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Research Article

Reflection of Plane Waves in Nonlocal Fractional-Order Thermoelastic Half Space

Surbhi Sharma and Sangeeta Kumari

Chandigarh University, Gharuan, Mohali, Punjab, India

Correspondence should be addressed to Sangeeta Kumari; sangwan.sangeeta.ss@gmail.com

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The problem of plane waves in nonlocal fractional-order thermoelasticity has been studied. We have considered the *x-y* plane for the governing equation of nonlocal fractional thermoelasticity and solved these governing equations to calculate the equation in terms of frequency. This frequency shows that three sets of waves exist, in which two are coupled and one is uncoupled. The reflection coefficient of plane waves for classical theory and LS theory has been calculated. The effect of phase speeds, specific losses, and attenuation coefficients with respect to the frequency and nonlocal parameter for the two theories (LS theory and the classical theory of thermoelasticity) has been studied numerically for all propagating waves, and the same has been plotted graphically and explained thoroughly.

1. Introduction

The mechanics of deformable bodies that restore their original shape once the forces that caused the deformation are removed is known as elasticity theory. The earliest significant attempts to create a theory of elasticity using the continuum method, in which speculations about the molecular structure of the body are avoided and macroscopic events are represented in terms of field variables, originate from the first part of the eighteenth century. Since then, a huge amount of research has gone into understanding the theory of elasticity and its applications in the areas of engineering and physics. Elastic characteristics are one of the most essential mineral characterizations for detecting the earth's physical and chemical condition and also defining interatomic forces. Acoustic velocities in single crystal samples provide the most complete and precise collection of elasticity data. Ultrasonic methods such as MC Skimin's [1961] pulse superposition method and Papadakis's [1967] pulse echo overlap method can produce acoustic velocities with fractions of percent uncertainty. However, due to the technique's sample size limitations and the additional challenge of poor crystal symmetries, the number of rock-forming minerals that have been described using this method is very minimal. Cutting and polishing are required to create surfaces that are correctly aligned with respect to crystallographic axes. In reference to the acoustic wavelength, the sample should be huge (typically in the range 30–300 mm). Moreover, the sample should be large enough for the transit time to be accurately measured and for individual echoes to be distinguished in time without causing difficulties due to nanosecond pulse resolutions. After parallel sides have been constructed, all of these criteria normally determine a minimum sample length of around 2 mm. Eringen[1] discussed the dispersion of plane waves and the nonlocal linear theory of elasticity. Mohamed and Song [2] studied the reflection of plane waves under hydrostatic initial stress from the elastic solid half space without energy loss.

Biot [3] proposed the theory of coupled thermoelasticity, which eliminates the contradiction of uncoupled theory, which states that elastic changes have no effect on temperature. In both theories, heat equations are of the diffusion type, which predicts that heat waves propagate at infinite speeds that contradict physical observations. During the past few years, thermoelasticity theories have been developed, which state that thermal signals propagate at a finite speed. These nonclassical theories, in contrast to traditional thermoelasticity theory, use a hyperbolic-type heat transport equation that shows the presence of wave-type heat transfer. The theory of thermoelasticity has been addressed by several authors in recent years. Hitnarski and Ignaczak [4] discussed generalized thermoelasticity, which shows the existence of different models such as the L-S model, G-L model, H-I model, G-N model, and C-T model. Lord and Shulman [5] discussed the generalized theory of dynamical thermoelasticity based on the equation of heat transfer. Dhaliwal and Sherief [6] derived a generalized equation of thermoelasticity for an isotropic material. Eringen [7] developed constitutive equations for nonlocal thermoelastic solids. Roy Choudhuri [8] discussed generalized thermoelasticity with relaxation times and rotation on plane waves. Roychoudhuri and Mukhopadhyay [9] examined generalized thermoviscoelasticity with rotation and relaxation times on plane waves. Mohamed [10] investigated two relaxation times for general thermoelasticity with a rotational effect on plane waves. Sherief and Anwar [11] studied the effect of temperature and thermal stresses on generalized thermoelasticity. Casas and Quintanilla [12] proved the uniqueness theorem for different thermoelastic theories. Paria [13] studied plane-wave propagation thermoelasticity in the presence of magnetic fields.

The fractional order has been used by several authors to describe the viscoelastic material property. Ahmed El-Sayed [14] studied the fractional-order diffusion wave equation. Ahmed and El-Sayed [15] discussed the fractional-order linear differential equation. Momani and Odibat [16] investigated the numerical solution of fractional-order differential equations. Zhang [17] discussed a solution to the fractional-order boundary value problem. Odibat and Momani [18] applied different numeric methods to the nonlinear partial differential equation of fractional order. Arara et al. [19] studied the boundary value problem on unbounded domains for fractional-order differential equations. Wang [20] studied a numerical method with constant and time-varying delay for nonlinear fractional-order differential equations. Demirci and Ozalp [21] studied a technique to solve the differential equation of fractional order. Shah et al. [22] discussed the application and the survey of single and multiple component fractional-order elements. Sherief et al. [23] discussed fractional-order thermoelasticity theory. Youssef [24] derived a new theory of thermoelasticity based on the Duhamel–Neumann fractional-order stress-strain relation. Lata and Kaur [25] studied the propagation of plane waves with fractional-order-generalized heat transfer in an isotropic magneto-thermoelastic rotating medium.

