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Research Article

Mathematical Model Analysis on the Diffusion of Violence

Birhanu Baye Terefe

Department of Mathematics, Collage of Natural and Computational Sciences, Debre Berhan University, Debre Berhan, Ethiopia

Correspondence should be addressed to Birhanu Baye Terefe; birhanu1baye@dbu.edu.et

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Recently, violence has been a very common and serious public health problem in the world. In this new mathematical modeling tactic study, we formulated and examined the firsthand violence mathematical model with five distinct classes of the human population (susceptible, violence-exposed, violence, negotiated, and reconciled). The model takes into account the diffusion of violence and infection. The violence-free and violence-dominance model equilibrium points are calculated, and their local and global stabilities are analyzed. The model threshold values are obtained. As a result of the model analysis, the violence diffusion is under control if the basic reproduction number is less than unity, and it diffuses through the community if this number exceeds unity. Besides, the sensitivity analysis of the parameter values of the basic reproduction number is demonstrated. We have applied the MATLAB ode45 solver to illustrate the numerical results of the model. Finally, from analytical and numerical solutions, we obtain jointly equivalent and consistent results.

1. Introduction

Violence is a major and serious behavioral problem that affects all societies, involving physical force intended to hurt, damage, or kill someone in the world [1–3]. Violence is a form of mental pain that causes distress to people physically, psychologically, sexually, emotionally, and economically. It is one of the most common human right violations and a serious global public health concern in developing countries [4–9]. It is a contagious disease among the greatest killers of human beings, which spread from one person to another due to direct or indirect contact quickly or slowly, depending on a host of factors in the world [10–13]. Violence is a political thing between two states that functions as a kind of disciplinary, regulatory, and hierarchical form of militarized law enforcement agents and maldistributions of economic resources to achieve political profit [14–16]. The current political administrative systems facilitate state violence encoded in laws, policies, and schemes that arrange and define people by categories of indigeneity, race, gender, ability, and national origin, which leads to community mobilization from a different perspective [17–19].

The mathematical modeling tactics using the deterministic techniques have been systematic efforts to discover real-world situations using mathematical models for the analysis of the dynamics of communicable diseases and can be used for analyzing a number of real-world physical dynamical situations [20–23]. Many researchers have applied the infectious disease dynamics model to analyze violence, corruption, and other social situations. Of those researchers, some applied mathematical modeling analysis on the dynamics of university student animosity towards mathematics with optimal control theory [24], some applied modeling for the dynamics of racism in cyberspace [25], and some applied modeling for violence [26–28], some applied modeling for the stability analysis of corruption [29], and others study universality of political corruption networks [30]. But, to the best of our knowledge, no one has developed and analyzed a mathematical model of the diffusion of violence. Therefore, in this first-hand proposed violence diffusion model, we are motivated and interested in filling the specified break, and we try to examine this connection by constructing a mathematical model of violence diffusion. The remaining part of this study is organized as follows. In Section 2, we formulate the compartmental mathematical

model of violence diffusion, equilibrium points, basic reproduction numbers, and the stability analysis of the model. Section 3 presents numerical simulations. Section 4 presents discussions. Finally, we drew conclusions in Section 5.

2. Compartmental Model Formulation

In this study, we considered the total number of population $N(t)$ in a given time t, and we divide it into five mutually exclusive classes. Those are susceptible, violence-exposed, violence, negotiated, and reconciled individuals denoted by S, E, P, H, and R, respectively. Those state variables are described as follows:

(i) Susceptible individual is the number of populations who are free from violence, while they can receive or observe the idea of violent, and we call it susceptible. It is denoted by $S(t)$.

(ii) Violence-exposed individuals are the group of population who have close contact with violence and observe different violent activities. These individuals may have been violent while they are in a pause state which does not spread violence, and we call it violently exposed. It is denoted by $E(t)$.

(iii) Violently infectious individuals are a group of population who use physical force to harm, injure, damage, or destroy someone to spread violence, and we call it violence. It is denoted by $P(t)$.

(iv) Negotiated individuals are a group of population who are going to reach an agreement with a formal discussion between people. It is denoted by $H(t)$.

(v) reconciled individual is the group of people who made compatible, consistent, or group decisions to become friendly again after an argument.It is denoted by $R(t)$.

Basic assumptions and parameter definitions of the model.

(1) Susceptible individuals increase with the recruitment Λ, decreased by the contact rate β and enter into the violence exposed classes, and decrease with the natural death rate μ.

(2) Violence-exposed classes increase by the contact rate β and decreased by the reconciled rate δ, the incubation period α, and the natural death rate μ.

(3) Violent individuals increased by the incubation period α and decreased by the negotiation rate γ_1, the reconciled rate γ_2, and the natural death rate μ.

(4) The reconciled individual increased by the reconciled rate ε from the negotiate classes, increased by the reconciled rate γ_2 from the violence classes, and decreased by the conversion rate δ and the natural death rate μ.

Based on the model assumptions and descriptions, the flow diagram of the model is as follows:

Using the basic assumptions, parameters definitions, and flow diagram of Figure 1, the dynamical system of the study is given by

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \beta SP - \mu S,\\
\frac{dE}{dt} &= \beta SP - (\mu + \delta + \alpha)E,\\
\frac{dP}{dt} &= \alpha E - (\mu + \gamma_1 + \gamma_2)P,\\
\frac{dH}{dt} &= \gamma_1 P - (\mu + \varepsilon)H.
\end{aligned} \tag{1}$$

We considered that all model parameter values are nonnegative and the initial condition $S(0) > 0, E(0) \geq 0, P(0) \geq 0, H(0) \geq 0, R(0) \geq 0$.

