## **Appendix I**

**Theorem 1 (Itô’s Lemma)**

Let be an Itô process,

where is a Wiener process and and are functions of and . If is any twice differentiable function, then satisfies,

**Proof**

By expanding in a Taylor series in and , we have

 (A.1)

where is the remaining terms of Taylor series expansion. Substituting into Eq. (A.1) gives

In the limit, as tends to zero, and , but terms tends to (see [26]). Therefore, in the limit,

which proves the theorem.

**Corollary 1**

To investigate the general solution of Eq. (2), let . Then,

 (A.2)

 (A.3)

 (A.4)

By substituting Eq. (A.2), (A.3) and (A.4) into Theorem 1, we obtain:

## **Appendix II**

Table S1: Color of cocoa powder in *L*\*, *a*\*, *b*\*.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| No. | L\* | a\* | b\* |  |
| 1 | 16.2490  | 15.1954  | 17.2128  |  |
| 2 | 15.9575  | 15.8699  | 18.0751  |  |
| 3 | 17.2473  | 15.7009  | 17.0994  |  |
| 4 | 16.7100  | 14.9916  | 17.4345  |  |
| 5 | 15.8852  | 15.1182  | 17.1249  |  |
| 6 | 15.6164  | 15.4263  | 17.7236  |  |
| 7 | 15.6754  | 15.3157  | 17.4364  |  |
| 8 | 16.5155  | 15.5621  | 17.8735  |  |
| 9 | 15.7521  | 15.8545  | 17.5665  |  |
| 10 | 16.0515  | 16.0343  | 18.8122  |  |
| 11 | 16.1436  | 15.8058  | 18.9943  |  |
| 12 | 15.5243  | 15.9423  | 17.0249  |  |
| 13 | 16.0825  | 15.9290  | 16.9012  |  |
| 14 | 16.6154  | 15.8767  | 18.0538  |  |
| 15 | 16.5131  | 15.6263  | 17.6106  |  |
| 16 | 15.4573  | 15.7403  | 16.5684  |  |
| 17 | 16.5203  | 15.4665  | 17.5544  |  |
| 18 | 16.2955  | 15.1965  | 18.4183  |  |
| 19 | 17.1261  | 15.6666  | 18.4984  |  |
| 20 | 17.0022  | 14.9280  | 17.8895  |  |
| 21 | 17.4206  | 14.5998  | 17.6493  |  |
| 22 | 17.3125  | 15.2958  | 18.2673  |  |
| 23 | 16.0839  | 15.8079  | 17.0504  |  |
| 24 | 15.2729  | 15.6060  | 16.6748  |  |
| 25 | 15.3417  | 15.4784  | 15.5543  |  |
| 26 | 16.7805  | 15.2652  | 15.1070  |  |
| 27 | 17.9587  | 14.9984  | 15.9616  |  |
| 28 | 18.1097  | 15.4542  | 16.7078  |  |
| 29 | 17.3686  | 15.3524  | 17.4470  |  |
| 30 | 17.1410  | 15.5138  | 17.5635  |  |
| 31 | 17.1030  | 15.4242  | 16.5834  |  |
| 32 | 18.0284  | 15.9045  | 17.8375  |  |
| 33 | 17.5280  | 16.2159  | 18.2315  |  |
| 34 | 17.4294  | 15.9649  | 18.0034  |  |
| 35 | 17.5145  | 15.8176  | 19.4181  |  |
| 36 | 16.5786  | 16.0266  | 19.4066  |  |
| 37 | 17.1880  | 15.5301  | 17.9864  |  |
| 38 | 16.4673  | 15.2870  | 17.2967  |  |
| 39 | 16.5933  | 15.7597  | 17.7316  |  |
| 40 | 17.7654  | 15.6770  | 17.4749  |  |
| 41 | 18.4256  | 15.8128  | 17.7958  |  |
| 42 | 18.2726  | 15.8171  | 17.7574  |  |
| 43 | 17.6376  | 15.5110  | 17.3934  |  |
| 44 | 16.5272  | 15.5174  | 16.8211  |  |
| 45 | 16.0695  | 15.1906  | 16.5142  |  |
| No. | L\* | a\* | b\* |  |
| 46 | 17.0118  | 15.5863  | 17.2827  |  |
| 47 | 17.7376  | 15.5518  | 17.2138  |  |
| 48 | 17.3226  | 15.5386  | 18.5782  |  |
| 49 | 17.6519  | 15.7553  | 18.1207  |  |
| 50 | 17.4370  | 15.2750  | 18.0773  |  |
| 51 | 16.7095  | 15.1978  | 17.9256  |  |
| 52 | 16.3263  | 14.8503  | 17.3255  |  |
| 53 | 16.5286  | 14.6654  | 17.0562  |  |
| 54 | 16.9367  | 14.9239  | 17.2237  |  |
| 55 | 16.9885  | 14.6802  | 16.7250  |  |
| 56 | 16.7486  | 14.7202  | 17.0541  |  |
| 57 | 17.1274  | 14.5063  | 16.1468  |
| 58 | 16.0607  | 14.3604  | 16.2220  |
| 59 | 16.3677  | 14.5109  | 16.5535  |
| 60 | 15.9986  | 14.9389  | 17.1464  |
| 61 | 17.0770  | 15.0013  | 16.9952  |
| 62 | 16.6761  | 14.6493  | 16.6461  |
| 63 | 16.7572  | 14.5356  | 15.5316  |
| 64 | 15.8894  | 14.4138  | 15.6724  |
| 65 | 15.5128  | 14.6101  | 17.4185  |
| 66 | 15.5045  | 14.7629  | 17.6655  |
| 67 | 15.5949  | 14.3620  | 16.5624  |
| 68 | 15.3002  | 14.6715  | 17.8113  |
| 69 | 15.7434  | 14.5254  | 17.6229  |
| 70 | 15.4540  | 14.7965  | 16.5726  |
| 71 | 15.9598  | 14.4028  | 17.1176  |
| 72 | 15.3347  | 14.5845  | 17.4562  |
| 73 | 15.2928  | 14.2200  | 16.7234  |
| 74 | 15.8238  | 14.2517  | 16.8601  |
| 75 | 14.8416  | 14.5257  | 17.5457  |
| 76 | 15.4418  | 14.5502  | 17.8160  |
| 77 | 15.2139  | 14.4077  | 17.8794  |
| 78 | 14.5855  | 14.8364  | 17.0768  |
| 79 | 14.1345  | 14.3970  | 16.6106  |
| 80 | 13.3181  | 14.5796  | 16.4056  |
| 81 | 13.6655  | 14.1645  | 15.5704  |
| 82 | 13.7415  | 13.9974  | 15.6318  |
| 83 | 12.9860  | 14.3948  | 14.7814  |
| 84 | 12.8644  | 14.6877  | 15.4609  |
| 85 | 13.6302  | 14.7849  | 16.2232  |
| 86 | 13.7980  | 14.8713  | 16.2419  |
| 87 | 14.6884  | 14.4239  | 16.7831  |
| 88 | 15.1300  | 14.2467  | 16.7967  |
| 89 | 15.5625  | 14.5320  | 15.5613  |
| 90 | 15.2829  | 14.8743  | 16.5629  |

