Hindawi
International Journal of Mathematics and Mathematical Sciences
Volume 2022, Article ID 1839796, 7 pages
https://doi.org/10.1155/2022/1839796

Research Article

Solving Multispecies Lotka–Volterra Equations by the Daftardar-Gejji and Jafari Method

Belal Batiha[1], **Firas Ghanim**[2], **O. Alayed**[1], **Ra'ed Hatamleh**[1], **Ahmed Salem Heilat**[1], **Hamzeh Zureigat**[1], **and Omar Bazighifan**[3]

[1]*Department of Mathematics, Faculty of Science and Information Technology, Jadara University, Irbid, Jordan*
[2]*College of Sciences, University of Sharjah, Sharjah, UAE*
[3]*Department of Mathematics, Faculty of Science, Hadhramout University, Hadhramout 50512, Yemen*

Correspondence should be addressed to Belal Batiha; b.bateha@jadara.edu.jo

Received 10 October 2022; Revised 2 December 2022; Accepted 3 December 2022; Published 26 December 2022

Academic Editor: Attila Gil nyi

In this article, we apply the Daftardar-Gejji and Jafari method (DJM) to solve the multispecies Lotka–Volterra equation. A comparison between the DJM, differential transformation method (DTM), the variational iteration method (VIM), and Adomian decomposition method (ADM) shows that the DJM is a reliable and powerful method for solving nonlinear equations. The efficiency and applicability of this method are confirmed by considering some examples. The proposed procedure provides better results in comparison to some existing methods.

1. Introduction

The area of mathematics called numerical analysis is in charge of coming up with practical methods for calculating answers to difficult computational calculations. The majority of mathematical issues in engineering and science are very challenging, and sometimes, there is no straightforward solution. To make a difficult mathematical problem simpler to solve, measurement is thus crucial. As a contemporary tool for scientists and engineers, numeracy has grown in popularity as a result of the tremendous developments in computing technology. As a consequence, a variety of software packages, including MATLAB, Mathematica, Maple, and others, are being created to solve even the most challenging issues quickly and simply. These programs provide features that make use of conventional numerical techniques, allowing the user to run a single command without entering any parameters and obtain the desired results. The creation, analysis, and application of algorithms for solving numerical problems in continuous mathematics are all made using the numerical analysis approach, which is mostly utilized in mathematics and computer science. These kinds of issues often come up in the actual world when algebra, geometry, and calculus are applied, and they also include continuous variables. These issues arise in all areas of study, including the scientific and social sciences, engineering, health care, and business [1–9]. Numerical analysis introduced realistic mathematical models which have become more prevalent in science and engineering over the last 50 years as a result of the expansion in the power and accessibility of digital computers. We shall learn more about numerical approaches and their analysis here. PDE solutions may be solved using the same numerical techniques used for ODEs. Many difficulties may be solved using the techniques mentioned for handling initial value concerns, for example, see references [6, 10–14].

The Lotka–Volterra equations describe the time history of a biological system [15]. The Lotka–Volterra equations are applied in a number of engineering areas. The one-species Lotka–Volterra equation is used to demonstrate a simple nonlinear control system [16].

The Lotka–Volterra equations were solved by many numerical methods like hybrid deep network [17], Gröbner bases elimination method [18], generalized backstepping control method [19], the differential transformation method (DTM) [20], the Adomian decomposition method (ADM), and variational iteration method (VIM) [21].

The differential transformation method (DTM) was first proposed by Zhou [22] (also check [23, 24]). The DTM is an iterative method that obtains the Taylor series solutions of different kinds of differential equations (see [20, 25–27]). The DTM can be applied directly to different kinds of DEs without requiring linearization, discretization, or perturbation, and it is a very accurate method with less computational work [28].

The Adomian decomposition method (ADM) was introduced by Adomian [29] to solve nonlinear differential equations and physical problems [30–33].

The VIM was first proposed by He [34] (see also [35–37]). The VIM has successfully been used for many ordinary and partial differential equations [21, 38–40].

In 2006, the DJM was first proposed by Daftardar-Gejji and Jafari [41]; the method can solve many nonlinear differential equations and physical problems [42–52]. Recently, the DJM was applied to create a quite new predictor-corrector method [53, 54]. Noor et al. [55–59] used the DJM to create numerical techniques to solve algebraic equations.

