Reflection of Plane Waves in Nonlocal Fractional-Order Thermoelastic Half Space

Surbhi Sharma and Sangeeta Kumari
Chandigarh University, Gharuan, Mohali, Punjab, India

Correspondence should be addressed to Sangeeta Kumari; sangwan.sangeeta.ss@gmail.com

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The problem of plane waves in nonlocal fractional-order thermoelasticity has been studied. We have considered the x-y plane for the governing equation of nonlocal fractional thermoelasticity and solved these governing equations to calculate the equation in terms of frequency. This frequency shows that three sets of waves exist, in which two are coupled and one is uncoupled. The reflection coefficient of plane waves for classical theory and LS theory has been calculated. The effect of phase speeds, specific losses, and attenuation coefficients with respect to the frequency and nonlocal parameter for the two theories (LS theory and the classical theory of thermoelasticity) has been studied numerically for all propagating waves, and the same has been plotted graphically and explained thoroughly.

1. Introduction

The mechanics of deformable bodies that restore their original shape once the forces that caused the deformation are removed is known as elasticity theory. The earliest significant attempts to create a theory of elasticity using the continuum method, in which speculations about the molecular structure of the body are avoided and macroscopic events are represented in terms of field variables, originate from the first part of the eighteenth century. Since then, a huge amount of research has gone into understanding the theory of elasticity and its applications in the areas of engineering and physics. Elastic characteristics are one of the most essential mineral characterizations for detecting the earth’s physical and chemical condition and also defining interatomic forces. Acoustic velocities in single crystal samples provide the most complete and precise collection of elasticity data. Ultrasonic methods such as MC Skimin’s [1961] pulse superposition method and Papadakis’s [1967] pulse echo overlap method can produce acoustic velocities with fractions of percent uncertainty. However, due to the technique’s sample size limitations and the additional challenge of poor crystal symmetries, the number of rock-forming minerals that have been described using this method is very minimal. Cutting and polishing are required to create surfaces that are correctly aligned with respect to crystallographic axes. In reference to the acoustic wavelength, the sample should be huge (typically in the range 30–300 mm). Moreover, the sample should be large enough for the transit time to be accurately measured and for individual echoes to be distinguished in time without causing difficulties due to nanosecond pulse resolutions. After parallel sides have been constructed, all of these criteria normally determine a minimum sample length of around 2 mm. Eringen[1] discussed the dispersion of plane waves and the nonlocal linear theory of elasticity. Mohamed and Song [2] studied the reflection of plane waves under hydrostatic initial stress from the elastic solid half space without energy loss.

Biot [3] proposed the theory of coupled thermoelasticity, which eliminates the contradiction of uncoupled theory, which states that elastic changes have no effect on temperature. In both theories, heat equations are of the diffusion type, which predicts that heat waves propagate at infinite speeds that contradict physical observations. During the past few years, thermoelasticity theories have been developed,


