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Research Article

t-Intuitionistic Fuzzy Structures on PMS-Ideals of a PMS-Algebra

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In this article, we apply the concept of a t-intuitionistic fuzzy set to PMS-ideals in PMS-algebras. The notion of the t-intuitionistic fuzzy PMS-ideal of PMS-algebra is introduced, and several related properties are studied. The relationships between a t-intuitionistic fuzzy PMS-ideal and a t-intuitionistic fuzzy PMS-subalgebra of a PMS-algebra, as well as the relationships between an intuitionistic fuzzy PMS-ideal and a t-intuitionistic fuzzy PMS-ideal are discussed in detail. A condition for an intuitionistic fuzzy set to be a t-intuitionistic fuzzy PMS-ideal is provided. The t-intuitionistic fuzzy PMS-ideals of PMS-algebra are described using their (α, β) level cuts. The homomorphism of a t-intuitionistic fuzzy PMS-ideal of a PMS-algebra is studied, and its homomorphic image and inverse image are explored. The Cartesian product of any two t-intuitionistic fuzzy PMS-ideals is discussed, and some related results are derived. The Cartesian product of the t-intuitionistic fuzzy PMS-ideals is also characterized using its (α, β) level cuts. The strongest t-intuitionistic fuzzy PMS-relation in a PMS-algebra is defined. Finally, the relationships between the strongest t-intuitionistic fuzzy PMS-relation and t-intuitionistic fuzzy PMS-ideal are studied.

1. Introduction

In 1965, Zadeh[1] introduced the concept of a fuzzy set for dealing with uncertainty and vagueness in real-world problems. Since then, several researchers have applied it to a wide range of algebraic structures, such as BCI-algebras, BCK-algebras, BG-algebras, KU-algebras, etc. Akram and Dar [2] introduced the notions of T-fuzzy subalgebras and T-fuzzy H-ideals in BCI-algebras using a t-norm T and investigated some of their properties. Akram and Zhan [3] introduced the notion of sensible fuzzy ideals of BCK-algebras with respect to a t-conorm and investigated some of their properties. Senapati et al. [4] introduced the notion of T-fuzzy subalgebras and T-fuzzy closed ideals of BG-algebras and investigated their related results. The notion of T-fuzzy KU-ideals of KU-algebras are introduced using t-norm T and their related results are investigated by Senapati [5]. He further investigated images and pre-images of KU-ideals under homomorphism and the Cartesian product and T-product of T-fuzzy KU-ideals of KU-algebras. After the introduction of fuzzy sets by Zadeh, many mathematicians have worked to extend this fundamental concept in a variety of ways. In 1975, Zadeh [6] developed a type-2 fuzzy set as an extension of the fuzzy set with a membership grade of fuzzy set in the unit interval [0, 1] rather than a point in [0, 1]. Torra and Narukawa [7, 8] developed the concept of a hesitant fuzzy set as one of the extensions of the fuzzy set to express hesitant information more thoroughly than other extensions of the fuzzy set, as it permits several possible values for the membership degree of an element.

Atanassov [9, 10] introduced the idea of an intuitionistic fuzzy set as a generalization of the fuzzy set. An intuitionistic fuzzy set is more effective than a fuzzy set in dealing with ambiguity and uncertainty since it assigns a membership and nonmembership degree to each element of a set. Since its appearance, mathematicians have applied this fundamental concept to a number of algebraic structures. Jun et al. [11] introduced the notion of an intuitionistic fuzzy quasi-associative ideal of a BCI-algebra and investigated some related properties. Many fundamental characteristics of intuitionistic fuzzy subgroups were also explored by Sharma [12, 13]. Panigrahi and Nanda [14] studied the idea of intuitionistic fuzzy relations over intuitionistic fuzzy subsets and found several interesting properties of intuitionistic fuzzy relations in intuitionistic fuzzy subsets. Peng [15] introduced the notion of intuitionistic fuzzy B-algebras in B-algebra and studied some properties of the homomorphic image and inverse image of intuitionistic fuzzy B-algebras. Jana et al. [16] introduced the concept of intuitionistic fuzzy set to G-subalgebras of G-algebras and investigated several properties. Sharma [17, 18] developed the concept of the t-intuitionistic fuzzy set as an extension of the intuitionistic fuzzy set to deal with uncertainty and vagueness within the context of some reference points in the unit interval [0,1] and then introduced the concepts of t-intuitionistic fuzzy subgroups and t-intuitionistic fuzzy subrings. Shuaib et al. [19] introduced the notion of η-intuitionistic fuzzy subgroup over η-intuitionistic fuzzy subset and studied some algebraic aspects of η-fuzzy subgroups. Barbhuiya [20] introduced the notion of t-intuitionistic fuzzy subalgebra and t-intuitionistic fuzzy normal subalgebra of BG-algebra and studied their properties. He also investigated the homomorphic image and inverse image of both t-intuitionistic fuzzy subalgebra and t-intuitionistic fuzzy normal subalgebra of a BG-algebra.

Iseki and Tanaka [21] introduced a class of abstract algebras called BCK-algebras. Iseki [22] introduced another class of abstract algebra called BCI-algebra as a generalization of BCK-algebra. In 2016, Sithar Selvam and Nagalakshmi [23] introduced a new algebraic structure, known as PMS-algebra, as a generalization of BCK \BCI \TM \PS-algebras and investigated various related results. In the same year, Sithar Selvam and Nagalakshmi [24] also fuzzified a PMS-ideal in a PMS-algebra and investigated its basic properties. The study of intuitionistic fuzzification of PMS-subalgebras and PMS-ideals of PMS-algebras was done by Derseh et al. [25]. The notion of t-intuitionistic fuzzy subalgebra has been studied in several algebraic structures (see [17–20, 26, 27]. However, to the best of our knowledge, no studies on t-intuitionistic fuzzy ideals of any algebraic structure, including PMS-algebra, are available. This motivated us to develop t-intuitionistic fuzzy PMS-ideals in PMS-algebra.

In this manuscript, we use the concept of a t-intuitionistic fuzzy set to PMS-ideals in PMS-algebras. We introduce the notion of a t-intuitionistic fuzzy PMS-ideal of PMS-algebra and study its properties. We consider the relationships between a t-intuitionistic fuzzy PMS-ideal and a t-intuitionistic fuzzy PMS-subalgebra of a PMS-algebra as well as the relationships between an intuitionistic fuzzy PMS-ideal and a t-intuitionistic fuzzy PMS-ideal of a PMS-algebra. We establish a condition for an intuitionistic fuzzy set in a PMS-algebra to be a t-intuitionistic fuzzy PMS-ideal of a PMS-algebra. We describe the t-intuitionistic fuzzy PMS-ideals of PMS-algebra using their (α,β) level cuts. We consider a t-intuitionistic fuzzy PMS-ideal in a PMS-algebra under homomorphism and explore its homomorphic image and inverse image in a PMS-algebra. Furthermore, we study the Cartesian product of any two t-intuitionistic fuzzy PMS-ideals of PMS-algebra and find some interesting results. We also characterize the Cartesian product of the t-intuitionistic fuzzy PMS-ideals using their (α,β) level cuts. We finally define the strongest t-intuitionistic fuzzy PMS-relation in a PMS-algebra and study the relationship between the strongest t-intuitionistic fuzzy PMS-relation and a t-intuitionistic fuzzy PMS-ideal.

2. Preliminaries

In this section, we consider some basic definitions, results, and some important concepts in PMS-algebras that are needed for our work.

Definition 1 (see [23]). A PMS-algebra is a nonempty set X with a constant 0 and a binary operation $*$ of type (2, 0) satisfying the following axioms:

(i) $0 * x = x$

(ii) $(y * x) * (z * x) = z * y$, for all $x, y, z \in X$

We can define a binary relation $\leq$ in X by $x \leq y$ if and only if $x * y = 0$.

Definition 2 (see [23]). A nonempty subset S of a PMS-algebra is called a PMS-subalgebra of X if $x * y \in S$, for all $x, y \in S$.

Definition 3 (see [23]). A nonempty subset I of a PMS-algebra $(X, *, 0)$ is said to be a PMS-ideal of X if it satisfies the following conditions:

(i) $0 \in \mathrm{I}$

(ii) $z * y, z * x \in \mathrm{I} \Rightarrow y * x \in \mathrm{I}$, for all $x, y \in \mathrm{I}$.

Proposition 1 (see [23]). *Let $(X, *, 0)$ be a PMS-algebra. Then the following properties hold for all $x, y, z \in X$,*

(i) $x * x = 0$

(ii) $(y * x) * x = y$

(iii) $x * (y * x) = y * 0$

(iv) $(y * x) * z = (z * x) * y$

(v) $(x * y) * 0 = y * x = (0 * y) * (0 * x)$

Definition 4 (see [1]). A fuzzy subset A in a nonempty set X is defined as $A = \{\langle x, \mu_A(x)\rangle | x \in X\}$, where the mapping $\mu_A\colon X \longrightarrow [0, 1]$ defines the degree of membership

Definition 5 (see [9, 10]). An intuitionistic fuzzy set A in a nonempty set X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$, where the functions $\mu_A\colon X \longrightarrow [0, 1]$ and $\nu_A\colon X \longrightarrow [0, 1]$ define the degree of membership and the degree of nonmembership of each element $x \in X$ to the set A, respectively, with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, $\forall x \in X$.

Definition 6 (see [9, 10]). Let A and B be two intuitionistic fuzzy subsets of the set X, where $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in X\}$, then

(i) $A \cap B = \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\}\rangle | x \in X\}$

(ii) $A \cup B = \{\langle x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\}\rangle | x \in X\}$
(iii) $\overline{A} = \{x, \nu_A(x), \mu_A(x) | x \in X\}$
(iv) $\square A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in X\}$
(v) $A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in X\}$

Definition 7 (see [25]). An intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of a PMS-algebra X is called an intuitionistic fuzzy PMS-subalgebra of X if $\mu_A(x * y) \geq \min\{\mu_A(x), \nu_A(y)\}$ and $\nu_A(x * y) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all $x, y \in X$.

Definition 8 (see [28]). An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ in X is called an intuitionistic fuzzy PMS-ideal of X if it satisfies the following conditions for all $x, y, z \in X$.

(i) $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$,
(ii) $\mu_A(y * x) \geq \min\{\mu_A(z * y), \mu_A(z * x)\}$,
(iii) $\nu_A(y * x) \leq \max\{\nu_A(z * y), \nu_A(z * x)\}$

Definition 9 (see [29]). Let A be a fuzzy set in a nonempty set X with membership function $\mu_A\colon X \longrightarrow [0,1]$ and let $t \in [0,1]$. Then the fuzzy set A^t in X is called the t-fuzzy subset of X whose membership function is μ_{A^t} (w.r.t fuzzy set A) and is defined by $\mu_{A^t}(x) = \min\{\mu_A(x), t\}$, for all $x \in X$.

Definition 10 (see [18]). Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set in a nonempty set X and $t \in [0,1]$. Then the t-intuitionistic fuzzy set (t-IFS) A^t in a nonempty set X is an object having the form $A^t = \{\langle x, \mu_{A^t}(x), \nu_{A^t}(x)\rangle | x \in X\}$, where the function $\mu_{A^t}\colon X \longrightarrow [0,1]$ and $\nu_{A^t}\colon X \longrightarrow [0,1]$ denote the degree of membership and degree of non-membership, respectively, such that $\mu_{A^t}(x) = \min\{\mu_A(x), t\}$ and $\nu_{A^t}(x) = \max\{\nu_A(x), 1 - t\}$ satisfying the condition $0 \leq \mu_{A^t}(x) + \nu_{A^t}(x) \leq 1$, for all $x \in X$.

