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Research Article

Global Asymptotic Stability of Nonlinear Strict Feedback System via Backstepping Control Approach

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The design of controller for nonlinear system in strict feedback form in which the equilibrium point is the origin is the focus of the study. Backstepping is the technique applied. Recursive design is used in the backstepping control technique. Using the backstepping approach, we are given a reasonably simple method to design the control law for a system with these properties. The closed-loop system's global asymptotic stability is attained by incorporating a Lyapunov function; in this case, a quadratic one was selected for the study. To demonstrate the viability of the technique, we also performed simulations of the system and the designed control with some initial values, and positive results were reported.

1. Introduction

Different control techniques have been developed over the past few years for both linear and nonlinear dynamic systems that have specific control design objectives like output regulation, synchronization, and local or global stabilization (see Zhao et al. [1]; Liu and Huang [2]; Niu et al. [3]; Bin and Marconi [4]; and Braverman and Rodkina [5]). Nonlinear control systems are frequently stabilized locally or globally using active state feedback control, also known as output feedback control.

Sliding mode control, a well-liked variable structure control technique, alters the dynamics of a nonlinear system by applying a discontinuous control signal which makes the system move along a surface known as the sliding manifold, and this is a cross section representation of the system's typical behavior (see Solis et al. [6]; Armghan et al. [7]; and Gang [8]).

Passive control is frequently employed for stabilizing the control systems in the control design of big and complex systems. There are numerous uses for passive control in the balancing and management of control systems (see Sambas et al. [9]; Sangpet and Kuntanapreeda [10]; Zhou et al. [11]; and Tian et al. [12]).

A helpful method in the control research is the construction of feedback controls using control Lyapunov functions (see Khalil [13]). Nevertheless, choosing an appropriate control Lyapunov function for an overall nonlinear control system is a challenging task. In these situations, one can use the Lyapunov stability theory to determine the stability of the closed-loop system upon implementing feedback control and attempting to establish local or global control [13].

The backstepping control technique is a recursive design process that ensures strict feedback systems' global asymptotic stability by connecting the selection of a control Lyapunov function alongside the creation of a feedback controller (see Kokotovic [14]; Kokotović and Arcak [15]; Krstić and Kokotović [16]; Vaidyanathan and Azar [17]; Shukla et al. [18]; and Vaidyanathan and Azar [17]). A more widely applicable backstepping control technique within the control papers is the block backstepping control approach (see Krstić and Kokotović [16]). The adaptive backstepping control technique is an improved form of the backstepping control technique that employs approximations for unknown variables within the control system (see Tran et al. [19]; Yang and Zheng [20]; Ali et al. [21]; Awan et al. [22]; and Ali et al. [23]).

Two of the more effective techniques are feedback linearization, also known as nonlinear dynamic inversion (NDI), and linear quadratic (LQ) control [24]. The suggested control (linear-quadratic-integral (LQI)) method aims to increase the adaptability of the controller by enabling flexible manipulation of the control rigidity, which helps in successfully discarding the bounded exogenous disturbances yet preserving the system's closed-loop stability and reducing the entire control energy expenditure [25].

2. Methodology

2.1. Lyapunov Stability Theory. Lyapunov theory is the foundation of backstepping control design. The goal is to create a control law that moves the system toward the target state or at least very close to it. In other words, we want to keep the closed-loop system's state in stable equilibrium. In this section, we define stability in the Lyapunov sense and then go over the key methods for demonstrating an equilibrium's stability. For the proof and stability theorems, see Slotin and Li [26] and Khalil [13].

Firstly, we have to describe the Lyapunov function and explain how to employ it in the stability analysis of controller design for straightforward first-order systems before introducing the typical backstepping method. In control theory and engineering, stability theory is crucial. Stability issues come up in the analysis of system dynamics in a variety of ways. The stability of equilibrium points is the main topic of discussion in this study.

A.M. Lyapunov introduced the first and second methods in 1892 as two approaches for finding the stability of dynamic systems given by ordinary differential equations which are often used to describe the stability of equilibrium points.

All approaches that use the explicit form of differential equation solutions for analysis fall under the first methodology. Contrarily, the second approach does not call for the resolution of the differential equations. That is, by applying the second Lyapunov approach, we can assess the system's stability without having to solve the state equation. This is highly helpful because it can be challenging to solve nonlinear and/or time-varying state equations.

