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Research Article

Roll of Newtonian and Non-Newtonian Motion in Analysis of Two-Phase Hepatic Blood Flow in Artery during Jaundice

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Biomathematics is an interdisciplinary subject consisting of mathematics and biology, which is widely applicable for the analysis of biological problems. In this paper, we provide a mathematical model of two-phase hepatic blood flow in a jaundice patient's artery. The blood flow is thought to be a two-phased process. The clinical data of a jaundice patient (blood pressure and hemoglobin) is gathered. To begin, hemoglobin is transformed into hematocrit, and blood pressure is turned to a decline in blood pressure. For the examination of hepatic arteries in Newtonian and non-Newtonian movements, a mathematical model is constructed. The relationship between two-phase blood flow flux and blood pressure reduction in the hepatic artery is established. For various hematocrit levels, the blood pressure decrease is determined. The patient's states are defined by the slope of the linear relationship between computed blood pressure decrease and hematocrit.

1. Introduction

Due to globalization and the multidisciplinary development of life science and mathematical science, mathematical modeling becomes a topic of interest for the scientific community. Mathematical modeling plays a significant role to understand the intricacies of infectious diseases [1–3]. It helps to study the mechanisms responsible for observed epidemiological patterns, assesses the efficiency of control policies, and predicts epidemiological inclinations. Nowadays, human beings suffer from many disorders due to global warming, pressure, and stress to lifestyle [4]. Some physiological elements that occur during exposure to weightlessness may include alteration in blood flow to the liver. So many disorders can be seen like diabetes, blood pressure, and other diseases estimation of hepatic blood flow.

Bio-mathematics is an emerging and dynamic field. The term bio-mathematics refers to the use of quantities and mathematical methods to solve biological problems [1, 2]. The bio-mathematics is an interdisciplinary subject to understand the mechanical properties of living tissues, anatomy and physiology in health and illness, and introductory biomechanics: from cell to organisms, blood flow, and microcirculation [5–7]. Bio-fluid mechanics is a well-established branch of bio-mathematics with the help of its normal functions, changed due to alternation via mathematical analysis [7].

It is well known that the heart circulates blood in the body using elastic tubes: the arteries, capillaries, and veins. A proper flow of blood is necessary for good health as it circulates oxygen and other essential nutrients to various parts of the body. Two main factors that affect the blood flow are known as the blood vessels and the properties of blood. Therefore, the modeling of blood flow is very complex as compared to other fluid flows. Blood is a non-Newtonian fluid; hence its viscosity is not constant, resulting in non-linearity of stress and strain rate relation [8]. However, this fact can be approximated by the power law or Casson fluid model [9]. The walls of blood vessels are elastic and disobey

Hooke's law. The curvature of the wall can also affect the properties of blood flow. Due to the composite elastic materials of the wall, a nonlinear stress-strain relation exists. Blood flow is pulsatile resulting in the periodic nature of pressure gradient and the fluid velocity [10]. It is caused due to the beating of the heart. The radii of the tube are variable and can be further divided into multiple veins. Most of the flow is not fully developed due to the high Reynolds's numbers and only inlet flow occurs. Blood is a non-homogeneous, unsteady, and pulsatile flow that flows from the elastic tube which is branched repeatedly.