The total number of populations in system (1) is given as

$$N(t) = S(t) + E(t) + P(t) + H(t) + R(t). \tag{2}$$

Then, the mathematically and biologically feasible domain of system (1) is

$$Q = (S, E, P, H, R) 0 < S(t) + E(t) + P(t) + H(t) + R(t) \leq \frac{\Lambda}{\mu}. \tag{3}$$

The model is both mathematically and biologically meaningful when the following important lemma holds.

Lemma 1. *-If $S(t) > 0$, $E(t) \geq 0$, $P(t) \geq 0$, $H(t) \geq 0$, $R(t) \geq 0$, then the solution $\{S(t), E(t), P(t), H(t), R(t)\}$ of the system (1) are positive for all time $t \geq 0$.*

Proof. Using the initial conditions, we can prove the components of the solutions of the system (1) are positive by contradiction, that is, there exists a time t_i for $i = 1, 2, 3, 4, 5$ such that

(1) t_1: $S(t_1) = 0, S'(t_1) < 0$, $S(t) > 0$, $E(t) > 0$, $P(t) > 0$, $(t) > 0$, $R(t) > 0$ 'for $0 < t < t_1$

(2) t_2: $E(t_2) = 0, E'(t_2) < 0$, $S(t) > 0$, $E(t) > 0$, $P(t) > 0$, $(t) > 0$, $R(t) > 0$ for $0 < t < t_2$

(3) t_3: $P(t_3) = 0, P'(t_3) < 0$, $S(t) > 0$, $E(t) > 0$, $(t) > 0$, $R(t) > 0$ for $0 < t < t_3$

(4) t_4: $H(t_4) = 0, H'(t_4) < 0$, $S(t) > 0$, $E(t) > 0$, $P(t) > 0$, $R(t) > 0$ for $0 < t < t_4$

(5) t_5: $R(t_5) = 0, R'(t_5) < 0$, $S(t) > 0$, $E(t) > 0$, $P(t) > 0$, $H(t) > 0$ for $0 < t < t_5$

Then, when we evaluate

(i) From the first equationof the system given in (1) at time t_1, we have obtained $S'(t_1) = \Lambda - \beta S(t_1)P(t_1) - \mu S(t_1) = \Lambda > 0$, which contradict our first assumption. It implies that $S(t) > 0$ for all $t \geq 0$.

(ii) The 2nd equation of system (1) at time t_2 we get $E'(t_2) = \beta S(t_2)P(t_2) - (\mu + \delta + \alpha)E(t_2) = \beta S(t_2)$

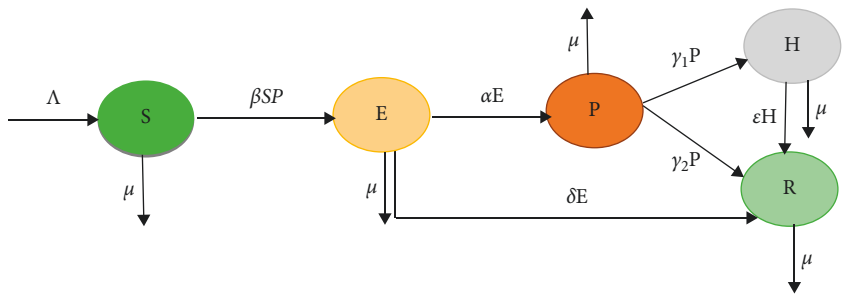

Figure 1: The flow diagram of violence diffusion.

$P(t_2) > 0$, which contradict our second assumption. It implies that $E(t) > 0$ for all $t \geq 0$.

(iii) The 3rd equation of the system (1) at time t_3 we get $P'(t_3) = \alpha E(t_3) - (\mu + \gamma_1 + \gamma_2)P(t_3) = \gamma E(t_3) > 0$, which contradict our third assumption. It implies that $P(t) > 0$ for all $t \geq 0$.

(iv) The 4th equation of the system (1) at time t_4 we get $H'(t_4) = \gamma_1 P(t_4) - (\mu + \varepsilon)H(t_4) = \gamma_1 P(t_4) > 0$, which contradict our fourth assumption. It implies that $H(t) > 0$ for all $t \geq 0$.

(v) The 5th equation of the system (1) at time t_5 we get $R'(t_5) = \gamma_2 P(t_5) + \varepsilon H(t_5) + \delta E(t_5) - \mu R(t_5) = \gamma_2 P(t_5) + \varepsilon H(t_5) + \delta E(t_5) > 0$, which contradict our fifth assumption. It implies that $A(t) > 0$ for all $t \geq 0$.

Therefore, from $(i - v)$, the solution $\{S(t), E(t), P(t), H(t), R(t)\}$ of system (1) are positive for all time $t \geq 0$. □

Lemma 2. *The total number of populations in system (1) is bounded on* $0 < N(t) \leq \Lambda/\mu$.

