studying combinatorics.

2. Identities on the Bernoulli and Genocchi Numbers and Polynomials

Let us consider the following fermionic p-adic integral on $\mathbb{Z}_p$ as follows:

$$\begin{aligned} I_1 &= \int_{\mathbb{Z}_p} B_n(x)d\mu_{-1}(x) = \sum_{l=0}^{n}\binom{n}{l}B_{n-l}\int_{\mathbb{Z}_p} x^l d\mu_{-1}(x) \\ &= \sum_{l=0}^{n}\binom{n}{l}B_{n-l}\frac{G_{l+1}}{l+1}, \quad \text{for } n \in \mathbb{Z}_+ = \mathbb{N} \cup \{0\}. \end{aligned} \tag{2.1}$$

On the other hand, by (1.14) and (1.15), we get

$$\begin{aligned}
I_1 &= (-1)^n \int_{\mathbb{Z}_p} B_n(1-x)\,d\mu_{-1}(x) \\
&= (-1)^n \sum_{l=0}^{n} \binom{n}{l} B_{n-l} \int_{\mathbb{Z}_p} (1-x)^l\,d\mu_{-1}(x) \\
&= (-1)^n \sum_{l=0}^{n} \binom{n}{l} B_{n-l} (-1)^l \frac{G_{l+1}(-1)}{l+1} \\
&= (-1)^n \sum_{l=0}^{n} \binom{n}{l} B_{n-l} \left(2 + \frac{G_{l+1}}{l+1} - 2\delta_{1,l+1}\right) \\
&= 2(-1)^n (B_n + \delta_{1,n}) + (-1)^n \sum_{l=0}^{n} \binom{n}{l} B_{n-l} \frac{G_{l+1}}{l+1} + 2(-1)^{n+1} B_n.
\end{aligned} \tag{2.2}$$

Equating (2.1) and (2.2), we obtain the following theorem.

Theorem 2.1. *For $n \in \mathbb{Z}_+$, one has*

$$\left(1 + (-1)^{n+1}\right) \sum_{l=0}^{n} \binom{n}{l} B_{n-l} \frac{G_{l+1}}{l+1} = 2(-1)^n \delta_{1,n}. \tag{2.3}$$

By using the reflection symmetric property for the Euler polynomials, we can also obtain some interesting identities on the Euler numbers.

Now, we consider the fermionic p-adic integral on $\mathbb{Z}_p$ for the polynomials as follows:

$$\begin{aligned}
I_2 &= \int_{\mathbb{Z}_p} G_n(x)\,d\mu_{-1}(x) \\
&= \sum_{l=0}^{n} \binom{n}{l} G_{n-l} \int_{\mathbb{Z}_p} x^l\,d\mu_{-1}(x) \\
&= \sum_{l=0}^{n} \binom{n}{l} G_{n-l} \frac{G_{l+1}}{l+1}, \quad \text{for } n \in \mathbb{Z}_+.
\end{aligned} \tag{2.4}$$

On the other hand, by (1.8), (1.10), and (1.11), we get

$$\begin{aligned}
I_2 &= (-1)^{n-1} \int_{\mathbb{Z}_p} G_n(1-x)\,d\mu_{-1}(x) \\
&= (-1)^{n-1} \sum_{l=0}^{n} \binom{n}{l} G_{n-l} \int_{\mathbb{Z}_p} (1-x)^l\,d\mu_{-1}(x) \\
&= (-1)^{n-1} \sum_{l=0}^{n} \binom{n}{l} G_{n-l} (-1)^l \frac{G_{l+1}(-1)}{l+1}
\end{aligned}$$

$$
\begin{aligned}
&= (-1)^{n-1}\sum_{l=0}^{n}\binom{n}{l}G_{n-l}\left(2+\frac{G_{l+1}}{l+1}-2\delta_{1,l+1}\right)\\
&= 2(-1)^{n-1}(2\delta_{1,n}-G_n)+2(-1)^{n}G_n\\
&\quad + (-1)^{n-1}\sum_{l=0}^{n}\binom{n}{l}G_{n-l}\frac{G_{l+1}}{l+1}.
\end{aligned}
\tag{2.5}
$$

Equating (2.4) and (2.5), we obtain the following theorem.

