Hindawi
International Journal of Mathematics and Mathematical Sciences
Volume 2022, Article ID 2297630, 24 pages
https://doi.org/10.1155/2022/2297630

Research Article

Approximate Solution of the Fractional Order Sterile Insect Technology Model via the Laplace–Adomian Decomposition Method for the Spread of Zika Virus Disease

William Atokolo[1] **Remigius Okeke Aja**[2] **Stephen Ekwueme Aniaku,**[3] **Ifeanyi Sunday Onah**[3] **and Godwin C. E. Mbah**[3]

[1]*Department of Mathematical Sciences, Kogi State University, Anyigba, Nigeria*
[2]*Department of Mathematics, Michael Okpara University of Agriculture, Umudike, Nigeria*
[3]*Department of Mathematics, University of Nigeria, Nsukka, Enugu State, Nigeria*

Correspondence should be addressed to Ifeanyi Sunday Onah; sunday.ifeanyi@unn.edu.ng

Received 16 February 2022; Revised 29 April 2022; Accepted 12 May 2022; Published 12 July 2022

Academic Editor: Chin-Chia Wu

Sterile insect technology (SIT) is an environmental-friendly method which depends on the release of sterile male mosquitoes that compete with the wild male mosquitoes and mate with wild female mosquitoes, which leads to the production of no offspring and as such reduces the population of Zika virus vector population over time, thereby eliminating the spread of Zika virus in a population. The fractional order sterile insect technology (SIT) model to reduce the spread of Zika virus disease is considered in this present work. We employed the use Laplace–Adomian decomposition method (LADM) to determine an analytical (approximate) solution of the model. The Laplace–Adomian decomposition method (LADM) produced a solution in form of an infinite series that further converges to the exact value. We compared solutions of the fractional model with the classical case using our plots and discovered that the fractional order has more degree of freedom and as such the system can be varied to get many preferred responses of the different classes of the model as the fraction (β) could be varied to the desired rate, say 0.7, 0.4, etc. We have been able to show that LADM can be used to solve an SIT model which has never been done before in literature.

1. Introduction

Zika is a viral infection that is usually spread in human population by the bite of infected mosquitoes. It was discovered in 1947 in Uganda [1]. The most common way to contact Zika virus is from the bites of an infected mosquito. Two species of mosquitoes spread the virus to people; the yellow fever mosquitoes (*Aedes aegypti*) and the Asian tiger mosquitoes (*Aedes albopictus*). Both are native to Texas [1, 2]. The sterile insect technology is an environmental-friendly insect pest control method involving the mass rearing and sterilization using radiation, of a target pest, followed by the systematic area-wide release of the sterile males by air over defined areas, where they mate with wild females, resulting in no offspring and a declining pest population [3, 4]. Mathematical modeling of disease transmission dates back to 1766 as was first presented by D. Bernoulli. This has metamorphosed to a deep study of disease transmission and control especially with infectious diseases like HIV/AIDS, Lassa fever, cholera, malaria, and others [5–10]. In recent years, fractional calculus has attracted great attention from researchers and different aspects of the said subject are under consideration for research; this is due to the fact that the fractional derivative is an important tool to explain the dynamical behaviour of various physical systems [11]. The strength of these differential operator as presented in [11] is their nonlocal characteristics which do not exist in the integer order differential operators and that fractional order models are more realistic and practical than the classical integer order model.

The technique of Laplace–Adomian decomposition method (LADM) involves the combination of the Adomian decomposition method (ADM) and the Laplace transforms. Adomian's is an effective technique for obtaining solutions of model or a system of ordinary differential equations. Laplace transform is an efficient method used in engineering and applied sciences. The coupling of these two methods leads to the Laplace–Adomian decomposition method (LADM). The Laplace–Adomian decomposition method has been applied to many problems in physics, biology, applied mathematics, and engineering [11]. The basic idea of this method is to assume an infinite solution of the kind: $q = \sum_{n=0}^{\infty} q_n$, then apply Laplace transformation to the differential equation. The nonlinear terms of the model are then decomposed in terms of the Adomian polynomials, and then an iterative procedure is formulated for the determination of the (q_n) in a recursive form. This method can be used for a system of linear and nonlinear ordinary and partial differential equations of the classical and fractional order. The method does not require any perturbation and also has no need for a predefined step size. The method is an effective method for numerical and explicit solutions of a system of differential equations representing physical problems [11].

Fazal et al. [11] presented a numerical solution of the fractional order smoking model via the Laplace–Adomian decomposition method; the model solution was obtained in form of an infinite series which converges rapidly to its exact value. Ogun [12] presented the Laplace–Adomian decomposition method to solve a model for HIV infection where the approximate solution of the model was determined. Adejoh and Mbah [13] also presented the application of fractional differential equations to obtain an approximate and numerical solution of a cancer disease model incorporating control measures. The Laplace–Adomian decomposition method was also used by Fazal et al. [14] to determine a numerical solution of a fractional order epidemic model of a childhood disease. The Laplace–Adomian decomposition method unlike other numerical methods requires no discretization and linearization and as such the results obtained from it are more effective and realistic. In fact, models such as the ones in [11–14] are veritable tools toward studying the application of LADM in solving linear and nonlinear differential equations.

The fractional order model gives a better description of the entire space of a system; unlike the integer model that describes only the local properties of a system, it also gives a better description of a real system with memory effects [15–18]. The Caputo derivative and the Riemann–Liouville derivative are regarded as singular kernels fractional derivative relative to biological problems, we also have others which are nonsingular such as Mittag-Leffler and the Atangana–Baleanu operators [17, 19]. The fractional order and classical derivatives have been used in studying transmission dynamics in SIR models and the like, but it has not been adopted in an SIT model. Also, an SIT model solution has not been determined using the LADM. So, in this work, we consider a fractional order model (using the Caputo derivative), which is an extension of the classical order sterile insect technology model for the control of Zika virus disease presented by Atokolo et al. [1], whose approximate solution would be determined using the Laplace–Adomian decomposition method (LADM).

2. Model Formulation and Procedures

The mosquito life cycle is generally divided into two stages, the aquatic and nonaquatic class. The aquatic class is denoted by a single compartment (A). The nonaquatic mosquito class is divided into seven compartments consisting of the male mosquitoes (M_M), female mosquitoes not yet laying eggs (F_M), female nonsterile mosquitoes, (F_{NM}), female sterile mosquitoes (F_{SM}), sterile male mosquitoes (M_S), female infected nonsterile mosquitoes, (F_{INM}), and female infected sterile mosquitoes (F_{ISM}). The human population is divided into susceptible human (S_H), exposed human (E_H), infected human (I_H), infected human but on treatment (I_{HT}), and recovered human (R_H). The aquatic stage of the mosquitoes which consists of eggs, larva, and pupae population increases from the oviposition by reproductive mosquitoes. It reduces due to natural death of the mosquitoes at the rate of (μ_A) and by density dependence death rate of (μ_ρ). The female mosquitoes (F_M) are recruited at the rate of $(A\phi\gamma)$, where (γ) is the maturity rate of aquatic mosquitoes to adult mosquitoes and (ϕ) is the proportion of emerging females; it is reduced by the mating rate at the level of (β) for female mosquitoes to be with wild male mosquitoes or sterile male mosquitoes with mating probabilities (ρ_ω) and (ρ_s), respectively. The population is reduced by death induced due to the attempt to seek for blood meals at the rate of (δ_M) and finally reduced by natural death at the rate of (μ_F). The male mosquitoes (M_M) are recruited by the proportion of the emerging male mosquitoes $(1 - \phi)$ that mature to adult mosquitoes at the rate of (γ) which also reduces by natural death at the rate of (μ_M). The female nonsterile mosquito (F_{NM}) population is increased by the female mosquitoes' probability to mate with the wild male mosquitoes which is given by the rate $(M_M/M_M + M_S)$, with a mating rate of (β_1). This population is reduced by (ω_1), the rate at which the female nonsterile mosquitoes (F_{NM}) are infected and moved to the female infected nonsterile mosquitoes class (F_{INM}). It is also reduced by the death induced due to the attempt to seek for blood meals at the rate of (δ_M);

this class is finally reduced by natural death rate at the rate of (δ_M). The female sterile mosquito (F_{SM}) population is increased by the wild female mosquitoes' probability to mate with the sterile mosquitoes which is given by the rate ($M_S/M_M + M_S$), with a mating rate of (β_2). The class reduces by (ω_2), the rate at which the (F_{SM}) becomes infected and moves to the (F_{ISM}) class. The class reduces by death induced due to the attempt to seek for blood meals at the rate of (δ_M) and reduces finally by natural death at the rate of (μ_F). The population of female infected nonsterile mosquitoes (F_{INM}) is recruited at the rate at which the female nonsterile mosquitoes (F_{NM}) are infected at the rate of (ω_1) The population is reduced by death induced due to the attempt to seek for blood meals at the rate of (δ_M) and finally reduced by natural death at the rate of (μ_F). The population of female infected sterile mosquitoes (F_{ISM}) is recruited at the rate at which the female sterile mosquitoes (F_{SM}) are infected at the rate of (ω_2). The population is reduced by death induced due to the attempt to seek for blood meals at the rate of (δ_M) and finally by natural death at the rate of (μ_F). The sterile male mosquitoes (M_S) are released into the population at the rate (Λ_2). However, due to some environmental and geographical factors that may affect the mixing of sterile and wild mosquitoes, such as location of mosquitoes breeding site, it is convenient to assume that only a fraction (p) of the released mosquitoes will join wild mosquitoes population. Second, because of the differences in physiology of wild and sterile mosquitoes, a parameter (q) is used to capture the mean mating competitiveness of sterile mosquitoes, so that the actual number of sterile male mosquitoes competing with wild mosquitoes is (pqM_S), and as such, the available injected sterile male mosquitoes (M_S) into the wild population of mosquitoes that can competitively mate with wild female mosquitoes is ($pq\Lambda_2$). This population is reduced by natural death at the rate of (μ_S). The susceptible human population is recruited at the level of (Λ_3), of which a fraction (l) of those infected at birth joined the infectious human population. The population reduces by the rate at which infectious mosquitoes (female infected nonsterile mosquitoes (F_{INM}) or female infected sterile mosquitoes (F_{ISM})) infects susceptible human at the levels of (α_1) and (α_2), respectively. Also, it reduces by the rate at which the infectious human (infected class (I_H), recovered class (R_H) or infected but on treatment class (I_{HT})](I_{HT}) infects susceptible human through sex at the level of or (α_3) or (α_4), (α_5), respectively. This is in line with the clinical studies that high viral load was found in the semen and saliva of recovered patients weeks after recovery, (WHO, 2016), which means, Zika can be transmitted sexually. The population finally reduces by natural death at the rate of (μ_H). The population of the exposed human (E_H) is generated by infection of susceptible individuals at the rate of (α). This population reduces by natural death at the of rate (μ_H) and by the rate at which the exposed are finally infectious at the rate of (σ). The infected human (I_H) class is generated by the incoming of infected babies from infected mothers at the rate of (Λ_3), due to vertical transmission. In addition, the population increases at the rate by which the exposed become infected at the level (σ). The class reduces at the rate (θ) by which the infected are taken for treatment and by natural recovery rate of (τ_1). This class reduces finally by both natural and disease-induced death rates at the levels of (μ_H) and (δ_1), respectively. The infected but on treatment class (I_{HT}) is recruited by the incoming of the infected who are taken for treatment at the rate of (θ); this class reduces at the rate by which the infected but on treatment class recovers due to supportive treatment at the rate of (τ_2). It reduces finally by natural death and disease induced death at the rates of (μ_H) and (δ_2), respectively, where (δ_2) is assumed to be less than (δ_1). The recovered human is recruited at the rate by which the infected human recovers naturally at the rate of (τ_1) or due to supportive treatment at the rate of (τ_2). The population reduces by natural death at the rate of (μ_H).

2.1. Assumptions of the Model

(1) Zika virus can be transmitted through the bite of Aedes mosquitoes or through sexual activities with an infected human.

(2) There is both vertical and horizontal transmission.

(3) The mating competitiveness of the sterile and nonsterile mosquitoes are not equal.

(4) Mosquitoes do not recover from infection.

(5) Aquatic mosquitoes have a density dependent death rate which is a nonlinear decreasing function.

(6) Aquatic and nonaquatic mosquitoes do not have the same death rate.

(7) Female mosquitoes do not have the same death rate with male mosquitoes.

(8) There is a disease-induced death rate by the female mosquitoes seeking blood meals.

2.2. Mathematical Model. The mathematical equations that incorporate the above formulations, assumptions, and from Figure 1, we have the following:

$$
\begin{aligned}
\frac{dA}{dt} &= \Lambda_1\left(F_{NM}+F_{INM}\right)-(1-\phi)\gamma A-\phi\gamma A-\mu_A A-\mu_\rho A^2,\\
\frac{dF_M}{dt} &= \phi\gamma A-\left[\frac{\beta_1 M_M}{M_M+M_S}+\frac{\beta_2 M_S}{M_M+M_S}\right]F_M-\delta_M F_M-\mu_F F_M,\\
\frac{dM_M}{dt} &= (1-\phi)\gamma A-\mu_M M_M,\\
\frac{dF_{SM}}{dt} &= \frac{\beta_1 M_M}{M_M+M_S}F_M-\omega_1 F_{NM}-\delta_M F_{NM}-\mu_F F_{NM},\\
\frac{dF_{INM}}{dt} &= \frac{\beta_2 M_M}{M_M+M_S}F_M-\omega_2 F_{SM}-\delta_M F_{SM}-\mu_F F_{SM},\\
\frac{dF_{ISM}}{dt} &= \omega_1 F_{NM}-\delta_M F_{INM}-\mu_F F_{INM},\\
\frac{dM_S}{dt} &= \omega_2 F_{SM}-\delta_M F_{ISM}-\mu_F F_{ISM},\\
\frac{dS_H}{dt} &= pq\Lambda_2-\mu_S M_S,\\
\frac{dE_H}{dt} &= (1-\ell)\Lambda_3-\alpha S_H-\mu_H S_H,\\
\frac{dE_H}{dt} &= \alpha S_H-\sigma E_H-\mu_H E_H,\\
\frac{dI_H}{dt} &= \ell\Lambda_3+\sigma E_H-\tau_1 I_H-\theta I_H-\partial_1 I_H-\mu_H I_H,\\
\frac{dI_{HT}}{dt} &= \theta I_H-\tau_2 I_{HT}-\partial_2 I_{HT}-\mu_H I_{HT},\\
\frac{dR_H}{dt} &= \tau_1 I_H-\tau_2 I_{HT}-\mu_H R_H,
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
\alpha &= \frac{\left(\alpha_1 F_{INM}+\alpha_2 F_{ISM}+\alpha_3 I_H+\alpha_4 I_{HT}+\alpha_5 R_H\right.}{N_H},\\
\omega &= \frac{\lambda_1 I_H+\alpha_2 I_{HT}}{N_H},\\
\omega &= \omega_1, = \omega_2.
\end{aligned}
\tag{2}
$$

From the earlier assumption, the mating competitiveness of both sterile and nonsterile mosquitoes are not equal, that is, $(\beta_1)\neq(\beta_2)$.