Proof. To show the boundedness of the solution, take the total population

$$N(t) = S(t) + E(t) + P(t) + H(t) + R(t), \tag{4}$$

by differentiating both sides with respect to time, we get

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dP}{dt} + \frac{dH}{dt} + \frac{dR}{dt}, \tag{5}$$

by substituting (1) in (5) and after simple simplification, we get

$$\frac{dN}{dt} = \Lambda - \mu N, \tag{6}$$

$dN/dt + \mu N = \Lambda$, which is first order ordinary differential equations, and its solution is$N(t) = \Lambda/\mu + ce^{-\mu t}$, then take limit on both sides

$$\lim_{t \longrightarrow \infty} N(t) = \lim_{t \longrightarrow \infty} \left(\frac{\Lambda}{\mu} + ce^{-\mu t} \right) = \frac{\Lambda}{\mu}. \tag{7}$$

Hence, the solution is bounded on $0 < N(t) \leq \Lambda/\mu$. □

2.1. Equilibrium Points of the Model

2.1.1. Violence-Free Equilibrium Point. In the absence of violence, the violence-free equilibrium point of the model (1) is obtained by making $dS/dt = dE/dt = dP/dt = dH/dt = dR/dt = 0$. At the violence-free equilibrium point they have no violently infectious. That is $P = 0$, and by substituting those in system (1), we get the violence-free equilibrium point is $E_0^* = (\Lambda/\mu, 0, 0, 0, 0)$.

2.1.2. Basic Reproduction Number of the Model. Take $X = (S, E, P, H, R)^T$, and system (1) can be rewritten as

$$\frac{dX}{dt} = f_i - v_i, \tag{8}$$

where

$$
\begin{aligned}
f_i &= \begin{bmatrix} \beta \mathrm{SP} \\ 0 \end{bmatrix}, \\
f &= \begin{pmatrix} 0 & \frac{\Lambda\beta}{\mu} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
v_i^+(x) &= \begin{pmatrix} 0 \\ \alpha E \\ \gamma_1 P \end{pmatrix}, \\
v_i^-(x) &= \begin{pmatrix} (\mu+\delta+\alpha)E \\ (\mu+\gamma_1+\gamma_2)P \\ (\mu+\varepsilon)H \end{pmatrix}, \\
v_i &= v_i^-(x) - v_i^+(x) \\
V &= \begin{pmatrix} \mu+\delta+\alpha & 0 & 0 \\ -\alpha & \mu+\gamma_1+\gamma_2 & 0 \\ 0 & -\gamma_1 & \mu+\varepsilon \end{pmatrix}, \\
V^{-1} &= \begin{pmatrix} \frac{1}{\mu+\delta+\alpha} & 0 & 0 \\ \frac{\alpha}{(\mu+\delta+\alpha)(\mu+\gamma_1+\gamma_2)} & \frac{1}{\mu+\gamma_1+\gamma_2} & 0 \\ \frac{\alpha\gamma_1}{(\mu+\delta+\alpha)(\mu+\gamma_1+\gamma_2)(\mu+\varepsilon)} & \frac{\gamma_1}{(\mu+\delta+\alpha)(\mu+\varepsilon)} & \frac{1}{(\mu+\varepsilon)} \end{pmatrix}, \\
fV^{-1} &= \begin{pmatrix} \frac{\Lambda\alpha\beta}{\mu(\mu+\delta+\alpha)(\mu+\gamma_1+\gamma_2)} & \frac{\Lambda\beta}{\mu(\mu+\gamma_1+\gamma_2)} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\end{aligned}
\tag{9}
$$

Then, the spectral radius of fV^{-1} is $\Lambda\alpha\beta/\mu(\mu+\delta+\alpha)(\mu+\gamma_1+\gamma_2)$.

$$
R_0 = \frac{\Lambda\alpha\beta}{\mu(\mu+\delta+\alpha)(\mu+\gamma_1+\gamma_2)}. \tag{10}
$$

Sensitivity Analysis. The basic reproduction number is the function of seven parameters $\beta, \delta, \alpha, \Lambda, \gamma_1, \gamma_2$, and μ. In order to reduce the diffusion of violence in the population, it is necessary to control the parameter values to make $R_0 > 1$. We are interested in finding the rate of change of R_0 due to each parameter value change. The rate of change of R_0 for a change in the value of parameters h can be estimated from a normalized sensitivity index, SI (h) defined as, SI $(h) = h/R_0 \partial R_0/\partial h$ [21, 24].

The normalized sensitivity indices of the reproduction number R_0 with respect to $\beta, \delta, \alpha, b, \gamma_1, \gamma_2$, and μ are given as

$$
\begin{aligned}
R_0 &= \frac{\Lambda\alpha\beta}{\mu(\mu+\delta+\alpha)(\mu+\gamma_1+\gamma_2)},\\
SI(\beta) &= \frac{\beta}{R_0}\frac{\partial R_0}{\partial\beta} = 1,\\
SI(\alpha) &= \frac{\alpha}{R_0}\frac{\partial R_0}{\partial\alpha} = \frac{(\mu+\delta)}{(\mu+\delta+\alpha)} > 0,\\
SI(\delta) &= \frac{\delta}{R_0}\frac{\partial R_0}{\partial\delta} = \frac{-\delta}{(\mu+\delta+\alpha)} < 0,\\
SI(\gamma_1) &= \frac{\gamma_1}{R_0}\frac{\partial R_0}{\partial\gamma_1} = \frac{-\gamma_1}{(\mu+\gamma_1+\gamma_2)} < 0,\\
SI(\gamma_1) &= \frac{\gamma_2}{R_0}\frac{\partial R_0}{\partial\gamma_2} = \frac{-\gamma_2}{(\mu+\gamma_1+\gamma_2)} < 0.
\end{aligned}
\tag{11}
$$

Then, the sensitivity indices SI (β) and SI (α) are positive, that is, the basic reproduction number R_0 is increased as the infectious rate β and incubation period α increase. The remaining indices are negative, which implies that the basic reproduction number R_0 is decreased as δ, α, and μ increase. Since all of the indices, except for SI (β), are functions of other parameters, the sensitivity indices will change with changes in values of these other parameters.