Nonlocal theory states that the stress of a continuum body depends on the strain at that particular point and its neighbourhood. When dealing with wave and vibration problems, the behavior of material is dependent on the internal characteristic length such as atomic size and the exterior characteristic length such as wavelength. When exterior and internal characteristic lengths are compared, the theory of nonlocal elasticity becomes useful. These characteristic lengths are comparable in the theory of micropolar materials; hence, the micropolar elastic model is suitable for the theory of nonlocal elasticity. Eringen [26, 27] discussed the continuum theory of nonlocal fluid dynamics and nonlocal polar bodies. Birman [28] studied the current developments in the area of nonlocal optics, which indicate the presence of four kinds of optical nonlocality phenomena. Wang et al. [29] discussed that deterministic rough surfaces can exhibit spatial dispersion in the presence of complete optical responses. Adolph et al. [30] discussed the optical properties of semiconductors in terms of nonlocality and many body effects. Frank and Gerhardts [31] discussed the applications of nonlocal metal optics. Singh et al. [32] examined the nonlocal elastic solid material with voids for the propagation of harmonic plane waves. Lata [33] discussed that in a layered nonlocal anisotropic and elastic-thermoelastic medium, plane waves reflect and refract. Sarkar and Tomar [34] discussed that in a nonlocal thermoelastic medium with void pores, a harmonic plane wave propagates. Das et al. [35] studied the reflection of harmonic plane waves in a nonlocal thermoelastic solid medium with stress-free-insulated and isothermal boundary conditions. Das et al. [36] studied propagation of plane waves with nonlocal effects based on G-N type-III. Patnaik and Semperlotti [37] discussed the propagation of elastic waves in nonlocal-attenuating materials using generalized elastodynamic theory based on fractional-order operators. Kaur and Singh [38] studied the three-phase lag fractional-order heat transfer and the Hall effect in a nonlocal semiconducting rotating medium in plane wave. Das et al. [39] discussed the propagation of plane waves in generalized thermoelasticity with nonlocal effects. Sarkar et al. [40] investigated the reflection of thermoelastic plane waves from homogeneous, isotropic, and thermally conducting elastic half space. Sheoran et al. [41] investigated the transmission and reflection of plane waves in a nonlocal thermoelastic and nonlocal micropolar thermoelastic solid half space with rotation. Using the dual-phase lag model, Kumar et al. [42] examined the reflection of plane harmonic waves in a nonlocal micropolar thermoelastic material with voids. In a rotating thermoelastic medium with temperature-dependent properties, Sheoran et al. [43] investigated nonlocal, homogeneous, isotropic deformations in two dimensions. With temperature-dependent properties, Deswal et al. [44] discussed the plane wave propagation in nonlocal, microstretch thermoelastic half space. With the effect of rotation, Kumar Kalkal et al. [45] studied the reflection of plane waves in nonlocal micropolar thermoelastic media.

2. Formulation of the Problem

We consider a constitutive relation and field equation for nonlocal fractional thermoelasticity. We consider a thermoelastic body occupying the region A in R^3 at time t, and with the volume V and the surface S. Let the position of a point of A in the unbounded state be represented by X_i and in the deformed state by x_i. The displacement u_i is represented by $u_i = x_i - X_i$.

(1) Let the strain tensor be denoted by e_{ij}

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \tag{1}$$

(2) The consecutive relation is as follows:

(a) The stress relation in terms of a nonlocal operator is

$$\left(1 - e^2\nabla^2\right)\sigma_{ij} = \left[2\mu e_{ij}(x) + \left[-\frac{\alpha}{K_T}T(x) + \lambda e_{kk}(x)\right]\delta_{ij}\right], \tag{2}$$

where $T = \vartheta - T_0$, the temperature of the body in a natural state is represented by T such that $|T/T_0| \leq 1$ and λ and μ are Lame's constant, K_T is the isothermal compressibility, and the amplitude temperature of the material is represented by θ, and $e = \epsilon_0 a_{\mathrm{cl}}$ where, ϵ_0 is a material constant and a_{cl} is the internal characteristic length.

(b) The energy equation for the linear theory of the thermoelastic material:

$$-\left(1 - e^2\nabla^2\right)\rho_0 T_0 \dot{\eta} = q_{i,i}, \tag{3}$$

where $q_{i,i} = -(\rho_0 C_e T + (\alpha T_0/K_T)e^{ij})$ is the equation of motion in the absence of body forces and C_e is the specific heat.

In the absence of body forces, the equation of motion for nonlocal isotopic thermoelastic solid can be written as

$$\sigma_{ij,j} = \rho_0(\ddot{u}_i), \tag{4}$$

where ρ_0 is the density of the material.

The modified Fourier law is $(1 - e^2\nabla^2)(q + \tau_0\dot{q}) = K\nabla T$.

The nonlocal heat conduction in thermoelastic material is

$$\left(1 - e^2\nabla^2\right)\left(1 + \tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)q_i = K\nabla T, \tag{5}$$

where K is the thermal conductivity, τ_0 is the relaxation time, and α is the fractional-order parameter such that

$$\frac{\partial^\alpha}{\partial t^\alpha} f(x,t) = \begin{cases} f(x,t) - f(x,0), & \alpha \longrightarrow 0, \\ l^{(1-\alpha)\partial f(x,t)/\partial t}, & 0 < \alpha < 1, \\ \dfrac{\partial f(x,t)}{\partial t}, & \alpha \longrightarrow 1. \end{cases} \tag{6}$$

With

$$l^\alpha f(x,t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-g)^{\alpha-1} f(x,g)dg. \tag{7}$$

Here, Γ is the gamma function and is constant such that $0 \leq \alpha \leq 1$.

When $\alpha \longrightarrow 0$, (5) reduces to the theory of classical coupled thermoelasticity, and when $\alpha \longrightarrow 1$, (5) reduces to the Lord and Shulman theory of thermoelasticity.