Also, from (1), we let $M_S/(M_M+M_S)=\rho_S$ and $M_M/(M_M+M_S)=\rho_\omega$.

Therefore, from the second equation of (1), we have as follows:

$$
\frac{\beta_1 M_M}{M_M+M_S}+\frac{\beta_2 M_S}{M_M+M_S}=\beta_1\rho_\omega+\beta_2\rho_s,
\tag{3}
$$

where (ρ_s) is the female mating probability with the sterile mosquitoes and (ρ_ω) is the female mating probability with the wild male mosquitoes. Moreso, the rate of infection of F_{NM} and F_{SM} after biting an infectious human and then moving to the F_{INM} and F_{ISM} classes, respectively, is equal, that is, to say $\omega=\omega_1=\omega_2$. We also decoupled the sterile male mosquitoes population (M_S) equation from the entire system since it is independent of other compartments and the size of its population is controlled by human intervention. Hence, we can rewrite the mathematical equations that represent the sterile insect technology (SIT) model for the control of Zika virus disease as presented by Atokolo et al. in [1] as follows:

$$
\begin{aligned}
\frac{dA}{dt} &= \Lambda_1\left(F_{NM} + F_{INM}\right) - (1-\phi)\gamma A - \phi\gamma A - \mu_A A - \mu_\rho A^2,\\
\frac{dF_M}{dt} &= \phi\gamma A - \left[\frac{\beta_1 M_M}{M_M + M_S} + \frac{\beta_2 M_S}{M_M + M_S}\right] F_M - \delta_M F_M - \mu_F F_M,\\
\frac{dM_M}{dt} &= (1-\phi)\gamma A - \mu_M M_M,\\
\frac{dF_{SM}}{dt} &= \frac{\beta_1 M_M}{M_M + M_S} F_M - \omega_1 F_{NM} - \delta_M F_{NM} - \mu_F F_{NM},\\
\frac{dF_{INM}}{dt} &= \frac{\beta_2 M_M}{M_M + M_S} F_M - \omega_2 F_{SM} - \delta_M F_{SM} - \mu_F F_{SM},\\
\frac{dF_{ISM}}{dt} &= \omega_1 F_{NM} - \delta_M F_{INM} - \mu_F F_{INM},\\
\frac{dM_S}{dt} &= \omega_2 F_{SM} - \delta_M F_{ISM} - \mu_F F_{ISM},\\
\frac{dS_H}{dt} &= pq\Lambda_2 - \mu_S M_S,\\
\frac{dE_H}{dt} &= (1-\ell)\Lambda_3 - \alpha S_H - \mu_H S_H,\\
\frac{dE_H}{dt} &= \alpha S_H - \sigma E_H - \mu_H E_H,\\
\frac{dI_H}{dt} &= \ell\Lambda_3 + \sigma E_H - \tau_1 I_H - \theta I_H - \partial_1 I_H - \mu_H I_H,\\
\frac{dI_{HT}}{dt} &= \theta I_H - \tau_2 I_{HT} - \partial_2 I_{HT} - \mu_H I_{HT},\\
\frac{dR_H}{dt} &= \tau_1 I_H - \tau_2 I_{HT} - \mu_H R_H,
\end{aligned}
\tag{4}
$$

where

$$
\begin{aligned}
\alpha &= \frac{\alpha_1 F_{INM} + \alpha_2 F_{ISM} + \alpha_3 I_H + \alpha_4 I_{HT} + \alpha_5 R_H}{N_H},\\
\omega &= \frac{\lambda_1 I_H + \lambda_2 I_{HT}}{N_H}.
\end{aligned}
\tag{5}
$$

The model variables and parameters descriptions are presented in Appendix.

2.3. Fractional Order Zika Model. The Caputo derivative is considered as a differential operator in our model. In a Caputo fractional initial value problem, the initial condition can be expressed with an initial integer order, whose physical interpretation is very easy for real life interpretation, and hence is suitable for the Zika virus model. We now present in this section, some well known definitions and results that we will use throughout this paper.

Definition 1. The Caputo fractional order derivative of a function (f) on the interval $[0, T]$ is defined as follows:

$$
\left[{}^{c}D_0^{\beta} f(t)\right] = \frac{1}{\Gamma(n-\beta)} \int_0^t (t-s)^{n-\beta-1} f^{(n)}(s) ds, \tag{6}
$$

where $n = [\beta] + 1$ and $[\beta]$ represent the integer parts of β. In particularly, for $0 < \beta < 1$, the Caputo derivative becomes as follows:

$$
\left[{}^{c}D_0^{\beta} f(t)\right] = \frac{1}{\Gamma(n-\beta)} \int_0^t \frac{f(s)}{(t-s)^{\beta}} ds. \tag{7}
$$

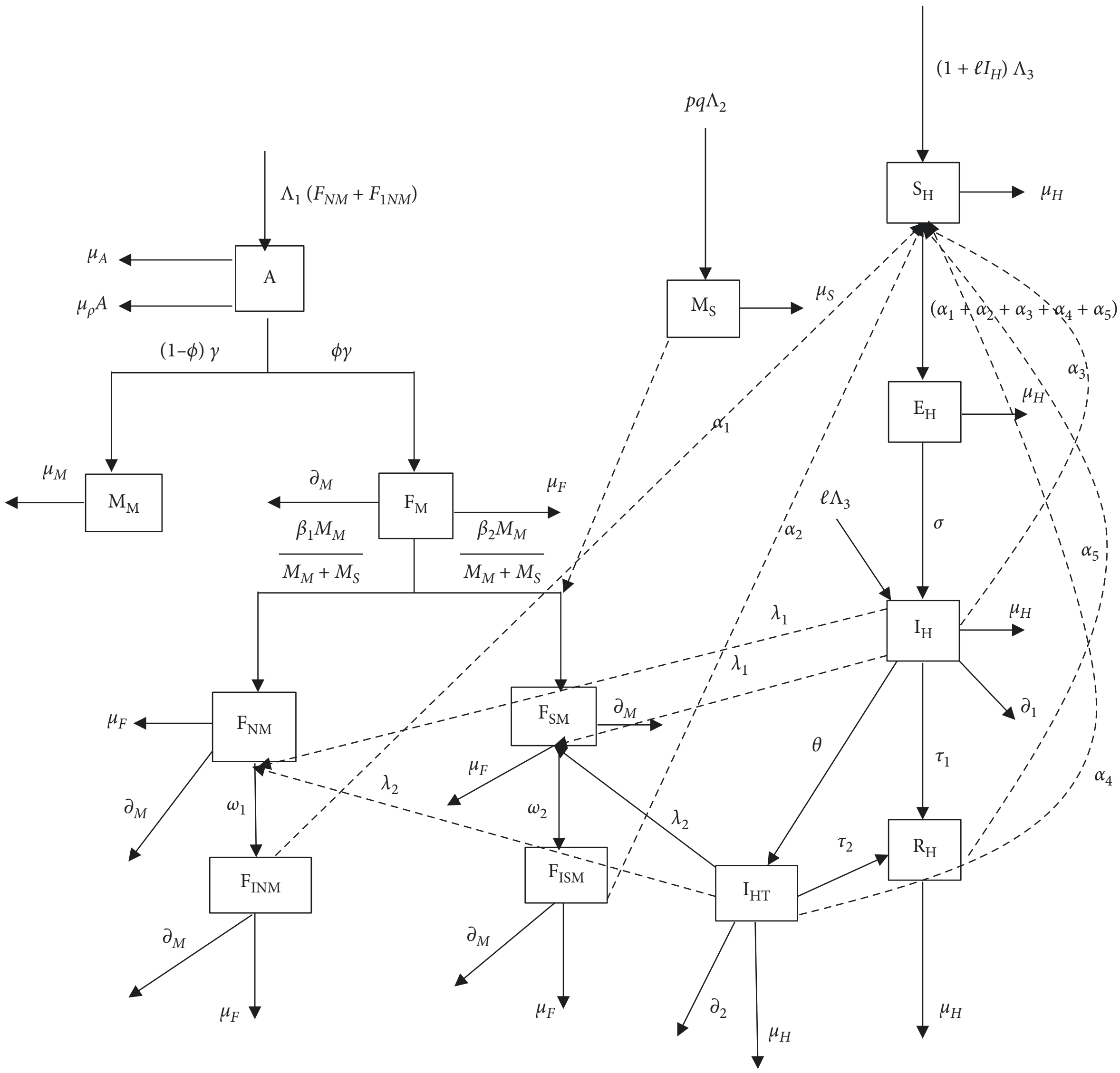


Figure 1: Flow diagram.

Definition 2. Laplace transform of Caputo derivatives is defined as follows:

$$\mathscr{L}\left[{}^{c}D^{\beta}q(t)\right] = S^{\beta}h(S) - \sum_{k=0}^{n} S^{\beta-i-1}y^{k}(0), \quad n-1<\beta<n, n\in N, \tag{8}$$

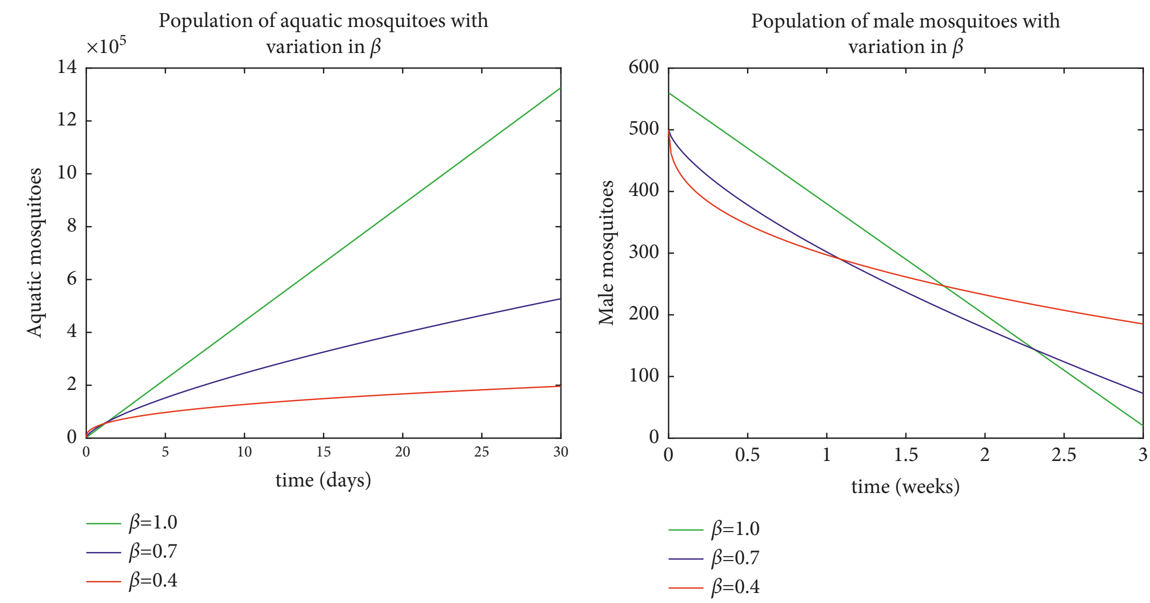
for arbitrary $c_i \in \mathbf{R}$, $i=0,1,2,\ldots,n-1$, where $n=[\beta]+1$ and $[\beta]$ represent the noninteger parts of β.

Lemma 1. *The following result holds for fractional differential equations:*

$$I^{\beta}\left[{}^{c}D^{\beta}h\right](t) = h(t) + \sum_{i=0}^{n-1}\frac{h^{(i)}(0)}{i!}t^{i}, \tag{9}$$

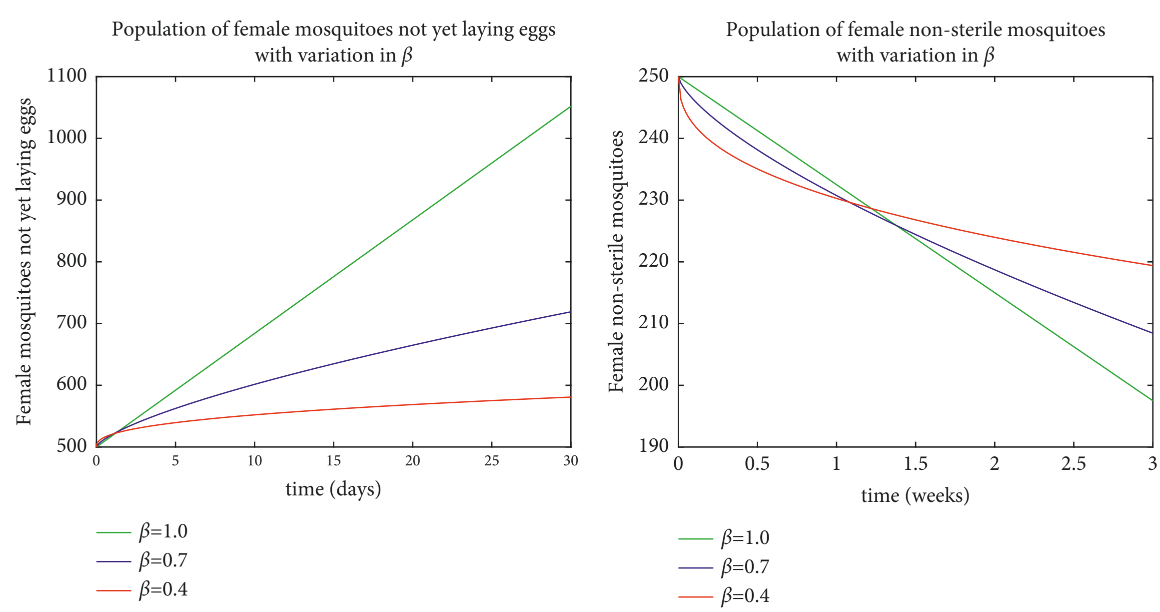
for arbitrary $\beta>0$, $i=0,1,2,\ldots,n-1$, *where* $n=[\beta]+1$ *and* $[\beta]$ *represent the integer parts of* β.