2.1.3. Violence Dominance Equilibrium Point. In the presence of violence in the population, the timely dependent solution of system (1) is said to be the violence-dominance equilibrium point, denoted by E^* and given by $e^* = (S^*, E^*, P^*, H^*, R^*)$, where after some simplification, we have got

$$
\begin{aligned}
S^* &= \frac{\Lambda}{\mu R_0},\\
E^* &= \frac{\mu(\mu+\gamma_1+\gamma_2)(R_0-1)}{\alpha\beta},\\
P^* &= \frac{\mu(R_0-1)}{\beta},\\
H^* &= \frac{\gamma_1\mu(R_0-1)}{\beta(\mu+\varepsilon)},\\
R^* &= \frac{((\mu+\gamma_1+\gamma_2)(\mu+\varepsilon)+\alpha(\mu+\varepsilon)+\alpha\gamma_1)(R_0-1)}{\alpha\beta(\mu+\varepsilon)}.
\end{aligned}
\tag{12}
$$

2.2. Stability Analysis of the Equilibrium Point

Theorem 1. *-The violence-free equilibrium point is locally asymptotically stable when $R_0 < 1$.*

Proof. The Jacobean matrix of system (1) at the violence-free equilibrium point is

$$
J\left(\frac{\Lambda}{\mu},0,0,0,0\right) = \begin{pmatrix}
-\mu & 0 & \frac{-\beta\Lambda}{\mu} & 0 & 0\\
0 & -(\mu+\delta+\alpha) & \frac{\beta\Lambda}{\mu} & 0 & 0\\
0 & \alpha & -(\mu+\gamma_1+\gamma_2) & 0 & 0\\
0 & 0 & \gamma_1 & -(\mu+\varepsilon) & 0\\
0 & \delta & \gamma_2 & \varepsilon & -\mu
\end{pmatrix}.
\tag{13}
$$

The characteristics equation is

$$\begin{vmatrix} -\mu-\lambda & 0 & \frac{-\beta\Lambda}{\mu} & 0 & 0 \\ 0 & -(\mu+\delta+\alpha)-\lambda & \frac{\beta\Lambda}{\mu} & 0 & 0 \\ 0 & \alpha & -(\mu+\gamma_1+\gamma_2)-\lambda & 0 & 0 \\ 0 & 0 & \gamma_1 & -(\mu+\varepsilon)-\lambda & 0 \\ 0 & \delta & \gamma_2 & \varepsilon & -\mu-\lambda \end{vmatrix} = 0. \tag{14}$$

After simple simplification, we have

$$\begin{aligned} &(-\mu-\lambda)(-(\mu+\delta+\alpha)-\lambda)\left[(-(\mu+\delta+\alpha)-\lambda)(-(\mu+\gamma_1+\gamma_2)-\lambda)(-(\mu+\varepsilon)-\lambda)(-\mu-\lambda)-\frac{\beta\alpha\Lambda}{\mu}(-(\mu+\varepsilon)-\lambda)(-\mu-\lambda)\right]=0, \\ &(-\mu-\lambda)(-(\mu+\delta+\alpha)-\lambda)(-(\mu+\varepsilon)-\lambda)(-\mu-\lambda) \\ &\cdot\left[\lambda^2+((\mu+\delta+\alpha)+(\mu+\gamma_1+\gamma_2))\lambda+\frac{\mu(\mu+\gamma_1+\gamma_2)(\mu+\delta+\alpha)}{\mu}\left(\left(1-\frac{\alpha\beta\Lambda}{\mu(\mu+\gamma_1+\gamma_2)(\mu+\delta+\alpha)}\right)\right)\right]=0. \end{aligned} \tag{15}$$

which gives negative eigenvalues $\lambda_1 = -\mu$, $\lambda_2 = -(\mu+\delta+\alpha)$, $\lambda_3 = -(\mu+\varepsilon)$ or

$$\begin{aligned} \lambda^2+((\mu+\delta+\alpha)+(\mu+\gamma_1+\gamma_2))\lambda+\frac{\mu(\mu+\gamma_1+\gamma_2)(\mu+\delta+\alpha)}{\mu}\left(1-\frac{\alpha\beta\Lambda}{\mu(\mu+\gamma_1+\gamma_2)(\mu+\delta+\alpha)}\right)=0, \\ \lambda^2+((\mu+\delta+\alpha)+(\mu+\gamma_1+\gamma_2))\lambda+\frac{\mu(\mu+\gamma_1+\gamma_2)(\mu+\delta+\alpha)}{\mu}(1-R_0)=0, \end{aligned} \tag{16}$$

which have negative root when $R_0 < 1$.

