Hindawi
International Journal of Mathematics and Mathematical Sciences
Volume 2023, Article ID 6689743, 14 pages
https://doi.org/10.1155/2023/6689743

Research Article

Unique Common Fixed Points for Expansive Maps

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Received 22 July 2023; Revised 15 August 2023; Accepted 28 August 2023; Published 11 September 2023

Academic Editor: Nawab Hussain

In this work, we establish three common fixed point results for expansive maps satisfying implicit relations in metric and dislocated metric spaces. We do this by utilizing recently developed concept of occasionally weakly biased maps of type $(\mathscr{A})$. These studies about the theory of common fixed points refine several earlier ones. Some illustrative examples are offered to support our theorems, and even better, a pertinent application is supplied to demonstrate the viability and applicability of one of these results.

1. Introduction and Preliminary Notes

In fixed point theory, many researchers investigated the existence and uniqueness of fixed and common fixed points for contractive maps, while several authors concentrated their investigations on expansive maps, and of course, some other mathematicians focused their inquiries on both contractive and expansive maps simultaneously. According to Chouhan and Malviya [1], the research about fixed points of expansive maps was initiated in 1967 by Machuca [2]. Later, many works searched fixed and common fixed points for expansive maps in different spaces.

On the other hand, in 1985, in his thesis, Matthews [3] proposed the kind of metric domains and he observed that there is a bijection between the family of metric domains and the one of metric spaces. He introduced this new space to show that fixed points can exist in other spaces under various contractive conditions.

Definition 1 (see [3]). A metric domain is a pair $\langle \Phi, \varphi \rangle$ where Φ is a nonempty set, and φ is a function from $\Phi \times \Phi$ to $\mathbb{R}_+$ such that

(1) $\forall \rho, \kappa \in \Phi$: $\varphi(\rho, \kappa) = 0 \Rightarrow \rho = \kappa$.
(2) $\forall \rho, \kappa \in \Phi$: $\varphi(\rho, \kappa) = \varphi(\kappa, \rho)$.
(3) $\forall \rho, \kappa, \varpi \in \Phi$: $\varphi(\rho, \kappa) \leq \varphi(\rho, \varpi) + \varphi(\varpi, \kappa)$.

In 2001, in his thesis, Hitzler [4] used metric domains under the name of dislocated metrics. In 2012, Amini-Harandi [5] suggested a new generalization of the metric space which is called a metric-like space. In fact, the notions of metric domains, metric-like spaces, and dislocated metric spaces are exactly the same, and these spaces are sometimes called as d-metric spaces.

Definition 2 (see [4, 5]). Let $\mathscr{X}$ be a nonempty set. A function d: $\mathscr{X} \times \mathscr{X} \longrightarrow [0, +\infty)$ is said to be a dislocated metric on $\mathscr{X}$ if for any ρ, $\kappa, \varpi \in \mathscr{X}$, the following conditions hold:

(1) $d(\rho, \kappa) = 0 \Rightarrow \rho = \kappa$.
(2) $d(\rho, \kappa) = d(\kappa, \rho)$.
(3) $d(\rho, \varpi) \leq d(\rho, \kappa) + d(\kappa, \varpi)$.

The pair $(\mathscr{X}, d)$ is then called a dislocated metric space.

Recently, in 2019, Markin and Sichel [6] introduced the notion of expansive maps and their types as follows.

Definition 3 (see [6]). Let $(\mathscr{X}, d)$ be a metric space. A map $\mathscr{M}$: $\mathscr{X} \longrightarrow \mathscr{X}$ on $(\mathscr{X}, d)$ such that

$$\forall \mu, \nu \in \mathcal{X}: d(\mathcal{M}\mu, \mathcal{M}\nu) \geq d(\mu, \nu), \quad (1)$$

is called an expansive map (or expansion).

Definition 4 (see [6]). Let $(\mathcal{X}, d)$ be a metric space.

(1) An expansion $\mathcal{E}: \mathcal{X} \longrightarrow \mathcal{X}$ such that

$$\forall \mu, \nu \in \mathcal{X}: d(\mathcal{E}\mu, \mathcal{E}\nu) = d(\mu, \nu), \quad (2)$$

is called an isometry, which is the weakest form of expansive maps.

(2) An expansion $\mathcal{E}: \mathcal{X} \longrightarrow \mathcal{X}$ such that

$$\exists \mu, \nu \in \mathcal{X}, \mu \neq \nu: d(\mathcal{E}\mu, \mathcal{E}\nu) > d(\mu, \nu), \quad (3)$$

and we call it a proper expansion.

(3) An expansion $\mathcal{E}: \mathcal{X} \longrightarrow \mathcal{X}$ such that

$$\forall \mu, \nu \in \mathcal{X}, \mu \neq \nu: d(\mathcal{E}\mu, \mathcal{E}\nu) > d(\mu, \nu), \quad (4)$$

and we call it a strict expansion.

(4) An expansion $\mathcal{E}: \mathcal{X} \longrightarrow \mathcal{X}$ such that

$$\exists \alpha > 1\, \forall \mu, \nu \in \mathcal{X}: d(\mathcal{E}\mu, \mathcal{E}\nu) \geq \alpha d(\mu, \nu), \quad (5)$$

and we call it an anti-contraction with expansion constant α.

Now, to combine the existing extensions and generalizations of Banach fixed-point theorem, different methods were used by many authors. Among them, Popa, in 1997 and 1999, in his articles [7, 8], established the implicit relation's idea. Afterwards, several researchers used this good combination for proving fixed and common fixed point theorems for single and multi-valued maps in various spaces (see for instance [9–26]). In this paper, we will introduce new kinds of implicit relations in order to use them to prove unified common fixed point theorems in metric and dislocated metric spaces.

In the sequel, our principal results will be presented and proved.

2. Unique Common Fixed Points for Quadruple Maps in Metric Spaces

In this section, we will present our new definitions and introduce some implicit relations in order to prove our first result.

2.1. New Concepts. In 2022, in [27], we initiated the notions of occasionally weakly $\mathcal{M}$-biased (respectively, $\mathcal{N}$-biased) maps of type $(\mathcal{A})$, and we revealed that these definitions coincide with our concepts: occasionally weakly $\mathcal{M}$-biased (respectively, $\mathcal{N}$-biased) maps given in [28]. Note that several authors proved the existence of fixed points for occasionally weakly biased, subweakly biased, and biased maps (see for instance [29–32]).

Definition 5 (see [27]). Let $\mathcal{M}$ and $\mathcal{N}$ be maps from a nonempty set $\mathcal{X}$ into itself. Maps $\mathcal{M}$ and $\mathcal{N}$ are called occasionally weakly $\mathcal{M}$-biased (respectively, $\mathcal{N}$-biased) of type $(\mathcal{A})$, iff there is an element p in $\mathcal{X}$ such that $\mathcal{M}p = \mathcal{N}p$ implies

$$d(\mathcal{M}\mathcal{M}p, \mathcal{N}p) \leq d(\mathcal{N}\mathcal{M}p, \mathcal{M}p),$$
$$d(\mathcal{N}\mathcal{N}p, \mathcal{M}p) \leq d(\mathcal{M}\mathcal{N}p, \mathcal{N}p), \quad (6)$$

respectively.

2.2. Implicit Relations. As we said above, in his papers, Popa [7, 8] unified several explicit contractions under the so-called implicit contraction. Motivated by Popa's technique, we instigate the following.

Let Γ be a set of functions $\gamma: [0, +\infty)^6 \longrightarrow \mathbb{R}$ such that γ is nondecreasing in r_2, r_3, r_4, r_5, and r_6 and satisfies the next condition:

$$\gamma(r, r, 0, 0, r, r),$$
$$\gamma(r, r, 2r, 0, r, r), \quad (7)$$
$$\gamma(r, r, 0, 2r, r, r),$$

are negative for all r positive.

Example 1. $\gamma(r_1, r_2, r_3, r_4, r_5, r_6) = -r_1 + \rho \max\{r_2, r_3, r_4\} + \varrho(r_5 + r_6)$, where ρ, $\varrho > 0$ and $2\rho + 2\varrho < 1$.

(i) Trivially, γ is nondecreasing in r_2,r_3,r_4,r_5, and r_6.

(ii) $\gamma(r, r, 0, 0, r, r) = -r + \rho \max\{r, 0, 0\} + \varrho(r + r) = r[\rho + 2\varrho - 1] < 0\, \forall r > 0$.

(iii) $\gamma(r, r, 2r, 0, r, r) = -r + \rho \max\{r, 2r, 0\} + \varrho(r + r) = r[2\rho + 2\varrho - 1] < 0\, \forall r > 0$.