In this paper, we apply the DJM to solve the multispecies Lotka–Volterra equation and compare the results obtained with DTM, VIM, ADM, and exact solution to show the simplicity and accuracy of this method. The efficiency and applicability of this method are confirmed by considering some examples. The proposed procedure provides better results in comparison to some existing methods. The DJM method will be implemented in a direct way without any linearization, perturbation, or restrictive assumptions.

2. The Daftardar-Gejji and Jafari Method (DJM)

Here, the DJM ([41]) will be described, which was successfully applied to solve nonlinear DEs of the following form:

$$v = f + L(v) + N(v), \tag{1}$$

where f is a function given, L is the linear operator and N is the nonlinear operator. The solution of equation (1) will be as follow:

$$v = \sum_{i=0}^{\infty} v_i. \tag{2}$$

Suppose

$$\begin{aligned} H_0 &= N(v_0),\\ H_m &= N\left(\sum_{i=0}^{m} v_i\right) - N\left(\sum_{i=0}^{m-1} v_i\right). \end{aligned} \tag{3}$$

So,

$$\begin{aligned} H_0 &= N(v_0),\\ H_1 &= N(v_0+v_1) - N(v_0),\\ H_2 &= N(v_0+v_1+v_2) - N(v_0+v_1),\\ H_3 &= N(v_0+v_1+v_2+v_3) - N(v_0+v_1+v_2). \end{aligned} \tag{4}$$

Thus, $N(v)$ is decomposed as follows:

$$\begin{aligned} N\left(\sum_{i=0}^{\infty} v_i\right) &= N(v_0) + N(v_0+v_1) - N(v_0) + N(v_0+v_1+v_2) - N(v_0+v_1)\\ &\quad + N(v_0+v_1+v_2+v_3) - N(v_0+v_1+v_2) + \cdots. \end{aligned} \tag{5}$$

So,

$$\begin{aligned} v_0 &= f,\\ v_1 &= L(v_0) + H_0,\\ v_{m+1} &= L(v_m) + H_m,\ m = 1, 2, \ldots. \end{aligned} \tag{6}$$

Since L is linear, then

$$\sum_{i=0}^{m} L(v_i) = L\left(\sum_{i=0}^{m} v_i\right). \tag{7}$$

Then,

$$\begin{aligned} \sum_{i=1}^{m+1} v_i &= \sum_{i=0}^{m} L(v_i) + N\left(\sum_{i=0}^{m} v_i\right)\\ &= L\left(\sum_{i=0}^{m} v_i\right) + N\left(\sum_{i=0}^{m} v_i\right),\ m = 1, 2, \ldots. \end{aligned} \tag{8}$$

Thus,

$$\sum_{i=0}^{\infty} v_i = f + L\left(\sum_{i=0}^{\infty} v_i\right) + N\left(\sum_{i=0}^{\infty} v_i\right). \tag{9}$$

The $k-$ term approximate solution is given as follows:

$$v = \sum_{i=0}^{k-1} v_i. \tag{10}$$

3. Convergence of the DJM

Theorem 1. *"For any n and for some real $\mathbb{L} > 0$ and $\|\mathbf{u}_i\| \leq M < 1/e$, $i = 1, 2, \ldots$, if $\mathbb{N}$ is $C^{(\infty)}$ in the neighborhood of $\mathbf{u}_0$ and $\|\mathbb{N}^{(n)}(\mathbf{u}_0)\| \leq \mathbb{L}$, then $\sum_{n=0}^{\infty} H_n$ is absolutely convergent and $\|H_n\| \leq \mathbb{L}M^n e^{n-1}(e-1)$, $n = 1, 2, \ldots$."*

Proof. Please see reference [46] for full details of the proof. □

Theorem 2. *"The series $\sum_{n=0}^{\infty} H_n$ is absolutely convergent if $\mathbb{N}$ is $C^{(\infty)}$ and $\|\mathbb{N}^{(n)}(\mathbf{u}_0)\| \leq M \leq e^{-1}$, $\forall n$."*