Therefore, the violence-free equilibrium point is locally stable when $R_0 < 1$. □

Theorem 2. *The violence-dominance equilibrium point is locally asymptotically stable if $R_0 > 1$.*

Proof. The Jacobean matrix of system (1) at violence-dominance equilibrium point e^* is

$$J(e^*) = \begin{pmatrix} -\mu(R_0-1)-\mu & 0 & -\frac{\beta\mu}{\Lambda R_0} & 0 & 0 \\ \mu(R_0-1) & -(\mu+\delta+\alpha) & \frac{\beta\mu}{\Lambda R_0} & 0 & 0 \\ 0 & \alpha & -(\mu+\sigma+\gamma_1+\gamma_2) & 0 & 0 \\ 0 & 0 & \gamma_1 & -(\mu+\varepsilon) & 0 \\ 0 & \delta & \gamma_2 & \varepsilon & -\mu \end{pmatrix}. \tag{17}$$

The characteristics equation of the Jacobean matrix is

$$\begin{vmatrix} -\mu(R_0-1)-\lambda & 0 & -\frac{\beta\mu}{\Lambda R_0} & 0 & 0 \\ \mu(R_0-1) & -(\mu+\delta+\alpha)-\lambda & \frac{\beta\mu}{\Lambda R_0} & 0 & 0 \\ 0 & \alpha & -(\mu+\sigma+\gamma_1+\gamma_2)-\lambda & 0 & 0 \\ 0 & 0 & \gamma_1 & -(\mu+\varepsilon)-\lambda & 0 \\ 0 & \delta & \gamma_2 & \varepsilon & -\mu-\lambda \end{vmatrix} = 0$$

$$\begin{aligned} &(-\mu(R_0-1)-\lambda)\left[(-(\mu+\delta+\alpha)-\lambda)(-(\mu+\gamma_1+\gamma_2)-\lambda)(-(\mu+\varepsilon)-\lambda)(-\mu-\lambda)-\left(\frac{\beta\alpha\mu}{\Lambda R_0}\right)(-(\mu+\varepsilon)-\lambda)(-\mu-\lambda)\right] \\ &\quad -\frac{\beta\mu}{\Lambda R_0}\left[(\mu\alpha(R_0-1))(-(\mu+\varepsilon)-\lambda)(-\mu-\lambda)\right]=0, \\ &(-\mu(R_0-1)-\lambda)\left[(-(\mu+\delta+\alpha)-\lambda)(-(\mu+\gamma_1+\gamma_2)-\lambda)(-(\mu+\varepsilon)-\lambda)(-\mu-\lambda)-\left(\frac{\beta\alpha\mu}{b\Lambda}\right)(-(\mu+\varepsilon)-\lambda)(-\mu-\lambda)\right] \\ &\quad -\frac{\alpha\beta\mu}{\Lambda R_0}\left[(\mu(R_0-1))(-(\mu+\varepsilon)-\lambda)(-\mu-\lambda)\right]=0, \\ &(-(\mu+\varepsilon)-\lambda)(-\mu-\lambda)\left[(-\mu(R_0-1)-\lambda)\left[(-(\mu+\delta+\alpha)-\lambda)(-(\mu+\gamma_1+\gamma_2)-\lambda)-\left(\frac{\beta\alpha\mu}{\Lambda R_0}\right)\right]-\frac{\alpha\beta\mu^2(R_0-1)}{\Lambda R_0}\right]=0. \end{aligned} \tag{18}$$

$(-(\mu+\varepsilon)-\lambda)(-\mu-\lambda)=0$, which implies that

$$\lambda_1=-\mu, \lambda_2=-(\mu+\varepsilon). \tag{19}$$

or

$$\begin{aligned} &\lambda^3+(((\mu+\delta+\alpha)+(\mu+\gamma_1+\gamma_2))+\mu(R_0-1))\lambda^2 \\ &\quad +\left(\mu(R_0-1)((\mu+\delta+\alpha)+(\mu+\gamma_1+\gamma_2))+\left((\mu+\delta+\alpha)(\mu+\gamma_1+\gamma_2)-\left(\frac{\beta\alpha\mu}{\Lambda R_0}\right)\right)\right)\lambda \\ &\quad +\left(\mu(R_0-1)\left((\mu+\delta+\alpha)(\mu+\gamma_1+\gamma_2)-\left(\frac{\beta\alpha\mu}{R_0}\right)\right)+\frac{\alpha\beta\mu^2(R_0-1)}{\Lambda R_0}\right)=0. \end{aligned} \tag{20}$$

Therefore, using the Routh–Hurwitz stability criteria, the violence-dominance equilibrium point is locally asymptotically stable when $R_0>1$. □

3. Numerical Simulations

In this section, we use the numerical simulations to deliver the analytical solution of our compartmental mathematical model (1). Mostly, some numerical explanations are considered to explain the analytical analysis and the results of numerical outputs. Here, we assume that the parameter values for numerical simulations are not from real data since there is a lack of mathematical modeling analysis literature which has been done to study the diffusion of violence. The initial values of model (1) are positive, i.e., $S(0)>0$, $E(0)\geq 0, P(0)\geq 0, H(0)\geq 0, H(0)\geq 0$. To understand the diffusion of violence, we need to assume parameter values and analyze the model (1) and describe how these parameters stimulate the diffusion of violence. In this case, let us consider the mathematical model (1) with the initial condition $(S(0), E(0), P(0), H(0), H(0)) = (100, 6, 4, 1, 0)$ and the parameter values as $\Lambda = 0.0042$, $\beta = 0.06$, $\alpha = 0.17$, $\delta = 0.03$, $\mu = 0.01$, $\gamma_1 = 0.12$, $\gamma_2 = 0.13$, $\varepsilon = 0.11$, and the basic reproduction number of model (1) with the estimated data is $R_0 = 0.085$. Theorem 1 confirms that the violence-free equilibrium point is locally asymptotically stable. Here, from