The first attempt to understand the mechanism of blood flow through various types of vessels was made by Leonardo da Vinci and Descartes [11]. Later, Aird and Silverman explained the nature of the circulation of blood in the cardiovascular system [11, 12]. In 1899, the first mathematical model of propagation of arterial pulse waves is given by Frank [13]. Sharan and Popel [14] explained the execution of the two-phase model for the blood flow in slim tubes and explained how the powerful extended viscosity near the wall affects blood flow [14]. The viscosity of human blood concerning its performance ability in a high static magnetic field is discussed by Haik et al. [15]. Formaggia et al. [16] have studied the one-dimensional nonlinear system, which defines the blood as compliant with arteries [16]. They used the Navier–Stoke′s equation, which shows the algebraic relation between intramural pressure and vessel. Jiang et al. [17] concentrated on the physical demonstration and propelled reproductions of fuel-fluid two-stage stream streams in atomization and splashes [17]. It was proposed that the large blood vessels behave like a Newtonian. The proposed model calculates the parameters of blood flow have no restrictions on the vessel size [18]. The precarious and incompressible stream of non-Newtonian liquid through composite stenosis was also considered [19]. The two phases Bingham model was proposed by Ahnert et al. [20, 21]. They used the differential equation for solutions and solve the mathematical problems. They also used the drift-flux model which defines the behavior of the model like the resistance of the flow and also use the different types of parameters. Kumawat et al. [22] mathematically analyze two-phase blood flow through a stenosed curved artery with hematocrit and temperature dependent viscosity [22]. A patient-specific artery geometry in the presence of stenosis (plaque) was considered. In 2016, Achaba et al. [23] studied the blood flow in arteries through a non-Newtonian viscosity model [23]. They define the two major difficulties. The first is a constitutive equation. No one model accepted the behavior of blood viscosity, and the second is the highly nonlinear equation for blood motion. They used a two-dimensional equation. The study reveals that the power law model is better for non-Newtonian blood flow. Mekheimer and their research group did extensive work on the nanoparticles drug delivery to blood hemodynamics in diseased organs [24–28]. Sharma et al. [29] proposed entropy analysis of thermally radiating MHD slip flow of hybrid nanoparticles (Au-Al_2O_3/Blood) through a tapered multi-stenosed artery [29]. Bhatti and Abdelsalam [30] studied the peristaltic propulsion of hybrid nanofluid flow with Tantalum and Gold nanoparticles under magnetic effects [30].

In this article, we consider the hepatic blood flow in arterioles with respect to the nature of the hepatic circulatory system in humans. The Herschel–Bulkley non-Newtonian model in Bio-fluid physiological is investigated.

2. Important Formulations

In this section, we discuss the meaning of Newtonian and non-Newtonian flow. The mathematical formulation can be found in our previous articles [31–35].

2.1. Covariant Vectors. A set of n function A_i of n coordinates in a coordinate system (x^i) are said to form the components of covariant vectors if they transform to another coordinate system $(\overline{x}^i)$ according to the following rule [36]:

$$\overline{A}_i = \frac{\partial x^k}{\partial \overline{x}^i} A_k. \tag{1}$$

2.2. Contravariant Vectors. A set of n function A^i of n coordinates in a coordinate system (x^i) are said to form the components of covariant vectors if they transform to another coordinate system $(\overline{x}^i)$ according to the following rule [36]:

$$\overline{A}^i = \frac{\partial \overline{x}^i}{\partial x^k} A^k. \tag{2}$$

2.3. Tensors of Second Order

2.3.1. Covariant Tensors of Order Two. A set A_{ij} of n^2 functions of n coordinates in a coordinates system (x^i) are said to form the components of a covariant tensor of order two if they transform to another coordinate system $(\overline{x}^i)$ as follows [37]:

$$\overline{A}_{ij} = \frac{\partial x^r}{\partial \overline{x}^i} \frac{\partial x^s}{\partial \overline{x}^j} A_{rs}. \tag{3}$$

2.3.2. Contravariant Tensors of Order Two. A set A^{ij} of n^2 functions of n coordinates x^i in a coordinates system (x^i) are said to form the components of a contravariant tensor of order two, if they transform to another coordinate system $(\overline{x}^i)$ as follows [37]:

$$\overline{A}_{ij} = \frac{\partial \overline{x}^i}{\partial x^r} \frac{\partial \overline{x}^j}{\partial x^s} A^{rs}. \tag{4}$$

2.3.3. Mixed Tensors of Order Two. A set A^{ij} of n^2 functions of n coordinates x^i in a coordinates system (x^i) are said to form the components of a mixed tensor of order two, if they transform to another coordinate system $(\overline{x}^i)$ as follows [37]:

$$\overline{A}^i_j = \frac{\partial \overline{x}^i}{\partial x^r} \frac{\partial x^s}{\partial \overline{x}^j} A^r_s. \tag{5}$$

2.4. Christoffel's Symbols

2.4.1. Christoffel Symbol of First Kind. The Christoffel 3 index symbol of the first kind are denoted by $[ij,k]$ and defined by the following equation [37]:

$$[ij,k] = \frac{1}{2}\left(\frac{\partial g_{jk}}{\partial x^i} + \frac{\partial g_{ik}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^k}\right), \quad (i,j,k = 1,2\ldots n). \tag{6}$$