The Lyapunov stability theorems provide sufficient conditions for stability as well as asymptotic stability and other types of stability. It is a necessary and sufficient condition for stability for several classes of ODEs that Lyapunov functions exist. There is no universal method for creating Lyapunov functions for ordinary differential equations.

While Lyapunov's direct method has emerged as the key technique for nonlinear system analysis and design, Lyapunov's first method now represents the theoretical basis of linear control. The so-called Lyapunov stability theorem combines the first approach with a direct method.

Consider an autonomous nonlinear system

$$\begin{aligned} \dot{x} &= \gamma(x), \\ x(0) &= x_{0,} \end{aligned} \tag{1}$$

where

$$x \in D \subseteq \mathbb{R}^{m}, \tag{2}$$

is a locally Lipschitz map from a domain D contained in R^{m} into R^{m}. Suppose γ has an equilibrium at $\tilde{x}$ so that $\gamma(\tilde{x}) = 0$.

The following definition describes the stability characteristics of this equilibrium.

Lyapunov's definition of stability (as shown in Figure 1). The equilibrium point $\tilde{x}$ of the above system is said to be

(i) Stable if for any given value $\varepsilon > 0$, $\exists \delta(\varepsilon)$ such that

$$\|x_0 - \tilde{x}\| < \delta \Rightarrow \|x(t) - \tilde{x}\| < \varepsilon, \forall t \geq 0. \tag{3}$$

(ii) Unstable if it is not stable.

(iii) Asymptotically stable (see Figure 2) if

(a) It is stable in the sense of Lyapunov.

(b) $\exists \delta(\varepsilon)$ such that

$$\|x_0 - \tilde{x}\| < \delta(\varepsilon) \Rightarrow \lim_{t \longrightarrow \infty} \|x(t) - \tilde{x}\| = 0. \tag{4}$$

(iv) Globally asymptotically stable if for any initial states, it is asymptotically stable, which means that

$$\begin{aligned} \lim_{t \longrightarrow \infty} x(t) &= \tilde{x} \\ &= 0. \end{aligned} \tag{5}$$

In these definitions, the trajectory $x(t)$ is the solution to the above system. In general, an analytical solution to $x(t)$ is difficult if not impossible. Fortunately, there are alternative ways to demonstrate stability.

2.2. Lyapunov's Direct Method. According to classical theory in mechanics, a vibrating system is steady when its amount of energy, which is a positive definite function, continuously decreases until its equilibrium state is attained. This requires that perhaps its derivative with respect to time of the amount of energy should be negative definite.

The second approach is based on a generalization of the following observation. If the system has an asymptotically stable equilibrium state, the system's stored energy that has been displaced within the domain of attraction decreases with the increase of time until it eventually presumes its minimum amount at the equilibrium state.

However, constructing an "energy function" for systems that are purely mathematical is not an easy task. The Lyapunov function, an artificial energy function, was developed by Lyapunov to get around this problem. But this concept is more universal and applicable than the one of energy. Actually, Lyapunov functions can be any scalar functions that satisfy this hypothesis of Lyapunov's stability theorem (see Theorem 2).

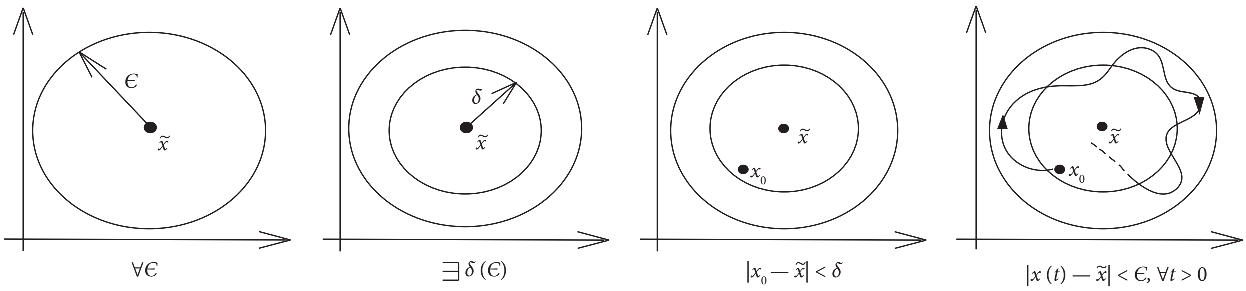

Figure 1: Stability in the sense of Lyapunov.