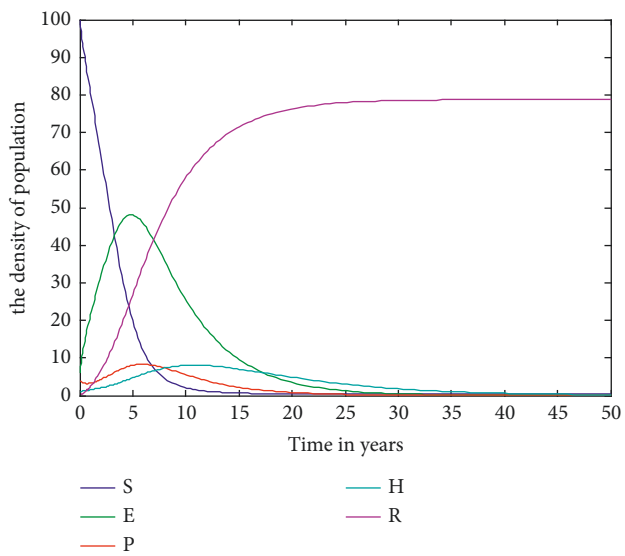

Figure 2: Solutions of model (1) when $R_0 < 1$.

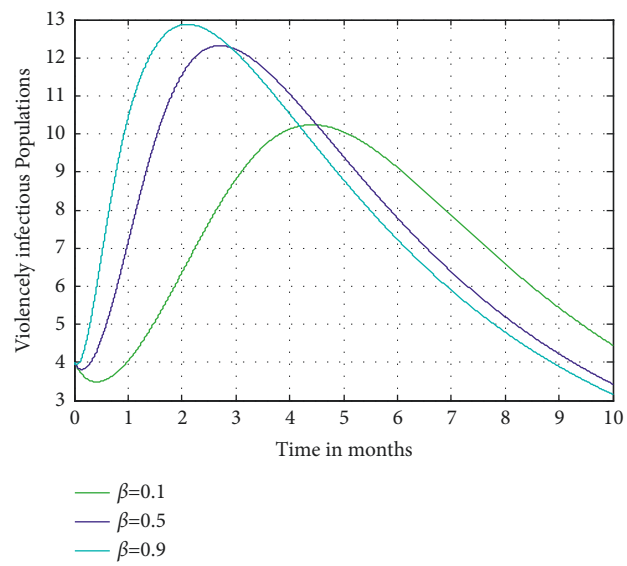

Figure 3: Effect of the diffusion rate β on the violently infectious individuals. P.

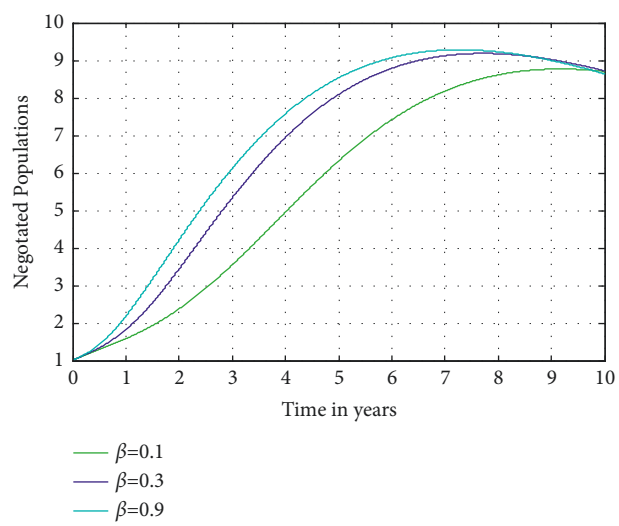

Figure 4: Effect of the diffusion rate β on the negotiated individuals. H.

Figure 2, we can explain that the violence-free equilibrium point is both locally and globally asymptotically stable whenever $R_0 = 0.085 < 1$, which means that violence will be eradicated. If we consider the parameter values $\Lambda = 0.055$, $\beta = 0.06$, $\alpha = 0.17$, $\delta = 0.03$, $\mu = 0.01$, $\gamma_1 = 0.12$, $\gamma_2 = 0.13$, $\varepsilon = 0.11$, then the basic reproduction number $R_0 = 1.1$. Theorem 2 confirms that the violence-dominance equilibrium point is locally asymptotically stable, which means that violence will be diffused. The effects of the transmission rate β on the violence-infected individual are given in Figure 3 by taking $\beta = 0.1, 0.3, 0.9$. Figure 3 shows that violence will be diffused in the population when the contact rate increases. The effects of the transmission rate β on the negotiated individual are given in Figure 4 by taking $\beta = 0.1, 0.3, 0.9$.Figure 4 shows that negotiated individuals will be an increase in the population when the contact rate increases.

Figure 2 shows us the path simulation of the violence diffusion model with assumed parameter values, where the violence diffusion model basic reproduction number is $R_0 = 0.085$. It shows that in the long run, we can see that the violently infectious state eliminates from the population. This means that the solutions of the model converge to the violence-free equilibrium point.

