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Research Article

N^{th} Composite Iterative Scheme via Weak Contractions with Application

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The main goal of this study is to formulate an effective iterative scheme, namely, an $n^{\text{th}}-$ composite iterative scheme for approximating the fixed point of a self-map $T\colon \mathscr{U} \longrightarrow \mathscr{U}$ with weak contraction property. We show that the $n^{\text{th}}-$ composite iterative scheme is faster than the scheme obtained by Sintunavarat–Pitea's iterative scheme. We present some examples using the MATLAB simulator to illustrate our results. Finally, we approximate the solution of some integral equations using our scheme and the Sintunavarat–Pitea scheme.

1. Introduction

The main concern of fixed point theory in distance spaces is to determine the existence and uniqueness of solutions to problems not only in mathematics but also in other fields of science. Many well-known equations can easily be converted to fixed-point equations. One of the important applications of fixed point theory is also the solution of integral and differential equations (existence and uniqueness). However, the challenges are to create some iteration to speed up the computation or approach the solution for such problems. Some creative researchers used iteration schemes to compute the fixed point numerically.

The result of Banach [1] is considered a principle in the theory of the fixed point. After that, many generalizations of this result were obtained by many researchers, see [2–20]. For example, Berinde [21] introduced the weak contraction as follows.

Definition 1 (See [21]). Suppose $(\mathscr{U}, \mathrm{d})$ is a metric space and $T\colon \mathscr{U} \longrightarrow \mathscr{U}$ is a self-mapping on $\mathscr{U}$. Then, we call T a weak contraction if for some $k \in 0,1$ and $\beta \geq 0$, the following inequality holds for all $u_1, u_2 \in \mathscr{U}$, we have the following equation:

$$d(Tu_1, Tu_2) \leq kd(u_1, u_2) + \beta d(u_1, Tu_2). \tag{1}$$

In normed, the weak contraction is as follows:

$$\|Tu_1 - Tu_2\| \leq k\|Tu_1 - Tu_2\| + \beta\|u_1 - Tu_2\|. \tag{2}$$

In this paper, we consider the weak contraction when $\beta = 0$.

In general, fixed point theory has studied the uniqueness and existence of fixed points for self-maps under certain conditions and has various applications in several fields of science such as economics, physics, applied mathematics, and some engineering subjects.

In mathematics, certain equations can easily be converted into fixed-point equations, for example, the integral equations,

$$\omega(t) = \omega_0 + \int_{t_0}^{t} H(r, \omega(r))dr, \quad (3)$$

where H is continuous with $H\colon I \times \mathscr{R} \longrightarrow \mathscr{R}$ and $\omega \in \mathscr{C}(I)$ where $\mathscr{C}(I)$ is the class of real valued and continuous functions and I is an interval in $\mathscr{R}$. This integral equation has identically a solution to which is the solution of the fixed point for the self-mapping $L\colon \mathscr{C}(I) \longrightarrow \mathscr{C}(I)$ which is defined as follows:

$$L\omega(t) = \omega_0 + \int_{t_0}^{t} H(r, \omega(r))dr. \quad (4)$$

Numerical analysis plays a key role in creating an iteration scheme for approximating fixed points for self-maps that require some constraints by less iterations. Many researchers have obtained several iteration schemes for approximating the existing fixed points for self-maps, see [22–24]. For example, Sintunavarat and Pitea [25] established an iteration process by which (S_n) is defined as follows:

$$\begin{cases} s_0 \in \mathscr{C}, \\ x_n = (1 - b_n)s_n + b_n T s_n, \\ y_n = (1 - c_n)s_n + c_n x_n, \\ s_{n+1} = (1 - a_n)T y_n + a_n T x_n. \end{cases} \quad (S_n) \quad (5)$$

Recently, Berinde [26] introduced some definitions for the convergence rate, which is important for our study.