Theorem 2.2. *For $n \in \mathbb{Z}_+$, one has*

$$
(1+(-1)^n)\sum_{l=0}^{n}\binom{n}{l}G_{n-l}\frac{G_{l+1}}{l+1} = 4(-1)^n G_n + 4(-1)^{n+1}\delta_{1,n}. \tag{2.6}
$$

Let us consider the fermionic p-adic integral on $\mathbb{Z}_p$ for the product of $B_n(x)$ and $G_n(x)$ as follows:

$$
\begin{aligned}
I_3 &= \int_{\mathbb{Z}_p} B_m(x)G_n(x)\,d\mu_{-1}(x)\\
&= \sum_{k=0}^{m}\sum_{l=0}^{n}\binom{m}{k}\binom{n}{l}B_{m-k}G_{n-l}\int_{\mathbb{Z}_p} x^{k+l}\,d\mu_{-1}(x)\\
&= \sum_{k=0}^{m}\sum_{l=0}^{n}\binom{m}{k}\binom{n}{l}B_{m-k}G_{n-l}\frac{G_{k+l+1}}{k+l+1}.
\end{aligned}
\tag{2.7}
$$

On the other hand, by (1.10) and (1.14), we get

$$
\begin{aligned}
I_3 &= \int_{\mathbb{Z}_p} B_m(x)G_n(x)\,d\mu_{-1}(x)\\
&= (-1)^{n+m-1}\int_{\mathbb{Z}_p} B_m(1-x)G_n(1-x)\,d\mu_{-1}(x)\\
&= (-1)^{n+m-1}\sum_{k=0}^{m}\sum_{l=0}^{n}\binom{m}{k}\binom{n}{l}B_{m-k}G_{n-l}\int_{\mathbb{Z}_p}(1-x)^{k+l}\,d\mu_{-1}(x)\\
&= 2(-1)^{n+m-1}B_m(1)G_n(1)+2(-1)^{m+n}B_mG_n\\
&\quad + (-1)^{n+m-1}\sum_{k=0}^{m}\sum_{l=0}^{n}\binom{m}{k}\binom{n}{l}B_{m-k}G_{n-l}\frac{G_{k+l+1}}{k+l+1}.
\end{aligned}
\tag{2.8}
$$

By (2.7) and (2.8), we easily see that

$$\begin{aligned}
&\left(1+(-1)^{n+m+1}\right)\sum_{k=0}^{m}\sum_{l=0}^{n}\binom{m}{k}\binom{n}{l}B_{m-k}G_{n-l}\frac{G_{k+l+1}}{k+l+1}\\
&\quad=2(-1)^{m+n-1}(\delta_{1,m}+B_m)(2\delta_{1,n}-G_n)+2(-1)^{m+n}B_mG_n\\
&\quad=4(-1)^{m+n-1}B_m\delta_{1,n}+2(-1)^{m+n}B_mG_n+4(-1)^{m+n-1}\delta_{1,m}\delta_{1,n}\\
&\qquad+2(-1)^{m+n}\delta_{1,m}G_n+2(-1)^{m+n}B_mG_n.
\end{aligned}\tag{2.9}$$

Therefore, by (2.9), we obtain the following theorem.

Theorem 2.3. *For $n,m\in\mathbb{Z}_+$, one has*

$$\begin{aligned}
&\left(1+(-1)^{n+m+1}\right)\sum_{k=0}^{m}\sum_{l=0}^{n}\binom{m}{k}\binom{n}{l}B_{m-k}\frac{G_{n-l+1}}{n-l+1}\frac{G_{k+l+1}}{k+l+1}\\
&\quad=4(-1)^{m+n}B_mG_n+4(-1)^{m+n-1}B_m\delta_{1,n}+4(-1)^{m+n-1}\delta_{1,m}\delta_{1,n}\\
&\qquad+2(-1)^{m+n}\delta_{1,m}G_n.
\end{aligned}\tag{2.10}$$