Figure 3 illustrates the impact of the violence diffusion rate β on violently infectious individuals P, which means that we observe the impact of the rate β by increasing their values 0.1, 0.3, and 0.9. Figure 3, shows that the number of violently infectious individuals increases as β increases.

Figure 4 shows the impact of the violence diffusion rate β on negotiated individuals H, and that means we set the impact of the rate β as we increase values 0.1, 0.3, and 0.9. In Figure 4, we see that the number of negotiated individuals increase as β increases.

Figure 5 shows the impact of the reconciled rate δ on violently infectious individuals P, which means we set the impact of the rate δ as we increase values 0.1, 0.3, and 0.9. In Figure 5, we see that the number of violently infectious individuals decreases as δ increases.

Figure 6 shows the impact of the reconciled rate δ on negotiated individuals H, and that means we set the impact of the rate δ as we increase values 0.1, 0.5, and 0.8. In Figure 6, we can see that the number of negotiated individuals decreases as δ increases.

Figure 7 shows the impact of the reconciled rate δ on reconciled individuals R, and that means we set the impact of the rate δ as we increase values 0.1, 0.5, and 0.9. In Figure 6, we can see that the number of reconciled individuals increase as δ increases.

Figure 8 shows the impact of the negotiation rate γ_1 on violently infectious individuals P, and that means we set the impact of the rate γ_1 as we increase values 0.1, 0.5, and 0.9. In

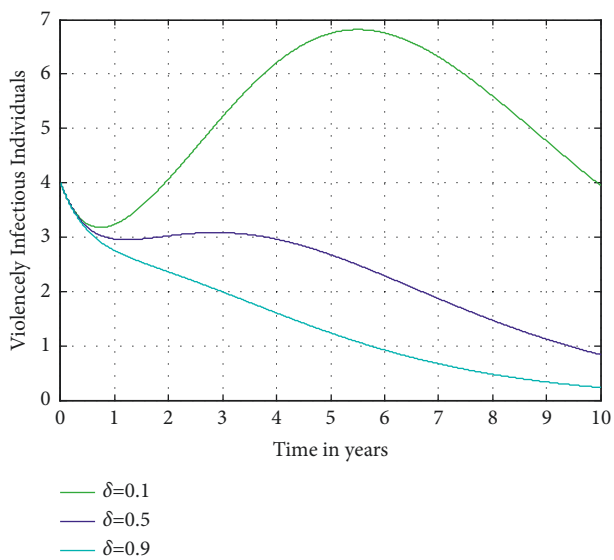

Figure 5: Effect of the reconciled rate δ on the violently infectious individuals. P.

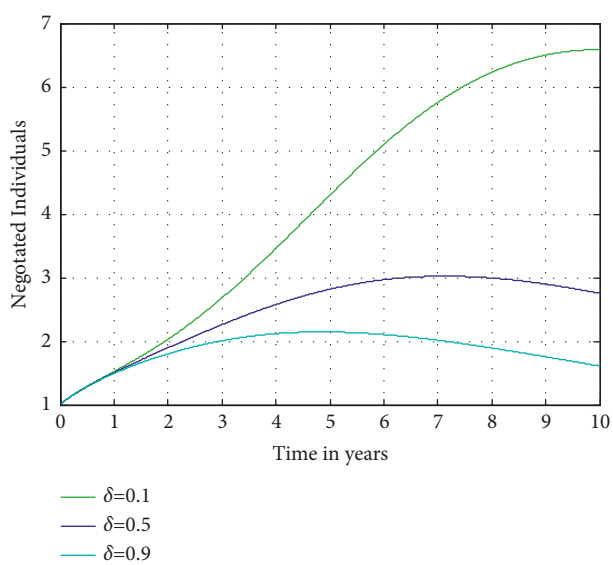

Figure 6: Effect of the reconciled rate δ on the negotiated individuals. H.

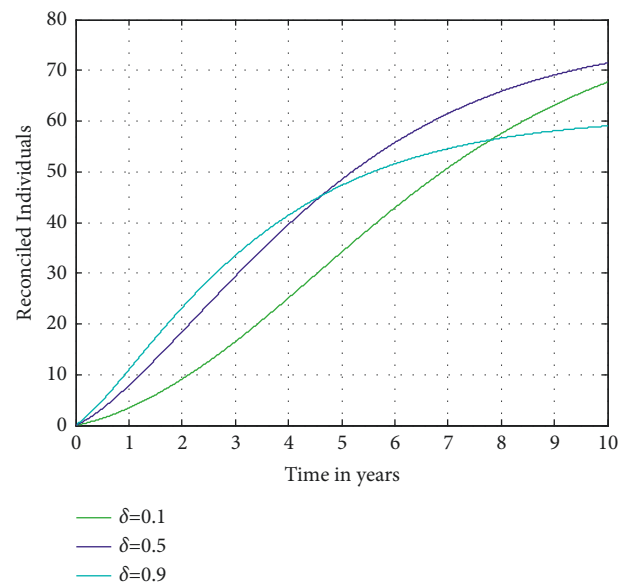

Figure 7: Effect of the reconciled rate δ on the reconciled individuals R.