Definition 2. [26] Suppose that (β_n) and (γ_n) are two sequences in $\mathbb{R}$ and $\beta, \gamma, r \in \mathbb{R}$ such that $\lim_{n \longrightarrow +\infty} \beta_n = \beta$ and $\lim_{n \longrightarrow +\infty} \gamma_n = \gamma$ assume that

$$\lim_{n \longrightarrow +\infty} \frac{\beta_n - \beta}{\gamma_n - \gamma} = r. \quad (6)$$

(1) If $r = 0$, then (β_n) converges to β faster than (γ_n) to γ

(2) If $r \in (0, +\infty)$, then (β_n) and (γ_n) have the same rate of convergence

Definition 3. [26] Suppose that $(\mathscr{U}, \|.\|)$ is a normed space and (u_n) and (t_n) are two sequences in $\mathscr{U}$. Also, assume that the two sequences (u_n) and (t_n) are converges to an element $u \in \mathscr{U}$. Then, the error estimates $\|u_n - u\| \leq \beta_n$ and $\|t_n - u\| \leq \gamma_n$ are available and (β_n) and (γ_n) are in $0, +\infty$ such that $\beta_n, \gamma_n \longrightarrow 0$.

If $\beta_n \longrightarrow 0$ faster than $\gamma_n \longrightarrow 0$, then $u_n \longrightarrow u$ faster than $t_n \longrightarrow u$.

2. Main Results

Next, we construct a new iterative scheme, namely, $n^{th}-$ composite iterative scheme for approximating the fixed point of a self-mapping T on $\mathscr{U}$ of weak contraction kind. Henceforth, we assume that $(\mathscr{U}, \|.\|)$ is a Banach space, $\varnothing \neq C \subseteq \mathscr{U}$ is closed and convex, and $T\colon \mathscr{C} \longrightarrow \mathscr{C}$ is a self-mapping. We define the $n^{th}-$ composite iterative scheme (BQ_n) by the following expression:

$$\begin{cases} b_0 \in \mathscr{C}, \\ w_n = (1 - a_n')T^n b_n + a_n' T^{n+1} b_n, \\ q_n = (1 - a_n'')T^n b_n + a_n'' T^n w_n, \\ b_{n+1} = (1 - a_n)T^{n+1} q_n + a_n T^{n+1} w_n. \end{cases} \quad (BQ_n) \quad (7)$$

Theorem 4. *Suppose that $T\colon \mathscr{C} \longrightarrow \mathscr{C}$ satisfies condition 1 and u is a fixed point of T. If (b_n) is a sequence in C defined by (QB_n), and (a_n), (a_n') and (a_n'') are sequences in C such that (a_n) in $[a, 1 - a]$, (a_n') in $[a', 1 - a']$, and (a_n'') in $[a', 1 - a'']$, with $a, a', a'' \in (0, 1/2)$. If $a \geq a''/2$, then $BQ_n \longrightarrow u$ faster than (S_n).*

Proof. For any positive integer n, utilizing (BQ_n), we obtain the following equation:

$$\begin{aligned} \|b_{n+1} - u\| &= \|(1 - a_n)T^{n+1} q_n + a_n T^{n+1} w_n - u\| \\ &\leq (1 - a_n)\|T^{n+1} q_n - u\| + a_n \|T^{n+1} w_n - u\| \\ &\leq k^{n+1}\left[(1 - a_n)\|q_n - u\| + a_n \|w_n - u\|\right]. \end{aligned} \quad (8)$$

So,

$$\|b_{n+1} - u\| \leq k^{n+1}\left[(1 - a_n)\|q_n - u\| + a_n \|w_n - u\|\right]. \quad (9)$$

Now,

$$\begin{aligned} \|q_n - u\| &= \|(1 - a_n'')T^n b_n + a_n'' T^n w_n - u\| \\ &\leq (1 - a_n'')\|T^n b_n - u\| + a_n'' \|T^n w_n - u\| \\ &\leq k^n\left[(1 - a_n'')\|b_n - u\| + a_n'' \|w_n - u\|\right]. \end{aligned} \quad (10)$$

Therefore,

$$\|q_n - u\| \leq k^n\left[(1 - a_n'')\|b_n - u\| + a_n'' \|w_n - u\|\right]. \quad (11)$$

In addition,

$$\begin{aligned} \|w_n - u\| &= \|(1 - a_n')T^n b_n + a_n' T^{n+1} b_n - u\| \\ &\leq (1 - a_n')\|T^n b_n - u\| + a_n' \|T^{n+1} b_n - u\| \\ &\leq k^n\left[(1 - a_n')\|b_n - u\| + k a_n' \|b_n - u\|\right] \\ &= k^n\left[(1 - (1 - k)a_n')\|b_n - u\|\right]. \end{aligned} \quad (12)$$