Note: For the sake of simplicity, we shall use the symbol $A^t = (\mu_{A^t}, \nu_{A^t})$, for t-IFS $A^t = \{\langle x, \mu_{A^t}(x), \nu_{A^t}(x)\rangle | x \in X\}$.

Remark 1. Let $A^t = \{\langle x, \mu_{A^t}(x), \nu_{A^t}(x)\rangle | x \in X\}$ be a t-IFSs of the set X. Then

$\square A^t = \{\langle x, \mu_{A^t}(x), 1 - \mu_{A^t}(x)\rangle | x \in X\} = \{\langle x, \mu_{A^t}(x), \overline{\mu}_{A^t}(x)\rangle | x \in X\}$ and

$$\begin{aligned} \Diamond A^t &= \{\langle x, 1 - \nu_{A^t}(x), \nu_{A^t}(x)\rangle | x \in X\} \\ &= \{\langle x, \overline{\nu}_{A^t}(x), \nu_{A^t}(x)\rangle | x \in X\}. \end{aligned} \tag{1}$$

Remark 2 (see [18, 27]). Let $A^t = (\mu_{A^t}, \nu_{A^t})$ and $B^t = (\mu_{B^t}, \nu_{B^t})$ be any two t-intuitionistic fuzzy subsets of any nonempty set X, then $(A \cap B)^t = A^t \cap B^t$ and $(A \cup B)^t = A^t \cup B^t$.

Definition 11 (see [23]). Let X and Y be PMS-algebras. The mapping $f\colon X \longrightarrow Y$ is called a homomorphism of PMS-algebras if $f(x * y) = f(x) * f(y)$, for all $x, y \in X$. A homomorphism $f\colon X \longrightarrow Y$ is called an epimorphism of PMS-algebras if $f(X) = Y$.

Note: If f is a homomorphism of PMS-algebras, then $f(0) = 0$.

Remark 3 (see [18]). Let $f\colon X \longrightarrow Y$ be a mapping and A such that B are any two t-IFSs of X and Y, respectively, then $f(A^t) = (f(A))^t$ and $f^{-1}(B^t) = (f^{-1}(B))^t$, for all $t \in [0,1]$.

Definition 12 (see [26]). Let $A^t = (\mu_{A^t}, \nu_{A^t})$ and $A^t = (\mu_{A^t}, \nu_{B^t})$ be two t-intuitionistic fuzzy subsets of X and Y, respectively. Then their Cartesian product of A^t and B^t denoted by $A^t \times B^t$ is defined as $A^t \times B^t = \{\langle (x, y), \mu_{A^t \times B^t}, \nu_{A^t \times B^t}\rangle \ | x \in X \text{and} \ y \in Y\}$, where $\mu_{A^t \times B^t}(x * y) = \min\{\mu_{A^t}(x), \mu_{B^t}(y))\}$ and $\nu_{A^t \times B^t}(x * y) = \max\{\nu_{A^t}(x), \nu_{B^t}(y)\}$, for all $x \in X$ and $y \in Y$.

Remark 4. Let X and Y be any two PMS-algebras, for every $(x, y), (u, v) \in X \times Y$, we define $' * '$ on $X \times Y$ by $(x, y) * (u, v) = (x * u, y * v)$. Clearly, $(X \times Y, *, (0, 0))$ is a PMS-algebra.

Definition 13 (see [26]). Let A^t be t-IFS of X w.r.t IFS A. Then the (α, β)-cut of A^t is a crisp subset $C_{(\alpha,\beta)}(A^t)$ of X and is given by $C_{(\alpha,\beta)}(A^t) = \{x \in X\colon \mu_{A^t}(x) \geq \alpha, \nu_{A^t}(x) \leq \beta\}$, where $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$.

3. t-Intuitionistic Fuzzy PMS-Ideals of a PMS-Algebra

In this section, we study the notion of a t-intuitionistic fuzzy PMS-ideal in a PMS-algebra and investigate several interesting results. In what follows, let X and Y denote PMS-algebra unless otherwise specified.

Definition 14 Let $t \in [0,1]$. A t-IFS A^t of X is called the t-intuitionistic fuzzy (t-IF) PMS-ideal of a PMS-algebra X if

(i) $\mu_{A^t}(0) \geq \mu_{A^t}(x)$ and $\nu_{A^t}(0) \leq \nu_{A^t}(x)$,
(ii) $\mu_{A^t}(y * x) \geq \min\{\mu_{A^t}(z * y), \mu_{A^t}(z * x)\}$,
(iii) $\nu_{A^t}(y * x) \leq \max\{\nu_{A^t}(z * y), \nu_{A^t}(z * x)\}$, for all $x, y, z \in X$.

Example 1. Consider $X = \{0, a, b\}$ such that $(X, *, 0)$ is a PMS-algebra with Table 1.

Define the intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ in X by $\mu_A(x) = \begin{cases} 0.8, & \text{if } x = 0, \\ 0.5, & \text{if } x = a, \\ 0.6, & \text{if } x = b \end{cases}$ and $\nu_A(x, q) = \begin{cases} 0.2, & \text{if } x = 0, \\ 0.4, & \text{if } x = a, \\ 0.3, & \text{if } x = b \end{cases}$ for all $x \in$ X.

If we take $t = 0.4$, then we have $\mu_{A^t}(x) = \min\{\mu_A(x), t\} = \min\{\mu_A(x), 0.4\} = 0.4$ and $\nu_{A^t}(x) = \max\{\nu_A(x), 1 - t\} = \max\{\nu_A(x), 1 - 0.4\} = \max\{\nu_A(x), 0.6\} = 0.6$, for all $x \in X$.

Then by routine calculation, we can see that $A^t = (\mu_{A^t}, \nu_{A^t})$ is a t-IF PMS-ideal of X.

Table 1: $(X, *, 0)$ is a PMS-algebra.

$*$	0	a	b
0	0	a	b
a	b	0	a
b	a	b	0

Theorem 1. *Every t-IF PMS-ideal of X is a t-IF PMS-subalgebra of X*

Proof. Let A^t be a t-IF PMS-ideal of X and $x, y \in X$. Then by Definition $14(ii, iii)$ and Definition $1(i)$, we have

$\mu_{A^t}(x * y) \geq \min\{\mu_{A^t}(0 * x), \mu_{A^t}(0 * y)\} = \min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$ and

$$\nu_{A^t}(x * y) \leq \max\{\nu_{A^t}(0 * x), \nu_{A^t}(0 * y)\} = \max\{\nu_{A^t}(x), \nu_{A^t}(y)\}. \tag{2}$$

Therefore, A^t is a t-IF PMS-subalgebra of X. □

Theorem 2. *Let A^t be a t-IF PMS-ideal of X. If $x \leq y$, then $\mu_{A^t}(x) \geq \mu_{A^t}(y)$ and $\nu_{A^t}(x) \leq \nu_{A^t}(y)$, for all $x, y \in X$*

Proof. Let $x, y \in X$ such that $x \leq y$. Then $x * y = 0$. By Definition $14(i)$, Proposition $1(iv)$, Definition 14, and Theorem 1, we have

$$\begin{aligned}
\mu_{A^t}(x) &= \mu_{A^t}(0 * x) \geq \min\{\mu_{A^t}(y * 0), \mu_{A^t}(y * x)\} \\
&= \min\{\mu_{A^t}(y * 0), (\mu_{A^t}(x * y) * 0)\} \\
&= \min\{\mu_{A^t}(y * 0), \mu_{A^t}(0 * 0)\} \\
&= \min\{\mu_{A^t}(y * 0), \mu_{A^t}(0)\} \\
&= \mu_{A^t}(y * 0) \geq \min\{\mu_{A^t}(y), \mu_{A^t}(0)\} = \mu_{A^t}(y), \\
\nu_{A^t}(x) &= \nu_{A^t}(0 * x) \leq \max\{\nu_{A^t}(y * 0), \nu_{A^t}(y * x)\} \\
&= \max\{\nu_{A^t}(y * 0), \nu_{A^t}(x * y) * 0\} \\
&= \max\{\nu_{A^t}(y * 0), \nu_{A^t}(0 * 0)\} \\
&= \max\{\nu_{A^t}(y * 0), \nu_{A^t}(0)\} \\
&= \nu_{A^t}(y * 0) \leq \max\{\nu_{A^t}(y), \nu_{A^t}(0)\} = \nu_{A^t}(y),
\end{aligned} \tag{3}$$

and, therefore, $\mu_{A^t}(x) \geq \mu_{A^t}(y)$ and $\nu_{A^t}(x) \leq \nu_{A^t}(y)$, for all $x, y \in X$. □

Theorem 3. *If A is an IF PMS-ideal of X, then A^t is also a t-IF PMS-ideal of X.*

Proof. Let A be an IF PMS-ideal of X and $x, y, z \in X$. Then by the definition of t-IFS and definition of intuitionistic fuzzy PMS-ideal, we have

$\mu_{A^t}(0) = \min\{\mu_A(0), t\} \geq \min\{\mu_A(x), t\} = \mu_{A^t}(x)$ and $\nu_{A^t}(0) = \max\{\nu_A(x), 1 - t\} \leq \max\{\nu_A(0), 1 - t\} = \nu_{A^t}(x)$

Also,

$$\begin{aligned}
\mu_{A^t}(y * x) &= \min\{\mu_A(y * x), t\} \\
&\geq \min\{\min\{\mu_A(z * y), \mu_A(z * x)\}, t\} \\
&= \min\{\min\{\mu_A(z * y), t\}, \min\{\mu_A(z * x), t\}\} \\
&= \min\{\mu_{A^t}(z * y), \mu_{A^t}(z * x)\} \\
&\Rightarrow \mu_{A^t}(y * x) \geq \min\{\mu_{A^t}(z * y), \mu_{A^t}(z * x)\},
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
\nu_{A^t}(y * x) &= \max\{\nu_A(y * x), 1 - t\} \\
&\leq \max\{\max\{\nu_A(z * y), \nu_A(z * x)\}, 1 - t\} \\
&= \max\{\max\{\nu_A(z * y), 1 - t\}, \\
&\quad \max\{\nu_A(z * x), 1 - t\}\} \\
&= \max\{\nu_{A^t}(z * y), \nu_{A^t}(z * x)\} \\
&\Rightarrow \nu_{A^t}(y * x) \leq \max\{\nu_{A^t}(z * y), \nu_{A^t}(z * x)\}.
\end{aligned} \tag{5}$$

Hence, $\mu_{A^t}(y * x) \geq \min\{\mu_{A^t}(z * y), \mu_{A^t}(z * x)\}$ and $\nu_{A^t}(y * x) \leq \max\{\nu_{A^t}(z * y), \nu_{A^t}(z * x)\}$, for all $x, y, z \in X$. Therefore, A^t is a t-IF PMS-ideal of X. □

Remark 5. The converse of above theorem need not be necessarily true. This fact is shown by the following example:

Example 2. Let $X = \{0, a, b\}$ be a set with Table 1 as in Example 1. Define the intuitionistic fuzzy set $B = (\mu_B, \nu_B)$ in X by

$$\mu_B(x) = \begin{cases} 0.4, & \text{if } x = 0, \\ 0.5, & \text{if } x = a, \\ 0.3, & \text{if } x = b \end{cases} \quad \text{and} \quad \nu_B(x, q) = \begin{cases} 0.5, & \text{if } x = 0, \\ 0.4, & \text{if } x = a, \text{ for all } x \in \text{X}. \\ 0.6, & \text{if } x = b \end{cases}$$

Since $\mu_B(0) = 0.4 < 0.5 = \mu_B(a)$ and $\nu_B(0) = 0.5 > 0.4 = \nu_B(a)$, it follows that B is not an intuitionistic fuzzy PMS-ideal of X as it does not satisfy Definition $14(i)$. If we take $t = 0.3$, then $\mu_{B^t}(x) = 0.3$ and $\nu_{B^t}(x) = 0.7$, for all $x \in X$. Therefore, by routine calculations, we get that

(i) $\mu_{B^t}(0) \geq \mu_{B^t}(x)$ and $\nu_{B^t}(0) \leq \mu_{B^t}(x)$

(ii) $\mu_{B^t}(y * x) \geq \min\{\mu_{B^t}(z * y), \mu_{B^t}(z * x)\}$, and

(iii) $\nu_{B^t}(y * x) \leq \max\{\nu_{B^t}(z * y), \nu_{B^t}(z * x)\}, \forall x, y, z \in X$.

