## **Appendix I**

**Theorem 1 (Itô’s Lemma)**

Let be an Itô process,

where is a Wiener process and and are functions of and . If is any twice differentiable function, then satisfies,

**Proof**

By expanding in a Taylor series in and , we have

(A.1)

where is the remaining terms of Taylor series expansion. Substituting into Eq. (A.1) gives

In the limit, as tends to zero, and , but terms tends to (see [26]). Therefore, in the limit,

which proves the theorem.

**Corollary 1**

To investigate the general solution of Eq. (2), let . Then,

(A.2)

(A.3)

(A.4)

By substituting Eq. (A.2), (A.3) and (A.4) into Theorem 1, we obtain:

## **Appendix II**

Table S1: Color of cocoa powder in *L*\*, *a*\*, *b*\*.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| No. | L\* | | a\* | | b\* | |  | |
| 1 | 16.2490 | | 15.1954 | | 17.2128 | |  | |
| 2 | 15.9575 | | 15.8699 | | 18.0751 | |  | |
| 3 | 17.2473 | | 15.7009 | | 17.0994 | |  | |
| 4 | 16.7100 | | 14.9916 | | 17.4345 | |  | |
| 5 | 15.8852 | | 15.1182 | | 17.1249 | |  | |
| 6 | 15.6164 | | 15.4263 | | 17.7236 | |  | |
| 7 | 15.6754 | | 15.3157 | | 17.4364 | |  | |
| 8 | 16.5155 | | 15.5621 | | 17.8735 | |  | |
| 9 | 15.7521 | | 15.8545 | | 17.5665 | |  | |
| 10 | 16.0515 | | 16.0343 | | 18.8122 | |  | |
| 11 | 16.1436 | | 15.8058 | | 18.9943 | |  | |
| 12 | 15.5243 | | 15.9423 | | 17.0249 | |  | |
| 13 | 16.0825 | | 15.9290 | | 16.9012 | |  | |
| 14 | 16.6154 | | 15.8767 | | 18.0538 | |  | |
| 15 | 16.5131 | | 15.6263 | | 17.6106 | |  | |
| 16 | 15.4573 | | 15.7403 | | 16.5684 | |  | |
| 17 | 16.5203 | | 15.4665 | | 17.5544 | |  | |
| 18 | 16.2955 | | 15.1965 | | 18.4183 | |  | |
| 19 | 17.1261 | | 15.6666 | | 18.4984 | |  | |
| 20 | 17.0022 | | 14.9280 | | 17.8895 | |  | |
| 21 | 17.4206 | | 14.5998 | | 17.6493 | |  | |
| 22 | 17.3125 | | 15.2958 | | 18.2673 | |  | |
| 23 | 16.0839 | | 15.8079 | | 17.0504 | |  | |
| 24 | 15.2729 | | 15.6060 | | 16.6748 | |  | |
| 25 | 15.3417 | | 15.4784 | | 15.5543 | |  | |
| 26 | 16.7805 | | 15.2652 | | 15.1070 | |  | |
| 27 | 17.9587 | | 14.9984 | | 15.9616 | |  | |
| 28 | 18.1097 | | 15.4542 | | 16.7078 | |  | |
| 29 | 17.3686 | | 15.3524 | | 17.4470 | |  | |
| 30 | 17.1410 | | 15.5138 | | 17.5635 | |  | |
| 31 | 17.1030 | | 15.4242 | | 16.5834 | |  | |
| 32 | 18.0284 | | 15.9045 | | 17.8375 | |  | |
| 33 | 17.5280 | | 16.2159 | | 18.2315 | |  | |
| 34 | 17.4294 | | 15.9649 | | 18.0034 | |  | |
| 35 | 17.5145 | | 15.8176 | | 19.4181 | |  | |
| 36 | 16.5786 | | 16.0266 | | 19.4066 | |  | |
| 37 | 17.1880 | | 15.5301 | | 17.9864 | |  | |
| 38 | 16.4673 | | 15.2870 | | 17.2967 | |  | |
| 39 | 16.5933 | | 15.7597 | | 17.7316 | |  | |
| 40 | 17.7654 | | 15.6770 | | 17.4749 | |  | |
| 41 | 18.4256 | | 15.8128 | | 17.7958 | |  | |
| 42 | 18.2726 | | 15.8171 | | 17.7574 | |  | |
| 43 | 17.6376 | | 15.5110 | | 17.3934 | |  | |
| 44 | 16.5272 | | 15.5174 | | 16.8211 | |  | |
| 45 | 16.0695 | | 15.1906 | | 16.5142 | |  | |
| No. | L\* | | a\* | | b\* | |  | |
| 46 | 17.0118 | | 15.5863 | | 17.2827 | |  | |
| 47 | 17.7376 | | 15.5518 | | 17.2138 | |  | |
| 48 | 17.3226 | | 15.5386 | | 18.5782 | |  | |
| 49 | 17.6519 | | 15.7553 | | 18.1207 | |  | |
| 50 | 17.4370 | | 15.2750 | | 18.0773 | |  | |
| 51 | 16.7095 | | 15.1978 | | 17.9256 | |  | |
| 52 | 16.3263 | | 14.8503 | | 17.3255 | |  | |
| 53 | 16.5286 | | 14.6654 | | 17.0562 | |  | |
| 54 | 16.9367 | | 14.9239 | | 17.2237 | |  | |
| 55 | 16.9885 | | 14.6802 | | 16.7250 | |  | |
| 56 | 16.7486 | | 14.7202 | | 17.0541 | |  | |
| 57 | | 17.1274 | | 14.5063 | | 16.1468 | |
| 58 | | 16.0607 | | 14.3604 | | 16.2220 | |
| 59 | | 16.3677 | | 14.5109 | | 16.5535 | |
| 60 | | 15.9986 | | 14.9389 | | 17.1464 | |
| 61 | | 17.0770 | | 15.0013 | | 16.9952 | |
| 62 | | 16.6761 | | 14.6493 | | 16.6461 | |
| 63 | | 16.7572 | | 14.5356 | | 15.5316 | |
| 64 | | 15.8894 | | 14.4138 | | 15.6724 | |
| 65 | | 15.5128 | | 14.6101 | | 17.4185 | |
| 66 | | 15.5045 | | 14.7629 | | 17.6655 | |
| 67 | | 15.5949 | | 14.3620 | | 16.5624 | |
| 68 | | 15.3002 | | 14.6715 | | 17.8113 | |
| 69 | | 15.7434 | | 14.5254 | | 17.6229 | |
| 70 | | 15.4540 | | 14.7965 | | 16.5726 | |
| 71 | | 15.9598 | | 14.4028 | | 17.1176 | |
| 72 | | 15.3347 | | 14.5845 | | 17.4562 | |
| 73 | | 15.2928 | | 14.2200 | | 16.7234 | |
| 74 | | 15.8238 | | 14.2517 | | 16.8601 | |
| 75 | | 14.8416 | | 14.5257 | | 17.5457 | |
| 76 | | 15.4418 | | 14.5502 | | 17.8160 | |
| 77 | | 15.2139 | | 14.4077 | | 17.8794 | |
| 78 | | 14.5855 | | 14.8364 | | 17.0768 | |
| 79 | | 14.1345 | | 14.3970 | | 16.6106 | |
| 80 | | 13.3181 | | 14.5796 | | 16.4056 | |
| 81 | | 13.6655 | | 14.1645 | | 15.5704 | |
| 82 | | 13.7415 | | 13.9974 | | 15.6318 | |
| 83 | | 12.9860 | | 14.3948 | | 14.7814 | |
| 84 | | 12.8644 | | 14.6877 | | 15.4609 | |
| 85 | | 13.6302 | | 14.7849 | | 16.2232 | |
| 86 | | 13.7980 | | 14.8713 | | 16.2419 | |
| 87 | | 14.6884 | | 14.4239 | | 16.7831 | |
| 88 | | 15.1300 | | 14.2467 | | 16.7967 | |
| 89 | | 15.5625 | | 14.5320 | | 15.5613 | |
| 90 | | 15.2829 | | 14.8743 | | 16.5629 | |

