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Research Article

Metric Induced Morphological Operators on Intuitionistic Fuzzy Hypergraphs

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A hypergraph consisting of hyperedges and nodes can be made intuitionistic fuzzy hypergraph (IFHG) by assigning membership and nonmembership degrees for both nodes and edges. Just as a hypergraph, an IFHG is also having hyperedges consisting of many nodes. Many flavours of a given IFHG can be created by applying morphological operators like dilation, erosion, adjunction, etc. The focus of this paper is to define morphological adjunction, opening, closing, and Alternative Sequential Filter (ASF) on IFHG. The system modeled in this way finds application in text processing, image processing, network analysis, and many other areas.

1. Introduction

Let $[H_{IF}, (\mu_n, \gamma_n), (\mu_e, \gamma_e), H^n, H^e]$ be an intuitionistic fuzzy hypergraph with nodes H^n and hyperedges H^e. H^n is the set of nodes with membership degree μ_n and non membership degree γ_n. Depending on the membership degree μ_n, the node can be treated as high priority, medium priority, and low priority. The non membership degree $\gamma_n <= 1 - \mu_n$. The sum of the membership degree and non membership degree of the node is less than or equal to 1 [1], i.e., $\mu_n + \gamma_n <= 1$. H^e is the set of hyperedges with membership degree μ_e and non membership degree γ_e. If all the nodes in a hyperedge have $\mu_n > 0.5$, then μ_e is the supremum of all μ_n in that edge. In such a case $\gamma_n <= 1 - \mu_n$. If there is at least one node with $\gamma_n > 0.5$, then γ_e of that edge is the supremum of all γ_n in that edge. In such a case $\mu_e <= 1 - \gamma_e$. If all the nodes of an edge are having $\mu_n = 0.5$, then its $\mu_e = \gamma_e = 0.5$. The sum of the membership degree and non membership degree of the hyperedge is less than or equal to 1 [1], i.e., $\mu_e + \gamma_e <= 1$. Depending on the membership degree μ_e, the hyperedge can be treated as high priority, medium priority, and low priority. Since μ_n can take any value from 0.0 to 1.0, priorities are not limited to the above three. Also range priorities can be set for nodes and edges. Morphological operations like dilation, erosion, adjunction are already defined on hypergraphs [2]. Dilation operation [3] and erosion [4] is already defined on IFHG. The purpose of this paper is to define adjunction, opening, closing, and ASF on IFHG.

2. Related Works

Many morphological operators [5] like dilation, erosion, adjunction, and duality were applied on lattice of all subgraphs, where these are further applied in binary and grey scale image denoising. The authors have designed filters for which graph is both the input and the output. Various edge-vertex correspondences [6], edge-vertex adjunctions, vertex dilation, vertex erosion, edge dilation, edge erosion, graph dilation, graph erosion, opening, closing filters, half opening, and half closing on graphs, are illustrated with examples which finds application in image filtering. Images were represented using set union [7] of hyperedges and were subjected to contraharmonic mean filter for salt and pepper noise removal. The method gave better results in terms of visual quality, peak signal to noise ratio, and mean absolute error. A framework was proposed to build

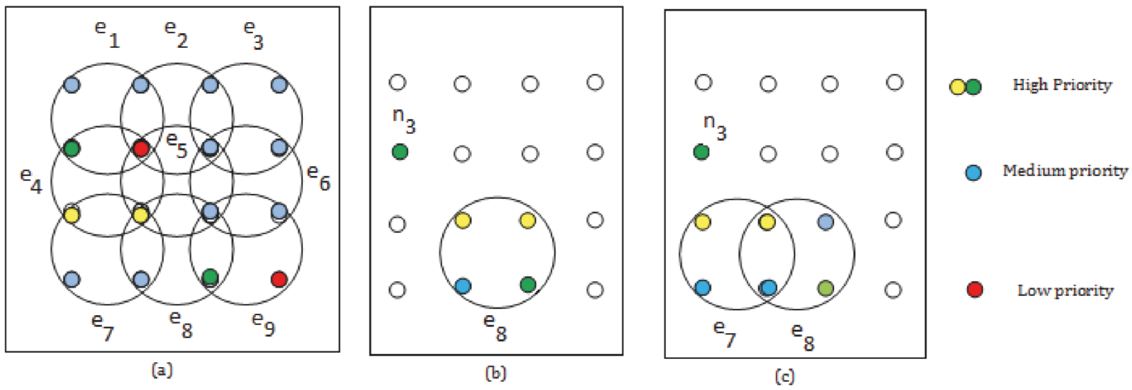


Figure 1: Intuitionistic fuzzy hypergraphs (a) H, (b) X, and (c) Y.

morphological operators for analyzing and filtering objects defined on simplicial complex spaces [8] where a simplex is any finite nonempty set. This finds application in mesh and image filtering. Applications of mathematical morphology to weighted graphs [9] were also developed. By converting the image to a tree representation, the authors have shown how to filter the image using connected filters. Mathematical morphology [10], which is the basis of morphological image processing, finds application in the field of digital image processing, area of graphs, surface mesh, solids, and other spatial structures.

Now coming to the field of hypergraphs, lattice structures on hypergraphs introduced in [11] have shown many properties like partial ordering, infimum, supremum, isomorphism, etc. The authors also introduced complete lattice, dualities, discrete probability distribution on vertices and hypergraph similarity based on dilation. Mathematical morphology of hypergraphs were also used for classification or matching problems [12] on data represented by hypergraphs. As an example, the authors have applied it on a 2D image and they proposed further applications of hypergraph in image analysis. New similarity measures and pseudometrics on lattices of hypergraphs are detailed in [13] which can be incorporated in existing system for hypergraph-based feature selection, indexing, retrieval, and matching.