Therefore, by Definition 14, $B^t = (\mu_{B^t}, \nu_{B^t})$ is a t-IF PMS-ideal of X.

The following theorem provides a condition for an intuitionistic fuzzy set in a PMS-algebra to be a t-intuitionistic fuzzy PMS-ideal.

Theorem 4. *Let A be an IFS in X and $t \in [0,1]$ such that $t \leq \min\{m, 1-n\}$, where $m = \min\{\mu_A(x) | x \in X\}$ and $n = \max\{\nu_A(x) | x \in X\}$. Then A^t is a t-IF PMS-ideal of X.*

Proof. Since $t \leq \min\{m, 1-n\}$, we have $m \geq t$ and $1-n \geq t$ $\Rightarrow m \geq t$ and $n \leq 1-t$, $\Rightarrow \min\{\mu_A(x) | x \in X\} \geq t$ and $\max\{\nu_A(x) | x \in X\} \leq 1-t$, $\Rightarrow \mu_A(x) \geq t$ and $\nu_A(x) \leq 1-t$, for all $x \in X$.

So, $\min\{\mu_A(x), t\} = t$ and $\max\{\nu_A(x), 1-t\} = 1-t$, $\Rightarrow \mu_{A^t}(x) = t$ and $\nu_{A^t}(x) = 1-t$, for all $x \in X$.

Therefore, $\mu_{A^t}(0) = t = \mu_{A^t}(x)$ and $\nu_{A^t}(0) = 1-t = \nu_{A^t}(x)$. This satisfies the condition $\mu_{A^t}(0) \geq \mu_{A^t}(x)$ and $\nu_{A^t}(0) \leq \nu_{A^t}(x)$, for all $x \in X$.

Also,
$\mu_{A^t}(y*x) = t \geq \min\{t,t\} = \min\{\mu_{A^t}(z*y), \mu_{A^t}(z*x)\}$ and $\nu_{A^t}(y*x) = 1-t \leq \max\{1-t, 1-t\} = \max\{\nu_{A^t}(z*y), \nu_{A^t}(z*x)\}$, for all $x, y, z \in X$.

Hence, $\mu_{A^t}(y*x) \geq \min\{\mu_{A^t}(z*y), \mu_{A^t}(z*x)\}$ and $\nu_{A^t}(y*x) \leq \max\{\nu_{A^t}(z*y), \nu_{A^t}(z*x)\}$, for all $x, y, z \in X$. □

Example 3. Let $\mathbb{Z}$ be the set of all integers. Let $*$ be a binary operation on $\mathbb{Z}$ defined by $x*y = y-x$ for all $x, y \in \mathbb{Z}$, where '$-$' is the usual subtraction of integers. Then $(\mathbb{Z}, *, 0)$ is a PMS-algebra since Definition 1 is satisfied as shown below.

(1) $0*x = x-0 = x$

(2) $(y*x)*(z*x) = (z*x)-(y*x) = (x-z)-(x-y) = y-z = z*y$.

Define an intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in \mathbb{Z}\}$ by

$$\mu_A(x) = \begin{cases} 0.8, & \text{if } x \in \langle 4\rangle, \\ 0.5, & \text{if } x \in \langle 2\rangle - \langle 4\rangle, \\ 0.3, & \text{otherwise}, \end{cases}$$
$$\nu_A(x) = \begin{cases} 0.2, & \text{if } x \in \langle 4\rangle, \\ 0.4, & \text{if } x \in \langle 2\rangle - \langle 4\rangle, \\ 0.6, & \text{otherwise}. \end{cases} \tag{6}$$

Since $m = 0.3$ and $n = 0.6$, then $\min\{m, 1-n\} = \min\{0.3, 1-0.6\} = 0.3$. So by Theorem 4 for $t \leq 0.3$, A^t is a t-IF PMS-ideal of $\mathbb{Z}$.

The subsequent result shows that the intersection of any two t-IF PMS-ideal is a t-IF PMS-ideal.

Theorem 5. *The intersection of any two t-IF PMS-ideals of X is also a t-IF PMS-ideal of X.*

Proof. Let A_1^t and A_2^t be any two t-IF PMS-ideals of X and $x, y, z \in X$. Then by Definition 14 and Definition 10, we have and

$$\begin{aligned}
\mu_{(A_1 \cap A_2)^t}(0) &= \min\{\mu_{A_1 \cap A_2}(0), t\} \\
&= \min\{\min\{\mu_{A_1}(0), \mu_{A_2}(0)\}, t\} \\
&= \min\{\min\{\mu_{A_1}(0), t\}, \min\{\mu_{A_2}(0), t\}\} \\
&= \min\left\{\mu_{A_1^t}(0), \mu_{A_2^t}(0)\right\} \\
&\geq \min\left\{\mu_{A_1^t}(x), \mu_{A_2^t}(x)\right\} \\
&= \min\{\min\{\mu_{A_1}(x), t\}, \min\{\mu_{A_2}(x), t\}\} \\
&= \min\{\min\{\mu_{A_1}(x), \mu_{A_2}(x)\}, t\} \\
&= \min\{\mu_{A_1 \cap A_2}(x), t\} = \mu_{(A_1 \cap A_2)^t}(x), \\
\nu_{(A_1 \cap A_2)^t}(0) &= \max\{\nu_{A_1 \cap A_2}(0), 1-t\} \\
&= \max\{\max\{\nu_{A_1}(0), \nu_{A_2}(0)\}, 1-t\} \\
&= \max\{\max\{\nu_{A_1}(0), 1-t\}, \max\{\nu_{A_2}(0), 1-t\}\} \\
&= \max\left\{\nu_{A_1^t}(0), \nu_{A_2^t}(0)\right\} \\
&\leq \max\left\{\nu_{A_1^t}(x), \nu_{A_2^t}(x)\right\} \\
&= \max\{\max\{\nu_{A_1}(x), 1-t\}, \min\{\nu_{A_2}(x), 1-t\}\} \\
&= \max\{\max\{\nu_{A_1}(x), \nu_{A_2}(x)\}, 1-t\} \\
&= \max\{\nu_{A_1 \cap A_2}(x), 1-t\} = \nu_{(A_1 \cap A_2)^t}(x),
\end{aligned} \tag{7}$$

thus, $\mu_{(A_1 \cap A_2)^t}(0) \geq \mu_{(A_1 \cap A_2)^t}(x)$ and $\nu_{(A_1 \cap A_2)^t}(0) \leq \nu_{(A_1 \cap A_2)^t}(x), \forall x \in X$

$$\begin{aligned}
\mu_{(A_1 \cap A_2)^t}(y*x) &= \min\{\mu_{A_1 \cap A_2}(y*x), t\} \\
&= \min\{\min\{\mu_{A_1}(y*x), \mu_{A_2}(y*x)\}, t\} \\
&= \min\{\min\{\mu_{A_1}(y*x), t\}, \\
&\quad \min\{\mu_{A_2}(y*x), t\}\} \\
&= \min\left\{\mu_{A_1^t}(y*x), \mu_{A_2^t}(y*x)\right\} \\
&\geq \min\left\{\min\left\{\mu_{A_1^t}(z*y), \mu_{A_1^t}(z*x)\right\},\right. \\
&\quad \left.\min\left\{\mu_{A_2^t}(z*y), \mu_{A_2^t}(z*x)\right\}\right\} \\
&= \min\left\{\min\left\{\mu_{A_1^t}(z*y), \mu_{A_2^t}(z*y)\right\},\right. \\
&\quad \left.\min\left\{\mu_{A_1^t}(z*x), \mu_{A_2^t}(z*x)\right\}\right\} \\
&= \min\left\{\mu_{A_1^t \cap A_2^t}(z*y), \mu_{A_1^t \cap A_2^t}(z*x)\right\} \\
&= \min\left\{\mu_{(A_1 \cap A_2)^t}(z*y), \mu_{(A_1 \cap A_2)^t}(z*x)\right\} \\
&\quad \text{(By remark 2.1)},
\end{aligned} \tag{8}$$

and

$$\begin{aligned}
\nu_{(A_1\cap A_1)^t}(y*x) &= \max\left\{\nu_{A_1\cap A_2}(y*x), 1-t\right\}\\
&= \max\left\{\max\left\{\nu_{A_1}(y*x), \nu_{A_2}(y*x)\right\}, 1-t\right\}\\
&= \max\left\{\max\left\{\nu_{A_1}(y*x), 1-t\right\},\right.\\
&\qquad \left.\max\left\{\nu_{A_2}(y*x), 1-t\right\}\right\}\\
&= \max\left\{\nu_{A_1^t}(y*x), \nu_{A_2^t}(y*x)\right\}\\
&\le \max\left\{\max\left\{\nu_{A_1^t}(z*y), \nu_{A_1^t}(z*x)\right\},\right.\\
&\qquad \left.\max\left\{\nu_{A_2^t}(z*y), \nu_{A_2^t}(z*x)\right\}\right\}\\
&= \max\left\{\max\left\{\nu_{A_1^t}(z*y), \nu_{A_2^t}(z*y)\right\},\right.\\
&\qquad \left.\max\left\{\nu_{A_1^t}(z*x), \nu_{A_2^t}(z*x)\right\}\right\}\\
&= \max\left\{\nu_{A_1^t\cap A_2^t}(z*y), \nu_{A_1^t\cap A_2^t}(z*x)\right\}\\
&= \max\left\{\nu_{(A_1\cap A_2)^t}(z*y), \nu_{(A_1\cap A_2)^t}(z*x)\right\}\\
&\qquad \text{(By remark 2).}
\end{aligned} \tag{9}$$

$$\Rightarrow \mu_{(A_1\cap A_2)^t}(y*x) \ge \min\left\{\mu_{(A_1\cap A_2)^t}(z*y),\ \mu_{(A_1\cap A_2)^t}(z*x)\right\}$$

and

$$\nu_{(A_1\cap A_2)^t}(y*x) \le \max\left\{\nu_{(A_1\cap A_2)^t}(z*y), \nu_{(A_1\cap A_2)^t}(z*x)\right\}. \tag{10}$$

Therefore, $A^t \cap B^t$ is a t-IF PMS-ideals of X.