Introducing the fractional order into the model (4), we now present a new model described by the following set of fractional differential equations of order (β), for $0<\beta<1$



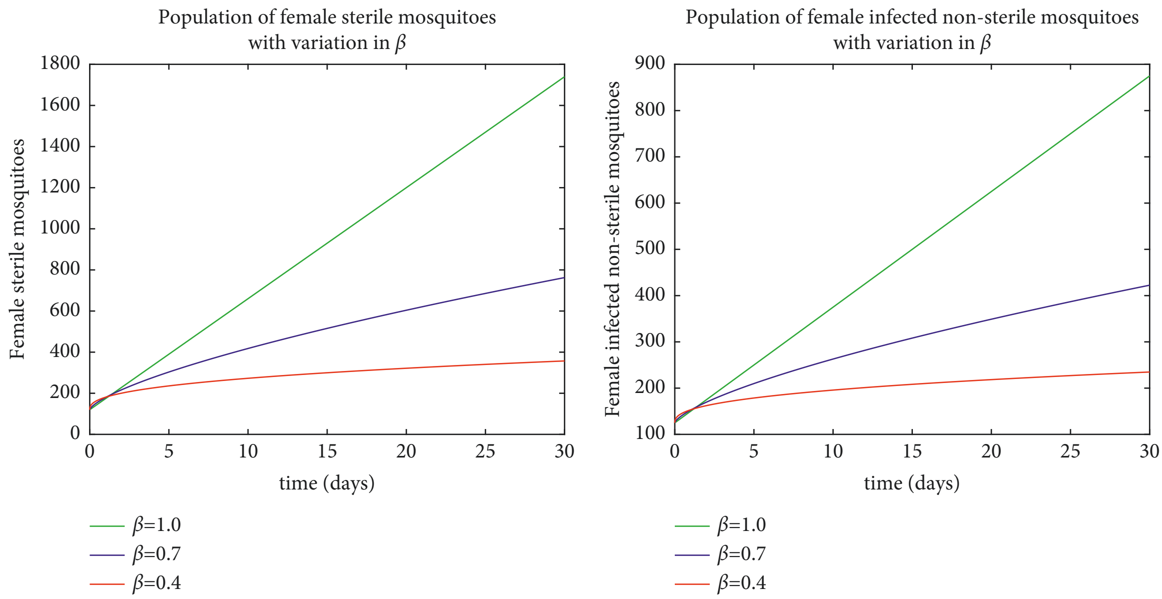
(a) (b)

Figure 2: .Population of aquatic mosquitoes and male mosquitoes with variations in values of β. (a) Population of aquatic mosquitoes with variation of β. (b) Population of male mosquitoes with different values of β.



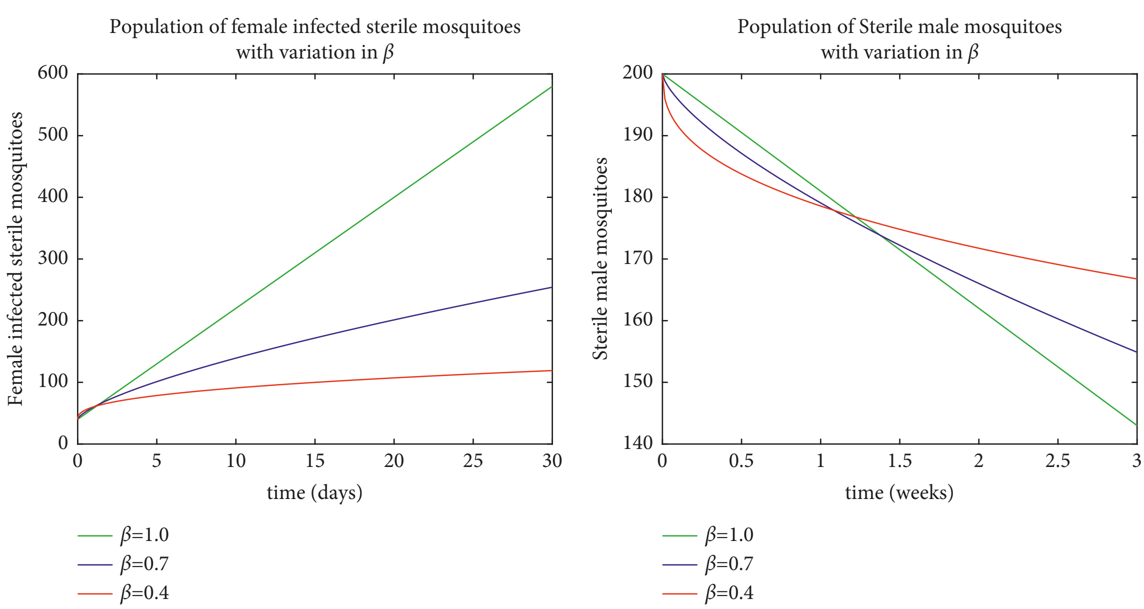
(a) (b)

Figure 3: .Population of female mosquitoes not yet laying eggs and female nonsterile mosquitoes with variation in β. (a) Population of female mosquitoes not yet laying eggs with variation of β. (b) Population of female nonsterile mosquitoes with different values of β.



(a) (b)

Figure 4: population of female sterile mosquitoes and female infected nonsterile mosquitoes with variation in β. (a) Population of female sterile mosquitoes with variation of β. (b) Population of female infected nonsterile mosquitoes with different values of β.



(a) (b)

Figure 5: Population of female infected sterile mosquitoes and sterile male mosquitoes with variation in β. (a) Population of female infected sterile mosquitoes with variation of β. (b) The population of sterile male mosquitoes with different values of β.

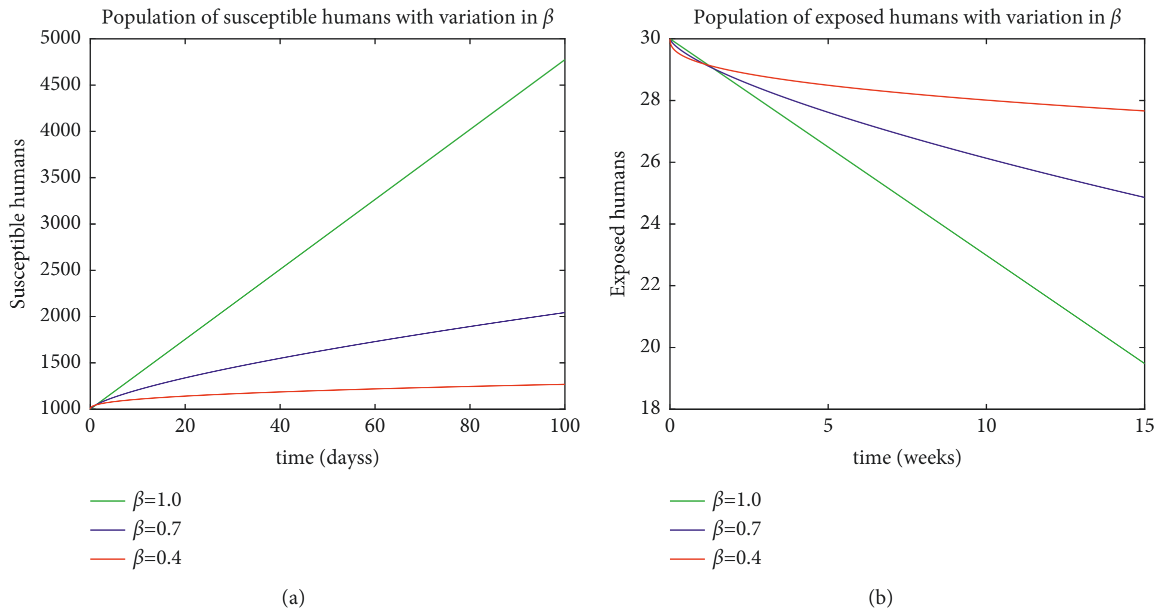


FIGURE 6: Population of susceptible humans and exposed humans with variation in β. (a) Population of susceptible humans with variation of β. (b) Population of exposed humans with different values of β.

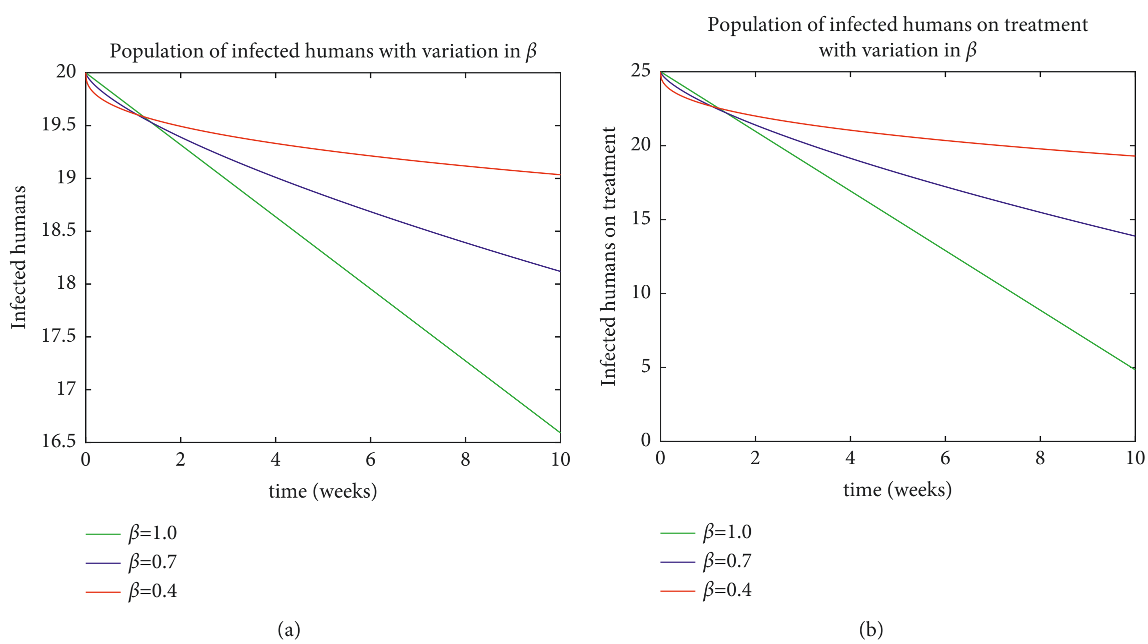


FIGURE 7: Population of infected humans and infected humans on treatment with variation in β. (a) Population of infected humans with variation of β. (b)Population of infected humans on treatment with different values of β.

Table 1: Variables of the model (2) and their meanings.

S/N	Variables	Descriptions
1	A	Aquatic mosquito
2	M_M	Male mosquitoes(wild)
3	F_M	Female mosquitoes not yet laying eggs
4	F_{NM}	Female non-sterile mosquitoes(can lay and hatch eggs)
5	F_{SM}	Female sterile mosquitoes(can lay but do not hatch)
6	M_S	Sterile male mosquitoes
7	F_{INM}	Female infected non-sterile mosquitoes
8	F_{ISM}	Female infected sterile mosquitoes
9	S_H	Susceptible human
10	E_H	Exposed human
11	I_H	Infected human
12	I_{HT}	Infected but on treatment human
13	R_H	Recovered human

Table 2: Parameters of the model (2) and their meanings.

Parameters	Meanings
Λ_1	Oviposition level of fertilized female mosquitoes
ϕ	Proportion of emerging female mosquitoes
$1-\phi$	Male mosquitoes emerging population
β_i	Mating rate, where $i = 1, 2$
γ	Maturity rate of mosquitoes
μ_M	Natural death rate of wild male mosquitoes
μ_S	Natural death rate of sterile mosquitoes
μ_ρ	Density dependent death rate of the aquatic mosquitoes class
μ_H	Natural death rate of human
μ_A	Natural death rate for aquatic mosquitoes
μ_F	Natural death rate for female mosquitoes
δ_M	Death induced rate due to attept by female mosquitoes seeking for blood
ρ_ω	Female mosquitoes probability to mate with wild male
ρ_s	Female mosquitoes probability to mate with sterile male
∂_1	Disease induced death rate for infected class
∂_2	Disease induced death rate for infected but on treatment class
τ_1	Natural recovery rate for human
θ	Rate at which the infected human are taken for treatment
τ_2	Recovery rate of the infected but on treatment due to supportive treatment
ℓ	Fraction of infected at birth that joined the susceptible class
σ	Rate at which the exposed becomes infectious
Λ_3	Recruitment level into the susceptible human class
p	Fraction of the released sterile mosquitoes that joined the wild male
q	Mean mating competitiveness of the sterile male mosquitoes
α	Force of infection for human population
α_1 & α_2	Rate at which the (F_{INM}) and the (F_{ISM}) infects susceptible humans, respectively.
α_3	Rate at which the infected human infects susceptible human through sex
α_4	Rate at which the recovered human infects susceptible human through sex
α_5	Rate at which the infected human infects susceptible human through sex
ω	Force of infection for mosquito population
ω_1 & ω_2	Rate at which the F_{NM} moves to F_{INM} and rate at which the F_{SM} moves to F_{ISM}, respectively.
λ_1	Rate at which (I_H) humans infects susceptible mosquitoes (F_{SM}
λ_2	Rate at which (I_{HT}) humans infects susceptible mosquitoes (F_{SM} and F_{NM}).

$$
\begin{aligned}
D^{\beta}(A) &= \Lambda_1\left(F_{NM}+F_{INM}\right)-\gamma A-\mu_A A-\mu_\rho A^2,\\
D^{\beta}\left(F_M\right) &= \phi\gamma A-\left[\beta_1\rho_\omega+\beta_2\rho_s\right]F_M-\delta_M F_M-\mu_F F_M,\\
D^{\beta}\left(M_M\right) &= (1-\phi)\gamma A-\mu_M M_M,\\
D^{\beta}\left(F_{NM}\right) &= \beta_1\rho_\omega F_M-\omega F_{NM}-\delta_M F_{NM}-\mu_F F_{NM},\\
D^{\beta}\left(F_{SM}\right) &= \beta_2\rho_s F_M-\omega F_{SM}-{}_{\delta}M F_{SM}-\mu_F F_{SM},\\
D^{\beta}\left(F_{INM}\right) &= \omega F_{NM}-\delta_M F_{INM}-\mu_F F_{INM},\\
D^{\beta}\left(F_{ISM}\right) &= \omega F_{SM}-\delta_M F_{ISM}-\mu_F F_{ISM},\\
D^{\beta}\left(S_H\right) &= (1-\ell)\Lambda_3-\alpha S_H-\mu_H S_H,\\
D^{\beta}\left(E_H\right) &= \alpha S_H-\sigma E_H-\mu_H E_H,\\
D^{\beta}\left(I_H\right) &= \ell\Lambda_3+\sigma E_H-\tau_1 I_H-\theta I_H-\partial_1 I_H-\mu_H I_H,\\
D^{\beta}\left(I_{HT}\right) &= \theta I_H-\tau_2 I_{HT}-\partial_2 I_{HT}-\mu_H I_{HT},\\
D^{\beta}\left(R_H\right) &= \tau_1 I_H-\tau_2 I_{HT}-\mu_H R_H,
\end{aligned}
\tag{10}
$$

where

$$
\begin{aligned}
\alpha &= \frac{\left(\alpha_1 F_{INM}+\alpha_2 F_{ISM}+\alpha_3 I_H+\alpha_4 I_{HT}+\alpha_5 R_H\right)}{N_H},\\
\omega &= \frac{\lambda_1 I_H+\lambda_2 I_{HT}}{N_H}.
\end{aligned}
\tag{11}
$$