Intuitionistic fuzzy graphs [14] are used for clustering with the help of many operations like complement, join, union, intersection, ringsum, Cartesian product, composition, etc. The concepts like (α, β) cut, edge strength, incidence matrix of an IFHG [1] are also used in clustering. Isomorphism between two IFHG and the Cartesian product of two IFS over the same universe are detailed in [15], where they also illustrated indegree and outdegree of vertex v, weak isomorphism, and coweak isomorphism. A hypernetwork [16] can be created with processors as vertices and connections between the processors modeled as hyperedge. Radio coverage networks in a geographic region can be modeled with radio receivers as vertices, where the membership values signify the quality of reception of a station/radio. The authors also proposed further research in intuitionistic fuzzy soft hypergraphs and rough hypergraphs. An application with intuitionistic fuzzy sets for career choice [17] which is a decision-making system was developed where the system represented the performance of students using membership, nonmembership, and hesitation margin. They applied normalized Euclidian distance to determine the apt career choice. Operations on transversals of IFDHG, their union, intersection, addition, structural subtraction, multiplication, and complement are defined and discussed in [18]. The authors also propose to work on application in coloring of IFHG. Generalized strong IFHG (GSIFHG), spanning IFHG, and generalized strong spanning IFHG were discussed in [19], which can be used to analyze the structure of a system and to represent a partition, covering, and clustering.

3. Preliminary Definitions

Let us define $H_{IF} = [H^n, H^e]$, where $H^n = [n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, n_{11}, n_{12}, n_{13}, n_{14}, n_{15}, n_{16}]$ and $H^e = [e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9]$ as given in Figure 1. Here nodes with low priority are having $(\mu_n < 0.5)$, nodes of medium priority are having $(\mu_n = 0.5)$, and nodes of high priority are with $(\mu_n > 0.5)$. Let X_{IF} be obtained by (α, β) cut on $H_{IF}/0.5 < \alpha \leq 0.7; \{\beta \leq 1 - \alpha\} \cap \{\beta \leq 0.3\}$. Let Y_{IF} be obtained by (α, β) cut on $H_{IF}/\alpha \geq 0.7; \{\beta \leq 1 - \alpha\} \cap \{\beta \leq 0.3\}$. Here α corresponds to membership degree and β corresponds to nonmembership degree. The details of the IFHGs H_{IF}, X_{IF}, and Y_{IF} are given in Tables 1, 2, and 3, respectively.

Given the above intuitionistic fuzzy hypergraphs, namely, H, X, and Y, we can apply morphological operations like dilation and erosion on them. These operations in turn retrieve sub-IFHGs which are of various priorities. Some of them will be high priority, medium priority, and low priority or different combinations of these. Here δ is the dilation operator. Let us define $\delta^n(X^e)$ as the set of all nodes within the hyperedges in X. This operation will retrieve only priority nodes (high/medium), since the hyperedges in X itself are of high priority. Now $\delta^e(X^n)$ is the set of hyperedges in H which consists of nodes in X. This operation can retrieve all hyperedges in X and also some low/medium priority edges in H which contain those high priority nodes in X. Let us define erosion with the help of the operator ε. Now $\varepsilon^e(X^n)$ is the set of all hyperedges in H which consists of nodes of X only. This

Table 1: Details of hypergraph H_{IF}.

Hyperedges	nodes				Edge priority
e_1 (0.3, 0.7)	n_1 (0.5, 0.5)	n_2 (0.5, 0.5)	n_3 (0.7, 0.3)	n_4 (0.3, 0.7)	Low
e_2 (0.3, 0.7)	n_2 (0.5, 0.5)	n_4 (0.3, 0.7)	n_5 (0.5, 0.5)	n_7 (0.5, 0.5)	Low
e_3 (0.5, 0.5)	n_5 (0.5, 0.5)	n_6 (0.5, 0.5)	n_7 (0.5, 0.5)	n_8 (0.5, 0.5)	Medium
e_4 (0.3, 0.7)	n_3 (0.7, 0.3)	n_4 (0.3, 0.7)	n_9 (0.6, 0.4)	n_{10} (0.6, 0.4)	Low
e_5 (0.3, 0.7)	n_4 (0.3, 0.7)	n_7 (0.5, 0.5)	n_{10} (0.6, 0.4)	n_{11} (0.5, 0.5)	Low
e_6 (0.5, 0.5)	n_7 (0.5, 0.5)	n_8 (0.5, 0.5)	n_{11} (0.5, 0.5)	n_{12} (0.5, 0.5)	Medium
e_7 (0.6, 0,4)	n_9 (0.6, 0,4)	n_{10} (0.6, 0,4)	n_{13} (0.5, 0.5)	n_{14} (0.5, 0.5)	High
e_8 (0.7, 0.3)	n_{10} (0.6, 0,4)	n_{11} (0.5, 0.5)	n_{14} (0.5, 0.5)	n_{15} (0.7, 0.3)	High
e_9 (0.4, 0,6)	n_{11} (0.5, 0.5)	n_{12} (0.5, 0.5)	n_{15} (0.7, 0.3)	n_{16} (0.4, 0.6)	Low

Table 2: Details of hypergraph X_{IF}.

Hyperedges	nodes				Edge priority
e_8 (0.7, 0.3)	n_{10} (0.6, 0,4)	n_{11} (0.5, 0.5)	n_{14} (0.5, 0.5)	n_{15} (0.7, 0.3)	High
Hyperedges	nodes				Node priority
Nil	n_3 (0.7, 0.3)				High

Table 3: Details of hypergraph Y_{IF}.

Hyperedges	nodes				Edge priority
e_7 (0.6, 0,4)	n_9 (0.6, 0,4)	n_{10} (0.6, 0,4)	n_{13} (0.5, 0.5)	n_{14} (0.5, 0.5)	High
e_8 (0.7, 0.3)	n_{10} (0.6, 0,4)	n_{11} (0.5, 0.5)	n_{14} (0.5, 0.5)	n_{15} (0.7, 0.3)	High
Hyperedges	nodes				Node priority
Nil	n_3 (0.7, 0.3)				High

will retrieve only priority edges. Similarly $\varepsilon^n(X^e)$ is the set of all nodes in X but not in $X^{e'}$. This will retrieve priority nodes within X only and not seen in any other edges.

4. Adjunction of IFHG

The adjunctions that we are going to state here are already defined on hypergraphs in [2]. We are extending these adjunctions to IFHG.

Proposition 1. *Let H be the intuitionistic fuzzy hypergraph and let X and Y be the sub-IFHGs, ε be the erosion operator, and δ be the dilation operator. We observe that $(\varepsilon^e, \delta^n)$ are adjunctions if the following holds*

$$X^e \subseteq \varepsilon^e\left(Y^n\right) \tag{1}$$

and

$$\delta^n\left(X^e\right) \subseteq Y^n; \quad X \subseteq Y. \tag{2}$$

Proof. Let us consider erosion operator ε^e; let e be an edge in X^e. That is, $e \subset X^e$. We know that $\varepsilon^e(Y^n) = Y^e$. Since $X \subset Y$, we get $e \subset X^e \subset Y^e$. Therefore $X^e \subseteq \varepsilon^e(Y^n)$. This edge is a priority edge in H. Now let us consider dilation operator δ^n.