Proof. Please see reference [46] for full details of the proof. □

4. Analysis of Multispecies Lotka–Volterra Equations

In this section, we will study the n^{th} general Lotka–Volterra system in the form as follows:

$$\frac{\mathrm{d}y_i}{\mathrm{d}t} = y_i\left(\beta_i + \sum_{j=1}^{n} \alpha_{ij} y_j\right), \quad i = 1, 2, \ldots, n. \tag{11}$$

To solve equation (11) with the initial condition $y(0) = y(0)$ by the Daftardar-Gejji and Jafari method (DJM), we write it in the following integral equation:

$$y(t) = y_0 + \int_0^t y_i\left(\beta_i + \sum_{j=1}^{n} \alpha_{ij} y_j\right)\mathrm{d}t, \quad i = 1, 2, \ldots, n. \tag{12}$$

Then, we will apply the DJM as in the previous section.

4.1. One Species. In this section, equation (11) is reduced to one species:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(\beta + \alpha y), \ \beta > 0, \ \alpha < 0, \ y(0) > 0, \tag{13}$$

where α and β are constants. With exact solution,

$$\begin{aligned} y(t) &= \frac{\beta e^{\beta t}}{\beta + \alpha y(0)/y(0) - \alpha e^{\beta t}} \text{ for } \beta \neq 0, \\ y(t) &= \frac{y(0)}{1 - \alpha y(0)t}, \text{ for } \beta = 0. \end{aligned} \tag{14}$$

To solve equation (13) with the initial condition $y(0) = 0.1$ by the Daftardar-Gejji and Jafari method (DJM), we write it in the following integral equation:

$$y(t) = 0 \cdot 1 + \int_0^t y(\beta + \alpha y)\mathrm{d}t. \tag{15}$$

By applying DJM, we obtain the following:

$$\begin{aligned} y_0 &= 0.1, \\ y_1 &= 0.07t, \\ y_2 &= -0.0049t^2(t - 2.857143), \\ &\vdots \end{aligned} \tag{16}$$

The four-term solution is as follows:

$$\begin{aligned} y(t) = {} & 0.1 + 0.07t - 0.0049t^2(t - 2.857143) \\ & - 0.00001029(t + 5.587558)t^3(t - 1.234763)(t - 3.493845)(t - 7.525617) \\ & - 2.117682 \times 10^{-11}(t + 5.877433)(t + 5.005629)t^4(t - 4.090877) \\ & \cdot (t - 6.871677)(t - 7.828486)\left(t^2 + 6.991252t + 21.95022\right) \\ & \cdot \left(t^2 - 2.520506t + 1.605527\right)\left(t^2 - 10.84848t + 38.63197\right). \end{aligned} \tag{17}$$

4.2. Two Species. In this section, equation (11) is reduced to two species:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = y_1(\beta_1 + \alpha_{11}y_1 + \alpha_{12}y_2), \tag{18}$$

$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = y_2(\beta_2 + \alpha_{21}y_1 + \alpha_{22}y_2), \tag{19}$$

where α_{11}, α_{12}, α_{21}, α_{22}, β_1, and β_2 are constants.

To solve equations (18) and (19) by DJM with initial conditions $y_1(0) = 4$ and $y_2(0) = 10$, we write it in the following integral equation:

$$\begin{aligned} y_{1,1} &= 4 + \int_0^t y_1(\beta_1 + \alpha_{11}y_1 + \alpha_{12}y_2)\mathrm{d}t, \\ y_{2,1} &= 10 + \int_0^t y_2(\beta_2 + \alpha_{21}y_1 + \alpha_{22}y_2)\mathrm{d}t. \end{aligned} \tag{20}$$

Table 1: Comparison study when $\beta = 1, \alpha = -3, y(0) = 0.1$.

t	Exact	DJM_4	ADM, ϕ_3 [21]	VIM_2 [21]	DTM_6 [20]
0.0	0.1000000	0.1000000	0.1000000	0.1000000	0.1000000
0.2	0.1145329	0.1145329	0.1145600	0.1145545	0.1145329
0.4	0.1300011	0.1300011	0.1302400	0.1302590	0.1300004
0.6	0.1461629	0.1461627	0.1470400	0.1474445	0.1461546
0.8	0.1627259	0.1627256	0.1649600	0.1671263	0.1626790
1.0	0.1793672	0.1793669	0.1840000	0.1915249	0.1791887