2.4.2. Christoffel Symbol of Second Kind. The Christoffel 3 index symbol of the second kind is denoted by $\left\{ \begin{matrix} l \\ i \quad j \end{matrix} \right\}$ and defined by the following equation [37]:

$$\left\{ \begin{matrix} l \\ i \quad j \end{matrix} \right\} = g^{lk}[ij,k]$$

$$= \frac{1}{2}g^{lk}\left(\frac{\partial g_{jk}}{\partial x^i} + \frac{\partial g_{ik}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^k}\right), \quad (i,j,k = 1,2\ldots n). \tag{7}$$

3. Modeling of Blood Flow through Vessels

3.1. Numerical Analysis for Hepatic Arteries. The power law equation of continuity is written as follows [38]:

$$\frac{1}{\sqrt{g\sqrt{(gv)^i}}} = 0. \tag{8}$$

The motion equation is expressed as follows [39]:

$$\rho_m \frac{\partial v^i}{\partial t} + \rho_m v^j v^i_j = T^{ij}_j, \tag{9}$$

where T^{ij} is power law constitutive equation. The blood density equation is

$$\rho_m = X\rho_c + (1 - X), \tag{10}$$

where $X = H/100$ is the volume ratio of blood cells and H is hematocrit. The viscosity of a mixture of blood is expressed as follows:

$$\eta_m = X\eta_c + (1 - X)\eta_p. \tag{11}$$

In cylindrical form,

$$\begin{aligned} x^1 &= r, \\ x^2 &= \theta, \\ x^3 &= z. \end{aligned} \tag{12}$$

Tonsorial form in cylindrical coordinates,

$$\left\{g_{ij}\right\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{13}$$

The conjugate metric tensor is as follows:

$$\left\{g^{ij}\right\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{14}$$

The nonvanishing Christoffel's symbols of 1st kind are as follows:

$$\begin{aligned} [22,1] &= -r[21,2] \\ &= [12,2] \\ &= r. \end{aligned} \tag{15}$$

And, all other Christoffel's symbols of 1st kind are zero. The nonvanishing Christoffel's symbols of 2nd kind are as follows:

$$\begin{aligned} \left\{ \begin{matrix} 1 \\ 2, \quad 2 \end{matrix} \right\} &= -r\left\{ \begin{matrix} 2 \\ 2, \quad 1 \end{matrix} \right\} \\ &= \left\{ \begin{matrix} 2 \\ 1, \quad 2 \end{matrix} \right\} \\ &= \frac{1}{r}. \end{aligned} \tag{16}$$

And, all other Christoffel's symbols of 2nd kind are zero. Relation between contravariant and component of velocity of-blood flow is as follows:

$$\begin{aligned} \sqrt{g_{11}v^1} &= v_r \Rightarrow v^1, \\ \sqrt{g_{22}v^2} &= v_\theta \Rightarrow rv^2, \\ \sqrt{g_{33}v^3} &= v_z \Rightarrow v^3. \end{aligned} \tag{17}$$

The component of: $p,_j g^{ij}$ are $\sqrt{g_{ij}}p,_j g^{ij}$. The components of shearing stress tensor,

$$\begin{aligned} T'^{ij} &= \eta_m\left(e^{ij}\right)^n \\ &= \eta_m\left(g^{jk}v^i_k + g^{ik}v^j_k\right)^n \\ &= \begin{bmatrix} 0 & 0 & \eta_m\left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^n \\ 0 & 1 & 0 \\ \eta_m\left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^n & 0 & 0 \end{bmatrix}. \end{aligned} \tag{18}$$

The covariant derivative of $T,^{ij}_j$ is

$$T,_j^{ij} = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^j}\left(T^{ij}\sqrt{g}\right) + T^{jk}\left\{\begin{array}{c} i \\ jk \end{array}\right\}. \tag{19}$$

3.2. Solution for Newtonian. The blood flow in the artery is symmetric with respect to the axis. Hence, v_z, v_r, and P do not depend upon θ, also $v_\theta = 0$. Since only one component of velocity, which is along the axis is effective. Now $v_r = 0$, $v_\theta = 0$, and $v_z = v$. Flow is steady, then obtain the following equation [39]:

$$\begin{aligned} \frac{\partial p}{\partial t} &= \frac{\partial v_r}{\partial t} \\ &= \frac{\partial v_\theta}{\partial t} \\ &= \frac{\partial v_z}{\partial t} \\ &= 0. \end{aligned} \tag{20}$$

Equation of continuity reduces to

$$\begin{aligned} \frac{\partial v_z}{\partial z} = 0 &\Rightarrow v_z \\ &= v(r). \end{aligned} \tag{21}$$