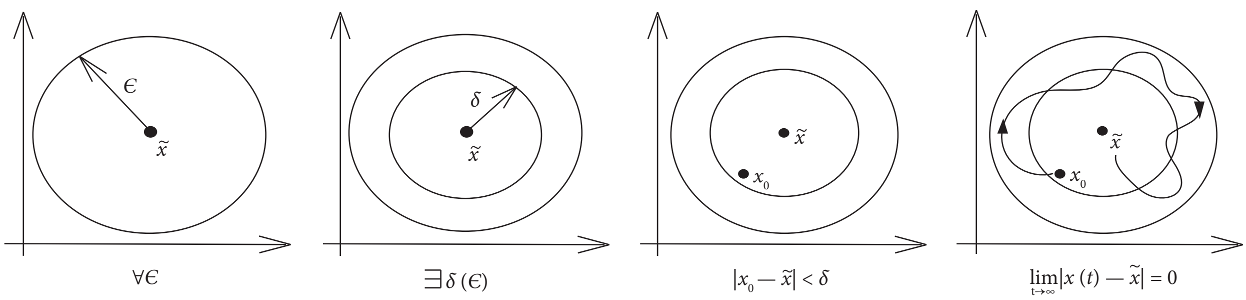

Figure 2: A pictorial overview of asymptotic stability.

Lyapunov functions depend on $y_1, y_2, \ldots, y_n$ and t. We denote them by $V(y_1, y_2, \ldots, y_n, t)$ or simply by $V(y, t)$. If a Lyapunov functions does not include t explicitly, we denote it by $V(y_1, y_2, \ldots, y_n)$ or $V(y)$ [27].

In the second Lyapunov technique, we can determine the stability and asymptotic stability of the equilibrium point without explicitly solving for the solution but by observing the sign behavior of $V(y, t)$ and that of the derivative with respect to time $\dot{V}(y,t) = \mathrm{d}V(y,t)/\mathrm{d}t$.

This method's fundamental idea is a mathematical elaboration of the subsequent physical system. Any electrical or mechanical system tends to approach a lesser power configuration if the amount of energy with positive sign is constantly reducing. In other words, it is not necessary to find the exact answer for the system in order to make conclusions about stability of such a system by looking at the changes of a particular scalar function known as the Lyapunov function. This really is precisely the method's strong feature because it eliminates the need to compute the motion equation of $x(t)$ in order to specify the development of the answer, and finding explicit solutions to nonlinear systems is challenging and occasionally impossible.

We begin by taking a look at a few of the functional characteristics that we will utilize to define Lyapunov functions.

Definition 1. Positive definiteness of scalar functions: in a region Ω (which contains the origin of the state space), a scalar function $V(x)$ is said to be *positive definite* if the following conditions are satisfied:

(i) If $V(x) > 0 \forall$ states x in the given region

(ii) $V(0) = 0$

Negative definiteness of scalar functions: $V(x)$ is said to be *negative definite* if $-V(x)$ is positive definite. A scalar function $V(x)$ is said to be *positive semidefinite* in a region Ω if it satisfies the following conditions:

(i) If $V(x) \geq 0 \forall$ states x in the given region

(ii) $V(0) = 0$

We have the following theorems to establish.

Theorem 2 (see Marquez [28]) (Lyapunov stability theorem). *Let* $\tilde{x} = 0$ *be an equilibrium point of* $\mathrm{d}x/\mathrm{d}t = g(x,t)$, $f\colon \Omega \longrightarrow \mathbb{R}^m$, *and* $V\colon \Omega \longrightarrow \mathbb{R}$ *be a continuously differentiable function. Then,* $\tilde{x} = 0$ *is stable in the sense of Lyapunov if the following are satisfied:*

(i) $V(0) = 0$

(ii) $V(x) > 0$ *in* $\Omega - \{0\}$

(iii) $\dot{V} \leq 0$ *in* $\Omega - \{0\}$

It is frequently crucial to understand the circumstances under which the initial state will tend toward the equilibrium state whenever the equilibrium is stable asymptotically. Every initial state will, in the best case scenario, tend to the equilibrium position. Globally asymptotically stable, it describes an equilibrium state that possesses this characteristic Marquez [28].