Table S1: Color of cocoa powder in *L*\*, *a*\*, *b*\*.

(Continue)

|  |  |  |  |
| --- | --- | --- | --- |
| No. | L\* | a\* | b\* |
| 91 | 14.0892 | 15.0548 | 16.4624 |
| 92 | 13.8099 | 14.8074 | 16.1881 |
| 93 | 14.4261 | 14.6542 | 16.3924 |
| 94 | 13.7831 | 14.3424 | 15.5216 |
| 95 | 13.9438 | 14.0967 | 15.7221 |
| 96 | 14.0380 | 14.2469 | 16.2585 |
| 97 | 15.0451 | 14.5689 | 16.6204 |
| 98 | 14.0181 | 14.8288 | 16.3531 |
| 99 | 14.6521 | 14.7224 | 16.9592 |
| 100 | 14.7613 | 14.2710 | 16.0417 |
| 101 | 14.2802 | 14.2646 | 15.0594 |
| 102 | 14.9029 | 14.5180 | 15.0484 |
| 103 | 15.0576 | 14.8164 | 15.7520 |
| 104 | 14.8809 | 14.8133 | 17.0078 |
| 105 | 15.8155 | 14.5547 | 16.4848 |
| 106 | 16.1368 | 14.7435 | 17.0572 |
| 107 | 15.8684 | 15.0032 | 17.6626 |
| 108 | 15.5041 | 14.7572 | 16.5927 |
| 109 | 15.5285 | 14.6325 | 16.9895 |
| 110 | 15.0230 | 14.4834 | 17.5119 |
| 111 | 14.0131 | 14.5241 | 16.7448 |
| 112 | 13.7367 | 14.2725 | 16.5753 |