Therefore,

$$\|w_n - u\| \leq k^n\left[(1 - (1 - k)a_n')\|b_n - u\|\right]. \quad (13)$$

From (11) and (13), we obtain the following equation:

$$\begin{aligned} \|q_n - u\| &\leq k^n\left\{(1 - a_n'')\|b_n - u\| + a_n'' \|w_n - u\|\right\} \\ &\leq k^n\left\{(1 - a_n'')\|b_n - u\| + k^n a_n''\left[1 - (1 - k)a_n'\right]\right\}\|b_n - u\| \\ &= k^n\left\{1 - (1 - k^n)a_n'' - (1 - k)k^n a_n'' a_n'\right\}\|b_n - u\|. \end{aligned} \quad (14)$$

Thus,

$$\|q_n - u\| \le k^n\{1 - (1 - k^n)a''_n - (1 - k)k^n a''_n a'_n\}\|b_n - u\|. \tag{15}$$

Now,

$$\begin{aligned}
\|b_{n+1} - u\| &\le k^{n+1}\left\{(1 - a_n)\|q_n - u\| + a_n\|w_n - u\|\right\}\\
&\le k^{n+1}\left\{(1 - a_n)k^n\{1 - (1 - k^n)a''_n - (1 - k)k^n a''_n a'_n\}\|b_n - u\| + a_n\|w_n - u\|\right\}\\
&\le k^{n+1}\{(1 - a_n)k^n\{1 - (1 - k^n)a''_n - (1 - k)k^n a''_n a'_n\}\\
&\quad + a_n k^n(1 - (1 - k)a'_n)\}\|b_n - u\|\\
&= k^{2n+1}\{(1 - a_n)[1 - (1 - k^n)a''_n - (1 - k)k^n a''_n a'_n]\\
&\quad + a_n(1 - (1 - k)a'_n)\}\|b_n - u\|\\
&= k^{2n+1}\{1 - (1 - k^n)(1 - a_n)a''_n - (1 - k)a'_n[k^n a'_n(1 - a_n) + a_n]\}\|b_n - u\|\\
&\le k^{2n+1}\left\{1 - (1 - k^n)aa'' - (1 - k)a'\left[k^n a'' a + a\right]\right\}\|b_n - u\|\\
&= k^{2n+1}\left\{1 - (1 - k^n)aa'' - (1 - k)aa'\left[1 + k^n a''\right]\right\}\|b_n - u\|.
\end{aligned} \tag{16}$$

Therefore,

$$\|b_n - u\| \le k^{2n}\left\{1 - \left(1 - k^{n-1}\right)aa'' - (1 - k)aa'\left[1 + k^{n-1}a''\right]\right\}^n\|b_0 - u\|. \tag{17}$$

But the iterative process t_n implies

$$\|s_n - u\| \le \|s_0 - u\|\left\{1 - a'(1 - k)\left(a'' - a\left(1 - a''\right)\right)\right\}^n. \tag{18}$$

Let $\alpha_n = k^{2n}\{1 - (1 - k^{n-1})aa'' - (1 - k)aa'\,[1 + k^{n-1}c]\}^n\,\|b_0 - u\|$ and

$$\beta_n = \left\{1 - (1 - k)a'\left(a'' - a\left(1 - a''\right)\right)\right\}^n\|s_0 - u\|. \tag{19}$$

Then, we obtain the following equation:

$$\lim_{n\longrightarrow+\infty}\|b_n - u\| \le \lim_{n\longrightarrow+\infty} k^{2n}\left\{1 - \left(1 - k^{n-1}\right)aa'' - (1 - k)aa'\left[1 + k^{n-1}a''\right]\right\}^n\|b_0 - u\| = 0, \tag{20}$$

and

$$\lim_{n\longrightarrow+\infty}\|s_n - u\| \le \lim_{n\longrightarrow+\infty}\left\{1 - (1 - k)a'\left(a'' - a\left(1 - a''\right)\right)\right\}^n\|s_0 - u\| = 0. \tag{21}$$

Since $\alpha_1 = k^2\{1 - (1 - k)aa'\,[1 + a'']\}\ \ \|b_0 - u\| < \beta_1 = \{1 - (1 - k)a'\,(a'' - a(1 - a''))\}\ \|s_0 - u\|$, we have

$$\lim_{n\longrightarrow+\infty}\left\{\frac{k^{2n}\{1 - (1 - k^{n-1})aa'' - (1 - k)aa'\left[1 + k^{n-1}a''\right]\}}{1 - (1 - k)a'\,(a'' - a(1 - a''))}\right\}^n = 0. \tag{22}$$