Theorem 3 (see Marquez [28]) (asymptotic stability theorem). *From the conditions stated in Theorem 4,* $\tilde{x} = 0$ *is asymptotically stable if* $V(.)$ *is such that*

(i) $V(0) = 0$

(ii) $V(x) > 0$ *in* $\Omega - \{0\}$

(iii) $\dot{V} < 0$ *in* $\Omega - \{0\}$

Theorem 4. *The origin $\tilde{x} = 0$ is globally asymptotically stable (stable in the large) if the following conditions are satisfied:*

(i) $V(0) = 0$

(ii) $V(x) > 0,\ x \neq 0$

(iii) $\dot{V} < 0,\ x \neq 0$

(iv) $\dot{V}(x)$ is radially unbounded i.e., $V(x) \longrightarrow \infty$ as $\|x \longrightarrow \infty\|$

3. Main Results

3.1. Backstepping. Backstepping is a method used in control theory, invented around 1990 by Kokotovic [14] and Lozano and Brogliato [29], for creating stabilizing control for a certain class of nonlinear dynamical systems. These systems are composed of subsystems that emanate from an irreducible subsystem that can be stabilized by other methods (because they backstep toward the control input beginning with the scalar equation that is distant from it by the greatest number of integration, they are known as backstepping designs). The designer can begin the design process just for known stable system and "back out" new controllers which gradually stabilize each outer subsystem because of the recursive nature. When the last external control is attained, the procedure is finished. As a result, this action is referred to as backstepping. Backstepping is a recursive process that integrates the design of the feedback control with the selection of a Lyapunov function. A design issue for the entire system is divided into a series of design issues involving lower order (including scalar) subsystems. Backstepping can frequently handle tracking, stabilization, and robust control issues under situations that are less constrained than those faced by other methods by taking use of the added flexibility presented with lower order and scalar subsystems. The backstepping method offers a strict feedback form of recursive stabilization of a system's origin. Specifically, it takes into account a system of the kind [30]

$$\begin{cases} \dot{y} = f_0(y) + \gamma_0(y)\xi_1, \\ \dot{\xi}_1 = f_1(y,\xi_1) + \gamma_1(y,\xi_1)\xi_2, \\ \dot{\xi}_2 = f_2(y,\xi_1,\xi_2) + \gamma_2(y,\xi_1,\xi_2)\xi_3, \\ \vdots \\ \dot{\xi}_i = f_i(y,\xi_1,\xi_2,\ldots,\xi_{i-1},\xi_i) + \gamma_i(y,\xi_1,\xi_2,\ldots,\xi_{i-1},\xi_i)\xi_{i+1}, \quad \text{for } 1 \leq i < k-1, \\ \vdots \\ \dot{\xi}_{k-1} = f_{k-1}(y,\xi_1,\xi_2,\ldots,\xi_{k-1}) + \gamma_{k-1}(y,\xi_1,\xi_2,\ldots,\xi_{i-1},\xi_{k-1})\xi_k, \\ \dot{\xi}_k = f_k(y,\xi_1,\xi_2,\ldots,\xi_{k-1},\xi_k) + \gamma_k(y,\xi_1,\xi_2,\ldots,\xi_{k-1},\xi_k)u, \end{cases} \tag{6}$$

where $y \in \mathbb{R}^m$ with $m \geq 1, \xi_1, \xi_2, \ldots, \xi_i, \ldots, \xi_{k-1}, \xi_k$ are scalars, u is a system's scalar input, $f_0, f_1, f_2, \ldots, f_i, f_{k-1}, f_k$ is such that $f_i(0,0,\ldots,0) = 0$, and $\gamma_1, \gamma_2, \ldots, \gamma_{k-1}, \gamma_k$ are nonzero across the field of interest (*i.e.*, $\gamma_i(y,\xi_1,\ldots,\xi_k) \neq 0$ for $i \leq i \leq k$).

Suppose that $\dot{\eta} = f_0(y) + \gamma_0(y)\xi_1$ has a known controller u_y which can stabilize the subsystem. Similarly, we suppose that a Lyapunov function V for the stable system exists. That is, we can stabilize the subsystem by other method while backstepping is used to extend the stability to the entire system.

The essence of referring to such systems as "strict feedback" is that the nonlinearity of f_i and γ_i in $\dot{\xi}_i (i = 1, 2, \ldots, k)$ only depends on $y, \xi_1, \xi_2, \ldots, \xi_i$, i.e., on the states which are "fed back to the subsystem" [31].

Most often, useful nonlinear terms may be canceled due to the feedback linearization method used to stabilize the strict feedback control system (6). However, since the backstepping approach does not necessitate that the final input-output dynamics is a linear system, it demonstrates greater flexibility compared to the feedback linearization approach [17].