Substituting equations (1)–(3) into equations (4) and (5), we get the equation of motion as

$$\overline{\mu}\nabla^2 u + (\mu + \lambda)\nabla(\nabla.u) - \frac{\alpha}{K_T}\nabla T = \rho_0\left(1 - e^2\nabla^2\right)\ddot{u}, \tag{8}$$

$$\left(1 + \tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\rho_0 C_e\dot{T} + \frac{\alpha T_0}{K_T}\nabla.\dot{u}\right) = K\nabla^2 T. \tag{9}$$

2.1. Remarks

Case 1: The L-S theory of classical coupled thermoelasticity is as follows:

If $e = 0$ in equations (1)–(4) and (8) and (9), we get

$$\sigma_{ij} = 2\mu e_{ij}(x) + \left[\lambda e_{kk}(x) - \frac{\alpha}{K_T}T(x)\right]\delta_{ij},$$

$$\rho_0\eta = \frac{\alpha}{K_T}e_{kk}(x) + \frac{\rho_0 C_e}{T_0}T(x),$$

$$\left(1 + \tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)u = K\nabla T. \tag{10}$$

And

$$\mu\nabla^2 u + (\mu + \lambda)\nabla(\nabla.u) - \frac{\alpha}{K_T}\nabla T = \rho_0\ddot{u},$$

$$\left(1 + \tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\rho_0 C_e\dot{T} + \frac{\alpha T_0}{K_T}\nabla.\dot{u}\right) = K\nabla^2.T. \tag{11}$$

Case 2: Classical thermoelasticity is as follows:

If $e = 0$ and $\alpha/K_T = 0$ in equations (1)–(4) and (8) and (9), we obtain

$$\sigma_{ij} = 2\mu e_{ij}(x) + \lambda e_{kk}(x)\delta_{ij},$$

$$\rho_0\eta = \frac{\rho_0 C_e}{T_0}T(x), \tag{12}$$

$$\left(1 + \tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)u = K\nabla T,$$

$$\mu\nabla^2 u + (\mu + \lambda)\nabla(\nabla.u) = \rho_0\ddot{u}, \quad \left(1 + \tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)\rho_0 C_e\dot{T} = K\nabla^2 T. \tag{13}$$

3. Wave Propagation

We consider the homogeneous thermoelastic medium rotating about the y-axis. The vector and scalar potential ψ and ϕ through the Helmholtz vector theorem can be represented as

$$\nabla.\psi = 0, u = \nabla\varphi + \nabla \times \psi. \tag{14}$$

By putting these potentials in (8) and (9), the absence of heat source density, and body forces, we get the following equations as

$$\overline{V}_T^2\nabla^2\varphi - \frac{\alpha}{K_T}T = \left(1 - e^2\nabla^2\right)\nabla^2\varphi, \quad (15)$$

$$\overline{V}_S^2\nabla^2\psi = \left(1 - e^2\nabla^2\right)\ddot{\psi}, \quad (16)$$

$$\left(1 + \tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(C_e\dot{T} + \frac{\alpha T_0}{K_T}\nabla^2.\dot{\varphi}\right) = \frac{K}{\rho_0}\nabla^2 T. \quad (17)$$

Here, (15) and (17) are coupled in the form of ϕ and T, whereas (16) is uncoupled. So, to get the solution of these equations, we take

$$(\varphi, T, \psi) = (A_1, B_1, C_1)\exp^{ik(x\sin\vartheta + y\cos\vartheta - vt)}, \quad (18)$$

where A_1, B_1, and C_1 are the constant amplitudes which can be complex numbers and k is the wave number and the vector constant, where $r = (xi + yj + zk)$ is the position vector. By putting equations (18) into equations (15) and (17), we get

$$\left(-\left(v^2 + e^2\omega^2\right) + \overline{V}_T^2\right)A_1 - \beta B_1 = 0, \quad (19)$$

$$-\omega^2\tau_0^*\beta T_0 A_1 + \left(\frac{K}{\rho_0} + C_e v^2\tau_0^*\right)B_1 = 0, \quad (20)$$

where the following variables are used in the equations,

$$\overline{V}_T^2 = \frac{2\overline{\mu} + \overline{\lambda}}{\rho_0}, \beta = \frac{\alpha}{\rho_0 K_T}, \overline{V}_S^2 = \frac{\overline{\mu}}{\rho_0}, \tau_0^* = \left(\frac{\iota}{\omega} + \overline{\tau}_0\right), \overline{\tau}_0 = \tau_0(-\iota\omega)^{\alpha-1}. \quad (21)$$

This system of homogeneous linear (19) and (20) has a nonvanishing solution for unknowns A_1 and B_1 when the determinant of their coefficient matrix vanishes,

$$P\left(v^2\right)^2 + Q\left(v^2\right) + R = 0, \quad (22)$$

where

$$P = C_e\tau_0^*,$$

$$Q = \frac{K}{\rho_0}\left(1 + e^2k^2\right) + e^2\omega^2 C_e\tau_0^* - C_e\tau_0^*\overline{V}_T^2 - \beta^2\tau_0^* T_0, \quad (23)$$

$$R = -\frac{K}{\rho_0}\overline{V}_T^2.$$

(22) is the dispersion relation for the propagation of plane waves in a nonlocal thermoelastic solid medium that gives the speeds of various wave propagation.