Equation of motion reduces to

$$\begin{aligned} -\frac{\partial p}{\partial z} = 0 &\Rightarrow p \\ &= p(z). \end{aligned} \tag{22}$$

Z-component,

$$-\frac{\partial p}{\partial z} + \frac{\eta_m}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial v_z}{\partial r}\right)\right] = 0. \tag{23}$$

Let us assume that the pressure gradient of blood flow in arteries is $p = \mathrm{d}p/\mathrm{d}z$.

$$\begin{aligned} p &= \frac{\eta_m}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[r\left(\frac{\partial v_z}{\partial r}\right)\right], \\ \frac{\mathrm{d}}{\mathrm{d}r}\left[r\left(\frac{\partial v_z}{\partial r}\right)\right] &= \frac{pr}{\eta_m}. \end{aligned} \tag{24}$$

Integration gives us,

$$r\frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{pr^2}{2\eta_m} + A, \tag{25}$$

where $r = 0, v = v_0$ are boundary conditions and A is constant.

$$r\frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{pr^2}{2\eta_m}. \tag{26}$$

Integration of equation (26) is as follows:

$$v = -\frac{pr^2}{4\eta_m} + B. \tag{27}$$

Again, using the second boundary condition $v = 0$, then from equation (27), we get the constant of integration value B is

$$B = \frac{pR^2}{4\eta_m}. \tag{28}$$

Putting the above-given value of B from equation (27), we obtain the following equation:

$$v = -\frac{pr^2}{4\eta_m} + \frac{pR^2}{4\eta_m} = \frac{\mathrm{p}}{4\eta_m}\left(R^2 - r^2\right), \tag{29}$$

where R is the radius of artery. If Q is the flux through the artery tube, then

$$\begin{aligned} Q &= \int_0^R 2\pi r v \mathrm{d}r \\ &= \int_0^R \frac{P(z)}{4\eta_m}\left(R^2 - r^2\right)2\pi r \mathrm{d}r, \\ Q &= \frac{P(z)}{4\eta_m}\left[\pi R^2 r^2 - \frac{\pi r^4}{2}\right]_0^R \\ &= \pi R^4 \frac{P(z)}{8\eta_m}. \end{aligned} \tag{30}$$

Therefore,

$$P(z) = -\frac{\partial p}{\partial z}. \tag{31}$$

Now,

$$Q = \frac{\pi R^4}{8\eta_m}\left(-\frac{\partial p}{\partial z}\right). \tag{32}$$

Integration of both sides with limit initial to final,

$$\begin{aligned} 8\eta_m \int_i^f Q\partial z &= -\pi R^4 \int_i^f \partial p, \\ 8\eta_m Q\{z\}_{z_i}^{z_f} &= -\pi R^4 \{P\}_{p_i}^{p_f}, \\ 8\eta_m Q\left\{z_f - z_i\right\} &= -\pi R^4\left\{p_f - p_i\right\}, \\ 8\eta_m Q\Delta z &= \pi R^4 \Delta p, \\ Q &= \frac{\pi R^4}{8\eta_m}\frac{\Delta p}{\Delta z}, \end{aligned} \tag{33}$$

3.3. Solution for Non-Newtonian. Now $v_r = 0$, $v_\theta = 0$, and $v_z = v$

The blood flow,

$$\begin{aligned}\frac{\partial p}{\partial t} &= \frac{\partial v_r}{\partial t}\\ &= \frac{\partial v_\theta}{\partial t}\\ &= \frac{\partial v_z}{\partial t}\\ &= 0.\end{aligned} \tag{34}$$

Equation of continuity reduces to,

$$\begin{aligned}\frac{\partial v_z}{\partial z} = 0 &\Rightarrow v_z\\ &= v(r).\end{aligned} \tag{35}$$

Equation of motion reduces to,

$$\begin{aligned}\frac{\partial p}{\partial z} = 0 &\Rightarrow p\\ &= p(z).\end{aligned} \tag{36}$$

Z-component,

$$-\frac{\partial p}{\partial z} + \frac{\eta_m}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial v}{\partial r}\right)^n\right] = 0. \tag{37}$$

Let us assume that the pressure gradient (p) of blood flow in arteries is $\mathrm{d}p/\mathrm{d}z$. Then,

$$\begin{aligned}p &= \frac{\eta_m}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[r\left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^n\right],\\ \frac{\mathrm{d}}{\mathrm{d}r}\left[r\left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^n\right] &= \frac{pr}{\eta_m}.\end{aligned} \tag{38}$$