Corollary 2.4. *For $n,m\in\mathbb{N}$, one has*

$$\sum_{k=0}^{2m}\sum_{l=0}^{2n}\binom{2m}{k}\binom{2n}{l}B_{2m-k}G_{2n-l}\frac{G_{k+l+1}}{k+l+1}=2B_{2m}G_{2n}.\tag{2.11}$$

Let us consider the fermionic p-adic integral on $\mathbb{Z}_p$ for the product of the Bernoulli polynomials and the Bernstein polynomials. For $n,k\in\mathbb{Z}_+$, with $0\le k\le n$, $B_{k,n}(x)=\binom{n}{k}x^k(1-x)^{n-k}$ are called the Bernstein polynomials of degree n, see [11]. It is easy to show that $B_{k,n}(x)=B_{n-k,n}(1-x)$,

$$\begin{aligned}
I_4&=\int_{\mathbb{Z}_p}B_m(x)B_{k,n}(x)d\mu_{-1}(x)\\
&=\binom{n}{k}\sum_{l=0}^{m}\binom{m}{l}B_{m-l}\int_{\mathbb{Z}_p}x^{k+l}(1-x)^{n-k}d\mu_{-1}(x)\\
&=\binom{n}{k}\sum_{l=0}^{m}\sum_{j=0}^{n-k}\binom{m}{l}\binom{n-k}{j}(-1)^jB_{m-l}\int_{\mathbb{Z}_p}x^{k+l+j}d\mu_{-1}(x)\\
&=\binom{n}{k}\sum_{l=0}^{m}\sum_{j=0}^{n-k}\binom{m}{l}\binom{n-k}{j}(-1)^jB_{m-l}\frac{G_{k+l+j+1}}{k+l+j+1}.
\end{aligned}\tag{2.12}$$

On the other hand, by (1.14) and (2.12), we get

$$\begin{aligned}
I_4 &= (-1)^m \int_{\mathbb{Z}_p} B_m(1-x) B_{n-k,n}(1-x)\, d\mu_{-1}(x) \\
&= (-1)^m \binom{n}{k} \sum_{l=0}^{m} \sum_{j=0}^{k} \binom{m}{l} \binom{k}{j} (-1)^j B_{m-l} \int_{\mathbb{Z}_p} (1-x)^{n-k+l+j}\, d\mu_{-1}(x) \\
&= (-1)^m \binom{n}{k} \sum_{l=0}^{m} \sum_{j=0}^{k} \binom{m}{l} \binom{k}{j} (-1)^j B_{m-l} \\
&\quad \times \left(2 - 2\delta_{1,n-k+l+j+1} + \frac{G_{n-k+l+j+1}}{n-k+l+j+1} \right) \\
&= 2(-1)^m \binom{n}{k} B_m(1)\delta_{0,k} + 2(-1)^{m+1} \binom{n}{k} B_m \delta_{k,n} \\
&\quad + (-1)^m \binom{n}{k} \sum_{l=0}^{m} \sum_{j=0}^{k} \binom{m}{l} \binom{k}{j} (-1)^j B_{m-l} \frac{G_{n-k+l+j+1}}{n-k+l+j+1}.
\end{aligned} \tag{2.13}$$

Equating (2.12) and (2.13), we see that

$$\begin{aligned}
&\sum_{l=0}^{m} \sum_{j=0}^{n-k} \binom{m}{l} \binom{n-k}{j} (-1)^j B_{m-l} \frac{G_{k+l+j+1}}{k+l+j+1} \\
&\quad = 2(-1)^m B_m(1)\delta_{0,k} + 2(-1)^{m+1} B_m \delta_{k,n} \\
&\qquad + (-1)^m \sum_{l=0}^{m} \sum_{j=0}^{k} \binom{m}{l} \binom{k}{j} (-1)^j B_{m-l} \frac{G_{n-k+l+j+1}}{n-k+l+j+1}.
\end{aligned} \tag{2.14}$$

Thus, from (2.14), we obtain the following theorem.