Table 2: Comparison study when $\beta = 1$, $\alpha = -3$, $y(0) = 0.1$.

t	Exact	DJM_4	VIM_2 [40]	ADM, ϕ_3 [21]	DTM_9 [20]
0.0	0.10000	0.10000	0.10000	0.10000	0.10000
0.5	0.13801	0.13801	0.13862	0.13850	0.13801
1.0	0.17936	0.17937	0.19152	0.18400	0.17937
1.5	0.21921	0.21921	0.29877	0.23650	0.21932
2.0	0.25333	0.25333	0.30286	0.29600	0.25442
2.5	0.27975	0.27969	−4.4899	0.36250	0.28519
3.0	0.29864	0.29824	−69.317	0.43600	0.31533

Table 3: Numerical comparison when $\beta_1 = 0.1, \alpha_{11} = -0.0014, \alpha_{12} = -0.0012, \beta_2 = 0.08, \alpha_{21} = -0.0009, \alpha_{22} = -0.001, y_1(0) = 4, y_2(0) = 10$, and $h = 0.001$.

	DJM		2-iterate VIM [*21*]		RK4	
t	y_1	y_2	y_1	y_2	y_1	y_2
0.0	4.00000	10.00000	4.00000	10.00000	4.00000	10.00000
0.1	4.03308	10.06657	4.03307	10.06657	4.033070	10.0665
0.2	4.06639	10.13348	4.06636	10.13349	4.066363	10.1334
0.3	4.09983	10.20075	4.09987	10.20075	4.099878	10.2007
0.4	4.13365	10.26835	4.13361	10.26836	4.13361	10.2683
0.5	4.16753	10.33631	4.16758	10.33632	4.167580	10.3363
0.6	4.20176	10.40460	4.20177	10.40462	4.201767	10.4046
0.7	4.23617	10.47324	4.23618	10.47328	4.236180	10.4732
0.8	4.27073	10.54223	4.27082	10.54228	4.270818	10.5422
0.9	4.30562	10.61156	4.30569	10.61163	4.305683	10.6116
1.0	4.34067	10.68124	4.34079	10.68133	4.340775	10.6813

Table 4: Numerical comparison when $\alpha = 0.1, \beta = 0.1, y_1(0) = 0.2, y_2(0) = 0.3, y_3(0) = 0.5$, and $h = 0.001$.

	DJM			4-Iterate VIM [*21*]			*RK4*		
t	y_1	y_2	y_3	y_1	y_2	y_3	y_1	y_2	y_3
0.0	0.20000	0.30000	0.50000	0.20000	0.30000	0.50000	0.20000	0.30000	0.50000
0.1	0.21475	0.31915	0.52233	0.21473	0.31914	0.52234	0.21473	0.31914	0.52234
0.2	0.23019	0.33886	0.54442	0.23010	0.33873	0.54429	0.23010	0.33873	0.54428
0.3	0.24623	0.35903	0.56627	0.24609	0.35867	0.56573	0.24609	0.35867	0.56572
0.4	0.26298	0.37977	0.58778	0.26265	0.37889	0.58662	0.26264	0.37888	0.58655
0.5	0.28034	0.40099	0.60902	0.27975	0.39931	0.60693	0.27973	0.39927	0.60667
0.6	0.29838	0.42264	0.62993	0.29734	0.41987	0.62679	0.29729	0.41974	0.62601
0.7	0.31714	0.44484	0.65062	0.31540	0.44054	0.64653	0.31527	0.44020	0.64449
0.8	0.33653	0.46763	0.67109	0.33393	0.46138	0.66683	0.33361	0.46054	0.66208
0.9	0.35658	0.49082	0.69122	0.35294	0.48254	0.68887	0.35222	0.48068	0.67871
1.0	0.37732	0.51467	0.71106	0.37256	0.50438	0.71455	0.37105	0.50053	0.69438

The rest components of the formulas (20) and (20) can be obtained using the computer algebra package Maple.