The roots of equation (22) are

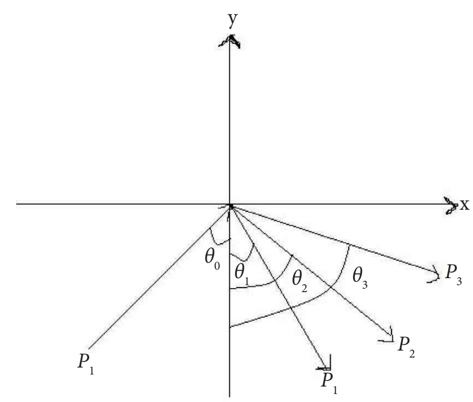


Figure 1: Diagram of the problem.

$$v_1^2 = \frac{-Q + \sqrt{Q^2 - 4PR}}{2P},$$
$$v_2^2 = \frac{-Q - \sqrt{Q^2 - 4PR}}{2P}. \quad (24)$$

The phase velocity (V_i), specific loss (S_i), and attenuation coefficient (Q_i) are represented in the following form [34]:

$$V_i = \frac{(\Re(v_i))^2 + (\Im(v_i))^2}{\Re(v_i)},$$
$$S_i = \left(\frac{\Delta W}{W}\right)_i = 4\pi\left|\frac{\Im(v_i)}{\Re(v_i)}\right|, \quad (25)$$
$$Q_i = \frac{-\omega\Im(v_i)}{(\Re(v_i))^2 + (\Im(v_i))^2},$$

where $\Re(v_i)$, $\Im(v_i)$ are the real and imaginary part of (v_i) where $i = 1, 2, 3$, respectively. To find the value of v_3, putting equation (18) into equation (16), we get,

$$v_3 = \sqrt{\overline{V}_S^2 - e^2\omega^2}. \quad (26)$$

(26) is the plane-wave propagation for the nonlocal thermoelastic medium that gives the speed of propagation for different waves, and for a given real value of ω lying within the range, we get

$$0 < \omega < \omega_c, \omega_c = \frac{\overline{V}_S}{e}. \quad (27)$$

From the expression, it has been noted that the speed of v_3 is that of an uncoupled wave that does not depend on thermal parameters. It travels slower than classical local elastic solid. The existence of e (nonlocal parameter) in the thermoelastic material results in the reduction of the phase speed of the uncoupled wave. As can be seen in (26), the phase speed of the uncoupled wave vanishes when $\omega = \omega_c$. This implies that for $\omega < \omega_c$, the speed of the phase velocity v_3 is real and that for $\omega > \omega_c$, it is complex. Thus, we can say that the uncoupled wave is a propagating wave in the frequency range: $0 < \omega < \omega_c$.

Based on the formula in (25), we can get the attenuation coefficient as well as the specific losses of the existing uncoupled wave as

$$Q_3 = 0, \left(\frac{\Delta W}{W}\right)_3 = 0. \tag{28}$$

4. Reflection at the Stress-Free Surface

In half space, the P_1 wave makes an angle of incidence (θ_0) with the normal, which yields three reflected waves, P_1, P_2, and P_3 as shown in Figure 1. The suitable potentials of incident and reflected waves are considered as

$$\varphi = A_2 \exp^{\left[ik_2 (x \sin \vartheta_2 + y \cos \vartheta_2) - i\omega t\right]} + A_1 \exp^{\left[ik_1 (x \sin \vartheta_1 + y \cos \vartheta_1) - i\omega t\right]} + A_2 \exp^{\left[ik_2 (x \sin \vartheta_2 - y \cos \vartheta_2) - i\omega t\right]}, \tag{29}$$

$$T = \xi_2 A_2 \exp^{\left[ik_2 (x \sin \vartheta_2 + y \cos \vartheta_2) - i\omega t\right]} + \xi_1 A_1 \exp^{\left[ik_1 (x \sin \vartheta_1 + y \cos \vartheta_1) - i\omega t\right]} + \xi_2 A_2 \exp^{\left[ik_2 (x \sin \vartheta_2 - y \cos \vartheta_2) - i\omega t\right]}, \tag{30}$$

$$\psi = B_1 \exp^{ik_3 (x \sin \vartheta_3 - y \cos \vartheta_3 - i\omega t)}, \tag{31}$$

where $\xi_i = (k_i^2 v_i^2 - k_i^2 \overline{V}_T^2)/\beta$ for $i = 1, 2$.

4.1. Boundary Condition. We now describe the following boundary conditions that must be satisfied for the proposed problem. Since the boundary surface at $y = 0$ is stress free, we have,

$$\sigma_{yx} = 0, \sigma_{yy} = 0, \frac{\partial T}{\partial y} = 0 \; at \; y = 0. \tag{32}$$

Taking equations (29)–(31) and making use of equation (2) in the boundary conditions, we get

$$\begin{aligned}
&\frac{A_1}{A_0} c_{11} + \frac{A_2}{A_0} c_{12} + \frac{B_1}{A_0} c_{13} = d_1, \\
&\frac{A_1}{A_0} c_{21} + \frac{A_2}{A_0} c_{22} + \frac{B_1}{A_0} c_{23} = d_2, \\
&\frac{A_1}{A_0} c_{31} + \frac{A_2}{A_0} c_{32} + \frac{B_1}{A_0} c_{33} = d_3,
\end{aligned} \tag{33}$$

where

$$\begin{aligned}
c_{11} &= \left(\overline{\lambda} + 2\overline{\mu}\cos^2\vartheta_1\right)k_1^2 - \beta\rho_0\xi_1, \\
c_{12} &= \left(\overline{\lambda} + 2\overline{\mu}\cos^2\vartheta_2\right)k_2^2 - \beta\rho_0\xi_2, \\
c_{13} &= -2\overline{\mu}k_3^2 \sin \vartheta_3 \cos \vartheta_3, \\
c_{21} &= \overline{\mu}k_1^2\left(\sin^2\vartheta_1 - \sin \vartheta_1 \cos \vartheta_1\right), \\
c_{22} &= \overline{\mu}k_2^2\left(\sin^2\vartheta_2 - \sin \vartheta_2 \cos \vartheta_2\right), \\
c_{23} &= \overline{\mu}k_3^2\left(\cos^2\vartheta_3 + \sin \vartheta_3 \cos \vartheta_3\right), \\
c_{31} &= k_1\xi_1 \cos \vartheta_1, \\
c_{32} &= k_2\xi_2 \cos \vartheta_2, \\
c_{33} &= 0, \\
d_1 &= \left(\overline{\lambda} + 2\overline{\mu}\cos^2\vartheta_2\right)k_2^2 - \beta\rho_0\xi_2, \\
d_2 &= \overline{\mu}k_2^2\left(\sin^2\vartheta_2 + \sin \vartheta_2 \cos \vartheta_2\right), \\
d_3 &= k_2\xi_2 \cos \vartheta_2.
\end{aligned} \tag{34}$$