To illustrate the backstepping procedure, we begin by examining the simplest instance of (6), for which $k = 1$. It is given by [13]

$$\dot{y} = f(y) + \gamma(y)\xi, \tag{7}$$

$$\dot{\xi} = u, \tag{8}$$

where $[y^T, \xi]^T \in \mathbb{R}^{m+1}$ is the real state and $u \in \mathbb{R}$ denotes the control to be designed. The functions $f\colon D \longrightarrow \mathbb{R}^m$ and $\gamma\colon D \longrightarrow \mathbb{R}^m$ are smooth (that is, it has continuous partial derivatives of any order) in a domain $D \subset \mathbb{R}^m$ that contains $y = 0$ and $f(0) = 0$. Suppose that f and γ are known.

In this paper, we focus on constructing feedback laws that stabilize system (6) to the origin. Now, we state and prove the backstepping theorem which will be utilized in this paper.

Theorem 5 (see Krstić and Kokotović [16] and Khalil [13]) (backstepping theorem). *Consider the system given in systems (7) and (8). Let $\phi(y)$ be a stabilizing state feedback controller for (7) with $\xi = \phi(y) = \phi(0) = 0$, and $V(y)$ be a Lyapunov candidate function that satisfies $\partial V/\partial\eta\dot{y} \leq -W(y)$, $\forall y \in D \subset \mathrm{R}^m$ where $W(y)$ is positive definite. Then, the state feedback control law stabilizes the origin of (7) and (8) with $V(y) + 1/2(\xi - \phi(y))^2$ as a Lyapunov candidate function.*

Moreover, if the assumptions are satisfied globally and $V(y)$ is radially unbounded, then the origin is globally asymptotically stable.

Proof. Our goal is to design a feedback controller to stabilize the origin (i.e., $y = 0, \xi = 0$) where ξ is called a virtual controller.

We presume that a smooth control law exists, i.e., $\xi = \phi(y)$ with $\phi(0) = 0$, which implies that the origin of

$$\dot{y} = f(y) + \gamma(y)\phi(y), \tag{9}$$

is stable asymptotically.

Moreover, suppose that we know a (positive definite, smooth) Lyapunov function $V(y)$ whereby the inequality

$$\dot{V} = \frac{\partial V}{\partial y}\dot{y} = \frac{\partial V}{\partial y}[f(y) + \gamma(y)\phi(y)] \leq -W(y), \quad y \in D, \tag{10}$$

is satisfied where $W(\eta)$ is a positive definite function on R^m.

Without changing the dynamics of the system, we add and subtract $\gamma(y)\phi(y)$ on the right hand side of equation (7), and we obtain

$$\begin{aligned} \dot{y} &= f(y) + \gamma(y)\xi + \gamma(y)\phi(y) - \gamma(y)\phi(y), \\ \dot{y} &= f(y) + \gamma(y)\phi(y) + \gamma(y)[\xi - \phi(y)], \\ \dot{\xi} &= u. \end{aligned} \tag{11}$$

We let

$$\begin{aligned} z &= \xi - \phi(y), \\ \xi &= z + \phi(y), \\ \dot{\xi} &= \dot{z} + \dot{\phi}, \end{aligned} \tag{12}$$

where z is corresponding *error variable*. Therefore, one of the design objectives of the backstepping control method is to determine u such that $z(t) \longrightarrow 0$ as $t \longrightarrow \infty$. We obtain

$$\dot{y} = f(y) + \gamma(y)\phi(y) + \gamma(y)z, \tag{13}$$

$$\dot{z} = u - \dot{\phi}. \tag{14}$$

Since $f, \phi,$ and γ are known, we can obtain $\dot{\phi}$ through the expression

$$\dot{\phi} = \frac{\partial \phi}{\partial y}\dot{y} = \frac{\partial \phi}{\partial y}[f(y) + \gamma(y)\xi]. \tag{15}$$

By letting $\tau = u - \dot{\phi}$, equations (13) and (14) become

$$\begin{aligned} \dot{y} &= [f(y) + \gamma(y)\phi(y)] + \gamma(y)z, \\ \dot{z} &= \tau. \end{aligned} \tag{16}$$