(iv) $\gamma(r, r, 0, 2r, r, r) = -r + \rho \max\{r, 0, 2r\} + \varrho(r + r) = r[2\rho + 2\varrho - 1] < 0\, \forall r > 0$.

Example 2. $\gamma(r_1, r_2, r_3, r_4, r_5, r_6) = -r_1 + \mu \max\{r_2, r_3, r_4\} + \nu \max\{r_5, r_6\}$, where μ, $\nu > 0$ and $2\mu + \nu < 1$.

(i) Clearly, γ is nondecreasing in variables r_2,r_3,r_4,r_5, and r_6.

(ii) $\gamma(r, r, 0, 0, r, r) = -r + \mu \max\{r, 0, 0\} + \nu \max\{r, r\} = r[\mu + \nu - 1] < 0\, \forall r > 0$.

(iii) $\gamma(r, r, 2r, 0, r, r) = -r + \mu \max\{r, 2r, 0\} + \nu \max\{r, r\} = r[2\mu + \nu - 1] < 0\, \forall r > 0$.

(iv) $\gamma(r, r, 0, 2r, r, r) = -r + \mu \max\{r, 0, 2r\} + \nu \max\{r, r\} = r[2\mu + \nu - 1] < 0\, \forall r > 0$.

Example 3. $\gamma(r_1, r_2, r_3, r_4, r_5, r_6) = -r_1 + \zeta[r_2 + r_3 + r_4] + \xi[r_5 + r_6]$, where ζ,$\xi > 0$ and $3\zeta + 2\xi < 1$.

(i) It is evident to see that γ is nondecreasing in variables r_2,r_3,r_4,r_5, and r_6.

(ii) $\gamma(r, r, 0, 0, r, r) = -r + \zeta[r + 0 + 0] + \xi[r + r] = r[\zeta + 2\xi - 1] < 0\, \forall r > 0$.

(iii) $\gamma(r, r, 2r, 0, r, r) = -r + \zeta[r + 2r + 0] + \xi[r + r] = r[3\zeta + 2\xi - 1] < 0\, \forall r > 0$.

(iv) $\gamma(r, r, 0, 2r, r, r) = -r + \zeta[r + 0 + 2r] + \xi[r + r] = r[3\zeta + 2\xi - 1] < 0\ \forall r > 0$.

Example 4. $\gamma(r_1, r_2, r_3, r_4, r_5, r_6) = -r_1 + \sigma[r_2 + r_3 + r_4] + \varsigma \max\{r_5, r_6\}$, where $\sigma, \varsigma > 0$ and $3\sigma + \varsigma < 1$.

(i) Evidently, γ is nondecreasing in variables r_2, r_3, r_4, r_5, and r_6.

(ii) $\gamma(r, r, 0, 0, r, r) = -r + \sigma[r + 0 + 0] + \varsigma \max\{r, r\} = r[\sigma + \varsigma - 1] < 0\ \forall r > 0$.

(iii) $\gamma(r, r, 2r, 0, r, r) = -r + \sigma[r + 2r + 0] + \varsigma \max\{r, r\} = r[3\sigma + \varsigma - 1] < 0\ \forall r > 0$.

(iv) $\gamma(r, r, 0, 2r, r, r) = -r + \sigma[r + 0 + 2r] + \varsigma \max\{r, r\} = r[3\sigma + \varsigma - 1] < 0\ \forall r > 0$.

Theorem 6. *Let $\mathscr{B}$, $\mathscr{C}$, $\mathscr{D}$, and $\mathscr{E}$ be maps from a metric space $(\mathscr{X}, d)$ into itself, such that, for all $x, y \in \mathscr{X}$, we have*

$$\gamma(d(\mathscr{B}x, \mathscr{C}y), d(\mathscr{D}x, \mathscr{E}y), d(\mathscr{B}x, \mathscr{D}x), d(\mathscr{C}y, \mathscr{E}y), d(\mathscr{D}x, \mathscr{C}y), d(\mathscr{B}x, \mathscr{E}y)) \geq 0, \tag{8}$$

where $\gamma \in \Gamma$. Assume that maps $\mathscr{B}$ and $\mathscr{D}$ (respectively, $\mathscr{C}$ and $\mathscr{E}$) are occasionally weakly $\mathscr{D}$-biased (respectively, $\mathscr{E}$-biased) of type $(\mathscr{A})$; then, $\mathscr{B}$, $\mathscr{C}$, $\mathscr{D}$, and $\mathscr{E}$ admit only one common fixed point.

Proof. According to the assumptions, we have the existence of two elements ρ and μ in $\mathscr{X}$ which verify $\mathscr{B}\rho = \mathscr{D}\rho$ implies $d(\mathscr{D}\mathscr{D}\rho, \mathscr{B}\rho) \leq d(\mathscr{B}\mathscr{D}\rho, \mathscr{D}\rho)$ and $\mathscr{C}\mu = \mathscr{E}\mu$ implies $d(\mathscr{E}\mathscr{E}\mu, \mathscr{C}\mu) \leq d(\mathscr{C}\mathscr{E}\mu, \mathscr{E}\mu)$.

Firstly, we will show that $\mathscr{B}\rho = \mathscr{C}\mu$. Let us assume that $\mathscr{B}\rho \neq \mathscr{C}\mu$, and the use of inequality (8) yields

$$\begin{aligned} &\gamma(d(\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{D}\rho, \mathscr{E}\mu), d(\mathscr{B}\rho, \mathscr{D}\rho), d(\mathscr{C}\mu, \mathscr{E}\mu), d(\mathscr{D}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{E}\mu)) \\ &= \gamma(d(\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{C}\mu), 0, 0, d(\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{C}\mu)) \geq 0, \end{aligned} \tag{9}$$

a contradiction, and thus $\mathscr{B}\rho = \mathscr{C}\mu$.

Secondly, we assure that $\mathscr{B}\mathscr{B}\rho = \mathscr{B}\rho$. Imagine we have the opposite; then, using assumption (8), we get

$$\begin{aligned} &\gamma(d(\mathscr{B}\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{D}\mathscr{B}\rho, \mathscr{E}\mu), d(\mathscr{B}\mathscr{B}\rho, \mathscr{D}\mathscr{B}\rho), d(\mathscr{C}\mu, \mathscr{E}\mu), d(\mathscr{D}\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{B}\mathscr{B}\rho, \mathscr{E}\mu)) \\ &= \gamma(d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho), d(\mathscr{D}\mathscr{B}\rho, \mathscr{B}\rho), d(\mathscr{B}\mathscr{B}\rho, \mathscr{D}\mathscr{B}\rho), 0, d(\mathscr{D}\mathscr{B}\rho, \mathscr{B}\rho), d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho)) \geq 0. \end{aligned} \tag{10}$$

As maps $\mathscr{B}$ and $\mathscr{D}$ are occasionally weakly $\mathscr{D}$-biased of type $(\mathscr{A})$, γ is nondecreasing in r_2, r_3, and r_5; using the triangle inequality, we obtain

$$\gamma(d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho), d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho), 2d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho), 0, d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho), d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho)) \geq 0, \tag{11}$$

and this contradiction implies that $\mathscr{B}\mathscr{B}\rho = \mathscr{B}\rho$ and so $\mathscr{D}\mathscr{B}\rho = \mathscr{B}\rho$.

Thirdly, suppose that $\mathscr{C}\mathscr{C}\mu \neq \mathscr{C}\mu$. Using inequality (8), we obtain

$$\begin{aligned} &\gamma(d(\mathscr{B}\rho, \mathscr{C}\mathscr{C}\mu), d(\mathscr{D}\rho, \mathscr{E}\mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{D}\rho), d(\mathscr{C}\mathscr{C}\mu, \mathscr{E}\mathscr{C}\mu), d(\mathscr{D}\rho, \mathscr{C}\mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{E}\mathscr{C}\mu)) \\ &= \gamma(d(\mathscr{C}\mu, \mathscr{C}\mathscr{C}\mu), d(\mathscr{C}\mu, \mathscr{E}\mathscr{C}\mu), 0, d(\mathscr{C}\mathscr{C}\mu, \mathscr{E}\mathscr{C}\mu), d(\mathscr{C}\mu, \mathscr{C}\mathscr{C}\mu), d(\mathscr{C}\mu, \mathscr{E}\mathscr{C}\mu)) \geq 0. \end{aligned} \tag{12}$$

Again, as γ is nondecreasing in r_2, r_4, and r_6 and maps $\mathscr{C}$ and $\mathscr{E}$ are occasionally weakly $\mathscr{E}$-biased of type $(\mathscr{A})$, by the triangle inequality, we find

$$\gamma(d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),0,2d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu))\geq 0, \tag{13}$$

which is a contradiction, and hence $\mathscr{C}\mathscr{C}\mu=\mathscr{C}\mu$ and so $\mathscr{E}\mathscr{C}\mu=\mathscr{C}\mu$, i.e., $\mathscr{C}\mathscr{B}\rho=\mathscr{B}\rho$ and $\mathscr{E}\mathscr{B}\rho=\mathscr{B}\rho$. Put $\mathscr{B}\rho=\mathscr{D}\rho=\mathscr{C}\mu=\mathscr{E}\mu=p$; therefore, p is a common fixed point of maps $\mathscr{B}$, $\mathscr{C}$, $\mathscr{D}$, and $\mathscr{E}$.