We arrive at,

$$r\left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^n = -\frac{pr^2}{2\eta_m} + A. \tag{39}$$

Let us consider boundary conditions; $r = 0$, $v = v_0$ and $A = 0$. We arrive at

$$\begin{aligned}r\left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^n &= -\frac{pr^2}{2\eta_m},\\ -\frac{\mathrm{d}v}{\mathrm{d}r} &= \left(\frac{pr}{2\eta_m}\right)^{(1/n)}.\end{aligned} \tag{40}$$

After integration,

$$v = -\left[\frac{p}{2\eta_m}\right]^{(1/n)}\frac{r^{(1/n)+1}}{(n+1/n)} + B. \tag{41}$$

Under no slip boundary condition ($v = 0$, $r = R$), equation (41) can be written as follows:

$$B = \left[\frac{p}{2\eta_m}\right]^{(1/n)}\frac{r^{(1/n)+1}}{(n+1/n)}. \tag{42}$$

By inserting the value of B in (41),

$$v = \left(\frac{p}{2\eta_m}\right)^{(1/n)}\frac{n}{n+1}\left\{R^{(1/n)+1} - r^{(1/n)+1}\right\}. \tag{43}$$

Equation (43) describes the velocity of blood flow in arteries. Total flux flow of blood (Q) through a tube of arteries is defined as follows:

$$\begin{aligned}Q &= \int_0^R v2\pi r\mathrm{d}r\\ &= \left(\frac{n}{n+1}\right)\int_0^R 2\pi r\left(\frac{P(z)}{2\eta_m}\right)^{(1/n)}\left\{R^{(1/n)+1} - r^{(1/n)+1}\right\}\mathrm{d}r,\end{aligned} \tag{44}$$

$$Q = \frac{n\pi}{3n+1}\left(\frac{P(z)}{2\eta_m}\right)^{(1/n)}R^{(1/n)+3}.$$

Thus,

$$\begin{aligned}Q^n &= \left(\frac{n\pi}{3n+1}\right)^n\left(\frac{P(z)}{2\eta_m}\right)R^{1+3n},\\ P(z) &= Q^n 2\eta_m\left(\frac{3n+1}{n\pi}\right)^n\frac{1}{R^{1+3n}}.\end{aligned} \tag{45}$$

Equation of motion,

$$\begin{aligned}P(z) &= \frac{\partial p}{\partial z}\\ &= Q^n 2\eta_m\left(\frac{3n+1}{n\pi}\right)^n\frac{1}{R^{1+3n}}.\end{aligned} \tag{46}$$

Integration of equation (46) gives us the pressure drop of blood.

$$\begin{aligned}\int_i^f \partial p &= Q^n 2\eta_m\left(\frac{3n+1}{n\pi}\right)^n\int_i^f\frac{\mathrm{d}z}{R^{1+3n}},\\ \Delta p &= Q^n 2\eta_m\left(\frac{3n+1}{n\pi}\right)^n\frac{\Delta z}{R^{1+3n}}.\end{aligned} \tag{47}$$

4. Analysis for Hepatic Arteries

4.1. For Newtonian Motion. If Q is the flux through the tube, then

$$\begin{aligned}Q &= \int_0^R\frac{P(z)}{4\eta_m}\left(R^2 - r^2\right)2\pi r\mathrm{d}r,\\ Q &= \frac{P(z)}{4\eta_m}\left[\pi R^2 r^2 - \frac{\pi r^4}{2}\right]_0^R\\ &= \pi R^4\frac{P(z)}{8\eta_m}.\end{aligned} \tag{48}$$

Table 1: Clinical data and calculated BP values of jaundice patient (name: *A*, age: 43, and sex: F).