3. Laplace–Adomian Decomposition Method(LADM) Implementation

In this section, we discuss the general procedure of this method with the given initial conditions. Applying the Laplace transform to both sides of (10), we obtain the following:

$$
\begin{aligned}
S^{\beta}\mathscr{L}(A)-S^{\beta-1}A(0) &= \mathscr{L}\left[\Lambda_1\left(F_{NM}+F_{INM}\right)-\gamma A-\mu_A A-\mu_\rho A^2\right],\\
S^{\beta}\mathscr{L}\left(F_M\right)-S^{\beta-1}\left(F_M\right)(0) &= \mathscr{L}\left[\phi\gamma A-\left[\beta_1\rho_\omega+\beta_2\rho_s\right]F_M-\delta_M F_M-\mu_F F_M\right],\\
S^{\beta}\mathscr{L}\left(M_M\right)-S^{\beta-1}\left(M_M\right)(0) &= \mathscr{L}\left[(1-\phi)\gamma A-\mu_M M_M\right],\\
S^{\beta}\mathscr{L}\left(F_{NM}\right)-S^{\beta-1}\left(F_{NM}\right)(0) &= \mathscr{L}\left[\beta_1\rho_\omega F_M-\frac{\left(\lambda_1 I_H+\lambda_2 I_{HT}\right)}{N_H}F_{NM}-\delta_M F_{NM}-\mu_F F_{NM}\right],\\
S^{\beta}\mathscr{L}\left(F_{SM}\right)-S^{\beta-1}\left(F_{SM}\right)(0) &= \mathscr{L}\left[\beta_2\rho_s F_M-\frac{\left(\lambda_1 I_H+\lambda_2 I_{HT}\right)}{N_H}F_{SM}-\delta_M F_{SM}-\mu_F F_{SM}\right],\\
S^{\beta}\mathscr{L}\left(F_{INM}\right)-S^{\beta-1}\left(F_{INM}\right)(0) &= \mathscr{L}\left[\frac{\left(\lambda_1 I_H+\lambda_2 I_{HT}\right)}{N_H}F_{NM}-\delta_M F_{INM}-\mu_F F_{INM}\right],\\
S^{\beta}\mathscr{L}\left(F_{ISM}\right)-S^{\beta-1}\left(F_{ISM}\right)(0) &= \mathscr{L}\left[\frac{\left(\lambda_1 I_H+\lambda_2 I_{HT}\right)}{N_H}F_{SM}-\delta_M F_{ISM}-\mu_F F_{ISM}\right],\\
S^{\beta}\mathscr{L}\left(S_H\right)-S^{\beta-1}\left(S_H\right)(0) &= \mathscr{L}\left[(1-\ell)\Lambda_3-\frac{\left(\alpha_1 F_{INM}+\alpha_2 F_{ISM}+\alpha_3 I_H+\alpha_4 I_{HT}+\alpha_5 R_H\right)}{N_H}S_H-\mu_H S_H\right],\\
S^{\beta}\mathscr{L}\left(E_H\right)-S^{\beta-1}\left(E_H\right)(0) &= \mathscr{L}\left[\frac{\left(\alpha_1 F_{INM}+\alpha_2 F_{ISM}+\alpha_3 I_H+\alpha_4 I_{HT}+\alpha_5 R_H\right)}{N_H}S_H-\sigma E_H-\mu_H E_H\right],\\
S^{\beta}\mathscr{L}\left(I_H\right)-S^{\beta-1}\left(I_H\right)(0) &= \mathscr{L}\left[\ell\Lambda_3+\sigma E_H-\tau_1 I_H-\theta I_H-\partial_1 I_H-\mu_H I_H\right],\\
S^{\beta}\mathscr{L}\left(I_{HT}\right)-S^{\beta-1}\left(I_{HT}\right)(0) &= \mathscr{L}\left[\theta I_H-\tau_2 I_{HT}-\partial_2 I_{HT}-\mu_H I_{HT}\right],\\
S^{\beta}\mathscr{L}\left(R_H\right)-S^{\beta-1}\left(R_H\right)(0) &= \mathscr{L}\left[\tau_1 I_H-\tau_2 I_{HT}-\mu_H R_H\right],
\end{aligned}
\tag{12}
$$

Table 3: Numerical values of variables and parameters used for implementation of LADM.

Variables/parameters	Value	Source
A	2500	[1]
M_M	160	[1]
F_M	500	[1]
F_{ISM}	40	[1]
S_H	1000	[1]
E_H	30	[1]
μ_ρ	$0.00002\, day^{-1}$	[15]
ϕ	$0.6\, day^{-1}$	[16]
β_1	$0.4\, day^{-1}$	Assumed
ρ_s	$0.4\, day^{-1}$	[3]
μ_F	$0.27\, day^{-1}$	Assumed
μ_M	$0.26\, day^{-1}$	Assumed
∂_1	$0.002\, day^{-1}$	[1]
δ_M	$0.03\, day^{-1}$	[1]
θ	$0.002\, day^{-1}$	Assumed
ℓ	$0.05\, day^{-1}$	Assumed
Λ_3	$40\, day^{-1}$	[1]
α_2	$0.0001\, day^{-1}$	[1]
α_4	$0.07\, day^{-1}$	[1]
λ_1	$0.09\, day^{-1}$	[21]
F_{NM}	250	[1]
F_{SM}	120	[1]
F_{INM}	125	[1]
I_H	20	[1]
I_{HT}	15	[1]
R_H	0	[1]
Λ_1	$120\, day^{-1}$	[16]
γ	$0.06\, day^{-1}$	[17]
ρ_ω	$0.6\, day^{-1}$	[1]
β_2	$0.6\, day^{-1}$	[3]
μ_A	$0.25\, day^{-1}$	[18]
μ_H	$0.00005\, day^{-1}$	[18]
∂_2	$0.001\, day^{-1}$	[1]
τ_1	$0.14\, day^{-1}$	[19]
τ_1	$0.16\, day^{-1}$	[19]
σ	$0.03\, day^{-1}$	[20]
α_1	$0.0002\, day^{-1}$	[1]
α_3	$0.09\, day^{-1}$	[16]
α_5	$0.05\, day^{-1}$	Assumed
λ_2	$0.07\, day^{-1}$	[1]

with initial conditions

$$\begin{aligned} A(0) &= n_1, & F_{ISM}(0) &= n_7, \\ F_M(0) &= n_2, & S_H(0) &= n_8, \\ M_M(0) &= n_3, & E_H(0) &= n_9, \\ F_{NM}(0) &= n_4, & I_H(0) &= n_{10}, \\ F_{SM}(0) &= n_5, & I_{HT}(0) &= n_{11}, \\ F_{INM}(0) &= n_6, & R_H(0) &= n_{12}. \end{aligned} \tag{13}$$

Dividing (12) by (S^{β}) yields

$$
\begin{aligned}
\mathscr{L}(A)(t) &= \frac{n_1}{S} + \frac{1}{S^{\beta}}\mathscr{L}\left[\Lambda_1\left(F_{NM} + F_{INM}\right) - \gamma A - \mu_A A - \mu_\rho A^2\right],\\
\mathscr{L}(F_M)(t) &= \frac{n_2}{S} + \frac{1}{S^{\beta}}\mathscr{L}\left[\phi\gamma A - [\beta_1\rho_\omega + \beta_2\rho_s]F_M - \delta_M F_M - \mu_F F_M\right],\\
\mathscr{L}(M_M)(t) &= \frac{n_3}{S} + \frac{1}{S^{\beta}}\mathscr{L}\left[(1-\phi)\gamma A - \mu_M M_M\right],\\
\mathscr{L}(F_{NM})(t) &= \frac{n_4}{S} + \frac{1}{S^{\beta}}\mathscr{L}\left[\beta_1\rho_\omega F_M - \frac{(\lambda_1 I_H + \lambda_2 I_{HT})}{N_H}F_{NM} - \delta_M F_{NM} - \mu_F F_{NM}\right],\\
\mathscr{L}(F_{SM})(t) &= \frac{n_5}{S} + \frac{1}{S^{\beta}}\mathscr{L}\left[\beta_2\rho_s F_M - \frac{(\lambda_1 I_H + \lambda_2 I_{HT})}{N_H}F_{SM} - \delta_M F_{SM} - \mu_F F_{SM}\right],\\
\mathscr{L}(F_{INM})(t) &= \frac{n_6}{S} + \frac{1}{S^{\beta}}\mathscr{L}\left[\frac{(\lambda_1 I_H + \lambda_2 I_{HT})}{N_H}F_{NM} - \delta_M F_{INM} - \mu_F F_{INM}\right],\\
\mathscr{L}(F_{ISM})(t) &= \frac{n_7}{S} + \frac{1}{S^{\beta}}\mathscr{L}\left[\frac{(\lambda_1 I_H + \lambda_2 I_{HT})}{N_H}F_{SM} - \delta_M F_{ISM} - \mu_F F_{ISM}\right],\\
\mathscr{L}(S_H)(t) &= \frac{n_8}{S} + \frac{1}{S^{\beta}}\mathscr{L}\left[(1-\ell)\Lambda_3 - \frac{(\alpha_1 F_{INM} + \alpha_2 F_{ISM} + \alpha_3 I_H + \alpha_4 I_{HT} + \alpha_5 R_H)}{N_H}S_H - \mu_H S_H\right],\\
\mathscr{L}(E_H)(t) &= \frac{n_9}{S} + \frac{1}{S^{\beta}}\mathscr{L}\left[\frac{(\alpha_1 F_{INM} + \alpha_2 F_{ISM} + \alpha_3 I_H + \alpha_4 I_{HT} + \alpha_5 R_H)}{N_H}S_H - \sigma E_H - \mu_H E_H\right],\\
\mathscr{L}(I_H)(t) &= \frac{n_{10}}{S} + \frac{1}{S^{\beta}}\mathscr{L}\left[\ell\Lambda_3 + \sigma E_H - \tau_1 I_H - \theta I_H - \partial_1 I_H - \mu_H I_H\right],\\
\mathscr{L}(I_{HT})(t) &= \frac{n_{11}}{S} + \frac{1}{S^{\beta}}\mathscr{L}\left[\theta I_H - \tau_2 I_{HT} - \partial_2 I_{HT} - \mu_H I_{HT}\right],\\
\mathscr{L}(R_H)(t) &= \frac{n_{12}}{S} + \frac{1}{S^{\beta}}\mathscr{L}\left[\tau_1 I_H - \tau_2 I_{HT} - \mu_H R_H\right].
\end{aligned}
\tag{14}
$$

We now decompose the nonlinear terms of system (9), we assume that the solutions of $A(t)$, $F_M(t)$, $M_M(t)$, $F_{NM}(t)$, $F_{SM}(t)$, $F_{INM}(t)$, $F_{ISM}(t)$, $S_H(t)$, $E_H(t)$, $I_H(t)$, $I_{HT}(t)$, $R_H(t)$ are in the form of infinite series given by:

$$\begin{aligned}
A(t) &= \sum_{n=0}^{\infty} A(n),\\
F_M(t) &= \sum_{n=0}^{\infty} F_M(n),\\
M_M(t) &= \sum_{n=0}^{\infty} M_M(n),\\
F_{NM}(t) &= \sum_{n=0}^{\infty} F_{NM}(n),\\
F_{SM}(t) &= \sum_{n=0}^{\infty} F_{SM}(n),\\
F_{INM}(t) &= \sum_{n=0}^{\infty} F_{INM}(n),\\
F_{ISM}(t) &= \sum_{n=0}^{\infty} F_{ISM}(n),\\
S_H(t) &= \sum_{n=0}^{\infty} S_H(n),\\
E_H(t) &= \sum_{n=0}^{\infty} E_H(n),\\
I_H(t) &= \sum_{n=0}^{\infty} I_H(n),\\
I_{HT}(t) &= \sum_{n=0}^{\infty} I_{HT}(n),\\
R_H(t) &= \sum_{n=0}^{\infty} R_H(n).
\end{aligned} \tag{15}$$

Moreover, the nonlinear terms involved in the models are as follows:

$$\begin{aligned}
&A(t)A(t), F_{INM}(t)S_H(t), F_{ISM}(t)S_H(t),\\
&I_H(t)S_H(t), I_{HT}(t)S_H(t),\\
&R_H(t)S_H(t), I_H(t)F_{NM}(t), I_{HT}(t)F_{NM}(t),\\
&I_H(t)F_{SM}(t), I_{HT}(t)F_{SM}(t).
\end{aligned} \tag{16}$$

The nonlinear terms in (16) are decomposed by the Adomian polynomial as follows:

$$\begin{aligned}
A(t)A(t) &= \sum_{n=0}^{\infty} B(n),\\
F_{INM}(t)S_H(t) &= \sum_{n=0}^{\infty} C(n),\\
F_{ISM}(t)S_H(t) &= \sum_{n=0}^{\infty} D(n),\\
I_H(t)S_H(t) &= \sum_{n=0}^{\infty} E(n),\\
I_{HT}(t)S_H(t) &= \sum_{n=0}^{\infty} F(n),\\
R_H(t)S_H(t) &= \sum_{n=0}^{\infty} G(n),\\
I_H(t)F_{NM}(t) &= \sum_{n=0}^{\infty} H(n),\\
I(t)F_{NM}(t) &= \sum_{n=0}^{\infty} I(n),\\
I_H(t)F_{SM}(t) &= \sum_{n=0}^{\infty} J(n),\\
I_{HT}(t)F_{SM}(t) &= \sum_{n=0}^{\infty} K(n).
\end{aligned} \tag{17}$$

where $B(n)$, $C(n)$, $D(n)$, $E(n)$, $F(n)$, $G(n)$, $H(n)$, $I(n)$, $J(n)$, $K(n)$ are the Adomian polynomial defined as follows:

$$B(n) = \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n} \lambda^k A(k) \sum_{k=0}^{n} \lambda^k A(k)\right\}|_{\lambda=0},$$

$$C(n) = \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n} \lambda^k F_{INM}(k) \sum_{k=0}^{n} \lambda^k S_H(k)\right\}|_{\lambda=0},$$

$$D(n) = \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n} \lambda^k F_{ISM}(k) \sum_{k=0}^{n} \lambda S_H(k)^k\right\}|_{\lambda=0},$$

$$E(n) = \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n} \lambda^k I_H(k) \sum_{k=0}^{n} \lambda^k F_H(k)\right\}|_{\lambda=0}.$$

$$F(n) = \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n} \lambda^k I_{HT}(k) \sum_{k=0}^{n} \lambda^k F_H(k)\right\}|_{\lambda=0}.$$

$$G(n) = \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n} \lambda^k I_H(k) \sum_{k=0}^{n} \lambda^k F_H(k)\right\}|_{\lambda=0}.$$

$$H(n) = \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n} \lambda^k I_H(k) \sum_{k=0}^{n} \lambda^k F_{NM}(k)\right\}|_{\lambda=0}.$$

$$I(n) = \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n} \lambda^k I_{HT}(k) \sum_{k=0}^{n} \lambda^k F_{NM}(k)\right\}|_{\lambda=0},$$

$$J(n) = \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n} \lambda^k I_H(k) \sum_{k=0}^{n} \lambda^k F_{SM}(k)\right\}|_{\lambda=0}.$$

$$I(n) = \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n}\left\{\sum_{k=0}^{n} \lambda^k I_{HT}(k) \sum_{k=0}^{n} \lambda^k F_{SM}(k)\right\}|_{\lambda=0}. \tag{18}$$