4.3. Three Species. In this section, equation (11) is reduced to three species:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = y_1\left(1 - y_1 - \alpha y_2 - \beta y_3\right), \tag{21}$$

$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = y_2\left(1 - \beta y_1 - y_2 - \alpha y_3\right), \tag{22}$$

$$\frac{\mathrm{d}y_3}{\mathrm{d}t} = y_3\left(1 - \alpha y_1 - \beta y_2 - y_3\right), \tag{23}$$

where α and β are constants.

To solve equations (21)–(23) by DJM with initial conditions $y_1(0) = 0.2$, $y_2(0) = 0.3$, and $y_3(0) = 0.5$, we write it in the following integral equation:

$$y_{1,1} = 0.2 + \int_0^t y_1\left(1 - y_1 - \alpha y_2 - \beta y_3\right)\mathrm{d}t, \tag{24}$$

$$y_{2,1} = 0.3 + \int_0^t y_2\left(1 - \beta y_1 - y_2 - \alpha y_3\right)\mathrm{d}t, \tag{25}$$

$$y_{3,1} = 0.5 + \int_0^t y_3\left(1 - \alpha y_1 - \beta y_2 - y_3\right)\mathrm{d}t. \tag{26}$$

Again, the rest components of the formulas (24)–(26) can be obtained using the computer algebra package Maple.

5. Discussion

We used Maple to code the DJM algorithm. Maple environment variable digits is set to 16 in all calculations done in this paper.

The numerical solutions obtained by using the DJM are compared with the exact solution and those obtained by ADM [21], DTM [20], and VIM [40]. Table 1 shows a comparison between the exact solution, the four iterations DJM with the DTM of order 6, two iterations of VIM, and 3-term ADM in the case $b = 1, a = -3$, and $y(0) = 0.1$ for $t \in [0,1]$; we can see the method is efficient to solve the one-species Lotka–Volterra equation. In Table 2, we compare four iteration DJM with DTM, VIM, ADM, and the exact solution where $b = 1, a = -3$, and $y(0) = 0.1$ for $t \in [0,3]$; in this table, we can prove the stability of DJM for large t. In Table 3, we perform the numerical comparison when $\beta_1 = 0.1, \alpha_{11} = -0.0014, \alpha_{12} = -0.0012, \beta_2 = 0.08, \alpha_{21} = -0.0009$, $\alpha_{22} = -0.001, y_1(0) = 4, y_2(0) = 10$, and $h = 0.001$. In Table 4, we compare DJM with 4-Iterate VIM and RK4 when $\alpha = 0.1, \beta = 0.1, y_1(0) = 0.2, y_2(0) = 0.3, y_3(0) = 0.5$, and $h = 0.001$

6. Conclusions

In this article, the DJM is used for solving the multispecies Lotka–Volterra equation. The Daftardar-Gejji and Jafari method was implemented in a direct way without any linearization, perturbation, or restrictive assumptions. Comparisons with the VIM, DTM, and ADM show that the DJM is a better method for solving nonlinear equations. We proved that DJM is a precise and efficient method to solve the multispecies Lotka–Volterra equation.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] F. Aslanova, "A comparative study of the hardness and force analysis methods used in truss optimization with meta-heuristic algorithms and under dynamic loading," *Journal of Research in Science, Engineering and Technology*, vol. 8, no. 1, pp. 25–33, 2020.

[2] B. Madina and L. N. Gumilyov, "Determination of the most effective location of environmental hardenings in concrete cooling tower under far-source seismic using linear spectral dynamic analysis results," *Journal of Research in Science, Engineering and Technology*, vol. 8, no. 1, pp. 22–24, 2020.

[3] N. B. Agarkov, V. V. Khaustov, A. M. Malikov, N. G. Karpenko, and I. M. Ignatenko, "Optimization of the drainage system of overburden dumps using geofiltration modeling," *Caspian Journal of Environmental Sciences (CJES)*, vol. 20, no. 1, pp. 171–175, 2022.

[4] S. Srinivasareddy, Y. V. Narayana, and D. Krishna, "Sector beam synthesis in linear antenna arrays using social group optimization algorithm," *Natl J Antennas Propag*, vol. 3, pp. 6–9, 2021.

[5] F. Didi and M. S. Chaouche, "Design and simulation of grid-connected photovoltaic SystemS performance analysis with optimal control of maximum power point tracking based on artificial intelligence," *Review of Computer Engineering Research*, vol. 9, no. 3, pp. 151–168, 2022.