Consequently, we obtain our result. □

3. Numerical Examples

Example 1. Suppose $\mathscr{U} = \mathbb{R}$ with the usual normed and suppose $\mathscr{C} = [0, 50]$. Assume that $T: \mathscr{C} \longrightarrow \mathscr{C}$ is defined by $T(x) = (5x^2 - 2x + 48)^{1/3}$. Let $a = 0.45, a' = 0.25$, and $a'' = 0.1$ and let $a_n = 0.45$, $a_n' = 0.75 - (1/(4 + n))$ and $a_n'' = 0.4 + (1/(2 + n)), n \in \mathbb{N} \cup \{0\}$. By mean value theorem, one can insures that T satisfies condition 1. Moreover, the sequences (a_n), (a_n'), and (a_n'') and the constants a, a', and a'' satisfy the conditions of Theorem 4 Thus, (BQ_n) is faster than (S_n). Table 1 illustrates the results obtained by using (BQ_n) and (S_n) for reckoning the approximated fixed point of T when we start from an arbitrary point $b_0 = 50$.

One can see in the above table that the solution was attained by BQ_n scheme at 8^{th} round while it needed 39^{th} round to be achieved through S_n scheme which means that BQ_n is faster and more effective than S_n under the assumed conditions.

Table 1: The results obtained by using (BQ_n) and (S_n) for reckoning the approximated fixed point of T when we start from $b_0 = 50$.

Steps	BQ_n	S_n
1	19.073217881051000	19.097061775637600
2	7.078653010637430	10.841595160852900
3	6.028447204181710	7.947865238145880
4	6.000212620262850	6.805241334232650
5	6.000000455488770	6.335322149502770
6	6.000000000282000	6.139844663267220
7	6.000000000000050	6.058315233977350
8	6	6.024304287747400
⋮	⋮	⋮
37	6	6.000000000000200
38	6	6.000000000000080
39	6	6

Example 2. Suppose $\mathscr{U} = \mathbb{R}$ with the usual normed and suppose $\mathscr{C} = [0, 2]$. Assume that $T: \mathscr{C} \longrightarrow \mathscr{C}$ is defined by $T(x) = e^{\sin x - 3/2}$. Let $a = 0.35, a' = 0.25$, and $a'' = 0.1$ and let $a_n = 0.6 - (2/(9 + n))$, $a_n' = 0.75 - (1/(4 + n))$ and $a_n'' = 0.2, n \in \mathbb{N} \cup \{0\}$. By mean value theorem, one can insures that T satisfies condition 1. Moreover, the sequences (a_n), (a_n'), and (a_n'') and the constants a, a', and a'' satisfy the conditions of Theorem 4. Thus, (BQ_n) is faster than (S_n). Table 2 illustrates the results obtained by using (BQ_n) and (S_n) for reckoning the approximated fixed point of T when we start from an arbitrary point $b_0 = 2$.

BQ_n is faster and more effective than S_n under the assumed conditions since one can see in the above table that the solution was attained by BQ_n scheme at 6^{th} round while it needed 22^{th} round to be achieved through S_n scheme.

Table 2: The results obtained by using (BQ_n) and (S_n) for reckoning the approximated fixed point of T when we start from $b_0 = 2$.

Steps	BQ_n	S_n
1	0.582038390039836	0.578010282006729
2	0.305245535842560	0.364824050158744
3	0.299794445763490	0.313785066271133
4	0.299786904117514	0.302692991943990
5	0.299786903282173	0.300379353073023
6	0.299786903282166	0.299906280405721
⋮	⋮	⋮
19	0.299786903282166	0.299786903282234
20	0.299786903282166	0.299786903282179
21	0.299786903282166	0.299786903282168
22	0.299786903282166	0.299786903282166
23	0.299786903282166	0.299786903282166

Example 3. Suppose $\mathscr{U} = \mathbb{R}$ with the usual normed and suppose $\mathscr{C} = [0, 1]$. Assume that $T: \mathscr{C} \longrightarrow \mathscr{C}$ is defined by $T(x) = 1 + x^2/7 - x^2$. Let $a = 0.35, a' = 0.25$, and $a'' = 0.1$ and let $a_n = 0.6 - (2/(9 + n))$, $a_n' = 0.75 - (1/(4 + n))$, and $a_n'' = 0.4 + (1/(2 + n)), n \in \mathbb{N} \cup \{0\}$. It follows by mean value theorem that T satisfies condition 1. Moreover, the sequences (a_n), (a_n'), and (a_n'') and the constants a, a', and a'' satisfy the conditions of Theorem 4. Thus, (BQ_n) is faster than (S_n). Table 3 illustrates the results obtained by using (BQ_n) and (S_n) for reckoning the approximated fixed point of T when we start from $b_0 = 0$.