We substitute (14) for $n = 0$ into (15) and (17) to have the following:

$$\begin{aligned}
\mathscr{L}A(0) &= \frac{n_1}{s},\\
\mathscr{L}F_M(0) &= \frac{n_2}{s},\\
\mathscr{L}M_M(0) &= \frac{n_3}{s},\\
\mathscr{L}F_{NM}(0) &= \frac{n_4}{s},\\
\mathscr{L}F_{SM}(0) &= \frac{n_5}{s},\\
\mathscr{L}F_{INM}(0) &= \frac{n_6}{s},\\
\mathscr{L}F_{ISM}(0) &= \frac{n_7}{s},\\
\mathscr{L}S_H(0) &= \frac{n_8}{s},\\
\mathscr{L}E_H(0) &= \frac{n_9}{s},\\
\mathscr{L}I_H(0) &= \frac{n_{10}}{s},\\
\mathscr{L}I_{HT}(0) &= \frac{n_{11}}{s},\\
\mathscr{L}R_H(0) &= \frac{n_{12}}{s}.
\end{aligned} \tag{19}$$

Similarly, for $n = 1$ to $n = n + 1$, we have as follows:

$$
\begin{aligned}
\mathscr{L}(A)(1) &= \frac{1}{S^{\beta}}\left\{\mathscr{L}\left[\Lambda_1\left(F_{NM}(0)+F_{INM}(0)\right)-\gamma A(0)-\mu_A A(0)-\mu_\rho B(0)\right]\right\},\\
\mathscr{L}(F_M)(1) &= \frac{1}{S^{\beta}}\{\mathscr{L}[\phi\gamma A(0)-(\beta_1\rho_\omega+\beta_2\rho_s)F_M(0)-\delta_M F_M(0)-\mu_F F_M(0)]\},\\
\mathscr{L}(M_M)(1) &= \frac{1}{S^{\beta}}\{\mathscr{L}[(1-\phi)\gamma A(0)-\mu_M M_M(0)]\},\\
\mathscr{L}(F_{NM})(1) &= \frac{1}{S^{\beta}}\left\{\mathscr{L}\left[\beta_1\rho_\omega F_M(0)-\frac{1}{N_H}[\lambda_1 H(0)+\lambda_2 I(0)]-\delta_M F_{NM}(0)-\mu_F F_{NM}(0)\right]\right\},\\
\mathscr{L}(F_{SM})(1) &= \frac{1}{S^{\beta}}\left\{\mathscr{L}\left[\beta_2\rho_s F_M(0)-\frac{1}{N_H}[\lambda_1 J(0)+\lambda_2 K(0)]-\delta_M F_{SM}(0)-\mu_F F_{SM}(0)\right]\right\},\\
\mathscr{L}(F_{INM})(1) &= \frac{1}{S^{\beta}}\left\{\mathscr{L}\left[\frac{1}{N_H}[\lambda_1 H(0)+\lambda_2 I(0)]-\delta_M F_{INM}(0)-\mu_F F_{INM}(0)\right]\right\},\\
\mathscr{L}(F_{ISM})(1) &= \frac{1}{S^{\beta}}\left\{\mathscr{L}\left[\frac{1}{N_H}[\lambda_1 J(0)+\lambda_2 K(0)]-\delta_M F_{ISM}(0)-\mu_F F_{ISM}(0)\right]\right\},\\
\mathscr{L}(S_H)(1) &= \frac{1}{S^{\beta}}\left\{\mathscr{L}\left[(1-\ell)\Lambda_3-\frac{1}{N_H}[(\alpha_1 C(0)+\alpha_2 D(0)+\alpha_3 E(0)+\alpha_4 F(0)+\alpha_5 G(0)]-\mu_H S_H(0)\right]\right\},\\
\mathscr{L}(E_H)(1) &= \frac{1}{S^{\beta}}\left\{\mathscr{L}\left[\frac{1}{N_H}[(\alpha_1 C(0)+\alpha_2 D(0)+\alpha_3 E(0)+\alpha_4 F(0)+\alpha_5 G(0)]-\sigma E_H(0)-\mu_H E_H(0)\right]\right\},\\
\mathscr{L}(I_H)(1) &= \frac{1}{S^{\beta}}\{\mathscr{L}[\ell\Lambda_3+\sigma E_H(0)-\tau_1 I_H(0)-\theta I_H(0)-\partial_1 I_H(0)-\mu_H I_H(0)],\\
\mathscr{L}(I_{HT})(1) &= \frac{1}{S^{\beta}}\{\mathscr{L}[\theta I_H(0)-\tau_2 I_{HT}(0)-\partial_2 I_{HT}(0)-\mu_H I_{HT}(0)],\\
\mathscr{L}(R_H)(1) &= \frac{1}{S^{\beta}}\{\mathscr{L}[\tau_1 I_H(0)-\tau_2 I_{HT}(0)-\mu_H R_H(0)],\\
\cdots &= \cdots\\
\mathscr{L}(A)(n+1) &= \frac{1}{S^{\beta}}\left\{\mathscr{L}\left[\Lambda_1\left(F_{NM}(n)t+nF_{INM}q(n)\right)-\gamma A(n)-\mu_A A(n)-\mu_\rho B(n)\right]\right\},\\
\mathscr{L}(F_M)(n+1) &= \frac{1}{S^{\beta}}\{\mathscr{L}[\phi\gamma A(n)-(\beta_1\rho_\omega+\beta_2\rho_s)F_M(n)-\delta_M F_M(n)-\mu_F F_M(n)]\},\\
\mathscr{L}(M_M)(n+1) &= \frac{1}{S^{\beta}}\{\mathscr{L}[(1-\phi)\gamma A(n)-\mu_M M_M(n)]\},\\
\mathscr{L}(F_{NM})(n+1) &= \frac{1}{S^{\beta}}\left\{\mathscr{L}\left[\beta_1\rho_\omega F_M(n)-\frac{1}{N_H}[\lambda_1 H(n)+\lambda_2 I(n)]-\delta_M F_{NM}(n)-\mu_F F_{NM}(n)\right]\right\},\\
\mathscr{L}(F_{SM})(n+1) &= \frac{1}{S^{\beta}}\left\{\mathscr{L}\left[\beta_2\rho_s F_M(n)-\frac{1}{N_H}[\lambda_1 J(n)+\lambda_2 K(n)]-\delta_M F_{SM}(n)-\mu_F F_{SM}(n)\right]\right\},\\
\mathscr{L}(F_{INM})(n+1) &= \frac{1}{S^{\beta}}\left\{\mathscr{L}\left[\frac{1}{N_H}[\lambda_1 H(n)+\lambda_2 I(n)]-\delta_M F_{INM}(n)-\mu_F F_{INM}(n)\right]\right\},\\
\mathscr{L}(F_{ISM})(n+1) &= \frac{1}{S^{\beta}}\left\{\mathscr{L}\left[\frac{1}{N_H}[\lambda_1 J(n)+\lambda_2 K(n)]-\delta_M F_{ISM}(n)-\mu_F F_{ISM}(n)\right]\right\},\\
\mathscr{L}(S_H)(n+1) &= \frac{1}{S^{\beta}}\left\{\mathscr{L}\left[(1-\ell)\Lambda_3-\frac{1}{N_H}[(\alpha_1 C(n)+\alpha_2 D(n)+\alpha_3 E(n)+\alpha_4 F(n)+\alpha_5 G(n)]-\mu_H S_H(n)\right]\right\},\\
\mathscr{L}(E_H)(n+1) &= \frac{1}{S^{\beta}}\left\{\mathscr{L}\left[\frac{1}{N_H}[(\alpha_1 C(n)+\alpha_2 D(n)+\alpha_3 E(n)+\alpha_4 F(n)+\alpha_5 G(n)]-\sigma E_H(n)-\mu_H E_H(n)\right]\right\},\\
\mathscr{L}(I_H)(n+1) &= \frac{1}{S^{\beta}}\{\mathscr{L}[\ell\Lambda_3+\sigma E_H(n)-\tau_1 I_H(n)-\theta I_H(n)-\partial_1 I_H(n)-\mu_H I_H(n)]\},\\
\mathscr{L}(I_{HT})(n+1) &= \frac{1}{S^{\beta}}\{\mathscr{L}[\theta I_H(n)-\tau_2 I_{HT}(n)-\partial_2 I_{HT}(n)-\mu_H I_{HT}(n)]\},\\
\mathscr{L}(R_H)(n+1) &= \frac{1}{S^{\beta}}\{\mathscr{L}[\tau_1 I_H(n)-\tau_2 I_{HT}(n)-\mu_H R_H(n)]\}.
\end{aligned}
\tag{20}
$$

Taking the inverse Laplace transform of (20), we have the following:

$$
\begin{aligned}
A(1) &= \left[\Lambda_1\left(F_{NM}(0) + F_{INM}(0)\right) - \gamma A(0) - \mu_A A(0) - \mu_\rho B(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
F_M(1) &= \left[\phi\gamma A(0) - (\beta_1\rho_\omega + \beta_2\rho_s)F_M(0) - \delta_M F_M(0) - \mu_F F_M(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
M_M(1) &= \left[(1-\phi)\gamma A(0) - \mu_M M_M(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
F_{NM}(1) &= \left[\beta_1\rho_\omega F_M(0) - \frac{1}{N_H}\left[\lambda_1 H(0) + \lambda_2 I(0)\right] - \delta_M F_{NM}(0) - \mu_F F_{NM}(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
F_{SM}(1) &= \left[\beta_2\rho_s F_M(0) - \frac{1}{N_H}\left[\lambda_1 J(0) + \lambda_2 K(0)\right] - \delta_M F_{SM}(0) - \mu_F F_{SM}(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
F_{INM}(1) &= \left[\frac{1}{N_H}\left[\lambda_1 H(0) + \lambda_2 I(0)\right] - \delta_M F_{INM}(0) - \mu_F F_{INM}(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
F_{ISM}(1) &= \left[\frac{1}{N_H}\left[\lambda_1 J(0) + \lambda_2 K(0)\right] - \delta_M F_{ISM}(0) - \mu_F F_{ISM}(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
S_H(1) &= \left[(1-\ell)\Lambda_3 - \frac{1}{N_H}\left[\left(\alpha_1 C(0) + \alpha_2 D(0) + \alpha_3 E(0) + \alpha_4 F(0) + \alpha_5 G(0)\right] - \mu_H S_H(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
E_H(1) &= \left[\frac{1}{N_H}\left[\left(\alpha_1 C(0) + \alpha_2 D(0) + \alpha_3 E(0) + \alpha_4 F(0) + \alpha_5 G(0)\right] - \sigma E_H(0) - \mu_H E_H(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
I_H(1) &= \left[\ell\Lambda_3 + \sigma E_H(0) - \tau_1 I_H(0) - \theta I_H(0) - \partial_1 I_H(0) - \mu_H I_H(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
I_{HT}(1) &= \left[\theta I_H(0) - \tau_2 I_{HT}(0) - \partial_2 I_{HT}(0) - \mu_H I_{HT}(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
R_H(1) &= \left[\tau_1 I_H(0) - \tau_2 I_{HT}(0) - \mu_H R_H(0)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
\cdots &= \cdots \\
A(n+1) &= \left[\Lambda_1\left(F_{NM}(n) + F_{INM}(n)\right) - \gamma A(n) - \mu_A A(n) - \mu_\rho B(n)\right], \\
F_M(n+1) &= \left[\phi\gamma A(n) - (\beta_1\rho_\omega + \beta_2\rho_s)F_M(n) - \delta_M F_M(n) - \mu_F F_M(n)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
M_M(n+1) &= \left[(1-\phi)\gamma A(n) - \mu_M M_M(n)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
F_{NM}(n+1) &= \left[\beta_1\rho_\omega F_M(n) - \frac{1}{N_H}\left[\lambda_1 H(n) + \lambda_2 I(n)\right] - \delta_M F_{NM}(n) - \mu_F F_{NM}(n)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
F_{SM}(n+1) &= \left[\beta_2\rho_s F_M(n) - \frac{1}{N_H}\left[\lambda_1 J(n) + \lambda_2 K(n)\right] - \delta_M F_{SM}(n) - \mu_F F_{SM}(n)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
F_{INM}(n+1) &= \left[\frac{1}{N_H}\left[\lambda_1 H(n) + \lambda_2 I(n)\right] - \delta_M F_{INM}(n) - \mu_F F_{INM}(n)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
F_{ISM}(n+1) &= \left[\frac{1}{N_H}\left[\lambda_1 J(n) + \lambda_2 K(n)\right] - \delta_M F_{ISM}(n) - \mu_F F_{ISM}(n)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
S_H(n+1) &= \left[(1-\ell)\Lambda_3 - \frac{1}{N_H}\left[\left(\alpha_1 C(n) + \alpha_2 D(n) + \alpha_3 E(n) + \alpha_4 F(n) + \alpha_5 G(n)\right] - \mu_H S_H(n)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
E_H(n+1) &= \left[\frac{1}{N_H}\left[\left(\alpha_1 C(n) + \alpha_2 D(n) + \alpha_3 E(n) + \alpha_4 F(n) + \alpha_5 G(n)\right] - \sigma E_H(n) - \mu_H E_H(n)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
I_H(n+1) &= \left[\ell\Lambda_3 + \sigma E_H(n) - \tau_1 I_H(n) - \theta I_H(n) - \partial_1 I_H(n) - \mu_H I_H(n)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
I_{HT}(n+1) &= \left[\theta I_H(n) - \tau_2 I_{HT}(n) - \partial_2 I_{HT}(n) - \mu_H I_{HT}(n)\right]\frac{t^\beta}{\Gamma(\beta+1)}, \\
R_H(n+1) &= \left[\tau_1 I_H(n) - \tau_2 I_{HT}(n) - \mu_H R_H(n)\right]\frac{t^\beta}{\Gamma(\beta+1)}.
\end{aligned}
\tag{21}
$$

Recalling the earlier stated initial conditions and substituting these conditions into (15), we have equation (22) as follows:

$A(0)=n_1,\quad F_M(0)=n_2,\quad M_M(0)=n_3,\quad F_{NM}(0)=n_4,$ $F_{SM}(0)=n_5,\quad F_{INM}(0)=n_6,\quad F_{ISM}(0)=n_7,\quad S_H(0)=n_8,$ $E_H(0)=n_9,\ I_H(0)=n_{10},\ I_{HT}(0)=n_{11},\ R_H(0)=n_{12}.$

$$
\begin{aligned}
A(t) &= n_1 + \left\{\left[\Lambda_1(n_4+n_6) - \gamma n_1 - \mu_A n_1 - \mu_\rho n_1 n_1\right]\frac{t^\beta}{\Gamma(\beta+1)}\right\} + \{\Lambda_1(\beta_1\rho_\omega n_2 - \lambda_1 n_{10} n_4 - \lambda_2 n_{11} n_4),\\
&\quad - \delta_M n_4 - \mu_F n_4 + \lambda_1 n_{10} n_4 + \lambda_2 n_{11} n_4 - \delta_M n_6 - \mu_F n_6)\frac{t^\beta}{\Gamma(\beta+1)} - [\gamma+\mu_A)\Lambda_1(n_4+n_6) - \gamma n_1 - \mu_A n_1,\\
&\quad -\mu_\rho n_1 n_1\Big]\frac{t^\beta}{\Gamma(\beta+1)} - \mu_\rho 2n_1\left[\Lambda_1(n_4+n_6) - \gamma n_1 - \mu_A n_1 - \mu_\rho n_1 n_1\right]\frac{t^\beta}{\Gamma(\beta+1)}\bigg\}\frac{t^\beta}{\Gamma(\beta+1)},\\
F_M(t) &= n_2 + \left\{[\phi\gamma n_1 - (\beta_1\rho_\omega + \beta_2\rho_s)n_2 - \delta_M n_2 - \mu_F n_2]\frac{t^\beta}{\Gamma(\beta+1)}\right\}\{[\phi\gamma\Lambda_1(n_4+n_6) - \gamma n_1 - \mu_A n_1 - \mu_\rho n_1 n_1\Big]\frac{t^\beta}{\Gamma(\beta+1)}\\
&\quad - [(\beta_1\rho_\omega + \beta_2\rho_s)(\phi\gamma n_1 - (\beta_1\rho_\omega - \beta_2\rho_s)n_2 - \delta_M n_2 - \mu_F n_2]\frac{t^\beta}{\Gamma(\beta+1)},\\
&\quad -[(\delta_M + \mu_F)(\beta_1\rho_\omega + \beta_2\rho_s)(\phi\gamma n_1 - (\beta_1\rho_\omega + \beta_2\rho_s)n_2 - \delta_M n_2 - \mu_F n_2]\frac{t^\beta}{\Gamma(\beta+1)}\bigg\}\frac{t^\beta}{\Gamma(\beta+1)},\\
M_M(t) &= n_3 + \left\{[(1-\phi)\gamma n_1 - \mu_M n_3]\frac{t^\beta}{\Gamma(\beta+1)}\right\} + \{(1-\phi)\gamma\Lambda_1(n_4+n_6) - \gamma n_1 - \mu_A n_1\\
&\quad -\mu_\rho n_1 n_1\Big]\frac{t^\beta}{\Gamma(\beta+1)} - \mu_M[(1-\phi)\gamma n_1 - \mu_M n_3]\frac{t^\beta}{\Gamma(\beta+1)}\bigg\}\frac{t^\beta}{\Gamma(\beta+1)},\\
F_{NM}(t) &= n_4 + \left\{\left[\beta_1\rho_\omega n_2 - \frac{1}{N_H}[\lambda_1 n_{10} n_4 + \lambda_2 n_{11} n_4] - \delta_M n_4 - \mu_F n_4\right]\frac{t^\beta}{\Gamma(\beta+1)}\right\} + \{\beta_1\rho_\omega[\phi\gamma n_1 - [\beta_1\rho_\omega]n_2\\
&\quad -\delta_M n_2 - \mu_F n_2]\frac{t^\beta}{\Gamma(\beta+1)} - \lambda_1(n_{10}+n_4)[\beta_1\rho_\omega n_2 - \lambda_1 n_{10} n_4 - \lambda_2 n_{11} n_4 - \delta_M n_4 - \mu_F n_4]\frac{t^\beta}{\Gamma(\beta+1)},\\
&\quad - \lambda_2(n_{11}+n_4)[\beta_1\rho_\omega n_2 - \lambda_1 n_{10} n_4 - \lambda_2 n_{11} n_4 - \delta_M n_4 - \mu_F n_4]\frac{t^\beta}{\Gamma(\beta+1)} - (\delta_M + \mu_F)[\beta_1\rho_\omega n_2,\\
&\quad -\lambda_1 n_{10} n_4 - \lambda_2 n_{11} n_4 - \delta_M n_4 - \mu_F n_4] - \delta_M n_4 - \mu_F n_4]\frac{t^\beta}{\Gamma(\beta+1)}\bigg\}\frac{t^\beta}{\Gamma(\beta+1)},\\
F_{SM}(t) &= n_5 + \left\{\left[\beta_2\rho_s n_2 - \frac{1}{N_H}[\lambda_1 n_{10} n_5 + \lambda_2 n_{11} n_5] - \delta_M n_5 - \mu_F n_5\right]\frac{t^\beta}{\Gamma(\beta+1)}\right\} + \{\beta_2\rho_s[\beta_1\rho_\omega n_2\\
&\quad -\lambda_1 n_{10} n_4 - \lambda_2 n_{11} n_4 - \delta_M n_4]\frac{t^\beta}{\Gamma(\beta+1)} - \lambda_1(n_{10}+n_5)[\beta_1\rho_\omega n_2 - \lambda_1 n_{10} n_4 - \lambda_2 n_{11} n_4 - \delta_M n_4,\\
&\quad -\mu_F n_4]\frac{t^\beta}{\Gamma(\beta+1)} - \lambda_2(n_{11}+n_5)[\beta_2\rho_s n_2 - \lambda_1 n_{10} n_5 - \lambda_2 n_{11} n_5 - \delta_M n_5 - \mu_F n_5]\frac{t^\beta}{\Gamma(\beta+1)},\\
&\quad -(\delta_M + \mu_F)[\beta_2\rho_s n_2 - \lambda_1 n_{10} n_5 + \lambda_2 n_{11} n_5 - \delta_M n_6 - \mu_F n_6]\frac{t^\beta}{\Gamma(\beta+1)}\bigg\}\frac{t^\beta}{\Gamma(\beta+1)},\\
F_{INM}(t) &= n_6 + \left\{\left[\frac{1}{N_H}[\lambda_1 n_{10} n_4 + \lambda_2 n_{11} n_4] - \delta_M n_6 - \mu_F n_6\right]\frac{t^\beta}{\Gamma(\beta+1)}\right\} + \left\{\left[\frac{1}{N_H}[\lambda_1(n_{10}+n_4)[\beta_1\rho_\omega n_2\right.\right.\\
&\quad -\lambda_2 n_{11} n_5 - \delta_M n_5 - \mu_F n_5]\frac{t^\beta}{\Gamma(\beta+1)} + (\lambda_2 n_{11} + n_5)[\beta_2\rho_s n_2 - \lambda_1 n_{10} n_5 - \lambda_2 n_{11} n_5 - \delta_M n_4,\\
&\quad \left.\left. -\delta_M n_4 - \mu_F n_4]\frac{t^\beta}{\Gamma(\beta+1)} - (\delta_M + \mu_F)[\lambda_1 n_{10} n_5 + \lambda_2 n_{11} n_5 - \delta_M n_6 - \mu_F n_6]\frac{t^\beta}{\Gamma(\beta+1)}\right]\right\}\frac{t^\beta}{\Gamma(\beta+1)},\\
F_{ISM}(t) &= n_7 + \left\{\left[\frac{1}{N_H}[\lambda_1 n_{10} n_5 + \lambda_2 n_{11} n_5] - \delta_M n_7 - \mu_F n_7\right]\frac{t^\beta}{\Gamma(\beta+1)}\right\} + \left\{\left[\frac{1}{N_H}\{\lambda_1(n_{10}+n_5)[\beta_2\rho_s n_2 - \lambda_1 n_{10} n_5\right.\right.\\
&\quad -\lambda_2 n_{11} n_5 - \delta_M n_5 - \mu_F n_5\}\frac{t^\beta}{\Gamma(\beta+1)} + (\lambda_2 n_{11} + n_5)[\beta_2\rho_s n_2 - \lambda_1 n_{10} n_5 - \lambda_2 n_{11} n_5 - \delta_M n_4,\\
&\quad \left.\left. -\mu_F n_5]\frac{t^\beta}{\Gamma(\beta+1)} - (\delta_M + \mu_F)[\lambda_1 n_{10} n_5 + \lambda_2 n_{11} n_5 - \delta_M n_6 - \mu_F n_6]\frac{t^\beta}{\Gamma(\beta+1)}\right]\right\}\frac{t^\beta}{\Gamma(\beta+1)},
\end{aligned}
$$

$$
\begin{aligned}
S_H(t) &= n_8 + \left\{\left[(1-\ell)\Lambda_3 - \frac{1}{N_H}\left[\left(\alpha_1 n_6 n_8 + \alpha_2 n_7 n_8 + \alpha_3 n_{10} n_8 + \alpha_4 n_{11} n_8 + \alpha_5 n_{12} n_8\right] - \mu_H n_8\right]\frac{t^\beta}{\Gamma(\beta+1)}\right\} \\
&\quad + \left\{(1-\ell)\Lambda_3 - \frac{1}{N_H}\left[\left(\alpha_1\left(n_6+n_8\right)\left[\lambda_1 n_{10} n_4 + \lambda_2 n_{11} n_4 - \delta_M n_6 - \mu_F n_6\right]\right.\right.\right. \\
&\quad - \frac{t^\beta}{\Gamma(\beta+1)}\alpha_2\left(n_7+n_8\right)\left[\lambda_1 n_{10} n_5 + \lambda_2 n_{11} n_5 - \delta_M n_7 - \mu_F n_7\right] \\
&\quad - \frac{t^\beta}{\Gamma(\beta+1)}\alpha_3\left[(1-\ell)\Lambda_3 - \alpha_1\left(n_{10}+n_8\right)n_6 n_8 - \alpha_2 n_7 n_8 - \alpha_3 n_{10} n_8 - \alpha_4 n_{11} n_8 - \alpha_5 n_{12} n_8 - \mu_H n_8\right] \\
&\quad - \frac{t^\beta}{\Gamma(\beta+1)}\alpha_4\left(n_{11}+n_8\right)n_6 n_8 - \alpha_2 n_7 n_8 - \alpha_3 n_{10} n_8 - \alpha_4 n_{11} n_8 - \alpha_5 n_{12} n_8 - \mu_H n_8] \\
&\frac{t^\beta}{\Gamma(\beta+1)} - \alpha_5\left(n_8+n_{10}\right)\left[\tau_1 n_{10} - \tau_2 n_{11} - \mu_H n_{12}\right] - \mu_H\left[1-\ell)\Lambda_3 - \alpha_1 n_6 n_8 - \alpha_2 n_7 n_8 - \alpha_3 n_{10} n_8 - \alpha_4 n_{11} n_8 - \alpha_5 n_{12} n_8\right]\}\frac{t^\beta}{\Gamma(\beta+1)}, \\
E_H(t) &= n_9 + \left\{\left[\frac{1}{N_H}\left[\left(\alpha_1 n_6 n_8 + \alpha_2 n_7 n_8 + \alpha_3 n_{10} n_8 + \alpha_4 n_{11} n_8 + \alpha_5 n_{12} n_8 - \sigma n_9 - \mu_H n_9\right]\frac{t^\beta}{\Gamma(\beta+1)}\right.\right.\right\} \\
&\quad + \frac{1}{N_H}\left\{\left(\alpha_1\left(n_6+n_8\right)\left[\lambda_1 n_{10} n_4 + \lambda_2 n_{11} n_4 - \delta_M n_6 - \mu_F n_6\right] + \frac{t^\beta}{\Gamma(\beta+1)}\alpha_2\left(n_7+n_8\right)\left[\lambda_1 n_{10} n_5 + \lambda_2 n_{11} n_5 - \delta_M n_7 - \mu_F n_7\right]\right.\right. \\
&\quad + \frac{t^\beta}{\Gamma(\beta+1)}\alpha_3\left[(1-\ell)\Lambda_3 - \alpha_1\left(n_{10}+n_8\right)n_6 n_8 - \alpha_2 n_7 n_8 - \alpha_3 n_{10} n_8 - \alpha_4 n_{11} n_8 - \alpha_5 n_{12} n_8 - \mu_H n_8\right] \\
&\quad \frac{t^\beta}{\Gamma(\beta+1)} + \alpha_4\left(n_{11}+n_8\right)n_6 n_8 - \alpha_2 n_7 n_8 - \alpha_3 n_{10} n_8 - \alpha_4 n_{11} n_8 + \alpha_5 n_{12} n_8 - \mu_H n_8\Big] + \alpha_5\left(n_8+n_{10}\right)\tau_1 n_{10} \\
&\quad -\tau_2 n_{11} - \mu_H n_{12}] - \left(\sigma+\mu_H\right)\left[\alpha_1 n_6 n_8 + \alpha_2 n_7 n_8 + \alpha_3 n_{10} n_8 + \alpha_4 n_{11} n_8 + \alpha_5 n_{12} n_8 - \sigma n_9 - \mu_H n_9\right]\}\frac{t^\beta}{\Gamma(\beta+1)}, \\
I_H(t) &= n_{10} + \left\{\left[\ell\Lambda_3 + \sigma n_9 - \tau_1 n_{10} - \theta n_{10} - \partial_1 n_{10} - \mu_H n_{10}\right]\frac{t^\beta}{\Gamma(\beta+1)}\right\} + \left\{\ell\Lambda_3 + \sigma\alpha_1 n_6 n_8 + \alpha_2 n_7 n_8 + \alpha_3 n_{10} n_8\right\} \\
&\quad +\alpha_4 n_{11} n_8 + \alpha_5 n_{12} n_8 - \sigma n_9 - \mu_H n_9] - \left(\tau_1+\theta+\partial_1+\mu_H\right)\left[\ell\Lambda_3 + \sigma n_9 - n_{10}\left(\tau_1+\theta+\partial_1+\mu_H\right)\right]\}\frac{t^\beta}{\Gamma(\beta+1)}, \\
I_{HT}(t) &= n_{11} + \left\{\left[n_{10} - \tau_2 n_{11} - \partial_2 n_{11} - \mu_H n_{11}\right]\right\}\frac{t^\beta}{\Gamma(\beta+1)} + \left\{\left[\theta\left\{\Lambda_3 + \sigma n_9 - \tau_1 n_{10} - \theta n_{10} - \partial_2 n_{10} - \mu_H n_{10}\right\}\right]\right. \\
&\quad - \left(\tau_2+\partial_2+\mu_H\right)\left[\phi n_{10} - \tau_2 n_{11} - \partial_2 n_{11} - \mu_H n_{11}\right]\}\frac{t^\beta}{\Gamma(\beta+1)}, \\
R_H(t) &= n_{12} + \left\{\left[\tau_1 n_{10} - \tau_2 n_{11} - \mu_H n_{12}\right]\right\}\frac{t^\beta}{\Gamma(\beta+1)} + \left\{\left[\tau_1\left[\ell\Lambda_3 + \sigma n_9 - \tau_1 n_{10} - \theta n_{10} - \partial_2 n_{10} - \mu_H n_{10}\right]\right.\right. \\
&\quad -\tau_2\left[\phi n_{10} - \tau_2 n_{11} - \partial_2 n_{11} - \mu_H n_{11}\right] - \mu_H\left[\tau_1 n_{10} - \tau_2 n_{11} - \mu_H n_{12}\right]\}\frac{t^\beta}{\Gamma(\beta+1)}.
\end{aligned}
\tag{22}
$$