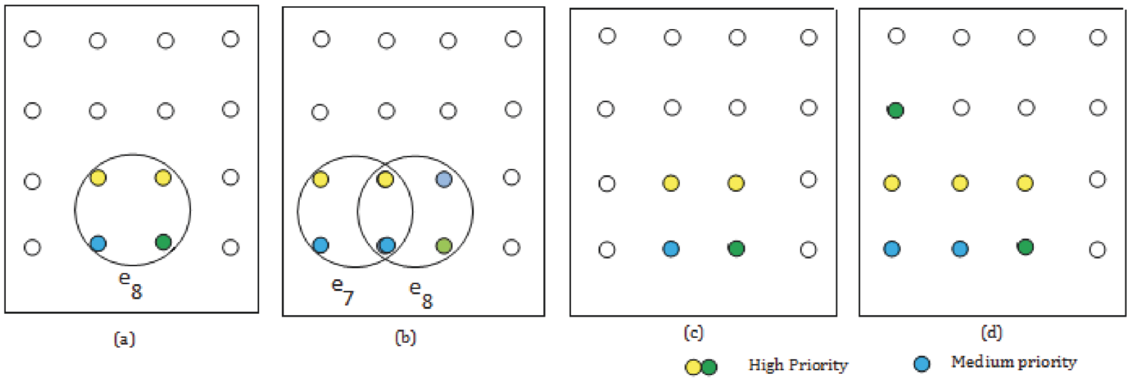


FIGURE 2: Results of adjunction(a) X^e, (b) $\varepsilon^e(Y^n)$, (c) $\delta^n(X^e)$, and (d) Y^n.

Let v be a node in $\delta^n(X^e)$, i.e., $v \subset X^n$. Since $x \subseteq Y$, $v \subset Y^n$. Therefore $v \subseteq X^n \subseteq Y^n$. Therefore $\delta^n(X^e) \subseteq Y^n$. This node v is definitely a priority node of H. □

Illustration. Let us check their results on IFHG by considering H, X, and Y IFHGs. Here, in R.H.S of (1), $\varepsilon^e(Y^n)$ means the set of edges in H, which consists of nodes in Y only. That is, $\varepsilon^e(Y^n) = \{e_7, e_8\}$. This operation returns the high priority edges in H. Now we know X^e as the hyperedges in X, i.e., $X^e = \{e_8\}$. Therefore $X^e \subseteq \varepsilon^e(Y^n)$. Now in L.H.S of (2), find $\delta^n(X^e)$ which is the set of nodes in edges of X. That is, $\delta^n(X^e) = \{n_{10}, n_{11}, n_{14}, n_{15}\}$. This operation returns priority nodes in H which are part of X. We get Y^n as the nodes in Y. That is, $Y^n = \{n_3, n_9, n_{10}, n_{11}, n_{13}, n_{14}, n_{15}\}$. We find that $\delta^n(X^e) \subset Y^n$. Therefore $(\varepsilon^e, \delta^n)$ are adjunctions and the results are shown in Figure 2.

Proposition 2. *Let H be the intuitionistic fuzzy hypergraph; let X and Y be the sub-IFHGs, ε be the erosion operator, and δ be the dilation operator. We observe that if $(\varepsilon^e, \delta^n)$ are adjunctions then*

$$\left(\delta^e\left(X^{n'}\right)\right)' = \varepsilon^e\left(X^n\right) \tag{3}$$

and

$$\left(\delta^n\left(X^{e'}\right)\right)' = \varepsilon^n\left(X^e\right) \tag{4}$$

Proof. Let e be an edge in $(\delta^e(X^{n'}))'$, i.e.,

$$e \in \left(\delta^e\left(X^{n'}\right)\right)' \tag{5}$$

then

$$\begin{gathered} e \notin \delta^e\left(X^{n'}\right); \\ e \notin X' \end{gathered} \tag{6}$$

i.e.,

$$e \in X \tag{7}$$

Let us consider R.H.S of (3). Let e be an edge of $\varepsilon^e(X^n)$, i.e., $e \in \varepsilon^e(X^n)$. That is,

$$e \in X \tag{8}$$

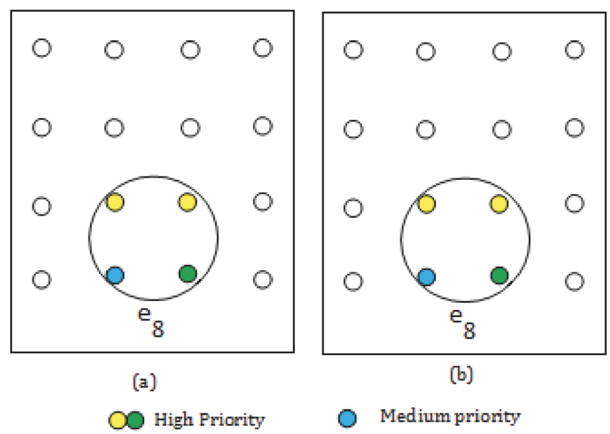


FIGURE 3: Complement results of adjunction (a) $(\delta^e(X^{n'}))'$ and (b) $\varepsilon^e(X^n)$.

Equation (3) is implied from (7) and (8). The edge e is a priority edge present in X.