Observe that under the assumed conditions BQ_n is faster and more effective than S_n since one can see in Tables 1–3 that the solution was attained by BQ_n scheme at 4^{th} round while it needed 10^{th} round to be achieved through S_n scheme.

Table 3: The results obtained by using (BQ_n) and (S_n) for reckoning the approximated fixed point of T when we start from $b_0 = 0$.

Steps	BQ_n	S_n
1	0.143592129157553	0.143586038445553
2	0.146365378902920	0.146295558732113
3	0.146365489032898	0.146363792725813
4	0.146365489032909	0.146365448771654
5	0.146365489032909	0.146365488088288
6	0.146365489032909	0.146365489010892
7	0.146365489032909	0.146365489032398
8	0.146365489032909	0.146365489032897
9	0.146365489032909	0.146365489032908
10	0.146365489032909	0.146365489032909
11	0.146365489032909	0.146365489032909

4. Applications

Next, we aim to give an application on our result in physics particularly the newton law of heat transfer. For that end, we need the following results.

Lemma 5 (See [27, 28]). *$w(x)$ is a solution for the I.V.P (the initial value problem)*

$$\zeta'(t) = H(t, \zeta(t)), \zeta(t_0) = \zeta_0; \tag{23}$$

$$\Leftrightarrow \zeta(t) = \zeta_0 + \int_{t_0}^{t} H(r, \zeta(r))dr. \tag{24}$$

Let I is an interval in $\mathbb{R}$ and let $\mathscr{C}(I)$ stands for the class of real-valued continuous functions $\zeta: I \longrightarrow \mathbb{R}$ with the sup-norm

$$\|\zeta\|_\infty = \sup_{t\in I}|\zeta(t)|. \tag{25}$$

The theorem below is obtained in [29].

Theorem 6. *[29] Suppose $H: I \times \mathbb{R} \longrightarrow \mathbb{R}$, where I is an interval in $\mathbb{R}$ and let t_0 be an interior point of I. Assume that $H(t, x)$ is a continuous function of (t, x) satisfying the following condition for some $\mu > 0$*

$$|H(t, x) - H(t, y)| \leq \mu|x - y|, \tag{26}$$

for all $x, y \in \mathbb{R}$, $t \in I$. Then, equation (23) has a unique continuously differentiable solution $w \in \mathscr{C}(I)$.

Newton's law of cooling is formed as a differential equation predicting the cooling of a warm body in a cold environment, which can be formed as follows:

$$x'(t) = -\mu\,(x(t) - x_\delta), \tag{27}$$

where $x(t)$ is temperature of the object, x_δ the environment temperature which is constant, and μ is a constant of proportionality.

If $x(t_0) = x_0$, then we get the I.V.P.

$$x'(t) = -\mu\,(x(t) - x_\delta), x(t_0) = x_0. \tag{28}$$

Suppose $H(t, x) = -\mu\,(x(t) - x_\delta)$. Then, it is easy to verify that H satisfies condition 13. Hence, by Theorem 10, there is a unique solution for 15.

In fact, one can solve 15 to find that the exact solution is

$$x(t) = x_\delta + (x_0 - x_\delta)\,e^{-\mu(t-t_0)}. \tag{29}$$

Now, we move on to show the usability of the $n^{th}-$ composite scheme (BQ_n) through the following example.

Example 4. A piece of iron with an initial temperature of 920° is removed from the furnace and then placed in a room with a temperature of 20° to cool. Suppose that the temperature of the piece of iron initially decreases at a rate of 9°/min. What is the relationship between the temperature of the piece of iron and time?