Fourthly, assume the existence of another common fixed point (say q). From (8), we have

$$\begin{aligned}&\gamma(d(\mathscr{B}p,\mathscr{C}q),d(\mathscr{D}p,\mathscr{E}q),d(\mathscr{B}p,\mathscr{D}p),d(\mathscr{C}q,\mathscr{E}q),d(\mathscr{D}p,\mathscr{C}q),d(\mathscr{B}p,\mathscr{E}q))\\&\quad=\gamma(d(p,q),d(p,q),0,0,d(p,q),d(p,q))\geq 0,\end{aligned} \tag{14}$$

which implies that $q=p$. □

The following example supports our result.

Example 5. Equip $\mathscr{X}=[0,10)$ with the usual metric $d(x,y)=|x-y|$ and set up the following maps:

$$\mathscr{B}x=\begin{cases}\dfrac{1}{2} \text{ for x is in } [0,1),\\ 1 \text{ for x is in } [1,10),\end{cases}\qquad \mathscr{C}x=\begin{cases}\dfrac{3}{4} \text{ for x is in } [0,1),\\ 1 \text{ for x is in } [1,10),\end{cases}$$
$$\mathscr{D}x=\begin{cases}3 \text{ for x is in } [0,1),\\ \dfrac{1}{x} \text{ for x is in } [1,10),\end{cases}\qquad \mathscr{E}x=\begin{cases}3 \text{ for x is in } [0,1),\\ \dfrac{1}{x^2} \text{ for x is in } [1,10).\end{cases} \tag{15}$$

Trivially, maps $\mathscr{B}$ and $\mathscr{D}$ (respectively, $\mathscr{C}$ and $\mathscr{E}$) are occasionally weakly $\mathscr{D}$-biased (respectively, $\mathscr{E}$-biased) of type $(\mathscr{A})$. Putting $\gamma(r_1,r_2,r_3,r_4,r_5,r_6)=-r_1+1/4\max\{r_2,r_3,r_4\}+1/5(r_5+r_6)$, we get

(1) Firstly, for x, $y\in[0,1)$, we have $\mathscr{B}x=1/2$, $\mathscr{C}y=3/4$, $\mathscr{D}x=3=\mathscr{E}y$, and

$$\begin{aligned}&\gamma(d(\mathscr{B}x,\mathscr{C}y),d(\mathscr{D}x,\mathscr{E}y),d(\mathscr{B}x,\mathscr{D}x),d(\mathscr{C}y,\mathscr{E}y),d(\mathscr{D}x,\mathscr{C}y),d(\mathscr{B}x,\mathscr{E}y))\\&\quad=\gamma\left(\frac{1}{4},0,\frac{5}{2},\frac{9}{4},\frac{9}{4},\frac{5}{2}\right)\\&\quad=-\frac{1}{4}+\frac{1}{4}\max\left\{0,\frac{5}{2},\frac{9}{4}\right\}+\frac{1}{5}\left(\frac{9}{4}+\frac{5}{2}\right)\\&\quad=\frac{53}{40}\geq 0.\end{aligned} \tag{16}$$

(2) Secondly, for x, $y\in[1,10)$, we have $\mathscr{B}x=\mathscr{C}y=1$, $\mathscr{D}x=1/x$, $\mathscr{E}y=1/y^2$, and

$$\begin{aligned}
&\gamma(d(\mathscr{B}x,\mathscr{C}y),d(\mathscr{D}x,\mathscr{E}y),d(\mathscr{B}x,\mathscr{D}x),d(\mathscr{C}y,\mathscr{E}y),d(\mathscr{D}x,\mathscr{C}y),d(\mathscr{B}x,\mathscr{E}y))\\
&=\gamma\left(0,\left|\frac{1}{x}-\frac{1}{y^2}\right|,1-\frac{1}{x},1-\frac{1}{y^2},1-\frac{1}{x},1-\frac{1}{y^2}\right)\\
&=\frac{1}{4}\max\left\{\left|\frac{1}{x}-\frac{1}{y^2}\right|,1-\frac{1}{x},1-\frac{1}{y^2}\right\}+\frac{1}{5}\left(1-\frac{1}{x}+1-\frac{1}{y^2}\right)\geq 0.
\end{aligned}\tag{17}$$

(3) Thirdly, for $x\in[0,1)$, $y\in[1,10)$, we have $\mathscr{B}x=1/2$, $\mathscr{C}y=1$, $\mathscr{D}x=3$, $\mathscr{E}y=1/y^2$, and

$$\begin{aligned}
&\gamma(d(\mathscr{B}x,\mathscr{C}y),d(\mathscr{D}x,\mathscr{E}y),d(\mathscr{B}x,\mathscr{D}x),d(\mathscr{C}y,\mathscr{E}y),d(\mathscr{D}x,\mathscr{C}y),d(\mathscr{B}x,\mathscr{E}y))\\
&=\gamma\left(\frac{1}{2},3-\frac{1}{y^2},\frac{5}{2},1-\frac{1}{y^2},2,\left|\frac{1}{2}-\frac{1}{y^2}\right|\right)\\
&=-\frac{1}{2}+\frac{1}{4}\max\left\{3-\frac{1}{y^2},\frac{5}{2},1-\frac{1}{y^2}\right\}+\frac{1}{5}\left(2+\left|\frac{1}{2}-\frac{1}{y^2}\right|\right)\geq 0.
\end{aligned}\tag{18}$$

(4) Fourthly, for $x\in[1,10)$, $y\in[0,1)$, we have $\mathscr{B}x=1$, $\mathscr{C}y=3/4$, $\mathscr{D}x=1/x$, $\mathscr{E}y=3$, and

$$\begin{aligned}
&\gamma(d(\mathscr{B}x,\mathscr{C}y),d(\mathscr{D}x,\mathscr{E}y),d(\mathscr{B}x,\mathscr{D}x),d(\mathscr{C}y,\mathscr{E}y),d(\mathscr{D}x,\mathscr{C}y),d(\mathscr{B}x,\mathscr{E}y))\\
&=\gamma\left(\frac{1}{4},3-\frac{1}{x},1-\frac{1}{x},\frac{9}{4},\left|\frac{3}{4}-\frac{1}{x}\right|,2\right)\\
&=-\frac{1}{4}+\frac{1}{4}\max\left\{3-\frac{1}{x},1-\frac{1}{x},\frac{9}{4}\right\}+\frac{1}{5}\left(\left|\frac{3}{4}-\frac{1}{x}\right|+2\right)\geq 0.
\end{aligned}\tag{19}$$

Thereby, all the theorem's conditions are fulfilled, and the four maps admit 1 as the sole common fixed point.

Remark 7. Note that Theorem 2 of [33] is inapplicable because the space is incomplete and the four maps are discontinuous. Also, we mention that Theorems 4.1 and 4.4 of [34] are not applicable because $\mathscr{E}(\mathscr{X})=(1/100,1]\cup\{3\}\nsubseteq\{1/2,1\}=B(\mathscr{X})$ and $\mathscr{D}(\mathscr{X})=(1/10,1]\cup\{3\}\nsubseteq\{3/4,1\}=\mathscr{C}(\mathscr{X})$.

3. Unique Common Fixed Points for Four Maps in Dislocated Metric Spaces

In this part, we will present a new type of implicit relations in order to use them for proving the existence and uniqueness of a common fixed point for two pairs of occasionally weakly biased maps of type $(\mathscr{A})$.

3.1. Implicit Relations. Now, let Γ be a set of functions γ: $[0,+\infty)^6\longrightarrow\mathbb{R}$ such that γ is nondecreasing in r_2, r_3, r_4, r_5, and r_6 and satisfies the next condition:

$$\gamma(r,r,2r,2r,r,r)<0,\tag{20}$$

for all $r>0$.