Here, the ratio of the amplitude of the reflected wave and incident wave represented by A_1/A_0,A_2/A_0 and B_1/A_0, which gives the reflection coefficient, where

$$\begin{aligned}
A_1 &= \left[d_1 (c_{23}c_{32} - c_{22}c_{33}) + d_2 (c_{12}c_{33} - c_{13}c_{32}) + d_3 (c_{13}c_{22} - c_{12}c_{23})\right], \\
A_2 &= \left[d_1 (c_{23}c_{31} - c_{21}c_{33}) + d_2 (c_{11}c_{33} - c_{13}c_{31}) - d_3 (c_{13}c_{21} - c_{11}c_{23})\right], \\
B_1 &= d_1 (c_{23}c_{31} - c_{21}c_{32}) + d_2 (c_{11}c_{32} - c_{12}c_{31}) + d_3 (c_{12}c_{21} - c_{11}c_{22}), \\
A_0 &= c_{11} (c_{23}c_{32} - c_{22}c_{33}) + c_{12} (c_{21}c_{33} - c_{23}c_{31}) + c_{13} (c_{31}c_{22} - c_{21}c_{32}).
\end{aligned} \tag{35}$$

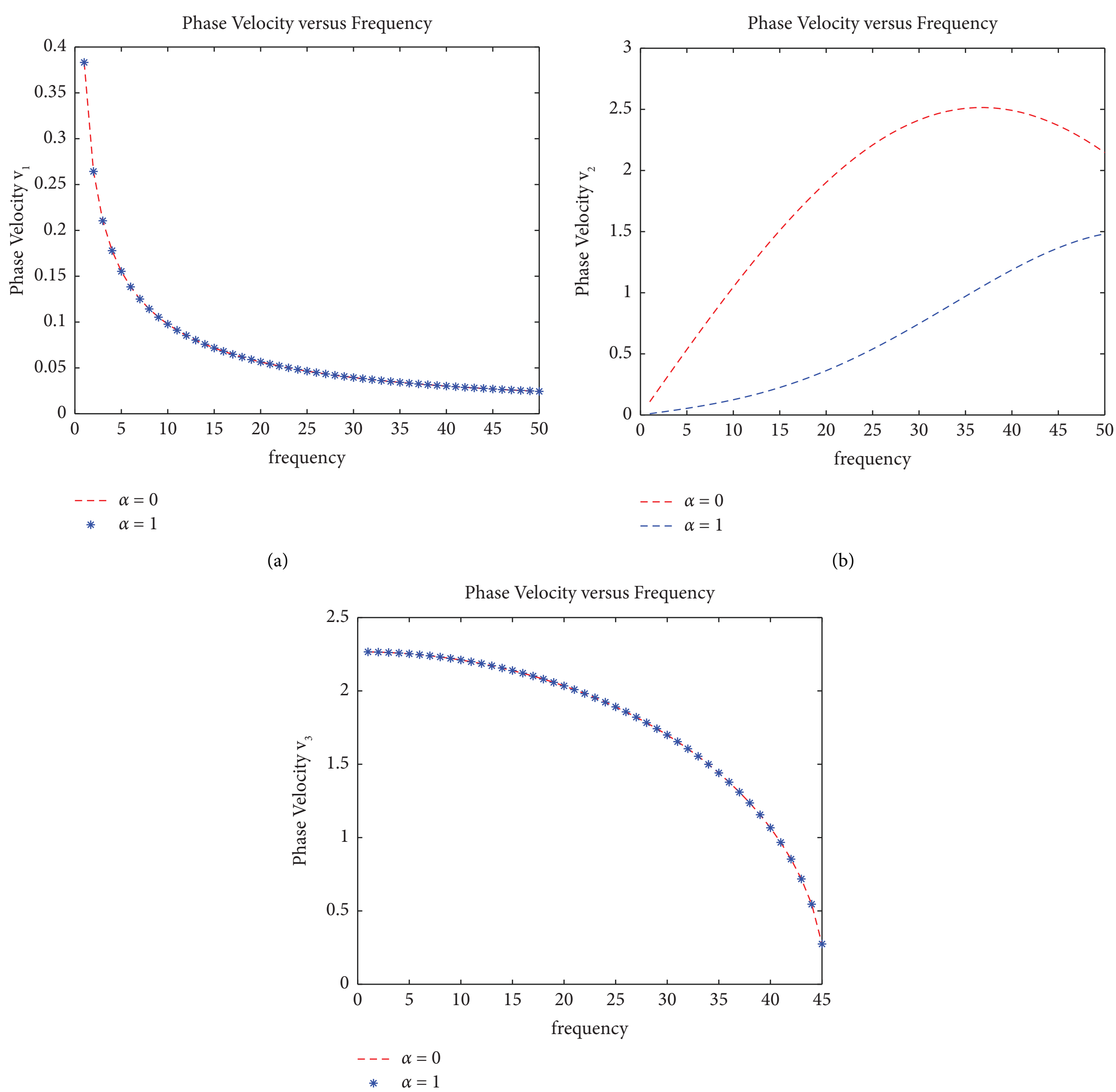
5. Numerical Results and Discussion

The values of the parameters mentioned in Table 1 have been used to find the numeric results taken from [2].

Figure 2 represents the variation of the phase velocity V_1, V_2, and V_3 with respect to the frequency for two different theories when $\alpha \longrightarrow 0$ and $\alpha \longrightarrow 1$. In Figure 2(a), it can be seen that the phase velocity first decreases sharply and then decreases slowly with the increase in the frequency for both theories. Figure 2(b) shows that the phase velocity increases at first and then decreases sharply for $\alpha \longrightarrow 0$ and that the phase velocity slightly decreases and then increases for

Table 1: Values of parameters.