Consider L.H.S of (4). Let v be a node in $(\delta^n(X^{e'}))'$, i.e., $v \in (\delta^n(X^{e'}))'$, i.e., $v \notin (\delta^n(X^{e'}))$. So we can write $v \notin X^{e'}$. That is,

$$v \in X^e \tag{9}$$

Now consider R.H.S of (4). Let v be a node of $\varepsilon^n(X^e)$, i.e., $v \in \varepsilon^n(X^e)$. We get

$$v \in X^e \tag{10}$$

Equation (4) is implied from (9) and (10). □

Illustration. In L.H.S of (3), $\delta^e(X^{n'})$ is the set of edges in H, which consists of any node $X^{n'}$. We know that $X^{n'} = \{n_1, n_2, n_4, n_5, n_6, n_7, n_8, n_9, n_{12}, n_{13}, n_{16}\}$. Thus $\delta^e(X^{n'}) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_9\}$. Therefore $(\delta^n(X^{n'}))' = \{e_8\} = \varepsilon^e(X^n)$. Both these operations return priority edges in X and the same are shown in Figures 3(a) and 3(b), respectively. Now consider L.H.S of (4), $\delta^n(X^{e'})$ which is the set of nodes in $X^{e'}$. That

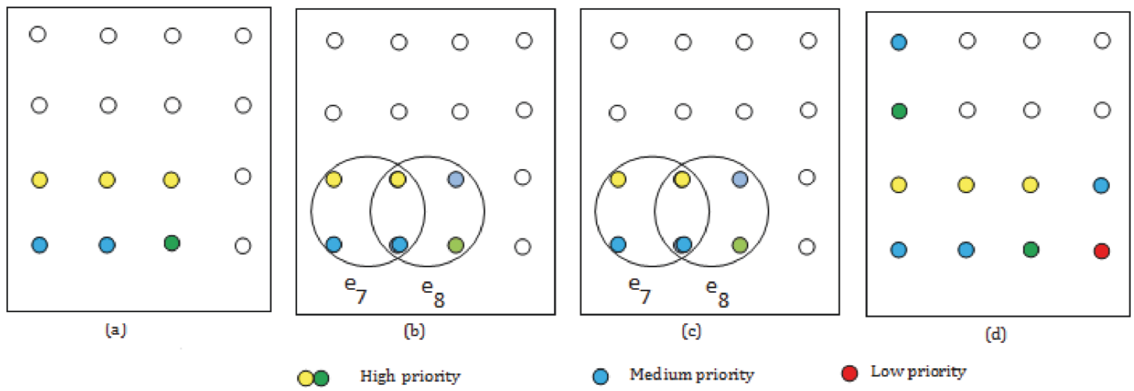


Figure 4: (a) $\delta^n(\varepsilon^e(Y^n))$, (b) $\delta^e(\varepsilon^n(Y^e))$, (c) $\varepsilon^e(\delta^n(Y^e))$, (d) $\varepsilon^n(\delta^e(Y^n))$.

is, $X^{e'} = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_9\}$. Now $\delta^n(X^{e'}) = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, n_{11}, n_{12}, n_{13}, n_{14}, n_{15}, n_{16}\}$. Now $(\delta^n(X^{e'}))' = \phi$. We get $\varepsilon^n(X^e)$ as the set of nodes in X which are not in $X^{e'}$. That is, $\varepsilon^n(X^e) = \phi$.

5. Materials and Methods-Construction of Various IFHG Filters

A filter is something which gives the same result if a function is repeatedly applied on it. Consider a water/sand filter where the filtrate on repeated passage through the same filter gives the same filtrate. Similarly in the case of a IFHG, a filter applied on a sub-IFHG should produce the same set of edges and nodes even if it is filtered many times. If δ is the dilation operator and ε is the erosion operator, $\gamma = \delta \circ \varepsilon$ is an opening filter and $\phi = \varepsilon \circ \delta$ is a closing filter.

5.1. Half Opening Filter with Respect to Nodes. If H is the parent IFHG, Y is the sub-IFHG, δ is the dilation operator, and ε is the erosion operator, then $\gamma_{1/2} = \delta^n(\varepsilon^e(Y^n))$ is a half opening filter with respect to the nodes in Y. Here $\varepsilon^e(Y^n)$ is the set of edges in H which consists of Y^n only. That is, $\varepsilon^e(Y^n) = \{e_7, e_8\}$. Now $\delta^n(\varepsilon^e(Y^n))$ is the set of nodes within those edges. That is, $\delta^n(\varepsilon^e(Y^n)) = \{n_9, n_{10}, n_{11}, n_{13}, n_{14}, n_{15}\}$. This will retrieve all nodes within all edges in Y. To this result if we apply half opening again, it will retrieve the same set of nodes. Thus we can prove that half opening $\gamma_{1/2} = \delta^n(\varepsilon^e(Y^n))$ is a filter. Here only a part of Y is retrieved as shown in Figure 4(a).

5.2. Half Opening Filter with Respect to Hyperedges. If H is the parent IFHG, Y is the sub-IFHG, δ is the dilation operator, and ε is the erosion operator, then $\gamma_{1/2} = \delta^e(\varepsilon^n(Y^e))$ is a half opening filter with respect to the hyperedges in Y. Here $\varepsilon^n(Y^e)$ is the set of all nodes in Y but not in $Y^{e'}$. That is, $\varepsilon^n(Y^e) = \{n_{13}, n_{14}\}$. Now $\delta^e(\varepsilon^n(Y^e))$ is the set of all hyperedges in H which consists of such nodes. That is, $\delta^e(\varepsilon^n(Y^e)) = \{e_7, e_8\}$. Here $\{e_7, e_8\}$ is the filtrate obtained. If we repeatedly apply $\delta^e \circ \varepsilon^n$ to this filtrate, we get the same results. Thus this half opening is a filter as shown in Figure 4(b).

5.3. Half Closing Filter with Respect to Hyperedges-$\varepsilon^e(\delta^n(Y^e))$. If H is the parent IFHG, Y is the sub-IFHG, δ is the dilation operator, and ε is the erosion operator, then $\phi_{1/2} = \varepsilon^e(\delta^n(Y^e))$ is a half closing filter with respect to the hyperedges in Y. Here $\delta^n(Y^e)$ is the set of nodes within the hyperedges of Y. That is, $\delta^n(Y^e) = \{n_9, n_{10}, n_{11}, n_{13}, n_{14}, n_{15}\}$. Now $\varepsilon^e(\delta^n(Y^e))$ is the set of all edges in H which consists of the above nodes only. That is, $\varepsilon^e(\delta^n(Y^e)) = \{e_7, e_8\}$. Here only a part of Y is retrieved as seen in Figure 4(c).