Example 6. $\gamma(r_1,r_2,r_3,r_4,r_5,r_6)=-r_1+\eta\max\{r_2,r_3,r_4,r_5,r_6\}$, where $\eta\in(0,1/2)$.

(i) It is trivial that γ is nondecreasing in r_2, r_3, r_4, r_5, and r_6.

(ii) $\gamma(r, r, 2r, 2r, r, r) = -r + \eta \max\{r, 2r, 2r, r, r\} = r(2\eta - 1) < 0\ \forall r > 0$.

Example 7. $\gamma(r_1, r_2, r_3, r_4, r_5, r_6) = -r_1 + \theta[r_2 + r_3 + r_4 + r_5 + r_6]$, where $\theta \in (0, 1/7)$.

(i) It is evident to see that γ is nondecreasing in variables r_2, r_3, r_4, r_5, and r_6.

(ii) $\gamma(r, r, 2r, 2r, r, r) = -r + \theta[r + 2r + 2r + r + r] = r(7\theta - 1) < 0\ \forall r > 0$.

Example 8. $\gamma(r_1, r_2, r_3, r_4, r_5, r_6) = -r_1 + \alpha r_2 + \beta r_3 + \delta r_4 + \lambda r_5 + \omega r_6$, where α, β, δ, λ, $\omega > 0$ and $\alpha + 2(\beta + \delta) + \lambda + \omega < 1$.

(i) Clearly, γ is nondecreasing in variables r_2, r_3, r_4, r_5, and r_6.

(ii) $\gamma(r, r, 2r, 2r, r, r) = -r + \alpha r + 2\beta r + 2\delta r + \lambda r + \omega r = r[\alpha + 2\beta + 2\delta + \lambda + \omega - 1] < 0\ \forall r > 0$.

Theorem 8. *Let $\mathscr{B}$, $\mathscr{C}$, $\mathscr{D}$, and $\mathscr{E}$ be four maps from a dislocated metric space $(\mathscr{X}, d)$ into itself satisfying*

$$\gamma(d(\mathscr{B}x, \mathscr{C}y), d(\mathscr{D}x, \mathscr{E}y), d(\mathscr{B}x, \mathscr{D}x), d(\mathscr{C}y, \mathscr{E}y), d(\mathscr{D}x, \mathscr{C}y), d(\mathscr{B}x, \mathscr{E}y)) \geq 0, \tag{21}$$

for all $x, y \in \mathscr{X}$, where $\gamma \in \Gamma$. Suppose that maps $\mathscr{B}$ and $\mathscr{D}$ (respectively, $\mathscr{C}$ and $\mathscr{E}$) are occasionally weakly $\mathscr{D}$-biased (respectively, $\mathscr{E}$-biased) of type $(\mathscr{A})$; then, maps $\mathscr{B}$, $\mathscr{C}$, $\mathscr{D}$, and $\mathscr{E}$ possess only one common fixed point.

Proof. As in the demonstration of the first theorem, since maps $\mathscr{B}$ and $\mathscr{D}$ as well as $\mathscr{C}$ and $\mathscr{E}$ are occasionally weakly $\mathscr{D}$-biased (respectively, $\mathscr{E}$-biased) of type $(\mathscr{A})$, there are two points ρ and μ in $\mathscr{X}$ such that $\mathscr{B}\rho = \mathscr{D}\rho$ implies $d(\mathscr{D}\mathscr{D}\rho, \mathscr{B}\rho) \leq d(\mathscr{B}\mathscr{D}\rho, \mathscr{D}\rho)$ and $\mathscr{C}\mu = \mathscr{E}\mu$ implies $d(\mathscr{E}\mathscr{E}\mu, \mathscr{C}\mu) \leq d(\mathscr{C}\mathscr{E}\mu, \mathscr{E}\mu)$. We need four steps to prove the existence and uniqueness of the common fixed point, as follows.

First step: We claim that $\mathscr{B}\rho = \mathscr{C}\mu$. Suppose that we have the contrary; using inequality (21), we get

$$\begin{aligned}&\gamma(d(\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{D}\rho, \mathscr{E}\mu), d(\mathscr{B}\rho, \mathscr{D}\rho), d(\mathscr{C}\mu, \mathscr{E}\mu), d(\mathscr{D}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{E}\mu))\\ &\quad = \gamma(d(\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{B}\rho), d(\mathscr{C}\mu, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{C}\mu)) \geq 0.\end{aligned} \tag{22}$$

Since γ is nondecreasing in the third and fourth variables, we get

$$\begin{aligned}0 &\leq \gamma(d(\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{B}\rho), d(\mathscr{C}\mu, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{C}\mu))\\ &\leq \gamma(d(\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{C}\mu), 2d(\mathscr{B}\rho, \mathscr{C}\mu), 2d(\mathscr{C}\mu, \mathscr{B}\rho), d(\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{C}\mu)),\end{aligned} \tag{23}$$

a contradiction, which implies that $d(\mathscr{B}\rho, \mathscr{C}\mu) = 0 \Rightarrow \mathscr{B}\rho = \mathscr{C}\mu$.

Second step: If $d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho) > 0$, the use of condition (21) gives

$$\begin{aligned}&\gamma(d(\mathscr{B}\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{D}\mathscr{B}\rho, \mathscr{E}\mu), d(\mathscr{B}\mathscr{B}\rho, \mathscr{D}\mathscr{B}\rho), d(\mathscr{C}\mu, \mathscr{E}\mu), d(\mathscr{D}\mathscr{B}\rho, \mathscr{C}\mu),\\ &d(\mathscr{B}\mathscr{B}\rho, \mathscr{E}\mu))\\ &\quad = \gamma(d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho), d(\mathscr{D}\mathscr{D}\rho, \mathscr{B}\rho), d(\mathscr{B}\mathscr{B}\rho, \mathscr{D}\mathscr{D}\rho), d(\mathscr{B}\rho, \mathscr{B}\rho), d(\mathscr{D}\mathscr{D}\rho, \mathscr{B}\rho),\\ &d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho)) \geq 0.\end{aligned} \tag{24}$$

Using the properties of γ, we get

$$\begin{aligned}&\gamma(d(\mathscr{B}\mathscr{B}\rho,\mathscr{B}\rho),d(\mathscr{D}\mathscr{D}\rho,\mathscr{B}\rho),d(\mathscr{B}\mathscr{B}\rho,\mathscr{B}\rho)+d(\mathscr{B}\rho,\mathscr{D}\mathscr{D}\rho),2d(\mathscr{B}\rho,\mathscr{B}\mathscr{B}\rho),\\&d(\mathscr{D}\mathscr{D}\rho,\mathscr{B}\rho),d(\mathscr{B}\mathscr{B}\rho,\mathscr{B}\rho))\geq 0.\end{aligned}\tag{25}$$

Again by the nondecreasing assumption of γ and using the relationship between maps $\mathscr{B}$ and $\mathscr{D}$, we find

$$\begin{aligned}&\gamma(d(\mathscr{B}\mathscr{B}\rho,\mathscr{B}\rho),d(\mathscr{B}\mathscr{D}\rho,\mathscr{D}\rho),d(\mathscr{B}\mathscr{B}\rho,\mathscr{B}\rho)+d(\mathscr{B}\mathscr{D}\rho,\mathscr{D}\rho),2d(\mathscr{B}\rho,\mathscr{B}\mathscr{B}\rho),\\&d(\mathscr{B}\mathscr{D}\rho,\mathscr{D}\rho),d(\mathscr{B}\mathscr{B}\rho,\mathscr{B}\rho))\geq 0,\end{aligned}\tag{26}$$

i.e.,

$$\begin{aligned}&\gamma(d(\mathscr{B}\mathscr{B}\rho,\mathscr{B}\rho),d(\mathscr{B}\mathscr{B}\rho,\mathscr{B}\rho),2d(\mathscr{B}\mathscr{B}\rho,\mathscr{B}\rho),2d(\mathscr{B}\rho,\mathscr{B}\mathscr{B}\rho),d(\mathscr{B}\mathscr{B}\rho,\mathscr{B}\rho),\\&d(\mathscr{B}\mathscr{B}\rho,\mathscr{B}\rho))\geq 0,\end{aligned}\tag{27}$$

which is a contradiction, and hence $d(\mathscr{B}\mathscr{B}\rho,\mathscr{B}\rho)=0\Rightarrow\mathscr{B}\mathscr{B}\rho=\mathscr{B}\rho$; consequently, $d(\mathscr{D}\mathscr{B}\rho,\mathscr{B}\rho)=0\Rightarrow\mathscr{D}\mathscr{B}\rho=\mathscr{B}\rho$.