Nonlocal theory states that the stress of a continuum body depends on the strain at that particular point and its neighbourhood. When dealing with wave and vibration problems, the behavior of material is dependent on the internal characteristic length such as atomic size and the exterior characteristic length such as wavelength. When exterior and internal characteristic lengths are compared, the theory of nonlocal elasticity becomes useful. These characteristic lengths are comparable in the theory of micropolar materials; hence, the micropolar elastic model is suitable for the theory of nonlocal elasticity. Eringen [26, 27] discussed the continuum theory of nonlocal fluid dynamics and nonlocal polar bodies. Birman [28] studied the current developments in the area of nonlocal optics, which indicate the presence of four kinds of optical nonlocality phenomena. Wang et al. [29] discussed that deterministic rough surfaces can exhibit spatial dispersion in the presence of complete optical responses. Adolph et al. [30] discussed the optical properties of semiconductors in terms of nonlocality and many body effects. Frank and Gerhardts [31] discussed the applications of nonlocal metal optics. Singh et al. [32] examined the nonlocal elastic solid material with voids for the propagation of harmonic plane waves. Lata [33] discussed that in a layered nonlocal anisotropic and elastic-thermoelastic medium, plane waves reflect and refract. Sarkar and Tomar [34] discussed that in a nonlocal thermoelastic medium with void pores, a harmonic plane wave propagates. Das et al. [35] studied the reflection of harmonic plane waves in a nonlocal thermoelastic solid medium with stress-free-insulated and isothermal boundary conditions. Das et al. [36] studied propagation of plane waves with nonlocal effects based on G-N type-III. Patnaik and Semperlotti [37] discussed the propagation of elastic waves in nonlocal-attenuating materials using generalized elastodynamic theory based on fractional-order operators. Kaur and Singh [38] studied the three-phase lag fractional-order heat transfer and the Hall effect in a nonlocal semiconducting rotating medium in plane wave. Das et al. [39] discussed the propagation of plane waves in generalized thermoelasticity with nonlocal effects. Sarkar et al. [40] investigated the reflection of thermoelastic plane waves from homogeneous, isotropic, and thermally conducting elastic half space. Sheoran et al. [41] investigated the transmission and reflection of plane waves in a nonlocal thermoelastic and nonlocal micropolar thermoelastic solid half space with rotation. Using the dual-phase lag model, Kumar et al. [42] examined the reflection of plane harmonic waves in a nonlocal micropolar thermoelastic material with voids. In a rotating thermoelastic medium with temperature-dependent properties, Sheoran et al. [43] investigated nonlocal, homogeneous, isotropic deformations in two dimensions. With temperature-dependent properties, Deswal et al. [44] discussed the plane wave propagation in nonlocal, microstretch thermoelastic half space. With the effect of rotation, Kumar Kalkal et al. [45] studied the reflection of plane waves in nonlocal micropolar thermoelastic media.

2. Formulation of the Problem

We consider a constitutive relation and field equation for nonlocal fractional thermoelasticity. We consider a thermoelastic body occupying the region $A$ in $R^n$ at time $t$, and with the volume $V$ and the surface $S$. Let the position of a point of $A$ in the unbounded state be represented by $X_i$, and in the deformed state by $x_i$. The displacement $u_i$ is represented by $u_i = x_i - X_i$. 

(1) Let the strain tensor be denoted by $e_{ij}$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}).$$  \hspace{1cm} (1)

(2) The consecutive relation is as follows:

(a) The stress relation in terms of a nonlocal operator is

$$-(1 - \alpha^2 \nabla^2)\rho_0 T \phi = \eta_{ij},$$ \hspace{1cm} (3)

where $\eta_{ij} = -(\rho_0 C_e T + (\alpha T_0/K_T)e^\alpha)$ is the equation of motion in the absence of body forces and $C_e$ is the specific heat.

In the absence of body forces, the equation of motion for nonlocal isotropic thermoelastic solid can be written as

$$\sigma_{ij} = \rho_0 (\ddot{u}_i),$$ \hspace{1cm} (4)

where $\rho_0$ is the density of the material.

The modified Fourier law is $(1 - \alpha^2 \nabla^2)(q + \tau_0 \dot{q}) = KVT$. The nonlocal heat conduction in thermoelastic material is

$$-(1 - \alpha^2 \nabla^2)(1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha})q = KVT,$$ \hspace{1cm} (5)

where $K$ is the thermal conductivity, $\tau_0$ is the relaxation time, and $\alpha$ is the fractional-order parameter such that

$$\frac{\partial^\alpha}{\partial t^\alpha} f(x,t) = \begin{cases} f(x,t) - f(x,0), & \alpha \to 0, \\ \frac{\Gamma(1-\alpha)}{\Gamma(\alpha)} \int_0^t (t - g)^{-\alpha} f(x, g) dg, & 0 < \alpha < 1, \\ \frac{\partial f(x,t)}{\partial t}, & \alpha \to 1. \end{cases}$$ \hspace{1cm} (6)

With

$$l^\alpha f(x,t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t - g)^{-\alpha-1} f(x, g) dg.$$ \hspace{1cm} (7)

Here, $\Gamma$ is the gamma function and is constant such that $0 \leq \alpha \leq 1$.