The new system is related to the original system, except for the fact that now the first system is asymptotically stable at the origin when the input is zero (i.e., by construction, when $z \longrightarrow 0$, at $y = 0$). Using this feature, we design τ for the stabilization of the entire system. Let

$$V(y, \xi) = V(y) + \frac{1}{2}z^2, \tag{17}$$

be our Lyapunov function. Then,

$$\dot{V} = \frac{\partial V}{\partial y}\dot{y} + z\dot{z}. \tag{18}$$

Substituting equations (13) and (16) into equation (18), we obtain

$$\begin{aligned} \dot{V} &= \frac{\partial V}{\partial y}[f(y) + \gamma(y)\phi(y) + \gamma(y)z] + z\tau \\ &= \frac{\partial V}{\partial y}[f(y) + \gamma(y)\phi(y)] + \frac{\partial V}{\partial y}\gamma(y)z + z\tau \\ &\leq -W(y) + \frac{\partial V}{\partial y}\gamma(y)z + z\tau \,(\text{from equation (10)}). \end{aligned} \tag{19}$$

As a special case, we choose τ so that we can cancel the positive terms to obtain negative definiteness in equation (19). We let

$$\tau = -\frac{\partial V}{\partial y}\gamma(y) - kz, \quad k > 0. \tag{20}$$

Substituting, we have

$$\begin{aligned} &= -W(y) + \frac{\partial V}{\partial y}\gamma(y)z + z\left(-\frac{\partial V}{\partial y}\gamma(y) - kz\right), \\ &\therefore \dot{V}(y, \xi) \leq -W(y) - kz^2, \end{aligned} \tag{21}$$

which shows that the origin $(y = 0, z = 0)$ is asymptotically stable. Since $\phi(0) = 0$, we conclude that the origin $(y = 0, \xi = 0)$ is asymptotically stable by Lyapunov stability theory [13].

Making u the subject of equation (14) and substituting for $\tau, z,$ and $\dot{\phi}$, we obtain

$$\begin{aligned} u &= \dot{z} + \dot{\phi} \\ &= \tau + \dot{\phi}, \\ u &= \frac{\partial \phi(y)}{\partial y}[f(y) + \gamma(y)\xi] - \frac{\partial V(y)}{\partial y}\gamma(y) - kz, \\ u &= \frac{\partial \phi(y)}{\partial y}[f(y) + \gamma(y)\xi] - \frac{\partial V(y)}{\partial y}\gamma(y) - k[\xi - \phi(y)]. \end{aligned} \tag{22}$$

Hence, (22) is the needed backstepping control law.

If all assumptions are globally correct and $V(\eta)$ is radially unbounded (i.e., $V(y) \longrightarrow \infty$ as $\|x \longrightarrow \infty\|$), then we can conclude that the origin is *globally asymptotically stable.*

In this section, we will provide a straightforward example to explain the common backstepping control scheme. □

Example 1. Consider the fifth-order nonlinear strict feedback system

$$\dot{y}_1 = y_1^2 - y_1^3 + y_2, \quad (23)$$

$$\dot{y}_2 = y_1^2 y_2 + y_3, \quad (24)$$

$$\dot{y}_3 = y_2^2 + y_4, \quad (25)$$

$$\dot{y}_4 = y_5, \quad (26)$$

$$\dot{y}_5 = u. \quad (27)$$

Our goal is to find a control $u(y_1, y_2, y_3, y_4, y_5)$ that will take the states y_1, y_2, y_3, y_4, y_5 to the origin as $t \longrightarrow \infty$ starting at any given initial value. For convenience, we transform the given system by considering the following.

For convenience, we define

$$\begin{aligned} \phi_1(y_1) &\triangleq \phi_1, \\ \phi_2(y_1, y_2) &\triangleq \phi_2, \\ \phi_3(y_1, y_2, y_3) &\triangleq \phi_3, \\ \phi_4(y_1, y_2, y_3, y_4) &\triangleq \phi_4. \end{aligned} \quad (28)$$