Third step: Now, assume that $d(\mathscr{C}\mathscr{C}\mu,\mathscr{C}\mu)$ is positive; then,

$$\begin{aligned}&\gamma(d(\mathscr{B}\rho,\mathscr{C}\mathscr{C}\mu),d(\mathscr{D}\rho,\mathscr{E}\mathscr{C}\mu),d(\mathscr{B}\rho,\mathscr{D}\rho),d(\mathscr{C}\mathscr{C}\mu,\mathscr{E}\mathscr{C}\mu),d(\mathscr{D}\rho,\mathscr{C}\mathscr{C}\mu),\\&d(\mathscr{B}\rho,\mathscr{E}\mathscr{C}\mu))\\&\quad=\gamma(d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),d(\mathscr{C}\mu,\mathscr{E}\mathscr{C}\mu),d(\mathscr{C}\mu,\mathscr{C}\mu),d(\mathscr{C}\mathscr{C}\mu,\mathscr{E}\mathscr{C}\mu),d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),\\&d(\mathscr{C}\mu,\mathscr{E}\mathscr{C}\mu))\geq 0.\end{aligned}\tag{28}$$

Using our hypotheses, we get

$$\begin{aligned}&\gamma(d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),d(\mathscr{C}\mu,\mathscr{E}\mathscr{E}\mu),2d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),d(\mathscr{C}\mathscr{C}\mu,\mathscr{C}\mu)+d(\mathscr{C}\mu,\mathscr{E}\mathscr{E}\mu),\\&d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),d(\mathscr{C}\mu,\mathscr{E}\mathscr{E}\mu))\geq 0.\end{aligned}\tag{29}$$

Then, we have

$$\begin{aligned}&\gamma(d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),d(\mathscr{E}\mu,\mathscr{C}\mathscr{E}\mu),2d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),d(\mathscr{C}\mathscr{C}\mu,\mathscr{C}\mu)+d(\mathscr{E}\mu,\mathscr{C}\mathscr{E}\mu),\\&d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),d(\mathscr{E}\mu,\mathscr{C}\mathscr{E}\mu))\\&=\gamma(d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),2d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),2d(\mathscr{C}\mathscr{C}\mu,\mathscr{C}\mu),d(\mathscr{C}\mu,\mathscr{C}\mathscr{C}\mu),\\&d(\mathscr{E}\mu,\mathscr{C}\mathscr{E}\mu))\geq 0,\end{aligned}\tag{30}$$

a contradiction, which implies that $d(\mathscr{C}\mathscr{C}\mu,\mathscr{C}\mu)=0\Rightarrow\mathscr{C}\mathscr{C}\mu=\mathscr{C}\mu$; consequently, $d(\mathscr{E}\mathscr{C}\mu,\mathscr{C}\mu)=0$ which implies that $\mathscr{E}\mathscr{C}\mathscr{B}\mu=\mathscr{C}\mu$.

Fourth step: Put $\mathscr{B}\rho=\mathscr{C}\mu=\zeta$ and assume the existence of another common fixed point (say ξ); utilizing (21), we obtain

$$\begin{aligned}&\gamma(d(\mathscr{B}\zeta,\mathscr{C}\xi),d(\mathscr{D}\zeta,\mathscr{E}\xi),d(\mathscr{B}\zeta,\mathscr{D}\zeta),d(\mathscr{C}\xi,\mathscr{E}\xi),d(\mathscr{D}\zeta,\mathscr{C}\xi),d(\mathscr{B}\zeta,\mathscr{E}\xi))\\&=\gamma(d(\zeta,\xi),d(\zeta,\xi),d(\zeta,\zeta),d(\xi,\xi),d(\zeta,\xi),d(\zeta,\xi))\geq 0.\end{aligned}\tag{31}$$

As γ is nondecreasing in the third and the fourth variables, we get

$$\gamma(d(\zeta,\xi),d(\zeta,\xi),2d(\zeta,\xi),2d(\xi,\zeta),d(\zeta,\xi),d(\zeta,\xi))\geq 0,\tag{32}$$

which is a contradiction; hence, $\xi=\zeta$, and this completes the proof. □

To support our result, we furnish the next example.

Example 9. Endow $\mathscr{X}=(-10,+\infty)$ with the dislocated metric $d(x,y)=\max\{|x|,|y|\}$ and establish the following maps:

$$\begin{aligned}&\mathscr{B}x=\begin{cases}x \text{ for x is in } (-10,0],\\ \dfrac{1}{9} \text{ for x is in } (0,+\infty),\end{cases}\quad \mathscr{C}x=\begin{cases}x \text{ for x is in } (-10,0],\\ \dfrac{1}{8} \text{ for x is in } (0,+\infty),\end{cases}\\&\mathscr{D}x=\begin{cases}-9x \text{ for x is in } (-10,0],\\ x+9 \text{ for x is in } (0,+\infty),\end{cases}\quad \mathscr{E}x=\begin{cases}-8x \text{ for x is in } (-10,0],\\ x+8 \text{ for x is in } (0,+\infty).\end{cases}\end{aligned}\tag{33}$$

First of all, the occasionally weakly biased of type $(\mathscr{A})$ assumption is satisfied. Define $\gamma(r_1,r_2,r_3,r_4,r_5,r_6)=-r_1+1/8(r_2+r_3+r_4+r_5+r_6)$, and we get

(1) For $x,\ y\in(-10,0]$, we have $\mathscr{B}x=x$, $\mathscr{C}y=y$, $\mathscr{D}x=-9x$, $\mathscr{E}y=-8y$, and

$$\begin{aligned}&\gamma(d(\mathscr{B}x,\mathscr{C}y),d(\mathscr{D}x,\mathscr{E}y),d(\mathscr{B}x,\mathscr{D}x),d(\mathscr{C}y,\mathscr{E}y),d(\mathscr{D}x,\mathscr{C}y),d(\mathscr{B}x,\mathscr{E}y))\\&=\gamma(\max\{|x|,|y|\},\ \max\{|-9x|,|-8y|\},\ \max\{|x|,|-9x|\},\\&\max\{|y|,|-8y|\},\ \max\{|-9x|,|y|\},\ \max\{|x|,|-8y|\})\\&=-\max\{|x|,|y|\}+\frac{1}{8}(\max\{|-9x|,|-8y|\}+\max\{|x|,|-9x|\}\\&+\max\{|y|,|-8y|\}+\max\{|-9x|,|y|\}+\max\{|x|,|-8y|\})\geq 0.\end{aligned}\tag{34}$$

(2) For $x,\ y\in(0,+\infty)$, we have $\mathscr{B}x=1/9$, $\mathscr{C}y=1/8$, $\mathscr{D}x=x+9$, $\mathscr{E}y=y+8$, and

$$\begin{aligned}
&\gamma(d(\mathscr{B}x,\mathscr{C}y),d(\mathscr{D}x,\mathscr{E}y),d(\mathscr{B}x,\mathscr{D}x),d(\mathscr{C}y,\mathscr{E}y),d(\mathscr{D}x,\mathscr{C}y),d(\mathscr{B}x,\mathscr{E}y))\\
&\quad=\gamma\Big(\max\left\{\frac{1}{8},\frac{1}{9}\right\},\ \max\{|x+9|,|y+8|\},\ \max\left\{\frac{1}{9},|x+9|\right\},\\
&\max\left\{\frac{1}{8},|y+8|\right\},\ \max\left\{|x+9|,\frac{1}{8}\right\},\ \max\left\{\frac{1}{9},|y+8|\right\}\Big)\\
&\quad=-\frac{1}{8}+\frac{1}{8}\Big(\max\{|x+9|,|y+8|\}+\max\left\{\frac{1}{9},|x+9|\right\}+\max\left\{\frac{1}{8},|y+8|\right\}\\
&\qquad+\max\left\{|x+9|,\frac{1}{8}\right\}+\max\left\{\frac{1}{9},|y+8|\right\}\Big)\geq 0.
\end{aligned}\tag{35}$$