The above theorem can also be generalized to any family of t-IF PMS-ideals in PMS-algebra as given in the next corollary. □

Corollary 1. *The intersection of a family of t-IF PMS-ideals of X is again a t-IF PMS-ideal of X.*

Remark 6. The union of any two t-IF PMS-ideals of X may not be a t-IF PMS-ideal of X. This is shown by the next example.

Example 4. Let $\mathbb{Z}$ be the set of all integers and $*$ is a binary operation on $\mathbb{Z}$ defined as in Example 3. Clearly, $(\mathbb{Z}, *, 0)$ is a PMS-algebra. Define the intuitionistic fuzzy sets $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in \mathbb{Z}\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in \mathbb{Z}\}$ in $\mathbb{Z}$ respectively by

$$\begin{aligned}
\mu_A(x) &= \begin{cases} 0.7, & \text{if } x \in \langle 2\rangle,\\ 0.3, & \text{otherwise,}\end{cases}\\
\nu_A(x) &= \begin{cases} 0.2, & \text{if } x \in \langle 2\rangle,\\ 0.5, & \text{otherwise,}\end{cases}
\end{aligned} \tag{11}$$

and

$$\begin{aligned}
\mu_B(x) &= \begin{cases} 0.5, & \text{if } x \in \langle 3\rangle,\\ 0.2, & \text{otherwise,}\end{cases}\\
\nu_B(x) &= \begin{cases} 0.3, & \text{if } x \in \langle 3\rangle,\\ 0.5, & \text{otherwise.}\end{cases}
\end{aligned} \tag{12}$$

Clearly, A and B are IF PMS-ideals of $\mathbb{Z}$. Thus, by Theorem 3A^t and B^t are t-IF PMS-ideals of $\mathbb{Z}$ for $t \in [0,1]$. If we take $t = 0.6$, then A^t and B^t are given by

$$\begin{aligned}
\mu_{A^t}(x) &= \begin{cases} 0.6, & \text{if } x \in \langle 2\rangle,\\ 0.3, & \text{otherwise,}\end{cases}\\
\nu_{A^t}(x) &= \begin{cases} 0.4, & \text{if } x \in \langle 2\rangle,\\ 0.5, & \text{otherwise,}\end{cases}
\end{aligned} \tag{13}$$

and

$$\begin{aligned}
\mu_{B^t}(x) &= \begin{cases} 0.5, & \text{if } x \in \langle 3\rangle,\\ 0.2, & \text{otherwise,}\end{cases}\\
\nu_{B^t}(x) &= \begin{cases} 0.4, & \text{if } x \in \langle 3\rangle,\\ 0.5, & \text{otherwise.}\end{cases}
\end{aligned} \tag{14}$$

Now $\mu_{A^t\cup B^t}(x) = \max\{\mu_{A^t}(x), \mu_{B^t}(x)\}$ and $\nu_{A^t\cup B^t}(x) = \min\{\nu_{A^t}(x), \nu_{B^t}(x)\}$. Therefore,

$$\begin{aligned}
\mu_{A^t\cup B^t}(x) &= \begin{cases} 0.6, & \text{if } x \in \langle 2\rangle,\\ 0.5, & \text{if } x \in \langle 3\rangle - \langle 2\rangle,\\ 0.3, & \text{if } x \notin \langle 2\rangle \text{and } x \notin \langle 3\rangle,\end{cases}\\
\nu_{A\cup B}(x) &= \begin{cases} 0.4 & \text{if } x \in \langle 2\rangle \cup \langle 3\rangle,\\ 0.5 & \text{if } x \notin \langle 2\rangle \text{and } x \notin \langle 3\rangle.\end{cases}
\end{aligned} \tag{15}$$

Take $x = 2, y = 3$, and $z = -1$, then $\mu_{A^t\cup B^t}(x) = 0.6, \mu_{A^t\cup B^t}(y) = 0.5, \mu_{A^t\cup B^t}(z) = 0.3$, and $\nu_{A^t\cup B^t}(x) = 0.4, \nu_{A^t\cup B^t}(y) = 0.4, \nu_{A^t\cup B^t}(z) = 0.5$.

Now $\mu_{A^t\cup B^t}(y*x) = \mu_{A^t\cup B^t}(x-y) = \mu_{A^t\cup B^t}(-1) = 0.3, \mu_{A^t\cup B^t}(z*y) = \mu_{A^t\cup B^t}(y-z) = \mu_{A^t\cup B^t}(4) = 0.6, \mu_{A^t\cup B^t}(z*x) = \mu_{A^t\cup B^t}(x-z) = \mu_{A^t\cup B^t}(3) = 0.5$ and $\min\{\mu_{A^t\cup B^t}(z*y), \mu_{A^t\cup B^t}(z*x)\} = \min\{0.6, 0.5\} = 0.5$.

$$\Rightarrow \mu_{A^t\cup B^t}(y*x) = 0.3 \ngeq 0.5 = \min\{\nu_{A^t\cup B^t}(z*y), \nu_{A^t\cup B^t}(z*x)\}. \tag{16}$$

Similarly, $\nu_{A^t\cup B^t}(y*x) = \nu_{A^t\cup B^t}(-1) = 0.5, \nu_{A^t\cup B^t}(z*y) = \nu_{A^t\cup B^t}(4) = 0.4, \nu_{A^t\cup B^t}(z*x) = \nu_{A^t\cup B^t}(3) = 0.4$ and

$\max\{\nu_{A^t\cup B^t}(z*y), \nu_{A^t\cup B^t}(z*x)\} = \max\{0.4, 0.5\} = 0.4$.

$$\nu_{A^t\cup B^t}(y*x) = 0.5 \nleq 0.4 = \max\{\nu_{A^t\cup B^t}(z*y), \nu_{A^t\cup B^t}(z*x)\}. \tag{17}$$

As a result of (16) and(17), we arrive at a contradiction with Definition 14.

Therefore, $A^t \cup B^t$ is not an IF-PMS-ideal of $\mathbb{Z}$.

Theorem 6. *If A^t is a t-IF PMS-ideal of X, then $\Box A^t$ is also a t-IF PMS-ideal of X.*

Proof. Suppose A^t is a t-IF PMS-ideal of X. Then by Definition 14, we have $\mu_{A^t}(0) \ge \mu_{A^t}(x)$ and $\mu_{A^t}(y*x) \ge \min\{\mu_{A^t}(z*y), \mu_{A^t}(z*x)\}$ for all $x, y, z \in X$. So, we need to show that $\overline{\mu}_{A^t}(0) \le \overline{\mu}_{A^t}(x)$ and $\overline{\mu}_{A^t}(y*x) \le \max\{\overline{\mu}_{A^t}(z*y), \overline{\mu}_{A^t}(z*x)\}$ for all $x, y, z \in X$. Now, $\overline{\mu}_{A^t}(0) = 1 - \mu_{A^t}(0) \le 1 - \mu_{A^t}(x) = \overline{\mu}_{A^t}(x) \Rightarrow \overline{\mu}_{A^t}(0) \le \overline{\mu}_{A^t}(x)$

Also,

$$\begin{aligned}\overline{\mu}_{A^t}(y*x) &= 1-\mu_{A^t}(y*x) \le 1-\min\{\mu_{A^t}(z*y),\mu_{A^t}(z*x)\}\\ &= \max\{1-\mu_{A^t}(z*y), 1-\mu_{A^t}(z*x)\}\\ &= \max\{\overline{\mu}_{A^t}(z*y),\overline{\mu}_{A^t}(z*x)\}\\ &\Rightarrow \overline{\mu}_{A^t}(y*x) \le \max\{\overline{\mu}_{A^t}(z*y),\overline{\mu}_{A^t}(z*x)\}.\end{aligned} \tag{18}$$

Hence, $\square A^t$ is a t-IF PMS-ideal of X. ☐

Theorem 7. *If A^t is a t-IF PMS-ideal of X, then $\diamond A^t$ is also a t-IF PMS-ideal of X.*

Proof. Suppose A^t is a t-IF PMS-ideal of X. Then by Definition 14, we have $\nu_{A^t}(0) \le \nu_{A^t}(x)$ and $\nu_{A^t}(y*x) \le \max\{\nu_{A^t}(z*y), \nu_{A^t}(z*x)\}$, for all $x, y, z \in X$. So, we have to show that $\overline{\nu}_{A^t}(0) \ge \overline{\nu}_{A^t}(x)$ and $\overline{\nu}_{A^t}(y*x) \ge \min\{\overline{\nu}_{A^t}(z*y), \overline{\nu}_{A^t}(z*x)\}$, for all $x, y, z \in X$.

Now, $\overline{\nu}_{A^t}(0) = 1-\nu_{A^t}(0) \ge 1-\nu_{A^t}(x) = \overline{\nu}_{A^t}(x) \Rightarrow \overline{\nu}_{A^t}(0) \ge \overline{\nu}_{A^t}(x)$.

And,

$$\begin{aligned}\overline{\nu}_{A^t}(x*y) &= 1-\nu_{A^t}(x*y) \ge 1-\max\{\nu_{A^t}(x),\nu_{A^t}(y)\}\\ &= \min\{1-\nu_{A^t}(x), 1-\nu_{A^t}(y)\}\\ &= \min\{\overline{\nu}_{A^t}(x),\overline{\nu}_{A^t}(y)\}\\ &\Rightarrow \overline{\nu}_{A^t}(x*y) \ge \min\{\overline{\nu}_{A^t}(x),\overline{\nu}_{A^t}(y)\}.\end{aligned} \tag{19}$$

Hence, $\diamond A^t$ is a t-IF PMS-ideal of X. ☐

Theorem 8. *Let A^t be a t-IFS of X. Then A^t is a t-IF PMS-ideal of X if and only if the nonempty subset $C_{\alpha,\beta}(A^t)$ of X is a PMS-ideal of X for all $\alpha, \beta \in [0,1]$ with $\alpha+\beta \le 1$.*

Proof. Since $C_{\alpha,\beta}(A^t) \neq \varnothing$, there exist $x \in X$ such that $x \in C_{\alpha,\beta}(A^t)$. Then, $\mu_{A^t}(x) \ge \alpha$ and $\nu_{A^t}(x) \le \beta$. Since A^t is a t-IF PMS-ideal of X, $\mu_{A^t}(0) \ge \mu_{A^t}(x)$ and $\nu_{A^t}(0) \le \nu_{A^t}(x)$ for all $x \in X$. Thus, $\mu_{A^t}(0) \ge \alpha$ and $\nu_{A^t}(0) \le \beta$. Therefore, $0 \in C_{\alpha,\beta}(A^t)$.

Let $x, y, z \in X$ such that $z*x, z*y \in C_{\alpha,\beta}(A^t)$. Then $\mu_{A^t}(z*y) \ge \alpha, \mu_{A^t}(z*x) \ge \alpha$ and $\nu_{A^t}(z*y) \le \beta$, $\nu_{A^t}(z*x) \le \beta$. Since A^t is a t-IF PMS-ideal of X, we have $\mu_{A^t}(y*x) \ge \min\{\mu_{A^t}(z*y), \mu_{A^t}(z*x)\} \ge \alpha$ and $\nu_{A^t}(y*x) \le \max\{\nu_{A^t}(z*y), \nu_{A^t}(z*x)\} \le \beta$. Thus, $y*x \in C_{\alpha,\beta}(A^t)$. Therefore, $C_{\alpha,\beta}(A^t)$ is a PMS-ideal of X.