Equation (22) can be further expressed as follows:

$$
\begin{aligned}
A(t) =& n_1 + \left\{ \left[\Lambda_1 (n_4 + n_6) - \gamma n_1 - \mu_A n_1 - \mu_\rho n_1 n_1\right] \frac{t^\beta}{\Gamma(\beta+1)} \right\} + \{\Lambda_1 (\beta_1 \rho_\omega n_2 - \lambda_1 n_{10} n_4 - \lambda_2 n_{11} n_4),\\
& -\delta_M n_4 - \mu_F n_4 + \lambda_1 n_{10} n_4 + \lambda_2 n_{11} n_4 - \delta_M n_6 - \mu_F n_6) - [\gamma + \mu_A)\Lambda_1 (n_4 + n_6) - \gamma n_1 - \mu_A n_1,\\
& -\mu_\rho n_1 n_1] - \mu_\rho 2 n_1 \left[\Lambda_1 (n_4 + n_6) - \gamma n_1 - \mu_A n_1 - \mu_\rho n_1 n_1\right]\} \frac{t^{2\beta}}{\Gamma(2\beta+1)},\\
F_M(t) =& n_2 + \left\{ [\phi \gamma n_1 - (\beta_1 \rho_\omega + \beta_2 \rho_s) n_2 - \delta_M n_2 - \mu_F n_2] \frac{t^\beta}{\Gamma(\beta+1)} \right\} \{ [\phi \gamma \Lambda_1 (n_4 + n_6) - \gamma n_1 - \mu_A n_1\\
& -\mu_\rho n_1 n_1] - [(\beta_1 \rho_\omega + \beta_2 \rho_s)(\phi \gamma n_1 - (\beta_1 \rho_\omega - \beta_2 \rho_s) n_2 - \delta_M n_2 - \mu_F n_2],\\
& \{-[(\delta_M + \mu_F)(\beta_1 \rho_\omega + \beta_2 \rho_s)(\phi \gamma n_1 - (\beta_1 \rho_\omega + \beta_2 \rho_s) n_2 - \delta_M n_2 - \mu_F n_2] \frac{t^{2\beta}}{\Gamma(2\beta+1)},\\
M_M(t) =& n_3 + \left\{ [(1-\phi)\gamma n_1 - \mu_M n_3] \frac{t^\beta}{\Gamma(\beta+1)} \right\} + \{(1-\phi)\gamma \Lambda_1 (n_4 + n_6) - \gamma n_1 - \mu_A n_1\\
& -\mu_\rho n_1 n_1] - \mu_M [(1-\phi)\gamma n_1 - \mu_M n_3]\} \frac{t^{2\beta}}{\Gamma(2\beta+1)},\\
F_{NM}(t) =& n_4 + \left\{ \left[\beta_1 \rho_\omega n_2 - \frac{1}{N_H} [\lambda_1 n_{10} n_4 + \lambda_2 n_{11} n_4] - \delta_M n_4 - \mu_F n_4\right] \frac{t^\beta}{\Gamma(\beta+1)} \right\} + \{\beta_1 \rho_\omega [\phi \gamma n_1 - [\beta_1 \rho_\omega] n_2\\
& -\delta_M n_2 - \mu_F n_2] - \lambda_1 (n_{10} + n_4)[\beta_1 \rho_\omega n_2 - \lambda_1 n_{10} n_4 - \lambda_2 n_{11} n_4 - \delta_M n_4 - \mu_F n_4] \frac{t^\beta}{\Gamma(\beta+1)},\\
& -\lambda_2 (n_{11} + n_4)[\beta_1 \rho_\omega n_2 - \lambda_1 n_{10} n_4 - \lambda_2 n_{11} n_4 - \delta_M n_4 - \mu_F n_4] - (\delta_M + \mu_F)[\beta_1 \rho_\omega n_2,\\
& -\lambda_1 n_{10} n_4 - \lambda_2 n_{11} n_4 - \delta_M n_4 - \mu_F n_4] - \delta_M n_4 - \mu_F n_4]\} \frac{t^{2\beta}}{\Gamma(2\beta+1)},\\
F_{SM}(t) =& n_5 + \left\{ \left[\beta_2 \rho_s n_2 - \frac{1}{N_H} [\lambda_1 n_{10} n_5 + \lambda_2 n_{11} n_5] - \delta_M n_5 - \mu_F n_5\right] \frac{t^\beta}{\Gamma(\beta+1)} \right\} + \{\beta_2 \rho_s [\beta_1 \rho_\omega n_2\\
& -\lambda_1 n_{10} n_4 - \lambda_2 n_{11} n_4 - \delta_M n_4] - \lambda_1 (n_{10} + n_5)[\beta_1 \rho_\omega n_2 - \lambda_1 n_{10} n_4 - \lambda_2 n_{11} n_4 - \delta_M n_4,\\
& -\mu_F n_4] - \lambda_2 (n_{11} + n_5)[\beta_2 \rho_s n_2 - \lambda_1 n_{10} n_5 - \lambda_2 n_{11} n_5 - \delta_M n_5 - \mu_F n_5] \frac{t^\beta}{\Gamma(\beta+1)},\\
& -(\delta_M + \mu_F)[\beta_2 \rho_s n_2 - \lambda_1 n_{10} n_5 + \lambda_2 n_{11} n_5 - \delta_M n_6 - \mu_F n_6]\} \frac{t^{2\beta}}{\Gamma(2\beta+1)},\\
F_{INM}(t) =& n_6 + \left\{ \left[\frac{1}{N_H} [\lambda_1 n_{10} n_4 + \lambda_2 n_{11} n_4] - \delta_M n_6 - \mu_F n_6\right] \frac{t^\beta}{\Gamma(\beta+1)} \right\} + \left\{ \left[\frac{1}{N_H} [\lambda_1 (n_{10} + n_4)[\beta_1 \rho_\omega n_2\right.\right.\\
& -\lambda_2 n_{11} n_5 - \delta_M n_5 - \mu_F n_5] + (\lambda_2 n_{11} + n_5)[\beta_2 \rho_s n_2 - \lambda_1 n_{10} n_5 - \lambda_2 n_{11} n_5 - \delta_M n_4,
\end{aligned}
$$

$$
\begin{aligned}
&-\delta_M n_4 - \mu_F n_4] - (\delta_M + \mu_F)[\lambda_1 n_{10} n_5 + \lambda_2 n_{11} n_5 - \delta_M n_6 - \mu_F n_6]]\}\frac{t^{2\beta}}{\Gamma(2\beta+1)},\\
F_{ISM}(t) =& n_7 + \left\{\left[\frac{1}{N_H}[\lambda_1 n_{10} n_5 + \lambda_2 n_{11} n_5] - \delta_M n_7 - \mu_F n_7\right]\frac{t^{\beta}}{\Gamma(\beta+1)}\right\} + \left\{\left[\frac{1}{N_H}\{\lambda_1 (n_{10} + n_5)[\beta_2 \rho_s n_2 - \lambda_1 n_{10} n_5\right.\right.\\
&-\lambda_2 n_{11} n_5 - \delta_M n_5 - \mu_F n_5] + (\lambda_2 n_{11} + n_5)[\beta_2 \rho_s n_2 - \lambda_1 n_{10} n_5 - \lambda_2 n_{11} n_5 - \delta_M n_4,\\
&-\mu_F n_5] - (\delta_M + \mu_F)[\lambda_1 n_{10} n_5 + \lambda_2 n_{11} n_5 - \delta_M n_6 - \mu_F n_6]]\}\frac{t^{2\beta}}{\Gamma(2\beta+1)},\\
S_H(t) =& n_8 + \left\{\left[(1-\ell)\Lambda_3 - \frac{1}{N_H}[[\alpha_1 n_6 n_8 + \alpha_2 n_7 n_8 + \alpha_3 n_{10} n_8 + \alpha_4 n_{11} n_8 + \alpha_5 n_{12} n_8] - \mu_H n_8\right]\frac{t^{\beta}}{\Gamma(\beta+1)}\right\}\\
&+\left\{(1-\ell)\Lambda_3 - \frac{1}{N_H}[[\alpha_1 (n_6 + n_8)[\lambda_1 n_{10} n_4 + \lambda_2 n_{11} n_4 - \delta_M n_6 - \mu_F n_6] - \alpha_2 (n_7 + n_8)[\lambda_1 n_{10} n_5 + \lambda_2 n_{11} n_5\right.\\
&-\delta_M n_7 - \mu_F n_7] - \alpha_3 [(1-\ell)\Lambda_3 - \alpha_1 (n_{10} + n_8) n_6 n_8 - \alpha_2 n_7 n_8 - \alpha_3 n_{10} n_8 - \alpha_4 n_{11} n_8 - \alpha_5 n_{12} n_8\\
&-\mu_H n_8] - \alpha_4 (n_{11} + n_8) n_6 n_8 - \alpha_2 n_7 n_8 - \alpha_3 n_{10} n_8 - \alpha_4 n_{11} n_8 - \alpha_5 n_{12} n_8 - \mu_H n_8]\\
&- \alpha_5 (n_8 + n_{10})[\tau_1 n_{10} - \tau_2 n_{11} - \mu_H n_{12}] - \mu_H [1-\ell)\Lambda_3 - \alpha_1 n_6 n_8 - \alpha_2 n_7 n_8 - \alpha_3 n_{10} n_8 - \alpha_4 n_{11} n_8 - \alpha_5 n_{12} n_8]\}\frac{t^{2\beta}}{\Gamma(2\beta+1)},\\
E_H(t) =& n_9 + \left\{\left[\frac{1}{N_H}[\{\alpha_1 n_6 n_8 + \alpha_2 n_7 n_8 + \alpha_3 n_{10} n_8 + \alpha_4 n_{11} n_8 + \alpha_5 n_{12} n_8 - \sigma n_9 - \mu_H n_9]\frac{t^{\beta}}{\Gamma(\beta+1)}\right\}\right.\\
&+\frac{1}{N_H}\{\{\alpha_1 (n_6 + n_8)[\lambda_1 n_{10} n_4 + \lambda_2 n_{11} n_4 - \delta_M n_6 - \mu_F n_6] + \alpha_2 (n_7 + n_8)[\lambda_1 n_{10} n_5 + \lambda_2 n_{11} n_5 - \delta_M n_7\\
&-\mu_F n_7] + \alpha_3 [(1-\ell)\Lambda_3 - \alpha_1 (n_{10} + n_8) n_6 n_8 - \alpha_2 n_7 n_8 - \alpha_3 n_{10} n_8 - \alpha_4 n_{11} n_8 - \alpha_5 n_{12} n_8 - \mu_H n_8]\\
&+\alpha_4 (n_{11} + n_8) n_6 n_8 - \alpha_2 n_7 n_8 - \alpha_3 n_{10} n_8 - \alpha_4 n_{11} n_8 + \alpha_5 n_{12} n_8 - \mu_H n_8] + \alpha_5 (n_8 + n_{10})\tau_1 n_{10}\\
&-\tau_2 n_{11} - \mu_H n_{12}] - (\sigma + \mu_H)[\alpha_1 n_6 n_8 + \alpha_2 n_7 n_8 + \alpha_3 n_{10} n_8 + \alpha_4 n_{11} n_8 + \alpha_5 n_{12} n_8 - \sigma n_9 - \mu_H n_9]\}\frac{t^{2\beta}}{\Gamma(2\beta+1)},\\
I_H(t) =& n_{10} + \left\{[\ell\Lambda_3 + \sigma n_9 - \tau_1 n_{10} - \theta n_{10} - \partial_1 n_{10} - \mu_H n_{10}]\frac{t^{\beta}}{\Gamma(\beta+1)}\right\} + \{\ell\Lambda_3 + \sigma[\alpha_1 n_6 n_8 + \alpha_2 n_7 n_8 + \alpha_3 n_{10} n_8\\
&+\alpha_4 n_{11} n_8 + \alpha_5 n_{12} n_8 - \sigma n_9 - \mu_H n_9] - (\tau_1 + \theta + \partial_1 + \mu_H)[\ell\Lambda_3 + \sigma n_9 - n_{10}(\tau_1 + \theta + \partial_1 + \mu_H)]\}\frac{t^{2\beta}}{\Gamma(2\beta+1)},\\
I_{HT}(t) =& n_{11} + \{[n_{10} - \tau_2 n_{11} - \partial_2 n_{11} - \mu_H n_{11}]\}\frac{t^{\beta}}{\Gamma(\beta+1)} + \{[\theta\{\ell\Lambda_3 + \sigma n_9 - \tau_1 n_{10} - \theta n_{10} - \partial_2 n_{10}\\
&-\mu_H n_{10}] - (\tau_2 + \partial_2 + \mu_H)[\phi n_{10} - \tau_2 n_{11} - \partial_2 n_{11} - \mu_H n_{11}]\}\frac{t^{2\beta}}{\Gamma(2\beta+1)},\\
R_H(t) =& n_{12} + \{[\tau_1 n_{10} - \tau_2 n_{11} - \mu_H n_{12}]\}\frac{t^{\beta}}{\Gamma(\beta+1)} + \{\{\tau_1[\ell\Lambda_3 + \sigma n_9 - \tau_1 n_{10} - \theta n_{10} - \partial_2 n_{10} - \mu_H n_{10}\\
&-\tau_2[\phi n_{10} - \tau_2 n_{11} - \partial_2 n_{11} - \mu_H n_{11}] - \mu_H[\tau_1 n_{10} - \tau_2 n_{11} - \mu_H n_{12}]\}\frac{t^{2\beta}}{\Gamma(2\beta+1)}.
\end{aligned}
\tag{23}
$$