Date	Hemoglobin (mmHg)	Hematocrit (kg/m3)	BP	BP drop	BP (Pa)	BP drop (using (64))
08/08/16	9.8	0.0277	150/70	40.0	5332.8	178.698
13/08/16	9.9	0.0281	144/70	37.0	4932.9	179.807
20/08/16	10.1	0.0286	140/75	32.5	4332.9	182.064
25/08/16	10.3	0.0292	135/78	28.5	3799.7	184.321
30/08/16	10.6	0.0300	125/80	22.5	2999.8	187.686

Using (21), equation (48) can be written as follows:

$$Q = \frac{\pi R^4}{8\eta_m}\left(-\frac{\partial p}{\partial z}\right). \tag{49}$$

Now,

$$8\eta_m Q\,\partial z = -\pi R^4 \partial p\,. \tag{50}$$

Integration of both sides with limit initial to final,

$$8\eta_m \int_i^f Q\partial z = -\pi R^4 \int_i^f \partial p,$$

$$8\eta_m Q\{z_f - z_i\} = -\pi R^4\{p_f - p_i\}, \tag{51}$$

$$\Delta p = \frac{8\eta_m Q}{\pi R^4}\Delta z.$$

Table 1 demonstrates the measured hemoglobin and standard BP of a jaundice patient (name: *A*, age: 43, and sex: F). Firstly, the hemoglobin is converted to hematocrit using the relation mentioned in Table 2. After that, the BP drop is assessed from standard BP using the following relation [31]:

$$\text{BP drop} = S - \frac{S+D}{2}, \tag{52}$$

where S is systolic and D is diastolic. Later, the BP drop is converted to BP (in Pa). Let us consider that $H = 0.02858$ and blood pressure drop is 4332.9. Equation (11) gives us,

$$\eta_c \frac{0.02858}{100} + 0.0013\left(1 - \frac{0.02858}{100}\right) = 0.0034,$$

$$\eta_c = 7.477795Pa. \tag{53}$$

The relation between flow flux and blood pressure drop of two-phase blood flow in the hepatic artery is expressed as follows:

$$\Delta p = \frac{8Q\Delta z}{\pi R^4}\eta_m. \tag{54}$$

Using (10) & (63), the solution of equation (54) is obtained as follows:

$$\Delta p = 3959.3742H + 68.9047. \tag{55}$$

Different values of H gives us the blood pressure drop (Table 1). Figure 1 demonstrates the BP drop as a function of hematocrit. It is observed that the BP drop is increased as hematocrit enhances, which means that the hemoglobin of the patient is normal.

Table 2: Clinical data of jaundice patients (name: *A*, age: 43, and sex: F).

Parameters	Magnitudes	Ref.
1 mmHg at 0°C	133.322 Pa	[40]
Hematocrit (three times of hemoglobin/Density of blood)	(1060 kg/m^3)	
Viscosity of mixture (η_m)	0.0034 Pa	[41]
Viscosity of plasma (η_p)	0.0013 Pa	[42]
Blood flow flux	Q = 800–1000 ml/min Q = 900 ml/min (average) $Q = 1.5 \times 10^{-5}$ (1000 ml/min = 1.666 $\times 10^{-8}$ m^3/s)	
Length of hepatic artery (Δz)	3.25 cm	[43]
Radius of hepatic artery (R)	0.0022 m	[44]

4.2. For Non-Newtonian Motion. The total flow-flux of blood through a tube of the arteries is Q defined by the following equation:

$$Q = \int_0^R 2\pi r V \mathrm{d}r,$$

$$Q = \left(\frac{n}{n+1}\right)\int_0^R 2\pi r\left(\frac{P(z)}{2\eta_m}\right)^{(1/n)}\left\{R^{(1/n)+1} - r^{(1/n)+1}\right\}\mathrm{d}r, \tag{56}$$

$$Q = \frac{n\pi}{3n+1}\left(\frac{P(z)}{2\eta_m}\right)^{(1/n)} R^{(1/n)+3}.$$

Both sides take power n,

$$Q^n = \left(\frac{n\pi}{3n+1}\right)^n\left(\frac{P(z)}{2\eta_m}\right)R^{1+3n},$$

$$P(z) = Q^n 2\eta_m\left(\frac{3n+1}{n\pi}\right)^n \frac{1}{R^{1+3n}}. \tag{57}$$

We know that the $P(z) = \partial p/\partial z$ for non-Newtonian motion. Hence,

$$\frac{\partial p}{\partial z} = Q^n 2\eta_m\left(\frac{3n+1}{n\pi}\right)^n \frac{1}{R^{1+3n}}. \tag{58}$$

Integration of both sides with limit initial to final,

$$\int_i^f \partial p = Q^n 2\eta_m\left(\frac{3n+1}{n\pi}\right)^n \int_i^f \frac{\mathrm{d}z}{R^{1+3n}}. \tag{59}$$