Assuming that the iron piece follows the Newton's law for cooling, so we obtain the following equation:

$$x'(t) = -\mu\,(x(t) - x_\delta), x(0) = 920, x'(0) = -9. \tag{30}$$

It is easy to figure out that $\mu = 0.01$, hence the solution is

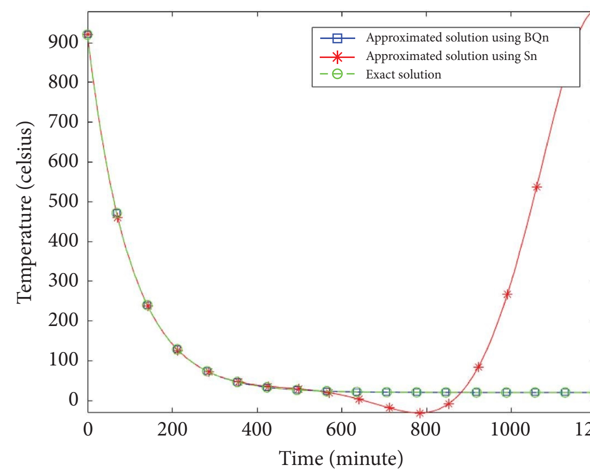

Figure 1: Comparative results between the exact solution and the 6^{th} term obtained by BQ_n and S_n schemes.

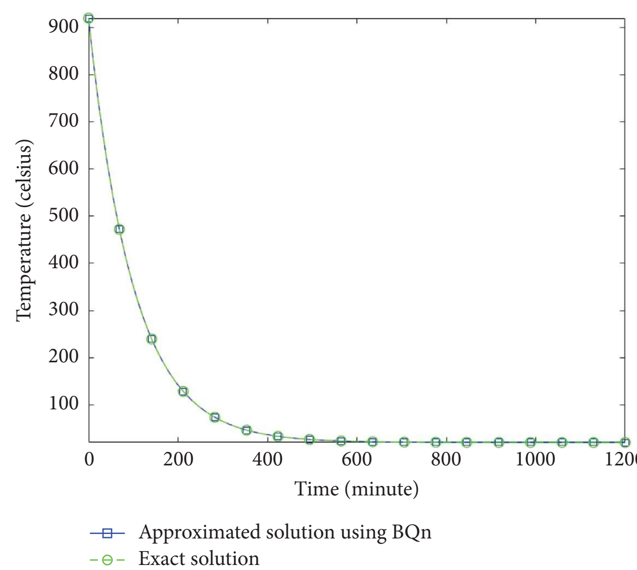

Figure 2: Comparative results between the exact solution and the 6^{th} term obtained by BQ_n scheme.

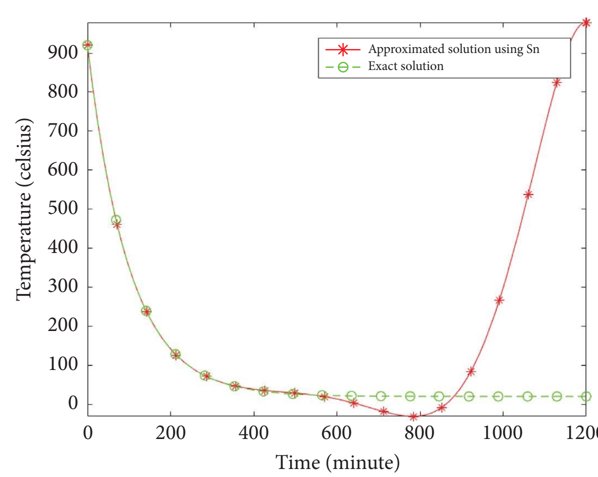

Figure 3: Comparative results between the exact solution and the 6^{th} term obtained by and S_n scheme.

$$x(t) = 900\, e^{-0.01t} + 20. \tag{31}$$

Next, suppose $\mathcal{T}x(t) = 920 + \int_0^t -0.01\,(x(r) - 20)dr$, where $H(t, x) = -0.01\,(x(t) - 20)$. Then, $\mathcal{T}$ has a unique fixed point after the argument in Theorem 10.

Figures 1–3 show the results for calculating the approximate fixed point of $\mathcal{T}$ when we start at $x(t) = \cos t$. Note that we only do 6 iterations with MATLAB.

5. Conclusions

In the presented study, we developed a new iterative scheme for approximating the fixed point for weak contraction-type mappings. In the numerical examples, we conclude that our iterative scheme computes the fixed point faster than that of Sintunavarat–Pitea. We approximated the function that described the relation between time and temperature of some materiel that obeys Newton's law for heat transfer by utilizing our significant scheme. This application shows the applicability of the fixed point theory in different scientific fields.

Data Availability

No underlying data were collected or produced in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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