3.1. Numerical Solution of the Laplace–Adomian Decomposition Method (LADM). In this section, we shall consider the numerical solution of the model. Using the initial conditions presented in Appendix, the Laplace–Adomian decomposition method (LADM) gives us an approximate solution in terms of an infinite series presented as follows:

$$\begin{aligned}
A(t) &= 2500 + 44100.3\frac{t^{\beta}}{\Gamma(\beta+1)} - 37045.1\frac{t^{2\beta}}{\Gamma(2\beta+1)}\cdots \\
F_M(t) &= 500 + 332\frac{t^{\beta}}{\Gamma(\beta+1)} + 1864.39\frac{t^{2\beta}}{\Gamma(2\beta+1)}\cdots \\
M_M(t) &= 160 + 18.43\frac{t^{\beta}}{\Gamma(\beta+1)} + 1053.16\frac{t^{2\beta}}{\Gamma(2\beta+1)}\cdots \\
F_{NM}(t) &= 250 + 74.331\frac{t^{\beta}}{\Gamma(\beta+1)} - 175.22\frac{t^{2\beta}}{\Gamma(2\beta+1)}\cdots \\
F_{SM}(t) &= 120 + 83.82\frac{t^{\beta}}{\Gamma(\beta+1)} - 100\frac{t^{2\beta}}{\Gamma(2\beta+1)}\cdots \\
F_{INM}(t) &= 125 - 36.835\frac{t^{\beta}}{\Gamma(\beta+1)} + 11.21\frac{t^{2\beta}}{\Gamma(2\beta+1)}\cdots \\
F_{ISM}(t) &= 40 - 11.672\frac{t^{\beta}}{\Gamma(\beta+1)} + 3.910\frac{t^{2\beta}}{\Gamma(2\beta+1)}\cdots \\
S_H(t) &= 1000 + 35.310\frac{t^{\beta}}{\Gamma(\beta+1)} + 36.46\frac{t^{2\beta}}{\Gamma(2\beta+1)}\cdots \\
E_H(t) &= 30 + 0.3052\frac{t^{\beta}}{\Gamma(\beta+1)} + 0.19\frac{t^{2\beta}}{\Gamma(2\beta+1)}\cdots \\
I_H(t) &= 20 + 0.020\frac{t^{\beta}}{\Gamma(\beta+1)} + 2.032\frac{t^{2\beta}}{\Gamma(2\beta+1)}\cdots \\
I_{HT}(t) &= 15 - 2.3752\frac{t^{\beta}}{\Gamma(\beta+1)} - 3.824\frac{t^{2\beta}}{\Gamma(2\beta+1)}\cdots \\
R_H(t) &= 0.412\frac{t^{\beta}}{\Gamma(\beta+1)} + 0.3848\frac{t^{2\beta}}{\Gamma(2\beta+1)}\cdots
\end{aligned} \tag{24}$$

The solution of the model for $\beta = 1$ is given as follows:

$$\begin{aligned}
A(t) &= 2500 + 44100.3t - 18522t^2\cdots \\
F_M(t) &= 500 + 332t - 932.19t^2\cdots \\
M_M(t) &= 160 + 18.42t + 526.58t^2\cdots \\
F_{NM}(t) &= 250 + 74.331t - 87.61t^2\cdots \\
F_{SM}(t) &= 120 + 83.82t - 50t^2\cdots \\
F_{INM}(t) &= 125 - 36.835t + 5.6t^2\cdots \\
F_{ISM}(t) &= 40 - 11.672t + 1.96t^2\cdots \\
S_H(t) &= 1000 + 35.310t + 18.23t^2\cdots \\
E_H(t) &= 30 + 0.3052t + 0.095t^2\cdots \\
I_H(t) &= 20 + 0.020t + 1.016t^2\cdots \\
I_{HT}(t) &= 15 - 2.3752t + 1.91t^2\cdots \\
R_H(t) &= 0.412t + 0.1924t^2\cdots
\end{aligned} \tag{25}$$

3.2. Numerical Simulation. Here, we present the numerical simulation of our fractional order sterile insect technology model so as to compare with the classical order. All the variables of our model are presented from Figures 2–7 with variation in the value of the fractional order β. Figures 2–7, shows that the fractional order sterile insect technology model has more degrees of freedom as such the order β can be varied to determine various degrees of responses of the different classes that makes up the model. This is seen when we compared the fractional order (ie $\beta = 0.4, 0.7$) with the classical case where $\beta = 1.0$. The effect of the sterile mosquitoes on the female mosquitoes are shown earlier when $\beta < 1$ than when $\beta = 1$, which agrees with our earlier result that the fractional order allows for determination of responses at different levels when compared with the classical case. The fractional order indeed helps us to get different responses in real time of the different classes of our model as shown in Figures 2–7. We also carefully chose our initial values and time to avoid having negative population of mosquitoes or human which might not be biologically meaningful.

3.3. Convergence Analysis for the Laplace–Adomian Decomposition Method (LADM). The solution of (17) is expressed in forms of infinite series as presented in equation (18) which converges uniformly to its exact solution. To verify the convergence of the series (17), we employ the method used in [20]. For sufficient conditions of convergence of the LADM, we present the following theorem:

Theorem 1 (see [11]). *Let X be a Banach space and $T: X \longrightarrow X$ be a constructive nonlinear operator such that for all $(x), (x)' \in X, \|T(x) - T(x')\| \leq k\|(x) - (x)'\|, 0 < k < 1$. Then, T has a unique point x such that $Tx = x$, where $x = (A, F_M, M_M, F_{NM}, F_{SM}, F_{INM}, F_{ISM}, S_H, E_H, I_H, I_{HT}, R_H)$.*

The series given in (?) can be written by applying the Adomian decomposition method as follows:

$$\begin{aligned} x_n &= Tx_{n-1}, x_{n-1}, \\ &= \sum_{i=1}^{n-1} x_i, n = 1, 2, 3, \ldots \end{aligned} \tag{26}$$

and we assume that $x_0 \in B_r(x)$, where $B_r(x) = \{x\prime \in X: \| x\prime - x\| < r\}$; then, we have as follows:

(i) $x_n \in B_r(x)$.

(ii) $\lim_{n\longrightarrow\infty} x_n = x$.

Proof. For condition (i), invoking mathematical induction for $n = 1$, we have as follows:

$$\left\|x_0 - x\right\| = \left\|T(x_0) - T(x)\right\| \leq \left\|x_0 - x\right\|. \tag{27}$$

If this is true for $m - 1$, then

$$\left\|x_0 - x\right\| \leq k^{m-1}\left\|x_0 - x\right\|. \tag{28}$$

This gives the following:

$$\left\|x_m - x\right\| = \left\|T(x_{m-1}) - T(x)\right\| \leq k\left\|x_{m-1} - x\right\| \leq k^n\left\|x_0 - x\right\|. \tag{29}$$

Therefore,

$$\left\|x_m - x\right\| \leq k^n\left\|x_0 - x\right\| \leq k^n r < r. \tag{30}$$

This directly implies that $x_n \in B_r(x)$.

Also, for (ii), we have that since $\|x_m - x\| \leq k^n\|x_0 - x\|$ and $\lim_{n\longrightarrow\infty} k^n = 0$, we can write $\lim_{n\longrightarrow\infty} x_n = x$. □

4. Conclusion

In this work, we extended the work of Atokolo et al. [1] by formulating a fractional order sterile insect technology model. To solve this model, we developed a numerical scheme that gives an analytical (approximate) solution of our model using the Laplace–Adomian decomposition method. Our solution showed quantitative agreement with other numerical solutions. We also showed that our approximate solution converges to an exact solution. Finally, we showed from our numerical simulation that the fractional order gives more degrees of freedom as the order can be varied to show responses by the different classes in real time. It is also seen that reduction of the fractional order β gives a corresponding reduction in the population value of each of the classes considered, which implies that the fractional order has a direct implication in our model which is aimed at reducing the mosquito population. We have been able to show that the LADM can be used to solve an SIT model which has never been done before in literature. We also hope to apply the homotopy perturbation method in solving our model and compare the result with the LADM so as to make strong recommendation on the best method to use and why.

We therefore conclude that the fractional order model solved via the Laplace–Adomian decomposition method gives a higher degree of freedom as compared to the integer order model, and as such, linear and nonlinear ordinary and partial differential equations of classical and fractional orders can be solved using the Laplace–Adomian decomposition method [21–27].

Appendix

Here, we present Table 1 showing definition of variables used in our model, Table 2 showing definition of parameters as used in our model equations, and Table 3 that shows initial conditions for our model variables and parameter values, respectively.

Data Availability

The data used to support the findings of this study are available from the corresponding author on request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] W. Atokolo, J. Akpa, M. A. Daniel, K. S. Olayemi, and G. C. E Mbah, "Modeling the impact of optimal control strategies on the dynamics of zika virus using sterile insect technology," *Journal of Advances in Mathematics and Computer Science*, vol. 35, no. 8, pp. 13–33, 2020.

[2] B. Luisa, T. Marta, S. Alexander, L. Enrico, and P. Giorgio, "Zika virus from pathogenesis to diseases control," *FEMS Microbiology Letters*, vol. 36, no. 4, pp. 202–206, 2016.

[3] W. Atokolo and G. Mbah Christopher Ezike, "Modeling the control of zika virus vector population using the sterile insect technology," *Journal of Applied Mathematics*, vol. 2020, pp. 1–12, 2020.

[4] W. Atokolo, R. O. Aja, D. Omale, B. S. Tenuche, and G. C. E. Mbah, "Numerical Solution of the Sterile Insect Technology model for the control of zika virus vector population," *Journal of Mathematical and Computational Science*, vol. 11, no. 2, pp. 1400–1412, 2021.

[5] F. Nyabadza, Z. Mukandavire, and S. D. Hove-Musekwa, "Modelling the HIV/AIDS epidemic trends in South Africa: insights from a simple mathematical model," *Nonlinear Analysis: Real World Applications*, vol. 12, no. 4, pp. 2091–2104, 2011.

[6] I. S. Onah, O. C. Collins, P.-G. U. Madueme, and G. C. E. Mbah, "Dynamical system analysis and optimal control measures of Lassa fever disease model," *International Journal of Mathematics and Mathematical Sciences*, vol. 2020, pp. 1–18, 2020.

[7] I. S. Onah and O. C. Collins, "Dynamical system Analysis of a Lassa fever model with varying socioeconomic classes," *Journal of Applied Mathematics*, vol. 2020, pp. 1–12, 2020.

[8] O. C. Collins and K. S. Govinder, "Incorporating heterogeneity into the transmission dynamics of a waterborne disease model," *Journal of Theoretical Biology*, vol. 356, pp. 133–143, 2014.

[9] S. I. Onah, O. C. Collins, C. Okoye, and G. C. E. Mbah, "Dynamics and control measures for malaria using a mathematical epidemiological model," *Electronic Journal of Mathematical Analysis and Applications*, vol. 7, no. 1, pp. 65–73, 2019.

[10] I. S. Onah, S. E. Aniaku, and O. M. Ezugorie, "Analysis and optimal control measures of diseases in cassava population," *Optimal Control Applications and Methods*, vol. 6, pp. 1–29, 2022.

[11] H. Fazal, S. Kamal, U. R Ghaus, and S. Muhammed, "Numerical solution of fractional order smoking model via Laplace Adomian Decomposition Method," *Alexandra Engineering Journal*, vol. 57, pp. 1061–1069, 2017.

[12] M. Y. Ongun, "The Laplace Adomian decomposition method for solving a model for HIV infection of CD4+ T cells," *Mathematical and Computer Modelling*, vol. 53, no. 5-6, pp. 597–603, 2011.

[13] P. Adejoh and G. C. E. Mbah, *Application of Fractional Differential Equations in Studying the Dynamics of Cancer Disease with its Control*, University of Nigeria, Nsukka, M. Sc, 2018.

[14] F. Haq, M. Shahzad, S. Muhammad, H. A. Wahab, and G. ur Rahman, "Numerical analysis of fractional order epidemic model of childhood diseases," *Discrete Dynamics in Nature and Society*, vol. 2017, pp. 1–7, 2017.

[15] B. Daşbaşı, "Stability analysis of an incommensurate fractional-order SIR model," *Mathematical Modelling and Numerical Simulation with Applications*, vol. 1, no. 1, pp. 44–55, 2021.

[16] E. K. Akgül, A. Akgül, and M. Yavuz, "New illustrative applications of integral transforms to financial models with different fractional derivatives," *Chaos, Solitons & Fractals*, vol. 146, Article ID 110877, 2021.

[17] N. Sene, "Second-grade fluid with Newtonian heating under Caputo fractional derivative: analytical investigations via Laplace transforms," *Mathematical Modelling and Numerical Simulation with Applications*, vol. 2, no. 1, pp. 13–25, 2022.

[18] M. Yavuz and N. Ozdemir, "Numerical inverse Laplace homotopy technique for fractional heat equations," *Thermal Science*, vol. 22, pp. 185–194, 2018.

[19] M. Yavuz and N. Sene, "Approximate solutions of the model describing fluid flow using generalized ρ-laplace transform method and heat balance integral method," *Axioms*, vol. 9, no. 4, p. 123, 2020.

[20] K. Shah, H. Khalil, and R. A. Khan, "Analytical solution of fractional order diffusion equations by natural transform method," *Iranian Journal of Science and Technology*, vol. 16, pp. 1–14, 2016.

[21] U. Suleiman, K. A. Ibrahim, and A. B. Huzifa, "Mathematical model for the transmission dynamics of zika virus infection with combined vaccination and treatment intervention," *Journal of Applied Mathematics and Physics*, vol. 5, pp. 1964–1978, 2017.

[22] U. A. Danbaba and S. M. Garba, "Analysis of model for the transmission dynamics of Zika with sterile insect technique," *Texts in Biomathematics*, vol. 1, pp. 81–99, 2018.

[23] N. Chitnis, J. M. Hyman, and J. M. Cushing, "Determining important parameters in the spread of malaria through the sensitivity analysis of a mathematical model," *Bulletin of Mathematical Biology*, vol. 70, no. 5, pp. 1272–1296, 2008.

[24] R. Anguelov, Y. Dumont, and J. Lubuma, "Mathematical modeling of sterile insect technology for control of anopheles mosquito," *Computers & Mathematics with Applications*, vol. 64, no. 3, pp. 374–389, 2012.

[25] G. Ademu, M. Bawa, M. Jiya, and U. Chadu, "Mathematical model for the dynamics of zika virus via Homotopy Perturbation Method," *Journal of Applied Sciences & Environmental Management*, vol. 21, no. 4, pp. 615–623, 2017.

[26] E. Bonyah and K. O. Okosun, "Mathematical modeling of zika virus," *Asian Pacific Journal of Tropical Disease*, vol. 6, no. 9, pp. 673–679, 2016.

[27] R. J. Patinvoh and A. A. Susu, "Mathematical modelling of sterile insect technology for mosquito control," *Advances in Entomology*, vol. 02, no. 04, pp. 180–193, 2014.