5.4. Half Closing Filter with Respect to Nodes-$\varepsilon^n(\delta^e(Y^n))$. If H is the parent IFHG, Y is the sub-IFHG, δ is the dilation operator, and ε is the erosion operator, then $\phi_{1/2} = \varepsilon^n(\delta^e(Y^n))$ is a half closing filter with respect to the nodes in Y. Here $\delta^e(Y^n)$ is the set of all edges in H which has nodes in Y. That is, $\delta^e(Y^n) = \{e_1, e_4, e_5, e_6, e_7, e_8, e_9\}$. Now $\varepsilon^n(\delta^e(Y^n))$ is the nodes not in $(\delta^e(Y^n))'$. From the given example, $(\delta^e(Y^n))' = \{e_2, e_3\}$. Now $\varepsilon^n(\delta^e(Y^n)) = \{n_1, n_3, n_9, n_{10}, n_{11}, n_{12}, n_{13}, n_{14}, n_{15}, n_{16}\}$. The result is shown in Figure 4(d).

6. Metric Induced Opening and Closing Filters

We can consider γ_λ as a metric induced opening where λ is a natural number which shows the number of edges/nodes to be included in the retrieved sub-IFHG after opening operation. That is, $\gamma_\lambda = (\delta \circ \varepsilon)_\lambda$. Similarly ϕ_λ is a metric induced closing, where λ is the number of edges/nodes to be included in the result after closing. Here λ should be from 1 to number of elements in γ. So let us see different flavours of γ_λ and ϕ_λ.

6.1. Metric Induced Opening (γ_λ) with Respect to Nodes. If H is a parent IFHG, Y is a sub-IFHG, δ is the dilation operator, and ε is the erosion operator, then $\gamma_\lambda = [\delta^n(\varepsilon^e(Y^n))]_\lambda$ is a metric induced opening with respect to the nodes where top λ nodes with high membership degrees are selected. Here not all nodes in Y are retrieved. Only top priority nodes are retrieved. The results of this opening are shown in Figures 5(a)–5(f). Here λ takes a maximum value of 6, since $\delta^n(\varepsilon^e(Y^n))$ returns a maximum of only 6 nodes with respect to IFHGs in Figure 1.

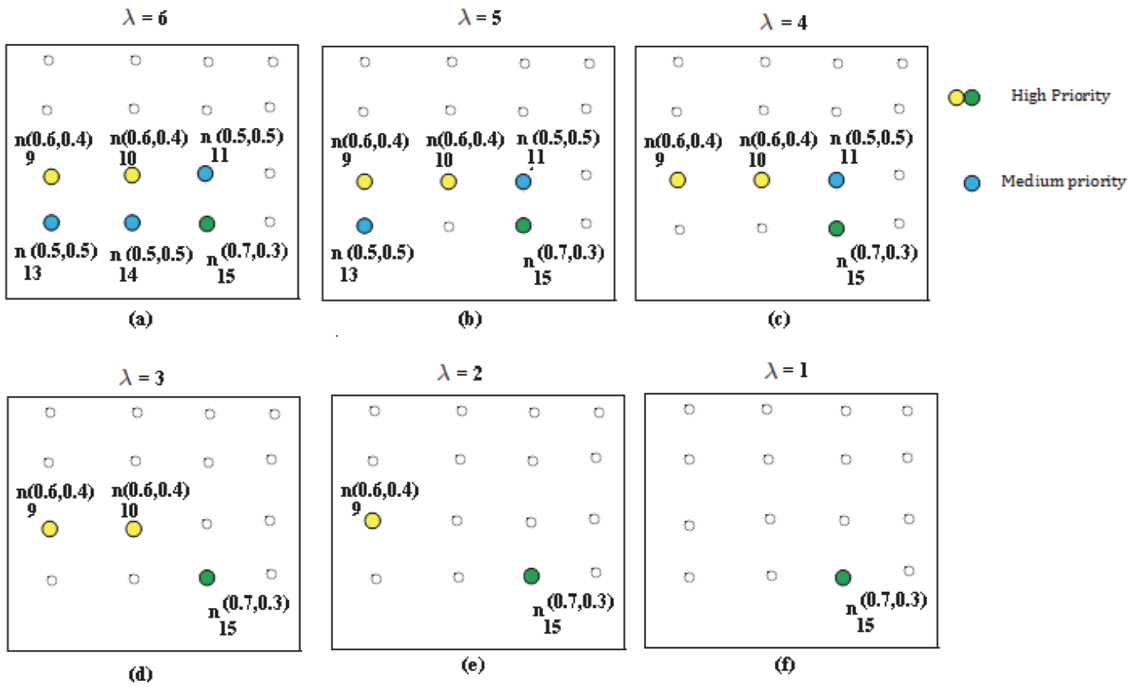


FIGURE 5: (a)-(f) Metric induced opening: $\gamma_\lambda = [\delta^n(\varepsilon^e(Y^n))]_\lambda$ for various λ.

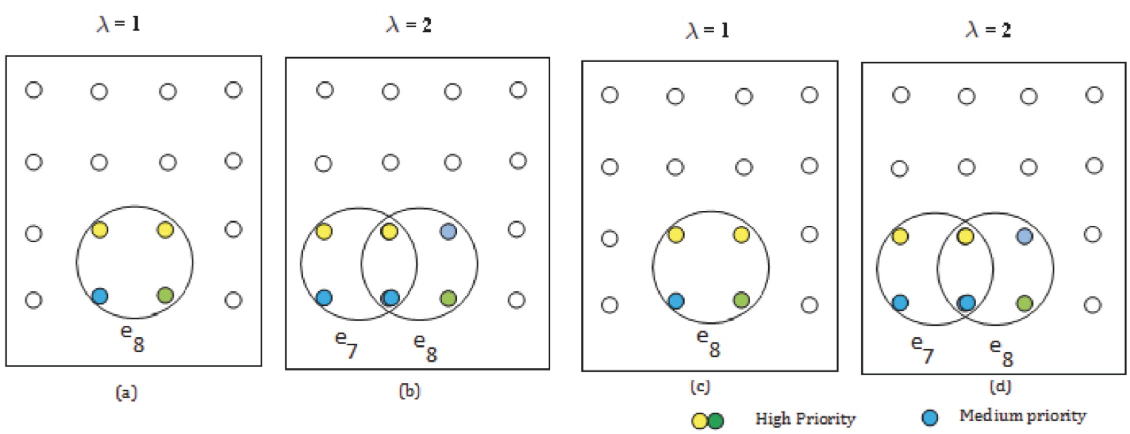


FIGURE 6: (a)-(b) Opening: $\gamma_\lambda = [\delta^e(\varepsilon^n(Y^e))]_\lambda$. (c)-(d) Closing: $\phi_\lambda = [\varepsilon^e(\delta^n(Y^e))]_\lambda$.