(3) For $x\in(-10,0]$, $y\in(0,+\infty)$, we have $\mathscr{B}x=x$, $\mathscr{C}y=1/8$, $\mathscr{D}x=-9x$, $\mathscr{E}y=y+8$, and

$$\begin{aligned}
&\gamma(d(\mathscr{B}x,\mathscr{C}y),d(\mathscr{D}x,\mathscr{E}y),d(\mathscr{B}x,\mathscr{D}x),d(\mathscr{C}y,\mathscr{E}y),d(\mathscr{D}x,\mathscr{C}y),d(\mathscr{B}x,\mathscr{E}y))\\
&\quad=\gamma\Big(\max\left\{|x|,\frac{1}{8}\right\},\ \max\{|-9x|,|y+8|\},\ \max\{|x|,|-9x|\},\\
&\max\left\{\frac{1}{8},|y+8|\right\},\ \max\left\{|-9x|,\frac{1}{8}\right\},\ \max\{|x|,|y+8|\}\Big)\\
&\quad=-\max\left\{|x|,\frac{1}{8}\right\}+\frac{1}{8}\Big(\max\{|-9x|,|y+8|\}+\max\{|x|,|-9x|\}\\
&\qquad+\max\left\{\frac{1}{8},|y+8|\right\}+\max\left\{|-9x|,\frac{1}{8}\right\}+\max\{|x|,|y+8|\}\Big)\geq 0.
\end{aligned}\tag{36}$$

(4) Finally, for $x\in(0,+\infty)$, $y\in(-10,0]$, we have $\mathscr{B}x=1/9$, $\mathscr{C}y=y$, $\mathscr{D}x=x+9$, $\mathscr{E}y=-8y$, and

$$\begin{aligned}
&\gamma(d(\mathscr{B}x,\mathscr{C}y),d(\mathscr{D}x,\mathscr{E}y),d(\mathscr{B}x,\mathscr{D}x),d(\mathscr{C}y,\mathscr{E}y),d(\mathscr{D}x,\mathscr{C}y),d(\mathscr{B}x,\mathscr{E}y))\\
&\quad=\gamma\Big(\max\left\{\frac{1}{9},|y|\right\},\ \max\{|x+9|,|-8y|\},\ \max\left\{\frac{1}{9},|x+9|\right\},\\
&\max\{|y|,|-8y|\},\ \max\{|x+9|,|y|\},\ \max\left\{\frac{1}{9},|-8y|\right\}\Big)\\
&\quad=-\max\left\{\frac{1}{9},|y|\right\}+\frac{1}{8}\Big(\max\{|x+9|,|-8y|\}+\max\left\{\frac{1}{9},|x+9|\right\}\\
&\qquad+\max\{|y|,|-8y|\}+\max\{|x+9|,|y|\}+\max\left\{\frac{1}{9},|-8y|\right\}\Big)\geq 0.
\end{aligned}\tag{37}$$

Thereby, all hypotheses of the theorem are satisfied; the four maps accept 0 as the only common fixed point.

Remark 9. Mention that $\mathscr{E}(\mathscr{X})=[0,+\infty)\nsubseteq(-10,0]\cup\{1/9\}=B(\mathscr{X})$ and $\mathscr{D}(\mathscr{X})=[0,+\infty)\nsubseteq(-10,0]\cup\{1/8\}=\mathscr{C}(\mathscr{X})$.

Using Theorem 8 and Example 6, we gain the next result.

Corollary 10. *Suppose that $\mathscr{B}$, $\mathscr{C}$, $\mathscr{D}$, and $\mathscr{E}$ are four maps from a dislocated metric space $(\mathscr{X}, d)$ into itself such that for all $x, y \in \mathscr{X}$, the following condition holds:*

$$d(\mathscr{B}x, \mathscr{C}y) \leq \eta \max\{d(\mathscr{D}x, \mathscr{E}y), d(\mathscr{B}x, \mathscr{D}x), d(\mathscr{C}y, \mathscr{E}y), d(\mathscr{D}x, \mathscr{C}y), d(\mathscr{B}x, \mathscr{E}y)\}, \quad (38)$$

where $\eta \in (0, 1/2)$. If $\mathscr{B}$ and $\mathscr{D}$, $\mathscr{C}$ and $\mathscr{E}$ are occasionally weakly $\mathscr{D}$-biased (respectively, $\mathscr{E}$-biased) of type $(\mathscr{A})$, then, there exists only one point (say θ) which verifies $\mathscr{B}\theta = \mathscr{C}\theta = \mathscr{D}\theta = \mathscr{E}\theta = \theta$.

Proof. Indeed, by assumptions, we have the existence of two elements ρ and μ in $\mathscr{X}$ which verify

$$\begin{aligned} &\mathscr{B}\rho = \mathscr{D}\rho \text{ implies } d(\mathscr{D}\mathscr{D}\rho, \mathscr{B}\rho) \leq d(\mathscr{B}\mathscr{D}\rho, \mathscr{D}\rho), \\ &\mathscr{C}\mu = \mathscr{E}\mu \text{ implies } d(\mathscr{E}\mathscr{E}\mu, \mathscr{C}\mu) \leq d(\mathscr{C}\mathscr{E}\mu, \mathscr{E}\mu). \end{aligned} \quad (39)$$

The proof needs four cases.

Case one: assume that $\mathscr{B}\rho \neq \mathscr{C}\mu$; the use of inequality (38) gives

$$\begin{aligned} d(\mathscr{B}\rho, \mathscr{C}\mu) &\leq \eta \max\{d(\mathscr{D}\rho, \mathscr{E}\mu), d(\mathscr{B}\rho, \mathscr{D}\rho), d(\mathscr{C}\mu, \mathscr{E}\mu), d(\mathscr{D}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{E}\mu)\} \\ &= \eta \max\{d(\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{B}\rho), d(\mathscr{C}\mu, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{C}\mu)\} \\ &\leq \eta \max\{d(\mathscr{B}\rho, \mathscr{C}\mu), 2d(\mathscr{B}\rho, \mathscr{C}\mu), 2d(\mathscr{C}\mu, \mathscr{B}\rho), d(\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{C}\mu)\} \\ &= 2\eta d(\mathscr{B}\rho, \mathscr{C}\mu) \\ &< d(\mathscr{B}\rho, \mathscr{C}\mu), \end{aligned} \quad (40)$$

a contradiction, and hence $\mathscr{B}\rho = \mathscr{C}\mu$.

Case two: suppose that $\mathscr{B}\mathscr{B}\rho \neq \mathscr{B}\rho$; using condition (38), we get

$$\begin{aligned} d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho) = d(\mathscr{B}\mathscr{B}\rho, \mathscr{C}\mu) &\leq \eta \max\{d(\mathscr{D}\mathscr{B}\rho, \mathscr{E}\mu), d(\mathscr{B}\mathscr{B}\rho, \mathscr{D}\mathscr{B}\rho), d(\mathscr{C}\mu, \mathscr{E}\mu), d(\mathscr{D}\mathscr{B}\rho, \mathscr{C}\mu), d(\mathscr{B}\mathscr{B}\rho, \mathscr{E}\mu)\} \\ &= \eta \max\{d(\mathscr{D}\mathscr{D}\rho, \mathscr{B}\rho), d(\mathscr{B}\mathscr{B}\rho, \mathscr{D}\mathscr{D}\rho), d(\mathscr{B}\rho, \mathscr{B}\rho), d(\mathscr{D}\mathscr{D}\rho, \mathscr{B}\rho), d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho)\} \\ &\leq \eta \max\{d(\mathscr{D}\mathscr{D}\rho, \mathscr{B}\rho), d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho) + d(\mathscr{B}\rho, \mathscr{D}\mathscr{D}\rho), 2d(\mathscr{B}\rho, \mathscr{B}\mathscr{B}\rho), d(\mathscr{D}\mathscr{D}\rho, \mathscr{B}\rho), d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho)\} \\ &\leq \eta \max\{d(\mathscr{B}\mathscr{D}\rho, \mathscr{D}\rho), d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho) + d(\mathscr{B}\mathscr{D}\rho, \mathscr{D}\rho), 2d(\mathscr{B}\rho, \mathscr{B}\mathscr{B}\rho), d(\mathscr{B}\mathscr{D}\rho, \mathscr{D}\rho), d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho)\} \\ &= \eta \max\{d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho), 2d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho), 2d(\mathscr{B}\rho, \mathscr{B}\mathscr{B}\rho), d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho), d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho)\} \\ &= 2\eta d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho) \\ &< d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho), \end{aligned} \quad (41)$$

which is a contradiction, and hence $d(\mathscr{B}\mathscr{B}\rho, \mathscr{B}\rho) = 0 \Rightarrow \mathscr{B}\mathscr{B}\rho = \mathscr{B}\rho$; consequently, $d(\mathscr{D}\mathscr{B}\rho, \mathscr{B}\rho) = 0 \Rightarrow \mathscr{D}\mathscr{B}\rho = \mathscr{B}\rho$.