Symbols	Value
E	6.9
Σ	10.1
ρ_0	2.7
C_e	0.236
K	0.0492
K_T	0.5
T_0	20
μ	1
Ω	2
E	0.1



(a) (b) (c)

Figure 2: Phase velocity w.r.t. frequency ω.

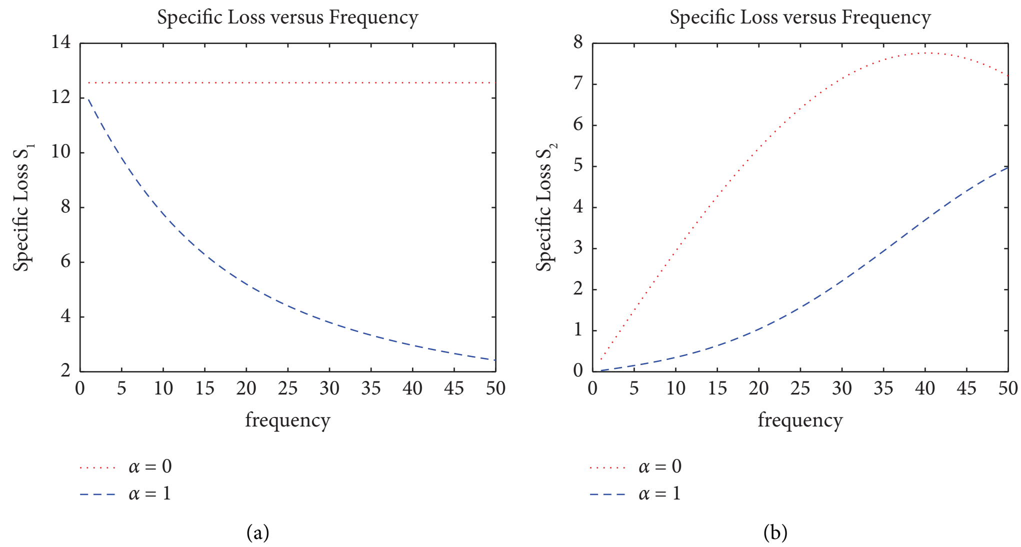


Figure 3: Specific loss w.r.t. frequency ω.

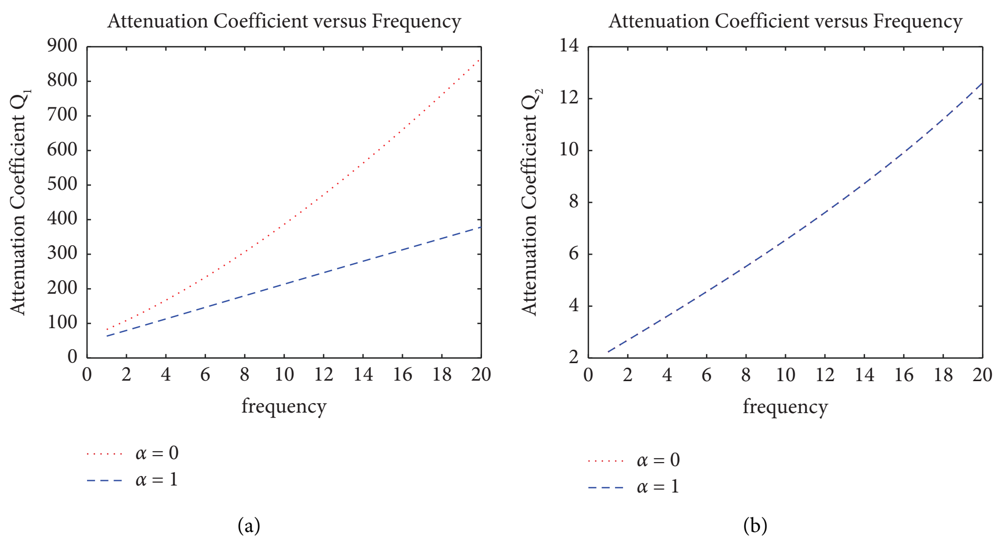


Figure 4: Attenuation coefficient w.r.t. frequency ω.

$\alpha \longrightarrow 1$. In Figure 2(c), it can be seen that the phase velocity decreases slowly with the increase in the frequency for both theories.

Figure 3 represents the variation of the specific loss S_1 and S_2 with respect to the frequency for two different theories when $\alpha \longrightarrow 0$ and $\alpha \longrightarrow 1$. Figure 3(a) represents that when the frequency increases, the specific loss remains constant for $\alpha \longrightarrow 0$. The specific loss decreases with the increase in the frequency for $\alpha \longrightarrow 1$. Figure 3(b) shows that the specific loss increases and then slightly decreases for theory when $\alpha \longrightarrow 0$ and for $\alpha \longrightarrow 1$, the specific loss slowly increases with the increase in the frequency.

Figure 4 represents the variation of the attenuation coefficient Q_1 and Q_2 with respect to the frequency for two different theories when $\alpha \longrightarrow 0$ and $\alpha \longrightarrow 1$. Figure 4(a) shows that the attenuation coefficient slightly increases with the increase in the frequency for $\alpha \longrightarrow 0$. For $\alpha \longrightarrow 1$, the attenuation coefficient sharply increases with the increase in the frequency. The attenuation coefficient slowly increases with the increase in the frequency for both theories. Figure 4(b) shows that the attenuation coefficient sharply increases with the increase in the frequency in both theories for $\alpha \longrightarrow 0$ and $\alpha \longrightarrow 1$.