Let

$$y_2 = \phi_1 + z_2, \quad (29)$$

$$z_2 = y_2 - \phi_1, \quad (30)$$

$$\dot{y}_2 = \dot{\phi}_1 + \dot{z}_2, \quad (31)$$

$$\dot{z}_2 = \dot{y}_2 - \dot{\phi}_1, \quad (32)$$

$$\dot{y}_1 = y_1^2 - y_1^3 + \phi_1 + z_2. \quad (33)$$

Let

$$y_3 = \phi_2 + z_3, \quad (34)$$

$$\dot{y}_3 = \dot{\phi}_2 + \dot{z}_3, \quad (35)$$

$$\dot{z}_2 = y_1^2 y_2 + y_3 - \dot{\phi}_1, \quad (36)$$

$$\dot{z}_2 = y_1^2 y_2 + \phi_2 + z_3 - \dot{\phi}_1. \quad (37)$$

Let

$$y_4 = \phi_3 + z_4, \quad (38)$$

$$\dot{y}_4 = \dot{\phi}_3 + \dot{z}_4, \quad (39)$$

$$\dot{z}_4 = \dot{y}_4 - \dot{\phi}_3. \quad (40)$$

From equation (35), we have

$$\begin{aligned} \dot{z}_3 &= y_2^2 + y_4 - \dot{\phi}_2, \\ \dot{z}_3 &= y_2^2 + \phi_3 + z_4 - \dot{\phi}_2. \end{aligned} \quad (41)$$

Let

$$\begin{aligned} y_5 &= \phi_4 + z_5, \\ \dot{y}_5 &= \dot{\phi}_4 + \dot{z}_5. \end{aligned} \quad (42)$$

From equation (40), substituting for $\dot{y}_4$, we have

$$\dot{z}_4 = y_5 - \dot{\phi}_3. \quad (43)$$

Let

$$y_5 = \phi_4 + z_5, \quad (44)$$

$$\dot{y}_5 = \dot{\phi}_4 + \dot{z}_5. \quad (45)$$

Substitute (44) into (43), we have

$$\begin{aligned} \dot{z}_4 &= \phi_4 + z_5 - \dot{\phi}_3, \\ \dot{z}_5 &= \dot{y}_5 - \dot{\phi}_4, \\ \dot{z}_5 &= u - \dot{\phi}_4. \end{aligned} \quad (46)$$

Hence, from the above equations, the given equation is transformed into

$$\begin{aligned} \dot{y}_1 &= y_1^2 - y_1^3 + \phi_1 + z_2, \\ \dot{z}_2 &= y_1^2 y_2 + \phi_2 + z_3 - \dot{\phi}_1, \\ \dot{z}_3 &= y_2^2 + \phi_3 + z_4 - \dot{\phi}_2, \\ \dot{z}_4 &= \phi_4 + z_5 - \dot{\phi}_3, \\ \dot{z}_5 &= u - \dot{\phi}_4. \end{aligned} \quad (47)$$

We choose the following Lyapunov candidate functions:

$$\begin{aligned} V(y_1, z_2, z_3, z_4, z_5) &= \frac{1}{2}y_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2 + \frac{1}{2}z_4^2 + \frac{1}{2}z_5^2, \\ \dot{V} &= y_1\dot{y}_1 + z_2\dot{z}_2 + z_3\dot{z}_3 + z_4\dot{z}_4 + z_5\dot{z}_5, \end{aligned} \quad (48)$$

$$\begin{aligned} \dot{V} = {} & y_1\left[y_1^2 - y_1^3 + \phi_1 + z_2\right] + z_2\left[y_1^2 y_2 + \phi_2 + z_3 - \dot{\phi}_1\right] \\ & + z_3\left[y_2^2 + \phi_3 + z_4 - \dot{\phi}_2\right] z_4\left[\phi_4 + z_5 - \dot{\phi}_3\right] + z_5\left[u - \dot{\phi}_4\right]. \end{aligned} \quad (49)$$

Rearranging terms,

$$\dot{V} = \underbrace{y_1\left[y_1^2 - y_1^3 + \phi_1\right]}_{\text{(i)}} + \underbrace{z_2\left[y_1 + y_1^2 y_2 + \phi_2 - \dot{\phi}_1\right]}_{\text{(ii)}} + \underbrace{z_3\left[y_2^2 + \phi_3 + z_2 - \dot{\phi}_2\right]}_{\text{(iii)}} + \underbrace{z_4\left[\phi_4 + z_3 - \dot{\phi}_3\right]}_{\text{(iv)}} + \underbrace{z_5\left[z_4 + u - \dot{\phi}_4\right]}_{\text{(v)}}. \tag{50}$$

Here, we aim to make $\dot{V}$ negative definite. In order to do so, we consider each term in equation (50). We consider equation (50) (i) separately. One of the ways to make it negative definite is to make

$$y_1^2 - y_1^3 + \phi_1 \triangleq -y_1 - y_1^3. \tag{51}$$

Then,

$$\begin{aligned} \phi_1 &\triangleq -y_1 - y_1^3 - y_1^2 + y_1^3, \\ \phi_1 &= -y_1 - y_1^2. \end{aligned} \tag{52}$$