Case three: now, if $d(\mathscr{C}\mathscr{C}\mu, \mathscr{C}\mu)$ is positive, then

$$\begin{aligned} d(\mathscr{C}\mu, \mathscr{C}\mathscr{C}\mu) &= d(\mathscr{B}\rho, \mathscr{C}\mathscr{C}\mu) \leq \eta \max\{d(\mathscr{D}\rho, \mathscr{E}\mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{D}\rho), d(\mathscr{C}\mathscr{C}\mu, \mathscr{E}\mathscr{C}\mu), d(\mathscr{D}\rho, \mathscr{C}\mathscr{C}\mu), d(\mathscr{B}\rho, \mathscr{E}\mathscr{C}\mu)\} \\ &= \eta \max\{d(\mathscr{C}\mu, \mathscr{E}\mathscr{C}\mu), d(\mathscr{C}\mu, \mathscr{C}\mu), d(\mathscr{C}\mathscr{C}\mu, \mathscr{E}\mathscr{C}\mu), d(\mathscr{C}\mu, \mathscr{C}\mathscr{C}\mu), d(\mathscr{C}\mu, \mathscr{E}\mathscr{C}\mu)\} \\ &\leq \eta \max\{d(\mathscr{C}\mu, \mathscr{E}\mathscr{E}\mu), 2d(\mathscr{C}\mu, \mathscr{C}\mathscr{C}\mu), d(\mathscr{C}\mathscr{C}\mu, \mathscr{C}\mu) + d(\mathscr{C}\mu, \mathscr{E}\mathscr{E}\mu), d(\mathscr{C}\mu, \mathscr{C}\mathscr{C}\mu), d(\mathscr{C}\mu, \mathscr{E}\mathscr{E}\mu)\} \\ &\leq \eta \max\{d(\mathscr{E}\mu, \mathscr{C}\mathscr{E}\mu), 2d(\mathscr{C}\mu, \mathscr{C}\mathscr{C}\mu), d(\mathscr{C}\mathscr{C}\mu, \mathscr{C}\mu) + d(\mathscr{E}\mu, \mathscr{C}\mathscr{E}\mu), d(\mathscr{C}\mu, \mathscr{C}\mathscr{C}\mu), d(\mathscr{E}\mu, \mathscr{C}\mathscr{E}\mu)\} \\ &= \eta \max\{d(\mathscr{C}\mu, \mathscr{C}\mathscr{C}\mu), 2d(\mathscr{C}\mu, \mathscr{C}\mathscr{C}\mu), 2d(\mathscr{C}\mathscr{C}\mu, \mathscr{C}\mu), d(\mathscr{C}\mu, \mathscr{C}\mathscr{C}\mu), d(\mathscr{E}\mu, \mathscr{C}\mathscr{E}\mu)\} \\ &= 2\eta d(\mathscr{C}\mu, \mathscr{C}\mathscr{C}\mu) \\ &< d(\mathscr{C}\mu, \mathscr{C}\mathscr{C}\mu), \end{aligned} \tag{42}$$

a contradiction, which implies that $d(\mathscr{C}\mathscr{C}\mu, \mathscr{C}\mu) = 0 \Rightarrow \mathscr{C}\mathscr{C}\mu = \mathscr{C}\mu$; consequently, $d(\mathscr{E}\mathscr{C}\mu, \mathscr{C}\mu) = 0$ which implies that $\mathscr{E}\mathscr{C}\mu = \mathscr{C}\mu$.

Case four: put $\mathscr{B}\rho = \mathscr{C}\mu = \theta$ and assume the existence of another element (say ϑ) which satisfies $\mathscr{B}\vartheta = \mathscr{C}\vartheta = \mathscr{D}\vartheta = \mathscr{E}\vartheta = \vartheta$; by inequality (38), we obtain

$$\begin{aligned} d(\theta, \vartheta) &= d(\mathscr{B}\theta, \mathscr{C}\vartheta) \leq \eta \max\{d(\mathscr{D}\theta, \mathscr{E}\vartheta), d(\mathscr{B}\theta, \mathscr{D}\theta), d(\mathscr{C}\vartheta, \mathscr{E}\vartheta), d(\mathscr{D}\theta, \mathscr{C}\vartheta), d(\mathscr{B}\theta, \mathscr{E}\vartheta)\} \\ &= \eta \max\{d(\theta, \vartheta), d(\theta, \vartheta), d(\theta, \theta), d(\vartheta, \vartheta), d(\theta, \vartheta), d(\theta, \vartheta)\} \\ &\leq \eta \max\{d(\theta, \vartheta), d(\theta, \vartheta), 2d(\theta, \vartheta), 2d(\vartheta, \theta), d(\theta, \vartheta), d(\theta, \vartheta)\} \\ &= 2\eta d(\theta, \vartheta) \\ &< d(\theta, \vartheta), \end{aligned} \tag{43}$$

a contradiction; hence, $\vartheta = \theta$, and the uniqueness of the common fixed point is satisfied; this achieves the proof. □

3.2. Unique Common Fixed Points for a Sequence of Maps

Theorem 11. *Suppose that $\mathscr{D}$, $\mathscr{E}$, and $\{\mathscr{B}_n\}_{n=1,2,\ldots}$ are maps from a metric space $(\mathscr{X}, d)$ into itself such that for all x, $y \in \mathscr{X}$, the next condition holds:*

$$\begin{aligned} &\gamma(d(\mathscr{B}_n x, \mathscr{B}_{n+1} y), d(\mathscr{D}x, \mathscr{E}y), d(\mathscr{B}_n x, \mathscr{D}x), d(\mathscr{B}_{n+1} y, \mathscr{E}y), d(\mathscr{D}x, \mathscr{B}_{n+1} y), \\ &d(\mathscr{B}_n x, \mathscr{E}y)) \geq 0, \end{aligned} \tag{44}$$

where $\gamma \in \Gamma$. In addition, assume that maps $\mathscr{B}_n$ and $\mathscr{D}$ (respectively, $\mathscr{B}_{n+1}$ and $\mathscr{E}$) are occasionally weakly $\mathscr{D}$-biased (respectively, $\mathscr{E}$-biased) of type $(\mathscr{A})$; then, there exists one element z which satisfies $\mathscr{D}z = \mathscr{E}z = \mathscr{B}_n z = z$ for $n = 1, 2, \ldots$.

Theorem 12. *Let $\mathscr{D}$, $\mathscr{E}$, and $\{\mathscr{B}_n\}_{n=1,2,\ldots}$ be maps from a dislocated metric space $(\mathscr{X}, d)$ into itself such that for all x, $y \in \mathscr{X}$, the following inequality holds:*

$$\begin{aligned} &\gamma(d(\mathscr{B}_n x, \mathscr{B}_{n+1} y), d(\mathscr{D}x, \mathscr{E}y), d(\mathscr{B}_n x, \mathscr{D}x), d(\mathscr{B}_{n+1} y, \mathscr{E}y), d(\mathscr{D}x, \mathscr{B}_{n+1} y), \\ &d(\mathscr{B}_n x, \mathscr{E}y)) \geq 0, \end{aligned} \tag{45}$$

where γ: $[0, +\infty]^6 \longrightarrow \mathbb{R}$ is a function, nondecreasing in variables r_2, r_3, r_4, r_5, and r_6, and satisfies $\gamma(r, r, 2r, 2r, r, r) < 0$ for all $r > 0$. If $\mathscr{B}_n$ and $\mathscr{D}$ (respectively, $\mathscr{B}_{n+1}$ and $\mathscr{E}$) are occasionally weakly $\mathscr{D}$-biased (respectively, $\mathscr{E}$-biased) of type $(\mathscr{A})$, then there is only one element z which verifies $\mathscr{D}z = \mathscr{E}z = \mathscr{B}_n z = z$ for $n \in \mathbb{N}$.

4. Application to an Integral Equation

Consider the next integral equation:

$$u(z) = f_n(u(z)) + \int_\alpha^z y(z,s)j_n(s,u(s))ds + \int_\alpha^\beta x(z,s)l_n(s,u(s))ds, \tag{46}$$

for all $z \in [\alpha,\beta]$, where

(1) f_n: $[\alpha,\beta] \longrightarrow \mathbb{R}$, $n = 1,2$, are continuous.

(2) $y(z,s)$, $x(z,s)$: $[\alpha,\beta] \times [\alpha,\beta] \longrightarrow \mathbb{R}_+$ are continuous functions.