Figure 5 represents the variation of the phase velocity V_1, V_2, and V_3 with respect to the nonlocal parameter e for two different theories when $\alpha \longrightarrow 0$ and $\alpha \longrightarrow 1$. In Figure 5(a), the phase velocity remains constant when the nonlocal parameter increases for both $\alpha \longrightarrow 0$ and $\alpha \longrightarrow 1$. Figure 5(b) represents that the phase velocity decreases for both theories with the increase in the nonlocal parameter. Figure 5(c) shows no effect

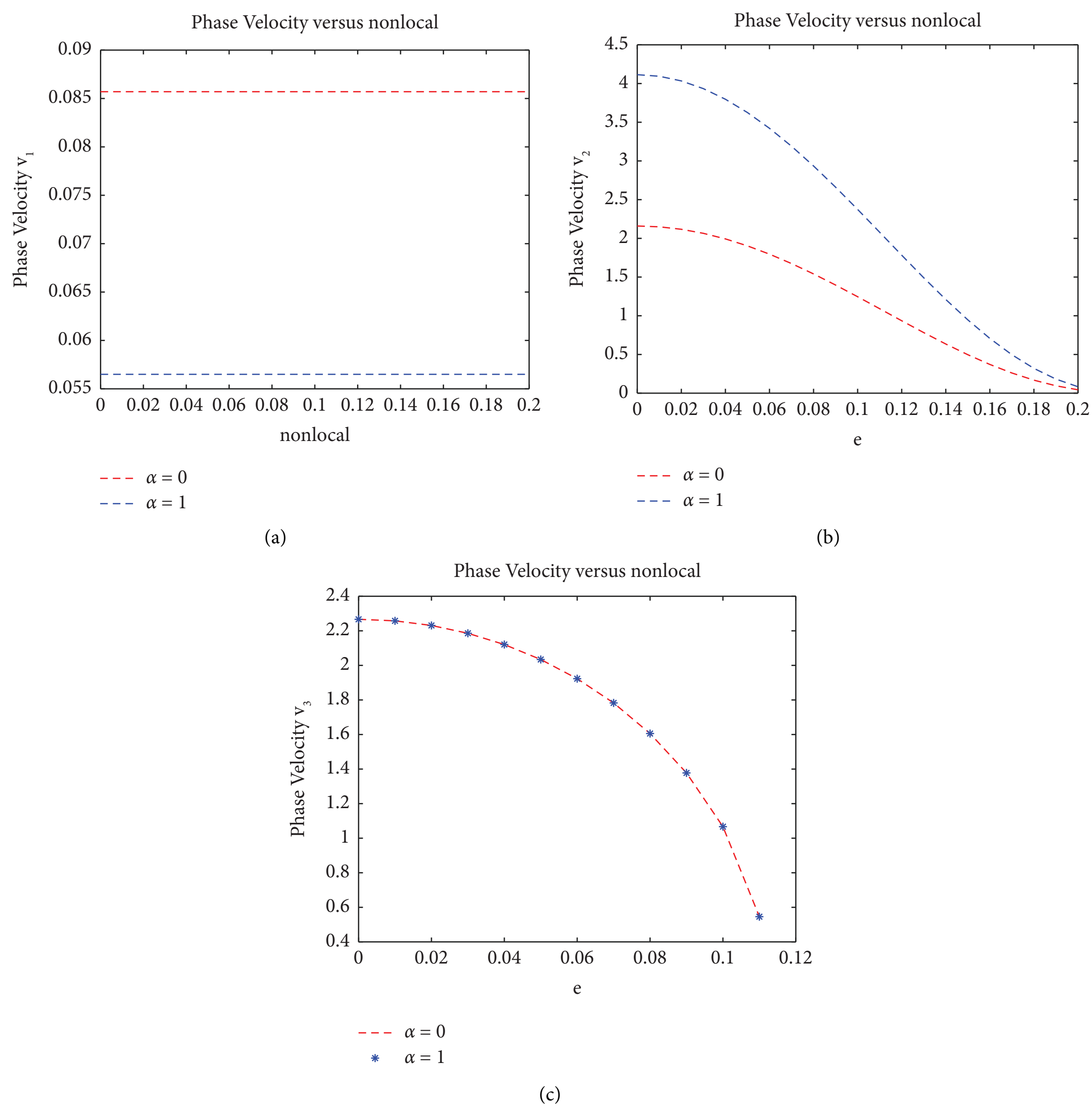


FIGURE 5: Phase velocity w.r.t. nonlocal parameter e.

of α on both theories. The phase velocity slowly decreases with the increase in the nonlocal parameter.

Figure 6 represents the variation of the specific loss S_1 and S_2 with respect to the nonlocal parameter e for two different theories when $\alpha \longrightarrow 0$ and $\alpha \longrightarrow 1$. In Figure 6(a), the specific loss remains constant for both theories when the nonlocal parameter increases. Figure 6(b) represents that the specific loss decreases sharply with the increase in the nonlocal parameter for the theory $\alpha \longrightarrow 0$.For $\alpha \longrightarrow 1$, with the increase in the nonlocal parameter, the phase velocity slowly decreases.