[6] A. Molajou, V. Nourani, A. Afshar, M. Khosravi, and A. Brysiewicz, "Optimal design and feature selection by genetic algorithm for emotional artificial neural network (EANN) in rainfall-runoff modeling," *Water Resources Management*, vol. 35, no. 8, pp. 2369–2384, 2021.

[7] B. Batiha, "Numerical solution of a class of singular second-order IVPs by variational iteration method," *International Journal of Mathematical Analysis*, vol. 3, no. 37-40, pp. 1953–1968, 2009.

[8] R. Hatamleh and V. A. Zolotarev, "On model representations of non-self-adjoint operators with infinitely dimensional imaginary component," *Zurnal matematiceskoj fiziki, analiza, geometrii*, vol. 11, no. 2, pp. 174–186, 2015.

[9] R. Hatamleh and V. A. Zolotarev, "On two-dimensional model representations of one class of commuting operators," *Ukrainian Mathematical Journal*, vol. 66, no. 1, pp. 122–144, 2014.

[10] N. Nabipour, M. Dehghani, A. Mosavi, and S. Shamshirband, "Short-term hydrological drought forecasting based on different nature-inspired optimization algorithms hybridized with artificial neural networks," *IEEE Access*, vol. 8, pp. 15210–15222, 2020.

[11] A. A. A. Ahmed, N. K. A. Dwijendra, N. Bynagari, A. K. Modenov, M. Kavitha, and E. Dudukalov, "Multi project scheduling and material planning using Lagrangian relaxation algorithm," *Industrial Engineering & Management Systems*, vol. 20, no. 4, pp. 580–587, 2021.

[12] S. M. Seyedhosseini, M. J. Esfahani, and M. Ghaffari, "A novel hybrid algorithm based on a harmony search and artificial bee colony for solving a portfolio optimization problem using a mean-semi variance approach," *Journal of Central South University*, vol. 23, no. 1, pp. 181–188, 2016.

[13] S. S. Band, A. Mohammadzadeh, P. Csiba, A. Mosavi, and A. R. Varkonyi-Koczy, "Voltage regulation for photovoltaics-battery-fuel systems using adaptive group method of data handling neural networks (GMDH-NN)," *IEEE Access*, vol. 8, pp. 213748–213757, 2020.

[14] H. R. Madvar, M. Dehghani, R. Memarzadeh, E. Salwana, A. Mosavi, and S. Shahab, "Derivation of optimized equations for estimation of dispersion coefficient in natural streams using hybridized ANN with PSO and CSO algorithms," *IEEE Access*, vol. 8, pp. 156582–156599, 2020.

[15] J. Hofbauer and K. Sigmund, *The Theory of Evolution and Dynamical Systems*, Cambridge University Press, London, UK, 1988.

[16] S. Olek, "An accurate solution to the multispecies Lotka-Volterra equations," *SIAM Review*, vol. 36, no. 3, pp. 480–488, 1994.

[17] Z.-F. Lin, Y.-M. Liang, J.-Li. Zhao, and J.-R. Li, "Predicting solutions of the Lotka Volterra equation using hybrid deep network," *Theoretical and Applied Mechanics Letters*, vol. 2022, Article ID 100384, 2022.

[18] Lu Tang and S. Chen, "Traveling wave solutions for the diffusive Lotka-Volterra equations with boundary problems," *Applied Mathematics and Computation*, vol. 413, Article ID 126599, 2022.

[19] M. K. Ghaffar, F. S. Fadhel, and N. E. Arif, "Application of the generalized backstepping control method for lotka-volterra prey-predator system with constant time delay," *Journal of Physics: Conference Series*, vol. 2322, no. 1, Article ID 012012, 2022.

[20] B. Batiha, "Comparison of numerical methods for solving one species lotka-volterra equation," *Analele Universit̆at¸ii Oradea, Fasc. Matematica, Tom XIX*, vol. 19, no. 1, pp. 243–253, 2012.

[21] B. Batiha, M. S. M. Noorani, and I. Hashim, "Variational iteration method for solving multispecies Lotka-Volterra equations," *Computers & Mathematics with Applications*, vol. 54, no. 7-8, pp. 903–909, 2007.