Theorem 2.5. *For $n, m \in \mathbb{N}$, one has*

$$\sum_{l=0}^{2m} \sum_{j=0}^{n} \binom{2m}{l} \binom{n}{j} (-1)^j B_{2m-l} \frac{G_{l+j+1}}{l+j+1} = 2B_{2m}(1) + \sum_{l=0}^{2m} \binom{2m}{l} B_{2m-l} \frac{G_{n+l+1}}{n+l+1}. \tag{2.15}$$

Finally, we consider the fermionic p-adic integral on $\mathbb{Z}_p$ for the product of the Euler polynomials and the Bernstein polynomials as follows:

$$\begin{aligned}
I_5 &= \int_{\mathbb{Z}_p} G_m(x) B_{k,n}(x)\, d\mu_{-1}(x) \\
&= \binom{n}{k} \sum_{l=0}^{m} \binom{m}{l} G_{m-l} \int_{\mathbb{Z}_p} x^{k+l} (1-x)^{n-k}\, d\mu_{-1}(x)
\end{aligned}$$

$$
\begin{aligned}
&= \binom{n}{k}\sum_{l=0}^{m}\sum_{j=0}^{n-k}\binom{m}{l}\binom{n-k}{j}(-1)^{j}G_{m-l}\int_{\mathbb{Z}_p} x^{k+l+j}\,d\mu_{-1}(x)\\
&= \binom{n}{k}\sum_{l=0}^{m}\sum_{j=0}^{n-k}\binom{m}{l}\binom{n-k}{j}(-1)^{j}G_{m-l}\frac{G_{k+l+j+1}}{k+l+j+1}.
\end{aligned}
\tag{2.16}
$$

On the other hand, by (1.10) and (2.12), we get

$$
\begin{aligned}
I_5 &= (-1)^{m-1}\int_{\mathbb{Z}_p} G_m(1-x)B_{n-k,n}(1-x)\,d\mu_{-1}(x)\\
&= (-1)^{m-1}\binom{n}{k}\sum_{l=0}^{m}\binom{m}{l}G_{m-l}\sum_{j=0}^{k}\binom{k}{j}(-1)^{j}\int_{\mathbb{Z}_p}(1-x)^{n-k+l+j}\,d\mu_{-1}(x)\\
&= (-1)^{m-1}\binom{n}{k}\sum_{l=0}^{m}\sum_{j=0}^{k}\binom{m}{l}\binom{k}{j}(-1)^{j}G_{m-l}\\
&\quad\times\left(2+\frac{G_{n-k+l+j+1}}{n-k+l+j+1}-2\delta_{1,n-k+l+j+1}\right)\\
&= 2(-1)^{m-1}\binom{n}{k}G_m(1)\delta_{0,k}+2(-1)^{m}\binom{n}{k}G_m\delta_{k,n}\\
&\quad+(-1)^{m-1}\binom{n}{k}\sum_{l=0}^{m}\sum_{j=0}^{k}\binom{m}{l}\binom{k}{j}(-1)^{j}G_{m-l}\frac{G_{n-k+l+j+1}}{n-k+l+j+1}.
\end{aligned}
\tag{2.17}
$$

Equating (2.16) and (2.17), we obtain

$$
\begin{aligned}
&\sum_{l=0}^{m}\sum_{j=0}^{n-k}\binom{m}{l}\binom{n-k}{j}(-1)^{j}G_{m-l}\frac{G_{k+l+j+1}}{k+l+j+1}\\
&\quad= 2(-1)^{m-1}G_m(1)\delta_{0,k}+2(-1)^{m}G_m\delta_{k,n}\\
&\qquad+(-1)^{m-1}\sum_{l=0}^{m}\sum_{j=0}^{k}\binom{m}{l}\binom{k}{j}(-1)^{j}G_{m-l}\frac{G_{n-k+l+j+1}}{n-k+l+j+1}.
\end{aligned}
\tag{2.18}
$$

Therefore, by (2.18), we obtain the following theorem.

Theorem 2.6. *For $n, m \in \mathbb{N}$, one has*

$$
\sum_{l=0}^{2m}\sum_{j=0}^{n}\binom{2m}{l}\binom{n}{j}(-1)^{j}G_{2m-l}\frac{G_{l+j+1}}{l+j+1} = -2G_{2m}(1)-\sum_{l=0}^{2m}\binom{2m}{l}G_{2m-l}\frac{G_{n+l+1}}{n+l+1}. \tag{2.19}
$$

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