Conversely, suppose $C_{\alpha,\beta}(A^t)$ is a PMS-ideal of X. Let $x \in X$ such that $\mu_A(x) = \alpha$ and $\nu_A(x) = \beta$. Since $C_{\alpha,\beta}(A^t)$ is a PMS-ideal of X, we have that $0 \in C_{\alpha,\beta}(A^t)$.

This implies $\mu_A(0) \ge \alpha = \mu_A(x)$ and $\nu_A(0) \le \beta = \nu_A(x)$.

Hence, $\mu_A(0) \ge \mu_A(x)$ and $\nu_A(0) \le \nu_A(x)$ for every $x \in X$.

Also, let $x, y, z \in X$ such that $\alpha = \min\{\mu_{A^t}(z*y), \mu_{A^t}(z*x)\}$ and $\beta = \max\{\nu_{A^t}(z*y), \nu_{A^t}(z*x)\}$. Then $\mu_{A^t}(z*y) \ge \alpha, \nu_{A^t}(z*y) \le \beta$ and $\mu_{A^t}(z*x) \ge \alpha$, $\nu_{A^t}(z*x) \le \beta$. Thus, $z*y, z*x \in C_{\alpha,\beta}(A^t)$. Since $C_{\alpha,\beta}(A^t)$ is a PMS-ideal of X, it follows that $y*x \in C_{\alpha,\beta}(A^t)$. So that we have $\mu_{A^t}(y*x) \ge \alpha = \min\{\mu_{A^t}(z*y), \mu_{A^t}(z*x)\}$ and $\nu_{A^t}(y*x) \le \beta = \max\{\nu_{A^t}(z*y), \nu_{A^t}(z*x)\}$. Hence, $\mu_{A^t}(y*x) \ge \min\{\mu_{A^t}(z*y), \mu_{A^t}(z*x)\}$ and $\nu_{A^t}(y*x) \le \max\{\nu_{A^t}(z*y), \nu_{A^t}(z*x)\}$, for all $x, y, z \in X$. Therefore, A^t is a t-IF PMS-ideal of X. ☐

4. Homomorphism of t-Intuitionistic Fuzzy PMS-Ideals

In this section, we discuss t-intuitionistic fuzzy PMS-ideals under homomorphism. We study the homomorphic images and inverse images of t-intuitionistic fuzzy PMS-ideals and find some related results.

Definition 15. Let X and Y be two nonempty sets and $f\colon X \longrightarrow Y$ be a mapping. Let A^t and B^t be t-IFSs of X and Y, respectively. Then the image of A^t under f is denoted by $f(A^t)$ and is defined as $f(A^t) = \{\langle y, \mu_{f(A^t)}(y), \nu_{f(A^t)}(y)\rangle \mid y \in Y\}$, where

$$\mu_{f(A^t)}(y) = \begin{cases}\sup \mu_{A^t}(x) :, & x \in f^{-1}(y),\\ 0, & \text{otherwise},\end{cases} \tag{20}$$

and

$$\nu_{f(A^t)}(y) = \begin{cases}\inf \nu_{A^t}(x) :, & x \in f^{-1}(y),\\ 1, & \text{otherwise}.\end{cases} \tag{21}$$

Also, the inverse image of B^t under f is denoted by $f^{-1}(B^t)$ and is defined as

$f^{-1}(B^t)(x) = \{\langle x, \mu_{f^{-1}(B^t)}(x), \nu_{f^{-1}(B)}(x)\rangle \mid x \in X\}$, where $\mu_{f^{-1}(B^t)}(x) = \mu_{B^t}(f(x))$ and $\nu_{f^{-1}(B^t))}(x) = \nu_{B^t}(f(x))$ for all $x \in X$ and $t \in [0,1]$.

Note: For any $x \in X$, we have $\mu_{f(A^t)}(f(x)) \ge \mu_{A^t}(x)$ and $\nu_{f(A^t)}(f(x)) \le \nu_{A^t}(x)$.

Theorem 9. *Let $f\colon X \longrightarrow Y$ be an epimorphism of PMS-algebras and $t \in [0,1]$. If A^t is a t-IF PMS-ideal of X, then $f(A^t)$ is a t-IF PMS-ideal of Y.*

Proof. Since $f\colon X \longrightarrow Y$ is an epimorphism of PMS-algebras, for each $y \in Y$, there exists $x \in X$, such that $f(x) = y$. Then $\mu_{\{f(A)^t(0)=\mu_{f(A^t)}(0)=\mu_{f(A^t)}(f(0)) \ge \mu_{A^t}(0) \ge \mu_{A^t}(x)=}$ $\mu_{f(A^t)}(f(x)) = \mu_{f(A^t)}(y) = \mu_{(f(A))^t}(y)$ and $\nu_{(f(A))^t}(0) = \nu_{f(A^t)}(0) = \nu_{f(A^t)}(f(0)) \le \nu_{A^t}(0) \le \nu_{A^t}(x) = \nu_{f(A^t)}(f(x)) = \nu_{f(A^t)}(y) = \nu_{(f(A))^t}(y)$.

Hence, $\mu_{(f(A))^t}(0) \ge \mu_{(f(A))^t}(y)$ and $\nu_{(f(A))^t}(0) \le \nu_{(f(A))^t}(y)$.

Also, let $x, y, z \in Y$. Since $f\colon X \longrightarrow Y$ is an epimorphism of PMS-algebras, there exist $a, b, c \in X$ such that $f(a) = x$, $f(b) = y$ and $f(c) = z$. So, using Definitions 14 and 15, we have,

$$\begin{aligned}\mu_{(f(A))^t}(y*x) &= \mu_{f(A^t)}(y*x)\\ &= \mu_{f(A^t)}(f(b)*f(a))\\ &= \mu_{f(A^t)}(f(b*a))\\ &\geq \mu_{A^t}(b*a)\\ &\geq \min\{\mu_{A^t}(c*b), \mu_{A^t}(c*a)\}, \forall a,b,c\in X \text{ such that } f(a)\\ &= x, f(b) = y \text{ and } f(c) = z\\ &= \min\left\{\mu_{f(A^t)}(f(c*b)), \mu_{f(A^t)}(f(c*a))\right\}\\ &= \min\left\{\mu_{f(A^t)}(f(c)*f(b)), \mu_{f(A^t)}(f(c)*f(a))\right\}\\ &= \min\left\{\mu_{f(A^t)}(z*y), \mu_{f(A^t)}(z*x)\right\}\\ &= \min\left\{\mu_{(f(A))^t}(z*y), \mu_{(f(A))^t}(z*x)\right\},\\ \nu_{(f(A))^t}(y*x) &= \nu_{f(A^t)}(y*x)\\ &= \nu_{f(A^t)}(f(b)*f(a))\\ &= \nu_{f(A^t)}(f(b*a))\\ &\leq \nu_{A^t}(b*a)\\ &\leq \max\{\nu_{A^t}(c*b), \nu_{A^t}(c*a)\}, \forall a,b,c\in X \text{ such that } f(a)\\ &= x, f(b) = y \text{ and } f(c) = z\\ &= \max\left\{\nu_{f(A^t)}(f(c*b)), \nu_{f(A^t)}(f(c*a))\right\}\\ &= \max\left\{\nu_{f(A^t)}(f(c)*f(b)), \nu_{f(A^t)}(f(c)*f(a))\right\}\\ &= \max\left\{\nu_{f(A^t)}(z*y), \nu_{f(A^t)}(z*x)\right\}\\ &= \max\left\{\nu_{(f(A))^t}(z*y), \nu_{(f(A))^t}(z*x)\right\},\end{aligned} \tag{22}$$

and, hence, $f(A^t)$ is a t-IF PMS-ideal of Y. □

Theorem 10. *Let $f\colon X \longrightarrow Y$ be a homomorphism of PMS-algebras and $t \in [0,1]$. If B^t is a t-IF PMS-ideal of Y, then $f^{-1}(B^t)$ is a t-IF PMS-ideal of X.*

Proof. Let B^t be a t-IF PMS-ideal of Y for $t \in [0,1]$ and let $x \in X$. Then

$\mu_{f^{-1}(B^t)}(0) = \mu_{B^t}(f(0)) \geq \mu_{B^t}(f(x)) = \mu_{f^{-1}(B^t)}(x)$ and $\nu_{f^{-1}(B^t)}(0) = \nu_{B^t}(f(0)) \leq \nu_{B^t}(f(x)) = \nu_{f^{-1}(B^t)}(x) \Rightarrow \mu_{f^{-1}(B^t)}(0) \geq \mu_{f^{-1}(B^t)}(x)$ and $\nu_{f^{-1}(B^t)}(0) \leq \nu_{f^{-1}(B^t)}(x)$, $\forall x \in X$.

Let $x, y, z \in X$. Then by Definition 14 and Definition 15, we have

$$\begin{aligned}\mu_{f^{-1}(B^t)}(y*x) &= \mu_{B^t}(f(y*x)) = \mu_{B^t}(f(y)*f(x))\\ &\geq \min\{\mu_{B^t}(f(z)*f(y)), \mu_{B^t}(f(z)*f(x))\}\\ &= \min\{\mu_{B^t}(f(z*y)), \mu_{B^t}(f(z*x))\}\\ &= \min\left\{\mu_{f^{-1}(B^t)}(z*y), \mu_{f^{-1}(B^t)}(z*x)\right\},\\ \nu_{f^{-1}(B^t)}(y*x) &= \nu_{B^t}(f(y*x)) = \nu_{B^t}(f(y)*f(x))\\ &\leq \max\{\nu_{B^t}(f(z)*f(y)), \nu_{B^t}(f(z)*f(x))\}\\ &= \max\{\nu_{B^t}(f(z*y)), \nu_{B^t}(f(z*x))\}\\ &= \max\left\{\nu_{f^{-1}(B^t)}(z*y), \nu_{f^{-1}(B^t)}(z*x)\right\}.\end{aligned} \tag{23}$$

Hence, $f^{-1}(B^t)$ is a t-IF PMS-ideal of X.

The converse of the above theorem is true if f is an epimorphism of PMS-algebras. □

Theorem 11. *Let $f\colon X \longrightarrow Y$ be an epimorphism of PMS-algebras and B^t is a t-IFS in Y. If $f^{-1}(B^t)$ is a t-IF PMS-ideal of X, then B^t is a t-IF PMS-ideal of Y for $t \in [0,1]$.*

Proof. Assume that $f\colon X \longrightarrow Y$ is an epimorphism of PMS-algebras and $f^{-1}(B^t)$ is a t-IF PMS-ideal of X. Since f is an epimorphism of PMS-algebras for any $x \in Y$, there exist $a \in X$ such that $f(a) = x$. Then,

$\mu_{B^t}(0) = \mu_{B^t}(f(0)) = \mu_{f^{-1}(B^t)}(0) \geq \mu_{f^{-1}(B^t)}(a) = \mu_{B^t}(f(a)) = \mu_{B^t}(x)$ and

$$\begin{aligned}\nu_{B^t}(0) &= \nu_{B^t}(f(0)) = \nu_{f^{-1}(B^t)}(0) \leq \nu_{f^{-1}(B^t)}(a)\\ &= \nu_{B^t}(f(a)) = \nu_{B^t}(x).\end{aligned} \tag{24}$$

Also, let $x, y, z \in Y$. Then there exist $a, b, c \in X$ such that $f(a) = x$, $f(b) = y$, and $f(c) = z$.