Table S1: Color of cocoa powder in *L*\*, *a*\*, *b*\*.

(Continue)

|  |  |  |  |
| --- | --- | --- | --- |
| No. | L\* | a\* | b\* |
| 91 | 14.0892  | 15.0548  | 16.4624  |
| 92 | 13.8099  | 14.8074  | 16.1881  |
| 93 | 14.4261  | 14.6542  | 16.3924  |
| 94 | 13.7831  | 14.3424  | 15.5216  |
| 95 | 13.9438  | 14.0967  | 15.7221  |
| 96 | 14.0380  | 14.2469  | 16.2585  |
| 97 | 15.0451  | 14.5689  | 16.6204  |
| 98 | 14.0181  | 14.8288  | 16.3531  |
| 99 | 14.6521  | 14.7224  | 16.9592  |
| 100 | 14.7613  | 14.2710  | 16.0417  |
| 101 | 14.2802  | 14.2646  | 15.0594  |
| 102 | 14.9029  | 14.5180  | 15.0484  |
| 103 | 15.0576  | 14.8164  | 15.7520  |
| 104 | 14.8809  | 14.8133  | 17.0078  |
| 105 | 15.8155  | 14.5547  | 16.4848  |
| 106 | 16.1368  | 14.7435  | 17.0572  |
| 107 | 15.8684  | 15.0032  | 17.6626  |
| 108 | 15.5041  | 14.7572  | 16.5927  |
| 109 | 15.5285  | 14.6325  | 16.9895  |
| 110 | 15.0230  | 14.4834  | 17.5119  |
| 111 | 14.0131  | 14.5241  | 16.7448  |
| 112 | 13.7367  | 14.2725  | 16.5753  |