When $\alpha \to 0$, (5) reduces to the theory of classical coupled thermoelasticity, and when $\alpha \to 1$, (5) reduces to the Lord and Shulman theory of thermoelasticity.

Substituting equations (1)–(3) into equations (4) and (5), we get the equation of motion as

$$\bar{\omega}^2 u + (\mu + \lambda)\nabla(\nabla \cdot u) - \frac{\alpha}{K_T} \nabla T = \rho_0(1 - \alpha^2 \nabla^2)\ddot{u},$$ \hspace{1cm} (8)

$$\left(1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\rho_0 C_e \dot{T} + \frac{\alpha T_0}{K_T}\nabla \cdot u\right) = K\nabla^2 T.$$ \hspace{1cm} (9)

2.1. Remarks

Case 1: The L-S theory of classical coupled thermoelasticity is as follows:

If $\epsilon = 0$ in equations (1)–(4) and (8) and (9), we get

$$\sigma_{ij} = 2\mu e_{ij}(x) + \left[\lambda e_{kk}(x) - \frac{\alpha}{K_T} T(x)\right] \delta_{ij},$$

$$\rho_0 \eta = \frac{\alpha}{K_T} e_{kk}(x) \delta_{ij},$$

$$\left(1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha}\right)u = K\nabla T.$$ \hspace{1cm} (10)

And

$$\mu\nabla^2 u + (\mu + \lambda)\nabla(\nabla \cdot u) - \frac{\alpha}{K_T} \nabla T = \rho_0 \ddot{u},$$

$$\left(1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\rho_0 C_e \dot{T} + \frac{\alpha T_0}{K_T}\nabla \cdot u\right) = K\nabla^2 T.$$ \hspace{1cm} (11)

Case 2: Classical thermoelasticity is as follows:

If $\epsilon = 0$ and $\alpha/K_T = 0$ in equations (1)–(4) and (8) and (9), we obtain

$$\sigma_{ij} = 2\mu e_{ij}(x) + \lambda e_{kk}(x) \delta_{ij},$$

$$\rho_0 \eta = \rho_0 C_e T(x),$$

$$\left(1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha}\right)u = K\nabla T,$$

$$\mu\nabla^2 u + (\mu + \lambda)\nabla(\nabla \cdot u) = \rho_0 \ddot{u},$$

$$\left(1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha}\right)\rho_0 C_e \dot{T} = K\nabla^2 T.$$ \hspace{1cm} (13)

3. Wave Propagation

We consider the homogeneous thermoelastic medium rotating about the y-axis. The vector and scalar potential $\psi$ and $\phi$ through the Helmholtz vector theorem can be represented as

$$\nabla \cdot \psi = 0, \quad \psi = \nabla \phi + \nabla \times \psi.$$ \hspace{1cm} (14)
By putting these potentials in (8) and (9), the absence of heat source density, and body forces, we get the following equations as

\[ \nabla^2_T \nabla^2 \phi - \frac{\alpha}{K_T} T = (1 - e^2 \nabla^2) \nabla^2 \phi, \]  
\[ \nabla^2_S \nabla^2 \psi = (1 - e^2 \nabla^2) \psi, \]  
\[ \left( 1 + \tau_0 \frac{\partial^2}{\partial t^2} \right) \left( C_e \dot{T} + \frac{\alpha}{K_T} \nabla^2 \phi \right) = \frac{K}{\rho_0} \nabla^2 T. \]  

Here, (15) and (17) are coupled in the form of \( \phi \) and \( T \), whereas (16) is uncoupled. So, to get the solution of these equations, we take

\[ (\phi, T, \psi) = (A_1, B_1, C_1) \exp \left(ik(x \sin \theta + y \cos \theta - vt)\right), \]  
where \( A_1, B_1, \) and \( C_1 \) are the constant amplitudes which can be complex numbers and \( k \) is the wave number and the vector constant, where \( r = (x + iy + z\bar{k}) \) is the position vector. By putting equations (18) into equations (15) and (17), we get