Now,

$$\begin{aligned}\mu_{B^t}(y*x) &= \mu_{B^t}(f(b)*f(a) = \mu_{B^t}(f(b*a))\\ &= \mu_{f^{-1}(B^t)}(b*a)\\ &\geq \min\left\{\mu_{f^{-1}(B^t)}(c*b), \mu_{f^{-1}(B^t)}(c*a)\right\}\\ &= \min\{\mu_{B^t}(f(c*b)), \mu_{B^t}(f(c*a))\}\\ &= \min\{\mu_{B^t}(f(c)*f(b)), \mu_{B^t}(f(c)*f(a))\}\\ &= \min\{\mu_{B^t}(z*y), \mu_{B^t}(z*x)\},\end{aligned} \tag{25}$$

and

$$\begin{aligned}\nu_{B^t}(y*x) &= \nu_{B^t}(f(b)*f(a) = \nu_{B^t}(f(b*a))\\ &= \nu_{f^{-1}(B^t)}(b*a)\\ &\leq \max\left\{\nu_{f^{-1}(B^t)}(c*b), \nu_{f^{-1}(B^t)}(c*a)\right\}\\ &= \max\{\nu_{B^t}(f(c*b)), \nu_{B^t}(f(c*a))\}\\ &= \max\{\nu_{B^t}(f(c)*f(b)), \nu_{B^t}(f(c)*f(a))\}\\ &= \max\{\nu_{B^t}(z*y), \nu_{B^t}(z*x)\}.\end{aligned} \tag{26}$$

Hence, B^t is a t-IF PMS-ideal of Y. □

Theorem 12. *Let A^t be a t-IFS of X and $f\colon X \longrightarrow Y$ be an epimorphism of PMS-algebras. Then the homomorphic image of the nonempty subset $C_{\alpha,\beta}(A^t)$ of X is a PMS-ideal of Y.*

Proof. Let A^t be a t-IF PMS-ideal of X and let $x, y, z \in Y$. Since f is an epimorphism of PMS-algebras, there exist $a, b, c \in X$ such that $f(a) = x$, $f(b) = y$, and $f(c) = z$. By Theorem 9, $f(A^t)$ is a t-IF PMS-ideal of Y, and by Theorem6, $C_{\alpha,\beta}(A^t)$ is a PMS-ideal of X, that is $0 \in C_{\alpha,\beta}(A^t)$ and $c*b, c*a \in C_{\alpha,\beta}(A^t) \Rightarrow b*a \in C_{\alpha,\beta}(A^t)$.

Then,

$\mu_{(f(A))^t}(0) = \mu_{f(A^t)}(0) = \mu_{f(A^t)}(f(0)) \geq \mu_{A^t}(0) \geq \alpha \Rightarrow \mu_{(f(A))^t}(0) \geq \alpha$ and

$$\nu_{(f(A))^t}(0) = \nu_{f(A^t)}(0) = \nu_{f(A^t)}(f(0)) \le \mu_{A^t}(0) \le \beta \Rightarrow \nu_{(f(A))^t}(0) \le \beta. \quad (27)$$

Hence, $0 \in f(C_{\alpha,\beta}(A^t))$.

Also, assume that $z * y, z * x \in f(C_{\alpha,\beta}(A^t))$, that is $f(c) * f(b), f(c) * f(a) \in f(C_{\alpha,\beta}(A^t))$.

So, we have

$$\begin{aligned}\mu_{(f(A))^t}(y * x) &= \mu_{f(A^t)}(y * x)\\ &= \mu_{f(A^t)}(f(b) * f(a))\\ &= \mu_{f(A^t)}(f(b * a)) \ge \mu_{A^t}(b * a) \ge \alpha,\end{aligned} \quad (28)$$

and

$$\begin{aligned}\nu_{(f(A))^t}(y * x) &= \nu_{f(A^t)}(y * x)\\ &= \nu_{f(A^t)}(f(b) * f(a))\\ &= \nu_{f(A^t)}(f(b * a)) \le \nu_{A^t}(b * a) \le \beta.\end{aligned} \quad (29)$$

Thus, $\mu_{(f(A))^t}(f(b) * f(a)) = \mu_{(f(A))^t}(y * x) \ge \alpha$ and

$$\nu_{(f(A))^t}(f(b) * f(a)) = \nu_{(f(A))^t}(y * x) \le \beta. \quad (30)$$

Therefore, $f(b) * f(a) \in f(C_{\alpha,\beta}(A^t))$. Hence, $f(C_{\alpha,\beta}(A^t))$ is a PMS-ideal of Y. □

Theorem 13. *Let B^t be a t-IFS of a PMS-algebra Y and $f\colon X \longrightarrow Y$ be an epimorphism of PMS-algebras. Then the homomorphic inverse image of the nonempty subset $C_{\alpha,\beta}(B^t)$ of Y is a PMS-ideal of X.*

Proof. Let B^t be a t-IF PMS-ideal of Y and let $x, y, z \in Y$. Since f is an epimorphism of PMS-algebras, there exist $a, b, c \in X$ such that $f(a) = x$, $f(b) = y$, and $f(c) = z$. By Theorem 10, $f^{-1}(B^t)$ is a t-IF PMS-ideal of Y, and by Theorem 8, $C_{\alpha,\beta}(B^t)$ is a PMS-ideal of Y, that is $0 \in C_{\alpha,\beta}(B^t)$ and $z * y, z * x \in C_{\alpha,\beta}(B^t) \Rightarrow y * x \in C_{\alpha,\beta}(B^t)$.

$\mu_{f^{-1}(B^t)}(0) = \mu_{B^t}(f(0)) = \mu_{B^t}(0) \ge \alpha$ and $\nu_{f^{-1}(B^t)}(0) = \nu_{B^t}(f(0)) = \nu_{B^t}(0) \le \beta$.

Hence, $0 \in f^{-1}(C_{\alpha,\beta}(B^t))$.

Now let $c * b, c * a \in f^{-1}(C_{\alpha,\beta}(B^t))$. So, we have

$\mu_{f^{-1}(B^t)}(b * a) = \mu_{B^t}(f(b * a)) = \mu_{B^t}(f(b) * f(a)) = \mu_{B^t}(y * x) \ge \alpha$ and

$$\begin{aligned}\nu_{f^{-1}(B^t)}(b * a) &= \nu_{B^t}(f(b * a))\\ &= \nu_{B^t}(f(b) * f(a)) = \nu_{B^t}(y * x) \le \beta.\end{aligned} \quad (31)$$

Hence, $\mu_{f^{-1}(B^t)}(b * a) \ge \alpha$ and $\nu_{f^{-1}(B^t)}(b * a) \le \beta \Rightarrow b * a \in f^{-1}(C_{\alpha,\beta}(B^t))$.

Therefore, $f^{-1}(C_{\alpha,\beta}(B^t))$ is a PMS-ideal of X. □

5. Cartesian Product of t-Intuitionistic Fuzzy PMS-Ideals

In this section, we consider the Cartesian product of a t-intuitionistic fuzzy PMS-ideal and investigate its related properties. We define the strongest t-intuitionistic fuzzy PMS-relation and study its relationship with a t-intuitionistic fuzzy PMS-ideal.

Theorem 14. *Let A^t and B^t be two t-IF PMS-ideals of X and Y, respectively. Then $A^t \times B^t$ is a t-IF PMS-ideal of $X \times Y$.*

Proof. Let $(x, y) \in X \times Y$. Then by Definition 14, we have

$\mu_{A^t\times B^t}(0,0) = \min\{\mu_{A^t}(0), \mu_{B^t}(0)\} \ge \min\{\mu_{A^t}(x), \mu_{B^t}(y)\} = \mu_{A^t\times B^t}(x, y)$ and

$$\begin{aligned}\nu_{A^t\times B^t}(0,0) &= \max\{\nu_{A^t}(0), \nu_{B^t}(0)\} \le \max\{\nu_{A^t}(x), \nu_{B^t}(y)\}\\ &= \nu_{A^t\times B^t}(x, y).\end{aligned} \quad (32)$$

Thus, $\mu_{A^t\times B^t}(0,0) \ge \mu_{A^t\times B^t}(x, y)$ and $\nu_{A^t\times B^t}(0,0) \le \nu_{A^t\times B^t}(x, y), \forall (x, y) \in X \times Y$.

Also, let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times Y$. Again by Definition 14, we have

$$\begin{aligned}\mu_{A^t\times B^t}((y_1, y_2) * (x_1, x_2)) &= \mu_{A^t\times B^t}(y_1 * x_1, y_2 * x_2)\\ &= \min\{\mu_{A^t}(y_1 * x_1), \mu_{B^t}(y_2 * x_2)\}\\ &\ge \min\{\min\{\mu_{A^t}(z_1 * y_1), \mu_{A^t}(z_1 * x_1)\}, \min\{\mu_{B^t}(z_2 * y_2), \mu_{B^t}(z_2 * x_2)\}\}\\ &= \min\{\min\{\mu_{A^t}(z_1 * y_1), \mu_{B^t}(z_2 * y_2)\}, \min\{\mu_{A^t}(z_1 * x_1), \mu_{B^t}(z_2 * x_2)\}\}\\ &= \min\{\mu_{A^t\times B^t}(z_1 * y_1, z_2 * y_2), \mu_{A^t\times B^t}(z_1 * x_1, z_2 * x_2)\}\\ &= \min\{\mu_{A^t\times B^t}((z_1, z_2) * (y_1, y_2)), \mu_{A^t\times B^t}((z_1, z_2) * (x_1, x_2))\}\\ &\Rightarrow \mu_{A^t\times B^t}((y_1, y_2) * (x_1, x_2)) \ge \min\{\mu_{A^t\times B^t}((z_1, z_2) * (y_1, y_2)), \mu_{A^t\times B^t}((z_1, z_2) * (x_1, x_2))\},\end{aligned} \quad (33)$$

and

$$\begin{aligned}
\nu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2)) &= \nu_{A^t\times B^t}(y_1*x_1,y_2*x_2)\\
&= \max\{\nu_{A^t}(y_1*x_1),\nu_{B^t}(y_2*x_2)\}\\
&\le \max\{\max\{\nu_{A^t}(z_1*y_1),\nu_{A^t}(z_1*x_1)\},\ \max\{\nu_{B^t}(z_2*y_2),\nu_{B^t}(z_2*x_2)\}\}\\
&= \max\{\max\{\nu_{A^t}(z_1*y_1),\nu_{B^t}(z_2*y_2)\},\ \max\{\nu_{A^t}(z_1*x_1),\nu_{B^t}(z_2*x_2)\}\}\\
&= \max\{\nu_{A^t\times B^t}(z_1*y_1,z_2*y_2),\nu_{A^t\times B^t}(z_1*x_1,z_2*x_2)\}\\
&= \max\{\nu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2)),\nu_{A^t\times B^t}((z_1,z_2)*(x_1,x_2))\},\\
&\Rightarrow \nu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2))\le \max\{\nu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2)),\nu_{A^t\times B^t}((z_1,z_2)*(x_1,x_2))\}.
\end{aligned} \tag{34}$$

Therefore, $A^t\times B^t$ is an intuitionistic fuzzy PMS-ideal of $X\times Y$. □

Theorem 15. *Let A^t and B^t be any two t-IF sets in X and Y, respectively. If $A^t\times B^t$ is a t-IF PMS-ideal of $X\times Y$, then either A^t is a t-IF PMS-ideal of X or B^t is a t-IF PMS-ideal of Y.*

Proof. Let A^t and B^t be t-IFSs of X and Y, respectively, such that $A^t\times B^t$ is a t-IF PMS-ideal of $X\times Y$. Then $\mu_{A^t\times B^t}(0,0)\ge\mu_{A^t\times B^t}(x,y)$, $\forall(x,y)\in X\times Y$.