[22] J. K. Zhou, *Differential Transformation and its Applications for Electrical Circuits*, Huazhong University Press, Wuhan, China, 1986.

[23] C. L. Chen, L. Sy-Hong, and C. K. Chen, "Application of Taylor transformation to nonlinear predictive control problem," *Applied Mathematical Modelling*, vol. 20, no. 9, pp. 699–710, 1996.

[24] C. L. Chen and Y. C. Liu, "Differential transformation technique for steady nonlinear heat conduction problems," *Applied Mathematics and Computation*, vol. 95, no. 2-3, pp. 155–164, 1998.

[25] F. Ayaz, "Applications of differential transform method to differential-algebraic equations," *Applied Mathematics and Computation*, vol. 152, no. 3, pp. 649–657, 2004.

[26] K. Batiha and B. Batiha, "A new algorithm for solving linear ordinary differential equations," *World Applied Sciences Journal*, vol. 15, no. 12, pp. 1774–1779, 2011.

[27] M. J. Jang, C. L. Chen, and Y. C. Liy, "On solving the initial value problems using the differential transformation method," *Applied Mathematics and Computation*, vol. 115, no. 2-3, pp. 145–160, 2000.

[28] S.-H. Chang and I.-L. Chang, "A new algorithm for calculating one-dimensional differential transform of nonlinear functions," *Applied Mathematics and Computation*, vol. 195, no. 2, pp. 799–808, 2008.

[29] G. Adomian, *Solving Frontier Problems of Physics: The Decomposition Method*, Kluwer Academic, Boston, Massachusetts, 1994.

[30] A. M. Wazwaz, "A new method for solving singular initial value problems in the second-order ordinary differential equations," *Applied Mathematics and Computation*, vol. 128, no. 1, pp. 45–57, 2002.

[31] W. Chen and Z. Lu, "An algorithm for Adomian decomposition method," *Applied Mathematics and Computation*, vol. 159, no. 1, pp. 221–235, 2004.

[32] J. Biazar, E. Babolian, R. Islam, and R. Islam, "Solution of a system of Volterra integral equations of the first kind by Adomian method," *Applied Mathematics and Computation*, vol. 139, no. 2-3, pp. 249–258, 2003.

[33] B. Batiha, M. S. M. Noorani, and I. Hashim, "Numerical simulation of the generalized Huxley equation by He's variational iteration method," *Applied Mathematics and Computation*, vol. 186, no. 2, pp. 1322–1325, 2007.

[34] J. H. He, "A new approach to nonlinear partial differential equations," *Communications in Nonlinear Science and Numerical Simulation*, vol. 2, no. 4, pp. 230–235, 1997.

[35] J. H. He, "Approximate analytical solution of Blasius' equation," *Communications in Nonlinear Science and Numerical Simulation*, vol. 3, no. 4, pp. 260–263, 1998.

[36] J. H. He, "Variational iteration method-a kind of non-linear analytical technique: some examples," *International Journal of Non-linear Mechanics*, vol. 34, no. 4, pp. 699–708, 1999.

[37] J. H. He, "Variational iteration method for autonomous ordinary differential systems," *Applied Mathematics and Computation*, vol. 114, no. 2-3, pp. 115–123, 2000.

[38] Z. Odibat and S. Momani, "Application of variational iteration method to nonlinear differential equations of fractional order," *International Journal of Nonlinear Sciences and Numerical Stimulation*, vol. 7, no. 1, pp. 27–34, 2006.

[39] B. Batiha, M. S. M. Noorani, and I. Hashim, "Application of variational iteration method to the generalized Burgers-Huxley equation," *Chaos, Solitons & Fractals*, vol. 36, no. 3, pp. 660–663, 2008.

[40] B. Batiha, M. S. M. Noorani, I. Hashim, and E. S. Ismail, "The multistage variational iteration method for a class of nonlinear system of ODEs," *Physica Scripta*, vol. 76, no. 4, pp. 388–392, 2007.

[41] V. Daftardar-Gejji and H. Jafari, "An iterative method for solving nonlinear functional equations," *Journal of Mathematical Analysis and Applications*, vol. 316, no. 2, pp. 753–763, 2006.