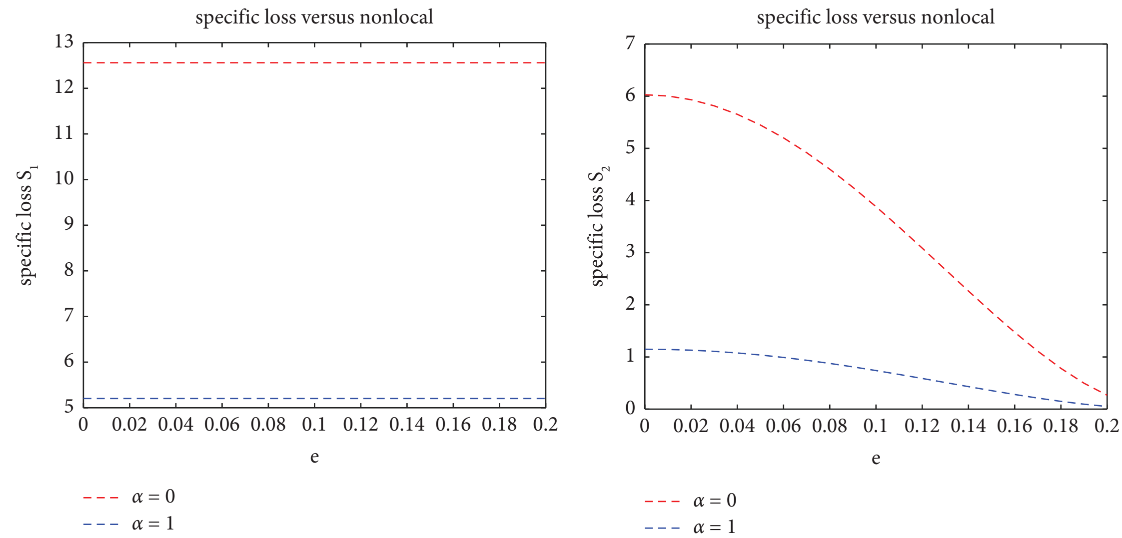
Figure 7 represents the attenuation coefficient Q_1 and Q_2 with respect to the nonlocal parameter e for two different theories when $\alpha \longrightarrow 0$ and $\alpha \longrightarrow 1$. Figure 7(a) shows that for both theories, the attenuation coefficient remains constant with the increase in the nonlocal parameter. Figure 7(b) shows that there is no effect of α on both theories seen in this case. The attenuation coefficient slowly increases with the increase in the nonlocal parameter.

5.1. Special Cases

Case 1. If the nonlocal effect is neglected from the medium, then we get a thermoelastic medium as $e = 0$ in equation (15),

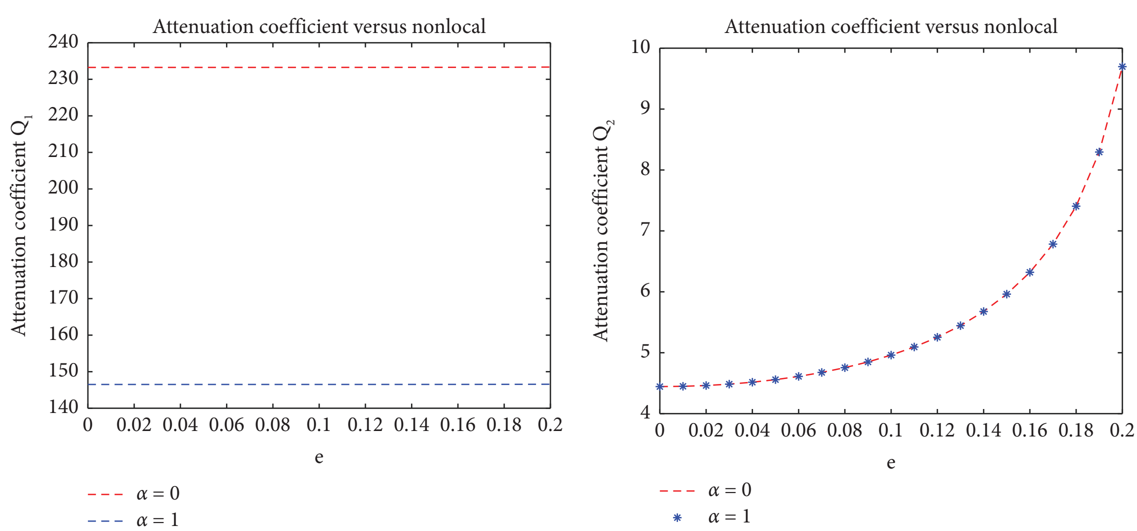
$$A\left(v^2\right)^2 + B\left(v^2\right) + C = 0, \tag{36}$$

where



(a) (b)

Figure 6: Specific loss w.r.t. nonlocal parameter e.



(a) (b)

Figure 7: Attenuation coefficient w.r.t. nonlocal parameter e.

$$A = \tau_0 C_e,$$
$$B = -\tau_0 C_e \overline{V}_T^2 - \tau_0^* \beta^2 T_0 + \frac{K}{\rho_0}, \qquad (37)$$
$$C = -\overline{V}_T^2 \frac{K}{\rho_0}.$$

(36) gives the speed of propagation of coupled waves in the thermoelastic medium. Similarly, when we use $e = 0$ in (26), the speed of transverse waves in the thermoelastic medium becomes the speed of a classical wave.

6. Conclusion

The propagation of plane waves in nonlocal fractional-order thermoelasticity has been studied. The constitutive relation for the propagation of plane waves in nonlocal fractional thermoelastic solid media is considered and solved. The specific loss, phase speed, and attenuation coefficient have been obtained for three waves. The effects of the specific loss, attenuation coefficient, and phase velocity on the frequency and nonlocal parameter for the two theories (classical theory and L-S theory) are shown graphically.

The following observations can be seen in the graphs:

(i) On applying the two theories $\alpha = 0$ and $\alpha = 1$ to the phase speeds, specific losses, and attenuation coefficients against frequency ω, it has been found that both theories have more effects on the phase velocities V_1 and V_2, specific losses S_1 and S_2, and attenuation coefficient Q_1.

(ii) On applying the two theories $\alpha = 0$ and $\alpha = 1$ to the phase speeds, specific losses, and attenuation coefficients against the nonlocal parameter e, it has been found that these theories have more significant effects on the phase velocities V_1 and V_2, specific losses S_1 and S_2, and attenuation coefficient Q_1.

Data Availability

The data used have been cited in [46].

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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