Assume $\mu_{A^t}(x)>\mu_{A^t}(0)$ and $\mu_{B^t}(y)>\mu_{B^t}(0)$ for some $(x,y)\in X\times Y$. Then,

$$\begin{aligned}
\mu_{A^t\times B^t}(x,y) &= \min\{\mu_{A^t}(x),\\
&\mu_{B^t}(y)>\min\{\mu_{A^t}(0),\mu_{B^t}(0)\}\\
&= \mu_{A^t\times B^t}(0,0),\ \text{which is a contradiction,}
\end{aligned}$$

$$\text{Thus, } \mu_{A^t}(0)\ge\mu_{A^t}(x)\text{or}\mu_{B^t}(0)\ge\mu_{B^t}(y),\forall(x,y)\in X\times Y. \tag{35}$$

Similarly, $\nu_{A^t}(0)\le\nu_{A^t}(x)$ or $\nu_{B^t}(0)\le\nu_{B^t}(y)$, $\forall(x,y)\in X\times Y$.

Let $(x_1,x_2),(y_1,y_2),(z_1,z_2)\in X\times Y$. Since $A^t\times B^t$ is a t-IF PMS-ideal of $X\times Y$, we have

$$\begin{aligned}
\mu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2)) &\ge \min\{\mu_{A^t\times B^t}(z_1,z_2)*(y_1,y_2)),\mu_{A^t\times B^t}((z_1,z_2)*(x_1,x_2))\}\\
&= \min\{\mu_{A^t\times B^t}(z_1*y_1,z_2*y_2),\mu_{A^t\times B^t}(z_1*x_1,z_2*x_2)\}\\
&= \min\{\min\{\mu_{A^t}(z_1*y_1),\mu_{B^t}(z_2*y_2)\},\ \min\{\mu_{A^t}(z_1*x_1),\mu_{B^t}(z_2*x_2)\}\}.
\end{aligned} \tag{36}$$

As $\mu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2))=\mu_{A^t\times B^t}(y_1*x_1,y_2*x_2)$, we have

$$\begin{aligned}
\mu_{A^t\times B^t}(y_1*x_1,y_2*x_2) &\ge \min\{\min\{\mu_{A^t}(z_1*y_1),\mu_{B^t}(z_2*y_2)\},\ \min\{\mu_{A^t}(z_1*x_1),\mu_{B^t}(z_2*x_2)\}\}\\
&\Rightarrow \min\{\mu_{A^t}(y_1*x_1),\mu_{B^t}(y_2*x_2)\}\\
&\ge \min\{\min\{\mu_{A^t}(z_1*y_1),\mu_{B^t}(z_2*y_2)\},\ \min\{\mu_{A^t}(z_1*x_1),\mu_{B^t}(z_2*x_2)\}\}\\
&= \min\{\min\{\mu_{A^t}(z_1*y_1),\mu_{A^t}(z_1*x_1)\},\ \min\{\mu_{B^t}(z_2*y_2),\mu_{B^t}(z_2*x_2)\}\}.
\end{aligned} \tag{37}$$

If we put $x_2=y_2=z_2=0$ (or resp. $x_1=y_1=z_1=0$), we have either

$\mu_{A^t}(y_1*x_1)\ge\min\{\mu_{A^t}(z_1*y_1),\mu_{A^t}(z_1*x_1)\}$ or $\mu_{B^t}(y_2*x_2)\ge\min\{\mu_{B^t}(z_2*y_2),\mu_{B^t}(z_2*x_2)\}$.

In similar way, we can show that either $\nu_{A^t}(y_1*x_1)\le\max\{\nu_{A^t}(z_1*y_1),\nu_{A^t}(z_1*x_1)\}$ or $\nu_{B^t}(y_2*x_2)\le\max\{\nu_{B^t}(z_2*y_2),\nu_{B^t}(z_2*x_2)\}$.

Therefore, A^t is a t-IF PMS-ideal of X or B^t is a t-IF PMS-ideal of Y. □

Definition 16. Let A^t and B^t be t-IFSs of X and Y w.r.t IFSs A and B. Then, (α,β)-cut of $A^t\times B^t$ is a crisp subset $C_{(\alpha,\beta)}(A^t\times B^t)$ of $X\times Y$ is given by

$C_{(\alpha,\beta)}(A^t\times B^t)=\{(x,y)\in X\times Y\colon\ \mu_{A^t}(x,y)\ge\alpha,\nu_{A^t}(x,y)\le\beta\}$, where $\alpha,\beta\in[0,1]$ with $\alpha+\beta\le 1$.

Theorem 16. *Let A^t and B^t be t-IFSs of X and Y respectively. Then $A^t\times B^t$ is a t-IF PMS-ideals of $X\times Y$ if and only if the nonempty subset $C_{(\alpha,\beta)}(A^t\times B^t)$ of $X\times Y$ is a PMS-ideal of $X\times Y$ for all $\alpha,\beta\in[0,1]$ with $\alpha+\beta\le 1$.*

Proof. Let $A^t=(\mu_{A^t},\nu_{A^t})$ and $B^t=(\mu_{B^t},\nu_{B^t})$ be t-IFSs of X and Y, respectively. Since $C_{\alpha,\beta}(A^t\times B^t)\ne\varnothing$, there exist $(x,y)\in X\times Y$ such that $(x,y)\in C_{\alpha,\beta}(A^t\times B^t)$. Then, $\mu_{A^t\times B^t}(x,y)\ge\alpha$ and $\nu_{A^t\times B^t}(x,y)\le\beta$. Since $A^t\times B^t$ is a t-IF PMS-ideal of $X\times Y$, $\mu_{A^t\times B^t}(0,0)\ge\mu_{A^t}(x,y)$ and

$\nu_{A^t\times B^t}(0,0)\le\nu_{A^t}(x,y)$ for all $(x,y)\in X\times Y$. Thus, it follows that $\mu_{A^t\times B^t}(0,0)\ge\alpha$ and $\nu_{A^t\times B^t}(0,0)\le\beta$. Therefore, $(0,0)\in C_{\alpha,\beta}(A^t)$

Let $(x_1,x_2),(y_1,y_2),(z_1,z_2)\in X\times Y$ such that $(z_1,z_2)*(y_1,y_2),(z_1,z_2)*(x_1,x_2)\in C_{(\alpha,\beta)}(A^t\times B^t)$ for $\alpha,\beta\in[0,1]$.Then, $\mu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2))\ge\alpha,\mu_{A^t\times B^t}((z_1,z_2)*(x_1,x_2))\ge\alpha$, and $\nu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2))\le\beta,\nu_{A^t\times B^t}((z_1,z_2)*(x_1,x_2))\le\beta$. Since $A^t\times B^t$ is a t-IF PMS-ideal of $X\times Y$, we have $\mu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2))\ge\min\{\mu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2)),\mu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2))\}\ge\min\{\alpha,\alpha\}=\alpha$ and $\nu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2))\le\max\{\nu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2)),\nu_{A^t\times B^t}((z_1,z_2)*(x_1,x_2))\}\le\max\{\beta,\beta\}=\beta$.

$\Rightarrow\mu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2))\ge\alpha$ and $\nu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2))\le\beta$.

Therefore, $(y_1,y_2)*(x_1,x_2)\in C_{(\alpha,\beta)}(A^t\times B^t)$.

Hence, $C_{(\alpha,\beta)}(A^t\times B^t)$ is a PMS-ideal of $X\times Y$.

Conversely, suppose $C_{(\alpha,\beta)}(A^t\times B^t)$ is a PMS-ideal of $X\times Y$ for all $\alpha,\beta\in[0,1]$ with $\alpha+\beta\le1$. Assume that $A^t\times B^t$ is not a t-IF PMS-ideal of $X\times Y$. Then there exist $(x_1,x_2),(y_1,y_2),(z_1,z_2)\in X\times Y$ such that $\mu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2))<\min\{\mu_{A^t\times B}{}^t((z_1,z_2)*(y_1,y_2)),\mu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2))\}$ and $\nu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2))>\max\{\nu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2)),\nu_{A^t\times B^t}((z_1,z_2)*(x_1,x_2))\}$. Then by taking

$\alpha_0=1/2\{\mu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2))+\min\{\mu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2)),\mu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2))\}\}$ and $\beta_0=1/2\{\nu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2))+\max\{\nu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2)),\nu_{A^t\times B^t}((z_1,z_2)*(x_1,x_2))\}\}$, $\mu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2))<\alpha_0<\min\{\mu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2)),\mu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2))\}$ and $\nu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2))>\beta_0>\max\{\nu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2)),\nu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2))\}$.

Hence, $((y_1,y_2)*(x_1,x_2))\notin C_{(\alpha,\beta)}(A^t\times B^t)$ but $(z_1,z_2)*(y_1,y_2)\in C_{(\alpha,\beta)}(A^t\times B^t)$ and $(z_1,z_2)*(x_1,x_2)\in C_{(\alpha,\beta)}(A^t\times B^t)$. This implies $C_{(\alpha,\beta)}(A^t\times B^t)$ is not a PMS-ideal of $X\times Y$, which is a contradiction.

Therefore, $\mu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2))\ge\min\{\mu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2)),\mu_{A^t\times B^t}((z_1,z_2)*(x_1,x_2))\}$ and $\nu_{A^t\times B^t}((y_1,y_2)*(x_1,x_2))\le\max\{\nu_{A^t\times B^t}((z_1,z_2)*(y_1,y_2)),\nu_{A^t\times B^t}((z_1,z_2)*(x_1,x_2))\}$. for all $(x_1,y_1),(x_2,y_2)\in X\times Y$.

Hence, $A^t\times B^t$ is an intuitionistic fuzzy PMS-ideal of $X\times Y$. □

Definition 17. Let $A^t=(\mu_{A^t},\nu_{A^t})$ be a t-intuitionistic fuzzy set in X and $R^t=(\mu_{R^t},\nu_{R^t})$ be a t-intuitionistic fuzzy relation on X. Then the strongest t-intuitionistic fuzzy PMS-relation $R^t_{A^t}$ on X, that is, a t-intuitionistic fuzzy PMS-relation R^t on A^t whose membership function $\mu_{R^t_{A^t}}: X\times X\longrightarrow[0,1]$ and whose nonmembership function $\nu_{R^t_{A^t}}: X\times X\longrightarrow[0,1]$ is given by $\mu_{R^t_{A^t}}(x,y)=\min\{\mu_{A^t}(x),\mu_{A^t}(y)\}$ and $\nu_{R^t_{A^t}}(x,y)=\max\{\nu_{A^t}(x),\nu_{A^t}(y)\}$, for all $x,y\in X$.