[42] V. Daftardar-Gejji and S. Bhalekar, "An Iterative method for solving fractional differential equations," *Proceedings in Applied Mathematics and Mechanics*, vol. 7, pp. 2050017-2050018, 2007.

[43] S. Bhalekar and V. Daftardar-Gejji, "New iterative method: application to partial differential equations," *Applied Mathematics and Computation*, vol. 203, no. 2, pp. 778–783, 2008.

[44] V. Daftardar-Gejji and S. Bhalekar, "Solving fractional diffusion-wave equations using a new iterative method,"

Fractional Calculus and Applied Analysis, vol. 11, no. 2, pp. 193–202, 2008.
[45] V. Daftardar-Gejji and S. Bhalekar, "Solving fractional boundary value problems with Dirichlet boundary conditions using a new iterative method," *Computers & Mathematics with Applications*, vol. 59, no. 5, pp. 1801–1809, 2010.
[46] S. Bhalekar and V. Daftardar-Gejji, "Convergence of the new iterative method," *International Journal of Differential Equations*, vol. 2011, Article ID 989065, 10 pages, 2011.
[47] S. Tauseef Mohyud-Din, A. Yildirim, and S. Mohammad Mahdi Hosseini, "Numerical comparison of methods for hirota-satsuma model," *Appl. Appl. Math.*vol. 5, no. 10, pp. 1558–1567, 2010.
[48] S. Tauseef Mohyud-Din, A. Yildirim, and M. M. Hosseini, "An iterative algorithm for fifth-order boundary value problems," *World Applied Sciences Journal*, vol. 8, no. 5, pp. 531–535, 2010.
[49] V. Srivastava and K. N. Rai, "A multi-term fractional diffusion equation for oxygen delivery through a capillary to tissues," *Mathematical and Computer Modelling*, vol. 51, no. 5-6, pp. 616–624, 2010.
[50] O. Solaymani Fard and M. Sanchooli, "Two successive schemes for numerical solution of linear fuzzy fredholm integral equations of the second kind," *Australian Journal of Basic and Applied Sciences*, vol. 4, no. 5, pp. 817–825, 2010.
[51] S. Bhalekar and V. Daftardar-Gejji, "Solving evolution equations using a new iterative method," *Numerical Methods for Partial Differential Equations*, vol. 26, no. 4, pp. 906–916, 2010.
[52] M. B. Ghori, M. Usman, and S. T. Mohyud-Din, "Numerical studies for solving a predictive microbial growth model using a new iterative Method," *International Journal of Modern Biology and Medicine*, vol. 5, no. 1, pp. 33–39, 2014.
[53] V. Daftardar-Gejji, Y. Sukale, and S. Bhalekar, "A new predictor-corrector method for fractional differential equations," *Applied Mathematics and Computation*, vol. 244, no. 2, pp. 158–182, 2014.
[54] V. Daftardar-Gejji, Y. Sukale, and S. Bhalekar, "Solving fractional delay differential equations: a new approach," *Fractional Calculus and Applied Analysis*, vol. 18, no. 2, pp. 400–418, 2015.
[55] M. Aslam Noor and K. Inayat Noor, "Three-step iterative methods for nonlinear equations," *Applied Mathematics and Computation*, vol. 183, no. 1, pp. 322–327, 2006.
[56] M. A. Noor, "New iterative schemes for nonlinear equations," *Applied Mathematics and Computation*, vol. 187, no. 2, pp. 937–943, 2007.
[57] K. I. Noor and M. A. Noor, "Iterative methods with fourth-order convergence for nonlinear equations," *Applied Mathematics and Computation*, vol. 189, no. 1, pp. 221–227, 2007.
[58] M. A. Noor and S. Tauseef Mohyud-Din, "An iterative method for solving helmholtz equations," *Arab Journal of Mathematics and Mathematical Sciences*, vol. 1, no. 1, pp. 13–18, 2007.
[59] M. A. Noor, K. I. Noor, E. Al-Said, and M. Waseem, "Some new iterative methods for nonlinear equations," *Mathematical Problems in Engineering*, vol. 2010, Article ID 198943, 12 pages, 2010.