6.2. Metric Induced Opening(γ_λ) with Respect to Hyperedges. If H is a parent IFHG, Y is a sub-IFHG, δ is the dilation operator, and ε is the erosion operator, then $\gamma_\lambda = [\delta^e(\varepsilon^n(Y^e))]_\lambda$ is a metric induced opening with respect to the hyperedges where top λ edges with high membership degrees are selected. Here λ takes a maximum value of 2, since $\delta^e(\varepsilon^n(Y^e))$ returns only maximum of 2 edges with respect to IFHGs in Figure 1. The results of this opening are shown in Figures 6(a) and 6(b) for different values of λ.

6.3. Metric Induced Closing(ϕ_λ) with Respect to Hyperedges. If H is a parent IFHG, Y is a sub-IFHG, δ is the dilation operator, and ε is the erosion operator, then $\phi_\lambda = [\varepsilon^e(\delta^n(Y^e))]_\lambda$ is a metric induced closing with respect to the hyperedges where top λ edges with high membership degrees are selected. Here λ takes a maximum value of 2, since $\varepsilon^e(\delta^n(Y^e))$ returns only 2 edges with respect to IFHGs in Figure 1. The results of this closing operation are shown in Figures 6(c) and 6(d) for different values of λ.

6.4. Metric Induced Closing(ϕ_λ) with Respect to Nodes. If H is a parent IFHG, Y is a sub-IFHG, δ is the dilation operator, and ε is the erosion operator, then $\phi_\lambda = [\varepsilon^n(\delta^e(Y^n))]_\lambda$ is a metric induced closing with respect to nodes where top λ nodes from edges which contain Y^n and which do not belong to the complement edges are selected. Here λ takes a maximum value of 10, since $\varepsilon^n(\delta^e(Y^n))$ returns a maximum of 10 nodes with respect to IFHGs in Figure 1. The results of this closing are shown in Figures 7(a)–7(j) for various values of λ.

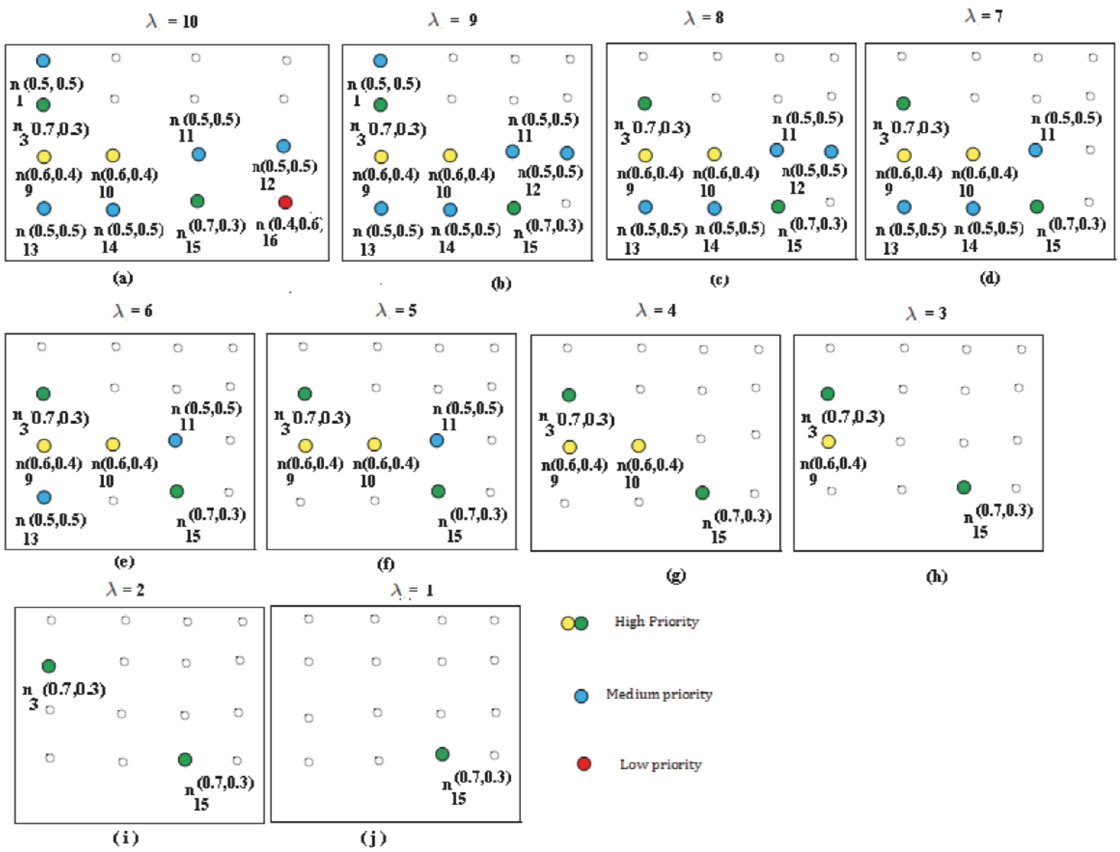


FIGURE 7: (a)-(j) Metric induced closing $\phi_\lambda = [\varepsilon^n(\delta^e(Y^n))]_\lambda$.

7. Alternate Sequential Filters

If H is a parent IFHG, Y is a sub-IFHG, γ_λ is an opening of the form $(\delta \circ \varepsilon)_\lambda$, and ϕ_λ is a closing operator of the form $(\varepsilon \circ \delta)_\lambda$, then $(\gamma_\lambda \circ \phi_\lambda)$ is also a filter. Now an alternate sequential filter can be obtained as $(\gamma_\lambda \circ \phi_\lambda) \circ (\gamma_\lambda \circ \phi_\lambda)$. The operations repeated n number of times will retrieve the same set of hyperedges/nodes for a particular value of λ, but we can have different results by varying the value of λ.