Theorem 17. *Let $A^t=(\mu_{A^t},\nu_{A^t})$ be a t-intuitionistic fuzzy subset of PMS-algebra X and let $R^t_{A^t}$ be the strongest t-intuitionistic fuzzy PMS-relation on X, then A^t is a t-intuitionistic fuzzy PMS-ideal of X if and only if $R^t_{A^t}$ is a t-intuitionistic fuzzy PMS-ideal of $X\times X$.*

Proof. Assume that A^t is a t-intuitionistic fuzzy PMS-ideal of X. Let $(x,y)\in X\times X$. Then

$\mu_{R^t_{A^t}}(0,0)=\min\{\mu_{A^t}(0),\mu_{A^t}(0)\}\ge\min\{\mu_{A^t}(x),\mu_{A^t}(y)\}=\mu_{R^t_{A^t}}(x,y)$ and

$\nu_{R^t_{A^t}}(0,0)=\max\{\nu_{A^t}(0),\nu_{A^t}(0)\}\le\max\{\nu_{A^t}(x),\nu_{A^t}(y)\}=\nu_{R^t_{A^t}}(x,y)$, for all $x,y\in X$, $\Rightarrow\mu_{R^t_{A^t}}(0,0)\ge\mu_{R^t_{A^t}}(x,y)$ and $\nu_{R^t_{A^t}}(0,0)\le\nu_{R^t_{A^t}}(x,y)$, for all $x,y\in X$.

Also, let $(x_1,x_2),(y_1,y_2),(z_1,z_2)\in X\times X$. Then

$$\begin{aligned}
\mu_{R^t_{A^t}}((y_1,y_2)*(x_1,x_2)) &= \mu_{R^t_{A^t}}(y_1*x_1,y_2*x_2)\\
&= \min\{\mu_{A^t}(y_1*x_1),\mu_A(y_2*x_2)\\
&\ge \min\{\min\{\mu_{A^t}(z_1*y_1),\mu_{A^t}(z_1*x_1)\},\ \min\{\mu_{A^t}(z_2*y_2),\mu_{A^t}(Z_2*x_2)\}\}\\
&= \min\{\min\{\mu_{A^t}(z_1*y_1),\mu_{A^t}(z_2*y_2)\},\ \min\{\mu_{A^t}(z_1*x_1),\mu_{A^t}(Z_2*x_2)\}\}\\
&= \min\left\{\mu_{R^t_{A^t}}(z_1*y_1,z_2*y_2),\mu_{R^t_{A^t}}(z_1*x_1,z_2*x_2)\right\}\\
&= \min\left\{\mu_{R^t_{A^t}}((z_1,z_2)*(y_1,y_2)),\mu_{R^t_{A^t}}((z_1,z_2)*(x_1,x_2))\right\}\\
\Rightarrow\mu_{R^t_{A^t}}((y_1,y_2)*(x_1,x_2)) &\ge \min\left\{\mu_{R^t_{A^t}}((z_1,z_2)*(y_1,y_2)),\mu_{R^t_{A^t}}((z_1,z_2)*(x_1,x_2))\right\},
\end{aligned} \tag{38}$$

and

$$\begin{aligned}
\nu_{R^t_{A^t}}((y_1,y_2)*(x_1,x_2)) &= \nu_{R^t_{A^t}}(y_1*x_1, y_2*x_2)\\
&= \max\{\nu_{A^t}(y_1*x_1), \nu_A(y_2*x_2)\\
&\le \max\{\max\{\nu_{A^t}(z_1*y_1), \nu_{A^t}(z_1*x_1)\}, \max\{\nu_{A^t}(z_2*y_2), \nu_{A^t}(Z_2*x_2)\}\}\\
&= \max\{\max\{\nu_{A^t}(z_1*y_1), \nu_{A^t}(z_2*y_2)\}, \max\{\nu_{A^t}(z_1*x_1), \nu_{A^t}(Z_2*x_2)\}\}\\
&= \max\left\{\nu_{R^t_{A^t}}(z_1*y_1, z_2*y_2), \nu_{R^t_{A^t}}(z_1*x_1, z_2*x_2)\right\}\\
&= \max\left\{\nu_{R^t_{A^t}}((z_1,z_2)*(y_1,y_2)), \nu_{R^t_{A^t}}((z_1,z_2)*(x_1,x_2))\right\}\\
&\Rightarrow \nu_{R^t_{A^t}}((y_1,y_2)*(x_1,x_2)) \le \max\left\{\nu_{R^t_{A^t}}((z_1,z_2)*(y_1,y_2)), \nu_{R^t_{A^t}}((z_1,z_2)*(x_1,x_2))\right\}.
\end{aligned} \tag{39}$$

Therefore, $R^t_{A^t}$ is a t-intuitionistic fuzzy PMS-ideal of $X \times X$.

Conversely, assume that $R^t_{A^t}$ is a t-intuitionistic fuzzy PMS-ideal of $X \times X$. Let $(x_1,x_2), (y_1,y_2), (z_1,z_2) \in X \times X$. Then we have

(i) $\min\{\mu_{A^t}(0), \mu_{A^t}(0)\} = \mu_{R^t_{A^t}}(0,0) \ge \mu_{R^t_{A^t}}(x_1,x_2) = \min\{\mu_{A^t}(x_1), \mu_{A^t}(x_2)\}$

$$\Rightarrow \min\{\mu_{A^t}(0), \mu_{A^t}(0)\} \ge \min\{\mu_{A^t}(x_1), \mu_{A^t}(x_2)\} \tag{40}$$

$\Rightarrow \mu_{A^t}(0) \ge \mu_{A^t}(x_1)$ or $\mu_{A^t}(0) \ge \mu_{A^t}(x_2)$

and

$$\begin{aligned}
&\max\{\nu_{A^t}(0), \nu_{A^t}(0)\}\\
&= \max\{\nu_{A^t}(x_1), \nu_{A^t}(x_2)\}\\
&\Rightarrow \max\{\nu_{A^t}(0), \nu_{A^t}(0)\}\\
&\le \max\{\nu_{A^t}(x_1), \nu_{A^t}(x_2)\}.
\end{aligned} \tag{41}$$

$\Rightarrow \nu_{A^t}(0) \le \nu_{A^t}(x_1)$ or $\nu_{A^t}(0) \le \nu_A(x_2)$

(ii) $\min\{\mu_{A^t}(y_1*x_1), \mu_{A^t}(y_2*x_2)\} = \mu_{R^t_{A^t}}(y_1*x_1, y_2*x_2) = \mu_{R^t_{A^t}}((y_1,y_2)*(x_1,x_2)) \ge \min\left\{\mu_{R^t_{A^t}}((z_1, z_2)*(y_1,y_2)), \mu_{R^t_{A^t}}((z_1,z_2)*(x_1,x_2))\right\} = \min\left\{\mu_{R_{A^t}}((z_1*y_1), (z_2*y_2)), \mu_{R_A}((z_1*x_1), (z_2*x_2))\right\} = \min\{\min\{\mu_{A^t}(z_1*y_1), \mu_{A^t}(z_2*y_2)\}, \min\{\mu_{A^t}(z_1*x_1), \mu_{A^t}(z_2*x_2)\}\} = \min\{\min\{\mu_{A^t}(z_1*y_1)\mu_{A^t}(z_1*x_1)\}, \min\{\mu_{A^t}(z_2*y_2)\mu_{A^t}(z_2*x_2)\}\}$

If we put $x_2 = y_2 = z_2 = 0$ (or resp. $x_1 = y_1 = z_1 = 0$), then we get

$\mu_{A^t}(y_1*x_1) \ge \min\{\mu_{A^t}(z_1*y_1), \mu_{A^t}(z_1*x_1)\}$ (or resp.

$\mu_{A^t}(y_2*x_2) \ge \min\{\mu_{A^t}(z_2*y_2), \mu_{A^t}(z_2*x_2)\}$)

(iii) $\max\{\nu_{A^t}(y_1*x_1), \nu_{A^t}(y_2*x_2)\} = \nu_{R^t_{A^t}}(y_1*x_1, y_2*x_2) = \nu_{R^t_{A^t}}((y_1,y_2)*(x_1,x_2)) \le \max\left\{\nu_{R^t_{A^t}}((z_1, z_2)*(y_1,y_2)), \nu_{R^t_{A^t}}((z_1,z_2)*(x_1,x_2))\right\} = \max\left\{\nu_{R^t_{A^t}}((z_1*y_1), (z_2*y_2)), \nu_{R^t_{A^t}}((z_1*x_1), (z_2*x_2))\right\} = \max\{\max\{\nu_{A^t}(z_1*y_1), \nu_{A^t}(z_2*y_2)\}, \max\{\nu_{A^t}(z_1*x_1), \nu_{A^t}(z_2*x_2)\}\} = \max\{\max\{\nu_{A^t}(z_1*y_1), \nu_{A^t}(z_1*x_1)\}, \max\{\nu_{A^t}(z_2*y_2)\nu_{A^t}(z_2*x_2)\}\}$

If we put $x_2 = y_2 = z_2 = 0$ (or resp. $x_1 = y_1 = z_1 = 0$), then we get

$\nu_{A^t}(y_1*x_1) \le \max\{\nu_{A^t}(z_1*y_1), \nu_{A^t}(z_1*x_1)\}$ (or resp. $\nu_{A^t}(y_2*x_2) \le \max\{\mu_{A^t}(z_2*y_2), \mu_{A^t}(z_2*x_2)\}$).

Hence, A^t is a t-intuitionistic fuzzy PMS-ideal of X. □

6. Conclusion

In this article, we used the concept of a t-intuitionistic fuzzy set to PMS-ideals in a PMS-algebra. We studied the notion of a t-intuitionistic fuzzy PMS-ideal of a PMS-algebra and explored some related properties. We provided the relationships between a t-intuitionistic fuzzy PMS-ideal and a t-intuitionistic fuzzy PMS-subalgebra of a PMS-algebra, as well as the relationships between an intuitionistic fuzzy PMS-ideal and a t-intuitionistic fuzzy PMS-ideal of a PMS-algebra. We established a condition for an intuitionistic fuzzy set in a PMS-algebra to be a t-intuitionistic fuzzy PMS-ideal of a PMS-algebra. We described the t-intuitionistic fuzzy PMS-ideals of PMS-algebra by their (α,β) level cuts. We studied a t-intuitionistic fuzzy PMS-ideal of a PMS-algebra under homomorphism and explored the homomorphic image and inverse image of the t-intuitionistic fuzzy PMS-ideal. Furthermore, we discussed the Cartesian product of any two t-intuitionistic fuzzy PMS-ideals of PMS-algebra and obtained some interesting results. We characterized the Cartesian product of the t-intuitionistic fuzzy PMS-ideals by their (α,β) level cuts. Finally, we defined the strongest t-intuitionistic fuzzy PMS-relation on a PMS-algebra and studied the relationship between the strongest t-intuitionistic fuzzy PMS-relation and a t-intuitionistic fuzzy PMS-ideal. We hope that the findings of this work will add other dimensions to the structures of t-intuitionistic fuzzy PMS-ideals based on t-intuitionistic fuzzy sets and serve as the foundation for further studies. In our future works, we will extend this concept to t-Q intuitionistic fuzzy PMS-ideals, t-intuitionistic multi-fuzzy, and anti-multi-fuzzy PMS-ideals to obtain additional novel results. Moreover, we will develop the neutro-algebraic structures with respect to the PMS-ideals of PMS-algebra.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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