Pressure drop of blood,

$$\Delta p = Q^n 2\eta_m\left(\frac{3n+1}{n\pi}\right)^n \frac{\Delta z}{R^{1+3n}}. \tag{60}$$

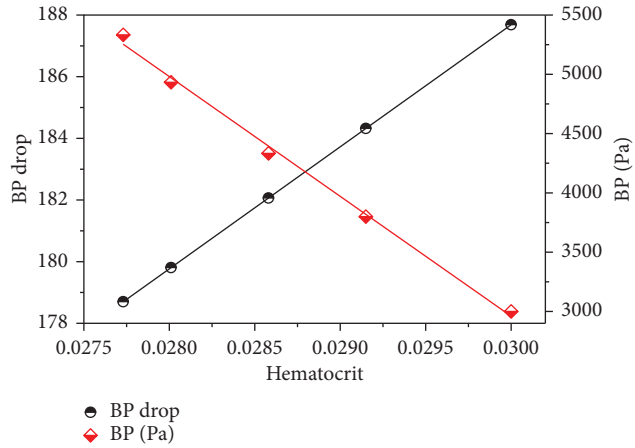

Figure 1: Mathematical data of BP drop (calculated using (66)) and BP (in Pa) as a function of hematocrit, by Table 1.

Table 3: Clinical data and calculated BP values of jaundice patient (name: *A*, age: 43, and sex: F).

Date	Hemoglobin (mmHg)	Hematocrit (kg/m3)	BP	BP drop	BP (Pa)	BP drop (using (66))
08/08/16	9.8	0.0277	150/70	40.0	5332.8	125.2133
13/08/16	9.9	0.0281	144/70	37.0	4932.9	126.3241
20/08/16	10.1	0.0286	140/75	32.5	4332.9	127.7126
25/08/16	10.3	0.0292	135/78	28.5	3799.7	129.3788
30/08/16	10.6	0.0300	125/80	22.5	2999.8	131.6004

Let us consider that $H = 0.02858$ and BP drop $= 4332.90$ and

$$P(z) = \frac{\Delta p}{\Delta z} = \frac{4332.90}{0.0325} = 133320. \quad (61)$$

Using equation (11),

$$\eta_c = 7.4778\ \text{Pa}. \quad (62)$$

Again, by using equation (11), we arrive at,

$$\eta_m = 0.07476H + 0.0013. \quad (63)$$

Rearrangement of (59) provides the blood flow flux (Q),

$$Q = \frac{n\pi}{3n+1}\left(\frac{P(z)}{2\eta_m}\right)^{(1/n)} R^{(1/n)+3}. \quad (64)$$

Solution of equation (64) gives the value of n,

$$n = 0.951. \quad (65)$$

Inserting the values of n, Q, η_m, and Δz in (60), we arrive at,

$$\Delta p = 2777.028H + 48.2896. \quad (66)$$

By inserting the value of H in equation (66), we get a BP drop (Table 3).

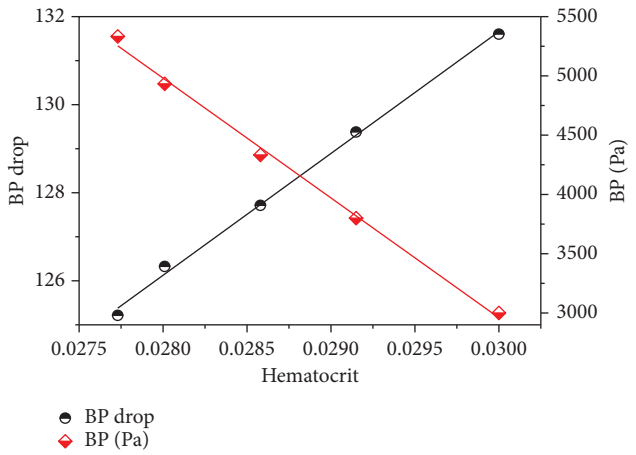

Figure 2: Mathematical data of BP drop (calculated using (66)) and BP (in Pa) as a function of hematocrit, by Table 3.

Figure 2 demonstrates the relation between the calculated BP drop (Δp) and hematocrit (H) for non-Newtonian motion. It is observed that the BP drop increases as the hematocrit increases.

Figures 3(a) and 3(b). Mathematical data of BP drop (calculated using (66)) and BP (in Q) as a function of Hematocrit for non-Newtonian motion. Solid lines represent the linear fitting.