7.1. Illustration. Consider H as a parent IFHG and Y as a sub-IFHG as shown in Figures 8(a) and 8(b), respectively. Let us apply $(\gamma_\lambda \circ \phi_\lambda) \circ (\gamma_\lambda \circ \phi_\lambda)$ on these IFHGs. In Figure 8, those marked in black represent the node numbers and those in red show the edge numbers. Let $\phi_\lambda = [\varepsilon^n(\delta^e(Y^n))]_\lambda$. Since the value of λ is determined by the maximum nodes retrieved by $\varepsilon^n(\delta^e(Y^n))$, we get $\lambda = 18$ in ϕ_λ. We get $\delta^e(Y^n) = \{e_2, e_3, e_4, e_6, e_7, e_8, e_{10}, e_{11}, e_{12}\}$. Now $[\varepsilon^n(\delta^e(Y^n))]_\lambda = \phi_\lambda = \{n_3, n_4, n_5, n_7, n_8, n_9, n_{12}, n_{13}, n_{14}, n_{16}, n_{17}, n_{18}, n_{21}, n_{22}, n_{23}, n_{25}, n_{26}, n_{27}\}$. Now $(\gamma_\lambda \circ \phi_\lambda) = [\delta^n(\varepsilon^e(\phi_\lambda))]_\lambda$. We get $\varepsilon^e(\phi_\lambda) = \{e_3, e_4, e_7, e_8\}$. Now $[\delta^n(\varepsilon^e(\phi_\lambda))]_\lambda = \{n_3, n_4, n_5, n_8, n_9, n_{12}, n_{13}, n_{14}, n_{17}, n_{18}, n_{21}, n_{22}, n_{23}\}$, where $\lambda = 13$. Applying $(\gamma_\lambda \circ \phi_\lambda)$ to these nodes will again retrieve the same set of nodes for the λ_{max} value. Different results can be obtained for $1 <= \lambda <= \lambda_{max}$ for which algorithm is shown above. The results of this ASF for λ_{max} value are shown in Figure 9 (Algorithm 1).

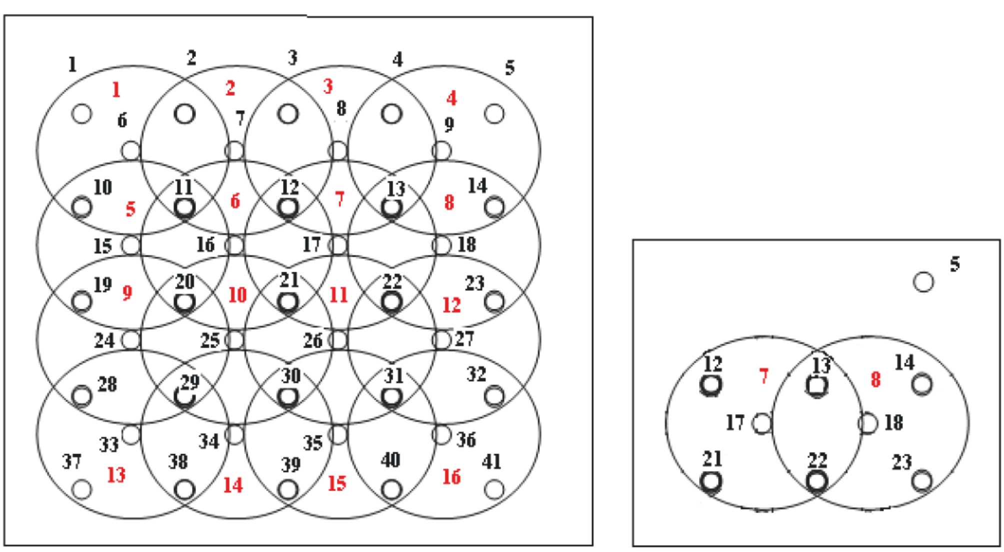
8. Applications

Modeling systems with intuitionistic fuzzy Hypergraphs find application in the field of medical report processing, where a patient can be modeled as a hyperedge and the symptoms can be modeled as nodes. When multiple patients are having the same symptom, such a node forms part of multiple edges. In Figure 10(a), symptom 5 is present in all the three patients. An IFHG constructed in this way can be subjected to many information retrieval operations. Membership and non membership values can be assigned to different nodes/symptoms based on the severity of the symptoms. Likewise membership and nonmembership values can be assigned to patients following the rules given in Section 1. IFHG modeling can be done in the area of social networking where a network group can be modeled as a hyperedge and the members/nodes of the network group can be converted to nodes. One member may be part of many network groups as shown in Figure 10(b). They can be assigned different membership value based on their life/character background.

The systems modeled in this way can be subjected to various morphological operations like dilation, erosion, adjunction, opening, closing, and filtering. An (α, β) cut can be applied on the medical report IFHGs to find sub IFHG X_1. Let us consider this sub-IFHG X_1 as the set of all patients with severe diseases and set of all severe symptoms. Let X_2 be the sub-IFHG of Figure 10(b), which consists of all blacklisted

P = number of nodes
J = number of hyperedges; Read α
for each $k = 1$ to P **do**
    Read μ_{n_k} of all nodes
    $\gamma_{n_k} = 1 - \mu_{n_k}$
end for
for each $m = 1$ to J **do**
    for each $i = 1$ to number of elements in an edge **do**
        if $\mu_{n_i} \geq 0.5$ **then**
            $\mu_{e_m} = \vee \mu_{n_i}$
            $\gamma_{e_m} = 1 - \mu_{e_m}$
        else
            $\gamma_{e_m} = \vee \gamma_{n_i}$
            $\mu_{e_m} = 1 - \gamma_{e_m}$
        end if
    end for
end for
Y = Read all edges with $\mu_{e_m} \geq \alpha$ and nodes with $\mu_{n_i} \geq \alpha$
$turn = 1$
do
    $Y_1 = \varepsilon^e(Y^n)$
    $Y_2 = \delta^n(Y_1)$
    if $turn = 1$ **then**
        λ = number of elements in Y_2
    else
        $\lambda = \lambda - 1$
    end if
    $\phi_\lambda = Y_2$
    $Y_3 = \varepsilon^e(Y_2)$
    $Y_4 = \delta^n(Y_3)$
    $\gamma_\lambda \circ \phi_\lambda = Y_4$
    $Y = Y_4$
    $turn = turn + 1$
while $\lambda > 0$

Algorithm 1: Metric induced alternate sequential filtering of IFHG.



(a) (b)

Figure 8: (a) H and (b) Y.

Table 4: Applications of IFHG.