\[ -\omega^2 \tau_0^* \beta T_0 A_1 + \left( \frac{K}{\rho_0} + C_e v^2 \tau_0^* \right) B_1 = 0, \]  
\[ -\omega^2 \tau_0^* \beta T_0 A_1 + \left( \frac{K}{\rho_0} + C_e v^2 \tau_0^* \right) B_1 = 0, \]  
where the following variables are used in the equations,

\[ \nabla^2_T = \frac{2\pi + \bar{T}}{\rho_0}, \beta = \frac{\alpha}{\rho_0 K_T}, \nabla^2_S = \frac{\bar{\theta}}{\rho_0}, \tau_0^* = \left( \frac{t + \bar{T}}{\bar{\theta}} \right), \tau_0 = \tau_0 (\bar{\theta} \alpha^{\omega^2 - 1}). \]  

This system of homogeneous linear (19) and (20) has a nonvanishing solution for unknowns \( A_1 \) and \( B_1 \) when the determinant of their coefficient matrix vanishes,

\[ P(v^2) + Q(v^2) + R = 0, \]  

where

\[ P = C_e \tau_0^*, \]
\[ Q = \frac{K}{\rho_0} \left( 1 + e^2 k^2 \right) + e^2 \omega^2 C_e \tau_0^* - C_e v^2 \nabla^2_S - \beta^2 \tau_0^* T_0, \]
\[ R = -\frac{K}{\rho_0} \nabla^2_T. \]  

(22) is the dispersion relation for the propagation of plane waves in a nonlocal thermoelastic solid medium that gives the speeds of various wave propagation.

The roots of equation (22) are

The phase velocity (\( V_p \)), specific loss (\( S_i \)), and attenuation coefficient (\( Q_i \)) are represented in the following form [34]:

\[ V_p = \frac{(\Re(v_i))^2 + (\Im(v_i))^2}{\Re(v_i)}, \]  
\[ S_i = \left( \frac{\Delta W_i}{W_i} \right) = 4\pi \left| \frac{\Im(v_i)}{\Re(v_i)} \right|, \]  
\[ Q_i = \frac{-\omega \Im(v_i)}{(\Re(v_i))^2 + (\Im(v_i))^2}, \]  
where \( \Re(v_i), \Im(v_i) \) are the real and imaginary part of \( v_i \) where \( i = 1, 2, 3 \), respectively. To find the value of \( v_3 \), putting equation (18) into equation (16), we get

\[ v_3 = \sqrt{\frac{V_p^2 - \omega^2}{\rho_0^2}}. \]  

(26) is the plane-wave propagation for the nonlocal thermoelastic medium that gives the speed of propagation for different waves, and for a given real value of \( \omega \) lying within the range, we get

\[ 0 < \omega < \omega_c, \omega_c = \frac{\nabla^2_S}{\rho_0}. \]  

From the expression, it has been noted that the speed of \( v_3 \) is that of an uncoupled wave that does not depend on thermal parameters. It travels slower than classical local elastic solid. The existence of \( \epsilon \) (nonlocal parameter) in the
thermoelastic material results in the reduction of the phase speed of the uncoupled wave. As can be seen in (26), the phase speed of the uncoupled wave vanishes when \( \omega = \omega_c \). This implies that for \( \omega < \omega_c \), the speed of the phase velocity \( v_j \) is real and that for \( \omega > \omega_c \), it is complex. Thus, we can say that the uncoupled wave is a propagating wave in the frequency range: \( 0 < \omega < \omega_c \).

Based on the formula in (25), we can get the attenuation coefficient as well as the specific losses of the existing uncoupled wave as

\[
\varphi = A_2 \exp \left[ ik_y (x \sin \theta + y \cos \theta) - i \omega t \right] + A_1 \exp \left[ ik_y (x \sin \theta + y \cos \theta) + i \omega t \right] + A_3 \exp \left[ ik_y (x \sin \theta + y \cos \theta) + i \omega t \right],
\]

\[
T = \xi_2 A_2 \exp \left[ ik_y (x \sin \theta + y \cos \theta) - i \omega t \right] + \xi_1 A_1 \exp \left[ ik_y (x \sin \theta + y \cos \theta) - i \omega t \right] + \xi_3 A_3 \exp \left[ ik_y (x \sin \theta + y \cos \theta) - i \omega t \right],
\]

\[
\psi = B_1 \exp ik_y (x \sin \theta - y \cos \theta - i \omega t),
\]

where \( \xi_i = \left( k_i^2 c_i^2 - k_i^2 v_i^2 \right) / \beta \) for \( i = 1, 2 \).