(3) j_n, l_n: $[\alpha,\beta] \times \mathbb{R} \longrightarrow [0,+\infty]$, $n = 1,2$, are continuous functions.

Let $\mathscr{X} = C[\alpha,\beta]$ be the set of real continuous functions on $[\alpha,\beta]$, endowed with the dislocated metric

$$\begin{aligned} d(u,v) &= \|u\|_\infty + \|v\|_\infty \\ &= \max_{z\in[\alpha,\beta]} u(z) + \max_{z\in[\alpha,\beta]} v(z), \end{aligned} \tag{47}$$

for all u, $v \in \mathscr{X}$. It is evident that $(\mathscr{X},d)$ is a dislocated metric space.

Theorem 13. *Integral equation (46) has only one solution in $\mathscr{X}$ for $\varrho\xi_2 < 1$ and $\chi + \rho\xi_1/1 - \varrho\xi_2 = \eta < 1/2$ if the next assumptions hold:*

(1) $\int_\alpha^\beta \max_{z\in[\alpha,\beta]} |y(z,s)|ds = \xi_1 < +\infty$.

(2) $\int_\alpha^\beta \max_{z\in[\alpha,\beta]} |x(z,s)|ds = \xi_2 < +\infty$.

(3) The functions commute at their each coincidence point.

(4) There is $0 < \rho < 1$ such that for all $s \in [\alpha,\beta]$ and $u \in \mathscr{X}$, $|j_n(s,u(s))| \le \rho|u(s)|$ for $n = 1,2$.

(5) There is $0 < \varrho < 1$ such that for all $s \in [\alpha,\beta]$ and $u \in \mathscr{X}$, $|l_n(s,u(s))| \le \varrho|u(s)|$ for $n = 1,2$.

(6) There is $0 < \chi < 1$ such that for all $s \in [\alpha,\beta]$, $|f_n(s)| \le \chi|s|$.

Proof. Define $\mathscr{P}$, $\mathscr{Q}$, $\mathscr{C}$, $\mathscr{D}$, $\mathscr{R}$, $\mathscr{S}$: $\mathscr{X} \longrightarrow \mathscr{X}$ by

$$\begin{aligned} \mathscr{P}u(z) &= f_1(u(z)) + \int_\alpha^z y(z,s)j_1(s,u(s))ds, \\ \mathscr{Q}u(z) &= f_2(u(z)) + \int_\alpha^z y(z,s)j_2(s,u(s))ds, \\ \mathscr{M}u(z) &= \int_\alpha^\beta x(z,s)l_1(s,u(s))ds, \\ \mathscr{N}u(z) &= \int_\alpha^\beta x(z,s)l_2(s,u(s))ds, \\ \mathscr{R}u(z) &= (\mathscr{I} - \mathscr{M})u(z), \\ \mathscr{S}u(z) &= (\mathscr{I} - \mathscr{N})u(z), \end{aligned} \tag{48}$$

where $\mathscr{I}$ is the identity function on $\mathscr{X}$.

By the virtue of condition 3, we can see that $\mathscr{P}$ and $\mathscr{R}$ as well as $\mathscr{Q}$ and $\mathscr{S}$ are occasionally weakly $\mathscr{R}$-biased (respectively, $\mathscr{S}$-biased) of type $(\mathscr{A})$.

Now, we prove that condition (38) of Corollary 10 is satisfied.

$$\begin{aligned} |\mathscr{P}u(z)| &= \left| f_1(u(z)) + \int_\alpha^z y(z,s)j_1(s,u(s))ds \right| \\ &\le |f_1(u(z))| + \left| \int_\alpha^z y(z,s)j_1(s,u(s))ds \right| \\ &\le |f_1(u(z))| + \int_\alpha^z |y(z,s)||j_1(s,u(s))|ds \\ &\le |f_1(u(z))| + \rho\int_\alpha^z |y(z,s)||u(s)|ds \\ &\le \chi \max_{z\in[\alpha,\beta]} |u(z)| + \rho\int_\alpha^\beta |y(z,s)| \max_{s\in[\alpha,\beta]} |u(s)|ds \\ &\le \chi\|u\|_\infty + \rho\|u\|_\infty \int_\alpha^\beta \max_{z\in[\alpha,\beta]} |y(z,s)|ds, \end{aligned} \tag{49}$$

which implies that

$$\|\mathscr{P}u\|_\infty \le (\chi + \rho\xi_1)\|u\|_\infty. \tag{50}$$

It follows that for all u, $v \in \mathscr{X}$,

$$d(\mathscr{P}u, \mathscr{Q}v) \le (\chi + \rho\xi_1)d(u,v). \tag{51}$$

Similarly, we have

$$\begin{aligned} |\mathscr{M}u(z)| &= \left| \int_\alpha^\beta x(z,s)l_1(s,u(s))ds \right| \\ &\le \int_\alpha^\beta |x(z,s)||l_1 s,u(s)|ds \\ &\le \varrho\int_\alpha^\beta |x(z,s)||u(s)|ds \\ &\le \varrho\int_\alpha^\beta |x(z,s) \max_{s\in[\alpha,\beta]} |u(s)|ds \\ &\le \varrho\|u\|_\infty \int_\alpha^\beta \max_{z\in[\alpha,\beta]} |x(z,s)|ds, \end{aligned} \tag{52}$$

which implies that

$$\|\mathscr{M}u\|_\infty \le \varrho\xi_2\|u\|_\infty. \tag{53}$$

It follows that for all u, $v \in \mathscr{X}$,

$$d(\mathscr{M}u, \mathscr{N}v) \le \varrho\xi_2 d(u,v). \tag{54}$$

Hence, we have

$$\begin{aligned} d(\mathscr{R}u, \mathscr{S}v) &= \|\mathscr{R}u\|_\infty + \|\mathscr{S}v\|_\infty \\ &= \max_{z\in[\alpha,\beta]} \mathscr{R}u(z) + \max_{z\in[\alpha,\beta]} \mathscr{S}v(z) \\ &= \max_{z\in[\alpha,\beta]} [\mathscr{R}u(z) + \mathscr{S}v(z)] \\ &= \max_{z\in[\alpha,\beta]} [(\mathscr{I} - \mathscr{M})u(z) + (\mathscr{I} - \mathscr{N})v(z)] \\ &= \max_{z\in[\alpha,\beta]} [u(z) + v(z)] - \max_{z\in[\alpha,\beta]} [\mathscr{M}u(z) + \mathscr{N}v(z)] \\ &= d(u, v) - d(\mathscr{M}u, \mathscr{N}v) \geq d(u, v) - \varrho\xi_2 d(u, v) \\ &= (1 - \varrho\xi_2) d(u, v), \end{aligned} \tag{55}$$

which implies that

$$d(u, v) \leq \left(\frac{1}{1 - \varrho\xi_2}\right) d(\mathscr{R}u, \mathscr{S}v). \tag{56}$$

From (51) and (56), we get

$$\begin{aligned} d(\mathscr{P}u, \mathscr{Q}v) &\leq \left(\frac{\chi + \rho\xi_1}{1 - \varrho\xi_2}\right) d(\mathscr{R}u, \mathscr{S}v) \\ &= \eta d(\mathscr{R}u, \mathscr{S}v) \\ &\leq \eta \max\{d(\mathscr{R}u, \mathscr{S}v), d(\mathscr{P}u, \mathscr{R}u), d(\mathscr{Q}v, \mathscr{S}v), d(\mathscr{R}u, \mathscr{Q}v), d(\mathscr{P}u, \mathscr{S}v)\}. \end{aligned} \tag{57}$$

As a result, the assumptions of Corollary 10 are fulfilled. Thus, there exists only one element $u' \in \mathscr{X}$ which satisfies $\mathscr{P}u' = \mathscr{Q}u' = \mathscr{R}u' = \mathscr{S}u'$; consequently, u' is a unique solution of (46). □

5. Conclusion

Three common fixed point results for expansive maps meeting implicit relations are proved in this article. Additionally, we have determined the basic characteristics of these maps in metric and dislocated metric spaces. Our recently created concept of occasionally weakly biased maps of type $(\mathscr{A})$ served as the foundation for this. These findings improve a number of previous results about the notion of common fixed points. Our theorems have been supported by some illustrated examples; also, a relevant application has been provided to show the viability and usefulness of one of these results. The relevant work shown and discussed in [4, 5] is expanded upon and improved by our results.

Data Availability

No real data were used to support this study. The data used in this study are hypothetical.

Disclosure

This work was carried out as part of the authors' duties at Hodeidah University.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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