Rheological properties of the Reiner–Rivlin fluid model for blood flow through a tapered artery with stenosis have been studied by Akbar et al. [45]. Elogail and Mekheimer [46] implemented a numerical study that

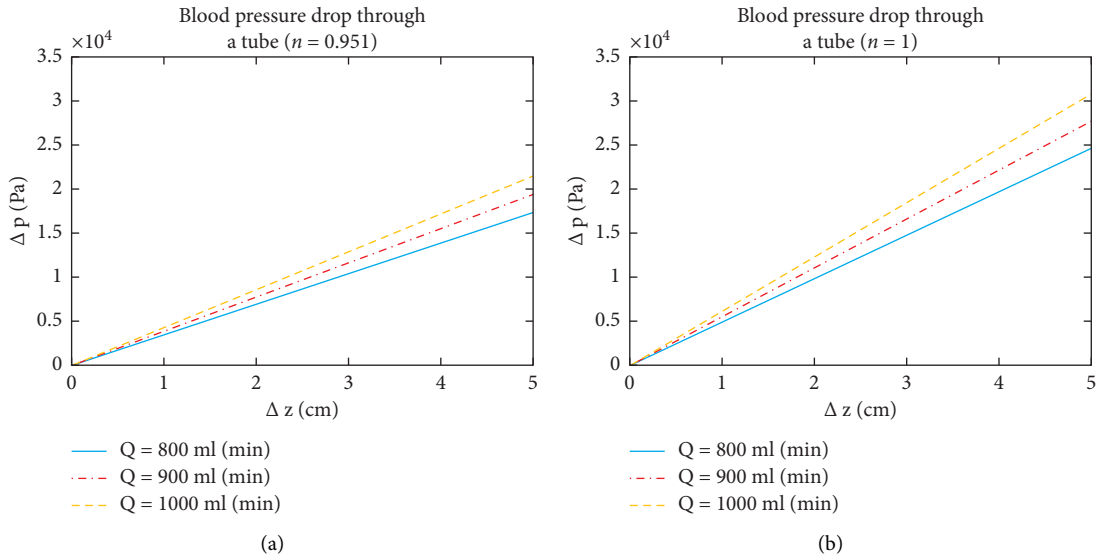

Figure 3: (a) Relation between Δp(Pa) and Δz(cm). (b) Relation between Δp(Pa) and Δz(cm).

simulates blood flow through a microvessel involving oxytactic microorganisms and nanoparticles [46]. The oxytactic microorganisms exhibit negative chemotaxis to gradients of oxygen (oxygen repellents). These microorganisms are to batter infected hypoxic tumor cells as drug carriers [46]. Awad et al. [47] studied the flow of a non-Newtonian fluid with nonzero yield stress [47]. Navier stokes equation is used to simulate this subject mathematically. The elasticity on the stenosis arterial walls is simulated by Rubinow and Keller model and the Mazumdar model [47]. Kumawat et al. [22] mathematically analyze two-phase blood flow through a stenosed curved artery with hematocrit and temperature dependent viscosity [22].

Last but not least, we developed mathematical for blood flow in arteries during jaundice. We collected the blood pressure data of jaundice patients from the hospital and later applied a mathematical model to analyze the data.

5. Conclusion

We developed a deterministic model of the link between hematocrit and blood pressure fluctuations in jaundice patients in this study. The mathematical analysis is validated on the clinical data and calculated BP values of the jaundice patient (name: *A*, age: 43, and sex: F). The role of blood flow and hemoglobin malfunction is investigated in this research. It is feasible to propose a patient for better therapy based on the trend line of the association between hematocrit and BP decline. With the use of model Newtonian motion and non-Newtonian motion, if the trend line exhibits positive slope, then the patient's hemoglobin is normal. If the trend line indicates a negative slope, however, the management of jaundice patients should be modified. This study is useful to predict the hemoglobin status of jaundice patients on the basis of blood pressure measurement.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Conceptualization was done by A. S. and R. A. K.; data curation was done by A. S., R. A. K., and T. A.; formal analysis was done by A. S. and R. A. K.; investigation was done by A. S., S. K., and R. A. K.; methodology was done by A. S. and R. A. K.; project administration and validation was done by A. S. and R. A. K; visualization was done by A. S., T. A., and R. A. K; writing-original draft was done by A. S., S. K., T. A., and R. A. K.; writing-review and editing was done by A. S., S. K., and R. A. K. All authors read and approved the final manuscript.

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