4.1. Boundary Condition. We now describe the following boundary conditions that must be satisfied for the proposed problem. Since the boundary surface at \( y = 0 \) is stress free, we have,

\[
\sigma_{yx} = 0, \sigma_{yy} = 0, \frac{\partial T}{\partial y} = 0 \text{ at } y = 0.
\] (32)

Taking equations (29)–(31) and making use of equation (2) in the boundary conditions, we get

\[
\frac{A_1}{A_0} c_{11} + \frac{A_2}{A_0} c_{12} + \frac{B_1}{A_0} c_{13} = d_1,
\]

\[
\frac{A_1}{A_0} c_{21} + \frac{A_2}{A_0} c_{22} + \frac{B_1}{A_0} c_{23} = d_2,
\]

\[
\frac{A_1}{A_0} c_{31} + \frac{A_2}{A_0} c_{32} + \frac{B_1}{A_0} c_{33} = d_3,
\] (33)

where

\[
A_1 = \left[ d_1 (c_{32} c_{32} - c_{22} c_{33}) + d_2 (c_{12} c_{32} - c_{13} c_{32}) + d_3 (c_{13} c_{22} - c_{12} c_{23}) \right],
\]

\[
A_2 = \left[ d_1 (c_{23} c_{31} - c_{21} c_{33}) + d_2 (c_{11} c_{33} - c_{13} c_{33}) - d_3 (c_{13} c_{21} - c_{11} c_{23}) \right],
\]

\[
B_1 = d_1 (c_{33} c_{31} - c_{21} c_{33}) + d_2 (c_{11} c_{33} - c_{13} c_{33}) + d_3 (c_{13} c_{21} - c_{11} c_{23}),
\]

\[
A_0 = c_{11} (c_{23} c_{32} - c_{22} c_{33}) + c_{12} (c_{23} c_{32} - c_{22} c_{33}) + c_{13} (c_{23} c_{32} - c_{22} c_{33}).
\] (35)

5. Numerical Results and Discussion

The values of the parameters mentioned in Table 1 have been used to find the numeric results taken from [2].

Figure 2 represents the variation of the phase velocity \( V_1 \), \( V_2 \), and \( V_3 \) with respect to the frequency for two different theories when \( \alpha \to 0 \) and \( \alpha \to 1 \). In Figure 2(a), it can be seen that the phase velocity first decreases sharply and then decreases slowly with the increase in the frequency for both theories. Figure 2(b) shows that the phase velocity increases at first and then decreases sharply for \( \alpha \to 0 \) and that the phase velocity slightly decreases and then increases for
Table 1: Values of parameters.

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<th>Symbols</th>
<th>Value</th>
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<tr>
<td>$\Omega$</td>
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<tr>
<td>$E$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(a) Phase Velocity $v_1$ versus Frequency

(b) Phase Velocity $v_2$ versus Frequency

(c) Phase Velocity $v_3$ versus Frequency

Figure 2: Phase velocity w.r.t. frequency $\omega$. 
In Figure 2(c), it can be seen that the phase velocity decreases slowly with the increase in the frequency for both theories.

Figure 3 represents the variation of the specific loss $S_1$ and $S_2$ with respect to the frequency for two different theories when $\alpha \to 0$ and $\alpha \to 1$. Figure 3(a) represents that when the frequency increases, the specific loss remains constant for $\alpha \to 0$. The specific loss decreases with the increase in the frequency for $\alpha \to 1$. Figure 3(b) shows that the specific loss increases and then slightly decreases for theory when $\alpha \to 0$ and for $\alpha \to 1$, the specific loss slowly increases with the increase in the frequency.