Notation	Operation	Result
Medical Report processing using IFHG		
Notation	Operation	Result
$\delta^n(X_1^e)$	Dilation w.r.to nodes	This operation will retrieve all the symptoms of patients with severe diseases.
$\delta^e(X_1^n)$	Dilation w.r.to hyperedges	This retrieves all the patients with at least one symptom common with severely diseased patients.
$\varepsilon^n(X_1^e)$	Erosion w.r.to nodes	Retrieve all the symptoms which are seen only in severely diseased patients.
$\varepsilon^e(X_1^n)$	Erosion w.r.to hyperedges	Retrieval of all patients with severe diseases.
Social Network analysis using IFHG		
Notation	Operation	Result
$\delta^n(X_2^e)$	Dilation w.r.to nodes	This operation will retrieve all the members of black listed groups
$\delta^e(X_2^n)$	Dilation w.r.to hyperedges	This retrieves all the groups with at least one criminal member
$\varepsilon^n(X_2^e)$	Erosion w.r.to nodes	Retrieve all the members which are seen only in black listed groups.
$\varepsilon^e(X_2^n)$	Erosion w.r.to hyperedges	Retrieval of all groups which are blacklisted.

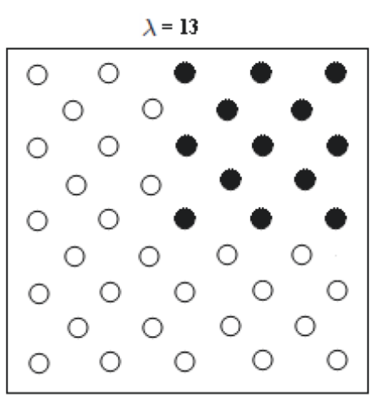


Figure 9: Node ASF $(\gamma_\lambda \circ \phi_\lambda) \circ (\gamma_\lambda \circ \phi_\lambda)$ for λ_{max}.

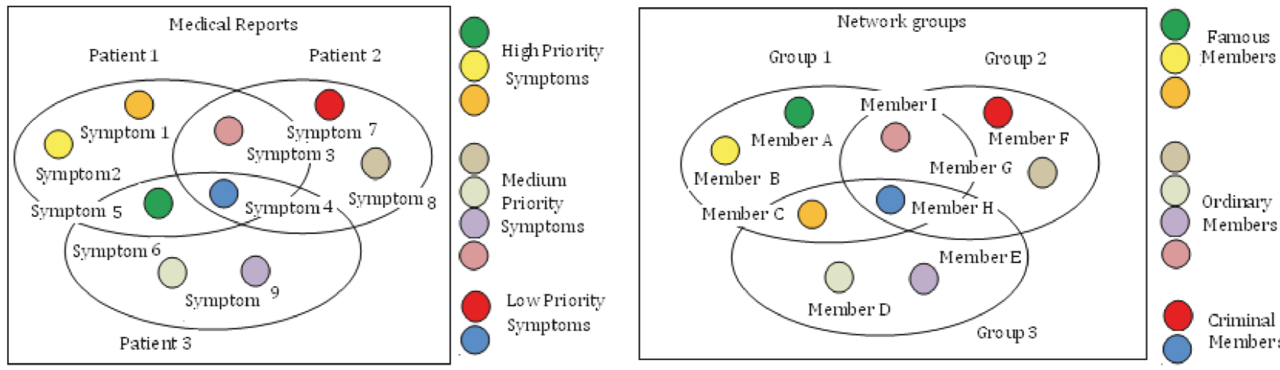
groups and low priority members. The operations applied to this X_1 and X_2 are given in Table 4. All these operations can be further expanded to opening, closing, filtering, etc. A detailed medical analysis of patients in a particular area can be done with such systems which opens a wide range of possibilities.

9. Data Availability, Results, and Discussion

The filters mentioned in this paper are tested on IFHGs consisting of maximum of 9,000 nodes. The method has shown 100% accurate results. The dataset can be produced on demand. The ASF algorithm designed on IFHG has a complexity of $O(n^2)$, since we are searching through the hyperedges and nodes within those hyperedges. The parameters α and β are working as filter parameters, since a high value of these parameters results in less amount of filtrate and low value of these parameters results in large amount of filtrate. With respect to text processing application using medical reports, patients with "minor," "moderate," "major," and "extreme" medical conditions are retrieved, when we vary the (α, β) cut. The algorithm to find the optimal number of nodes/hyperedges in ASF iterates till the following condition is satisfied:

$$\left|\gamma_\lambda \circ \phi_\lambda\right| \geq \epsilon \tag{11}$$

where γ_λ is an opening filter, ϕ_λ is a closing filter, $\gamma_\lambda \circ \phi_\lambda$ is an ASF, and $|\gamma_\lambda \circ \phi_\lambda|$ is the cardinality of the filter. In (11), ϵ is a positive number. The algorithm converges when $\epsilon \longrightarrow 0$. Also the algorithm may exit without an output if no sub-IFHG is obtained after (α, β) cut. That is, in terms of medical report analysis we can say that if we have set (α, β) cut such as to retrieve patients with "extreme" medical condition and if the area considered for analysis is not having such patients, then the algorithm exits without generating an output. In such a case we have to reduce the level of (α, β) cut such as to retrieve all patients in that area with "major" medical conditions. Likewise when we set the level of (α, β) cut such as to retrieve patients in the area with "minor" medical conditions and



(a)

(b)

Figure 10: (a) Medical report processing. (b) Social network analysis.

get an empty sub-IFHG, this implies that the area under consideration is the one with "good" medical conditions.

10. Conclusion

Here we have successfully defined the morphological operations like adjunction, opening, closing, half opening, half closing, and alternate sequential filter on intuitionistic fuzzy hypergraph. The results are substantiated with sample parent IFHG and sub-IFHG. Such filter designs find application in image processing, text processing, computer networks, etc. The (α, β) cut used to generate the subhypergraphs can be varied with different values of (α, β). Different subhypergraphs with varying (α, β) cuts when applied with the above morphological operators will produce results accordingly with various priority ranges of hyperdges/nodes. One who is working with text/image processing and network analysis can find numerous applications with these operations. A filter designed on text results in text summary. Such applications are left as future enhancements of this paper.

Data Availability

The filters mentioned in this article are tested on IFHGs consisting of maximum 9,000 nodes. The method has shown 100% accurate results. The dataset can be produced on demand.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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