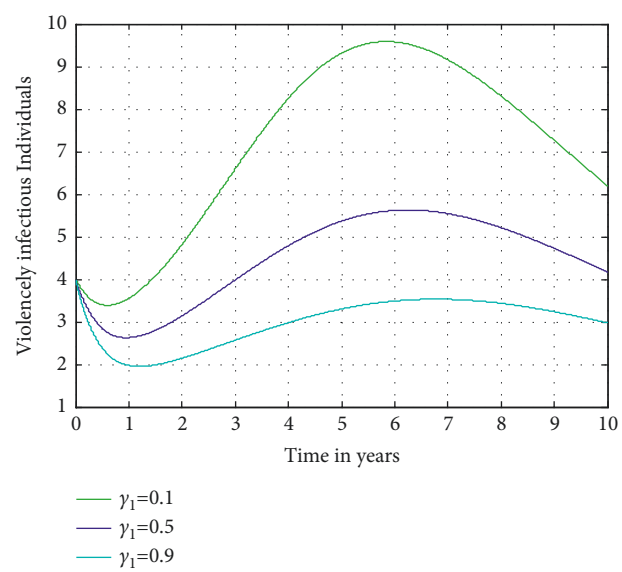

Figure 8: Effect of the negotiation rate γ_1 on the violently infectious individuals P.

Figure 8, we can see that the number of violently infectious individuals decreases as γ_1 increases.

Figure 9 shows the impact of the negotiation rate γ_1 on negotiated individuals H, and that means we set the impact of the rate γ_1 as we increase values 0.1, 0.5, and 0.9. In Figure 9, we can see that the number of negotiated individuals increase as γ_1 increases.

4. Discussion

We have seen the numerical reflection with the variation of model parameters in each state variable using the estimated data. Figure 2 reflects that violence is diffused in the population, the number of populations who are believed by negotiations increases, and the reconciled population also increases over a long period of time. Figure 3 shows that when the diffusion rate of violence increases, the violently infected individual increase within a long period of time. Figure 4 shows when the contact rate of violence increases, the number of negotiated individuals increases. Figure 5 shows that the violently infected individuals decrease when the reconciled rate δ of exposed individuals without negotiation increases. Figure 6 shows that the number of negotiated populations decreased when the reconciled rate δ of exposed individuals without negotiation increases. Figure 7

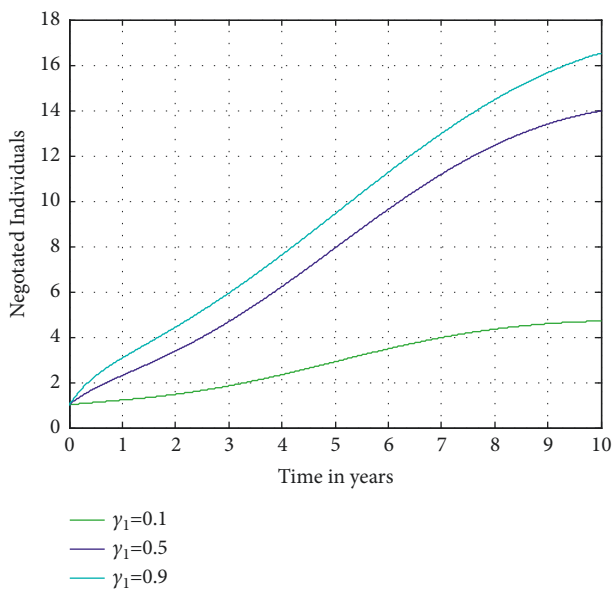

Figure 9: Effect of the negotiation rate γ_1 on the negotiated individuals H.

shows that the number of reconciled individuals increases when the reconciled rate δ of exposed individuals without negotiation increases. Figure 8 reflects that the violently infected population decreases when the negotiation rate γ_1 of violently infectious increases. Figure 9 reflects that the number of negotiated populations increases when the negotiation rate γ_1 of violently infectious increases.

5. Conclusion

In this paper, we formulated a first-hand compartmental model to study the diffusion of violence with the help of a nonlinear system of differential equations. A new SEPHR model is formulated to analyze analytically and numerically. We have shown the positivity and boundedness of the model. The violence-free and violence-dominance equilibrium points are calculated. The basic reproduction number of the model is calculated to determine the diffusion of violence. As the analytically analysis output, the violence-free equilibrium point is locally and globally asymptotically stable when $R_0 < 1$, and violence-dominance equilibrium is also locally and globally asymptotically stable when $R_0 < 1$. The sensitivity analysis of the parameters on the basic reproduction number is performed to determine the more sensitive parameter for the diffusion or elimination of violence. To eliminate violence from the population under the study we have to decrease the most sensitive parameterβ since it is a sensitive parameter. According to the numerical output of figures 3, 4, the violently infectious and negotiated population will decrease by decreasing the contact rate $\bar{\bar{\beta}}$. According to Figures 5–9, the population will live together and eliminate violence by increasing the negotiation rate and the reconciled rate with negotiation and without negotiation.

5.1. Limitation of the Study. There was a lack of literature study which took numerical simulation of modeling on the diffusion of violence and well-organized standard parameter values for the determination of model parameters. Due to the lack of literature and the current war in the Ethiopian population, we take estimated data because it is difficult to integrate experimental data into the study.

Data Availability

No data were used to support the findings of this study.

Conflicts of Interest

The author declares that there are no conflicts of interest.

Authors' Contributions

The author has read and approved the final manuscript.

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