Figure 4 represents the variation of the attenuation coefficient $Q_1$ and $Q_2$ with respect to the frequency for two different theories when $\alpha \to 0$ and $\alpha \to 1$. Figure 4(a) shows that the attenuation coefficient slightly increases with the increase in the frequency for $\alpha \to 0$. For $\alpha \to 1$, the attenuation coefficient sharply increases with the increase in the frequency. The attenuation coefficient slowly increases with the increase in the frequency for both theories. Figure 4(b) shows that the attenuation coefficient sharply increases with the increase in the frequency in both theories for $\alpha \to 0$ and $\alpha \to 1$.

Figure 5 represents the variation of the phase velocity $V_1$, $V_2$, and $V_3$ with respect to the nonlocal parameter $\varepsilon$ for two different theories when $\alpha \to 0$ and $\alpha \to 1$. In Figure 5(a), the phase velocity remains constant when the nonlocal parameter increases for both $\alpha \to 0$ and $\alpha \to 1$. Figure 5(b) represents that the phase velocity decreases for both theories with the increase in the nonlocal parameter. Figure 5(c) shows no effect.
of $\alpha$ on both theories. The phase velocity slowly decreases with the increase in the nonlocal parameter.

Figure 6 represents the variation of the specific loss $S_1$ and $S_2$ with respect to the nonlocal parameter $e$ for two different theories when $\alpha \rightarrow 0$ and $\alpha \rightarrow 1$. In Figure 6(a), the specific loss remains constant for both theories when the nonlocal parameter increases. Figure 6(b) represents that the specific loss decreases sharply with the increase in the nonlocal parameter for the theory $\alpha \rightarrow 0$. For $\alpha \rightarrow 1$, with the increase in the nonlocal parameter, the phase velocity slowly decreases.

Figure 7 represents the attenuation coefficient $Q_1$ and $Q_2$ with respect to the nonlocal parameter $e$ for two different theories when $\alpha \rightarrow 0$ and $\alpha \rightarrow 1$. Figure 7(a) shows that for both theories, the attenuation coefficient remains constant with the increase in the nonlocal parameter. Figure 7(b) shows that there is no effect of $\alpha$ on both theories seen in this case. The attenuation coefficient slowly increases with the increase in the nonlocal parameter.

5.1. Special Cases

Case 1. If the nonlocal effect is neglected from the medium, then we get a thermoelastic medium as $e = 0$ in equation (15),

$$A\left(v^2\right)^2 + B\left(v^2\right) + C = 0,$$

where
\begin{align}
A &= \tau_0 C_e, \\
B &= -\tau_0 C_e \nabla_T^2 - \tau_0 \beta^2 T_0 + \frac{K}{\rho_0}, \\
C &= -\nabla_T^2 \frac{K}{\rho_0}.
\end{align}

(36) gives the speed of propagation of coupled waves in the thermoelastic medium. Similarly, when we use \( e = 0 \) in (26), the speed of transverse waves in the thermoelastic medium becomes the speed of a classical wave.

\section{Conclusion}

The propagation of plane waves in nonlocal fractional-order thermoelasticity has been studied. The constitutive relation for the propagation of plane waves in nonlocal fractional thermoelastic solid media is considered and solved. The specific loss, phase speed, and attenuation coefficient have been obtained for three waves. The effects of the specific loss, attenuation coefficient, and phase velocity on the frequency and nonlocal parameter for the two theories (classical theory and L-S theory) are shown graphically.

The following observations can be seen in the graphs:
(i) On applying the two theories $\alpha = 0$ and $\alpha = 1$ to the phase speeds, specific losses, and attenuation coefficients against frequency $\omega$, it has been found that both theories have more effects on the phase velocities $V_1$ and $V_2$, specific losses $S_1$ and $S_2$, and attenuation coefficient $Q_1$.

(ii) On applying the two theories $\alpha = 0$ and $\alpha = 1$ to the phase speeds, specific losses, and attenuation coefficients against the nonlocal parameter $c$, it has been found that these theories have more significant effects on the phase velocities $V_1$ and $V_2$, specific losses $S_1$ and $S_2$, and attenuation coefficient $Q_1$.

Data Availability

The data used have been cited in [46].

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References