Substituting (52) into (50) (i), we have

$$\begin{aligned} y_1\left[y_1^2 - y_1^3 - y_1 - y_1^2\right] &= y_1\left[-y_1^3 - y_1\right] \\ &= -\left[y_1^2 + y_1^4\right]^2 < 0, \quad \forall y_1 \neq 0. \end{aligned} \tag{53}$$

Next, we consider (50) (ii). We can achieve negative definiteness by letting

$$y_1 + y_1^2 y_2 + \phi_2 - \dot{\phi}_1 \triangleq -z_2. \tag{54}$$

Then,

$$\phi_2 = -z_2 + \dot{\phi}_1 - y_1 - y_1^2 y_2. \tag{55}$$

From (30) and (52), we have

$$\begin{aligned} \dot{\phi}_1 &= (-1 - 2y_1)\left(y_1^2 - y_1^3 + y_2\right), \\ z_2 &= y_2 - \phi_1 \\ &= y_2 - \left(-y_1 - y_1^2\right), \\ z_2 &= y_2 + y_1 + y_1^2, \\ \phi_2 &= 2y_1^4 - 2y_2 - 2y_1y_2 - y_1^2y_2 - 2y_1^2 - y_1^3 - 2y_1, \\ &\quad \left[y_2 + y_1 + y_1^2\right]\left(y_1 + y_1^2y_2 + 2y_1^4 - 2y_2 - 2y_1y_2 - y_1^2y_2 - 2y_1^2 - y_1^3 - 2y_1 - (-1 - 2y_1)\left(y_1^2 - y_1^3 + y_2\right)\right) \\ &= \left[y_2 + y_1 + y_1^2\right]^2 < 0, \forall y_1, y_2 \neq 0. \end{aligned} \tag{56}$$

Next, we consider (50) (iii). This can be negative definite by letting

$$\begin{aligned} y_2^2 + z_2 + \phi_3 - \dot{\phi}_2 &\triangleq z_3, \\ \phi_3 &= -z_3 - y_2^2 - z_2 + \dot{\phi}_2. \end{aligned} \tag{57}$$

From (34), we have

$$\begin{aligned} z_3 &= y_3 - 2y_1^4 + 2y_2 + 2y_1y_2 + y_1^2y_2 + 2y_1^2 + y_1^3 + 2y_1, \\ \dot{\phi}_2 &= \frac{\partial\phi_2}{\partial y_1}\dot{y}_1 + \frac{\partial\phi_2}{\partial y_2}\dot{y}_2, \\ \dot{\phi}_2 &= \frac{\partial\left[2y_1^4 - 2y_2 - 2y_1y_2 - y_1^2y_2 - 2y_1^2 - y_1^3 - 2y_1\right]}{\partial y_1}\left[y_1^2 - y_1^3 - y_2\right] + \frac{\partial\left[2y_1^4 - 2y_2 - 2y_1y_2 - y_1^2y_2 - 2y_1^2 - y_1^3 - 2y_1\right]}{\partial y_2}\left[y_1^2y_2 + y_3\right], \\ \dot{\phi}_2 &= 6y_1^3y_2 - 2y_3 - 4y_1y_2 - 2y_1y_3 - 2y_1y_2^2 - 7y_1^2y_2 - y_1^2y_3 - 2y_2 + y_1^4y_2 - 2y_1^2 - 2y_1^3 - 2y_2^2 + y_1^4 + 11y_1^5 - 8y_1^6, \\ \phi_3 &= 6y_1^3y_2 - 3y_2 - 3y_3 - 6y_1y_2 - 2y_1y_3 - 2y_1y_2^2 - 8y_1^2y_2 - y_1^2y_3 - y_1 - y_1^4y_2 - 3y_1^2 - 3y_1^3 - 3y_2^2 + 3y_1^4 + 11y_1^5 - 8y_1^6. \end{aligned} \tag{58}$$

Making appropriate substitutions, we obtain

$$\left[y_2^2 + \phi_3 + z_2 - \dot{\phi}_2\right] = -\left(2y_1 + 2y_2 + y_3 + 2y_1y_2 + y_1^2y_2 + 2y_1^2 + y_1^3 - 2y_1^4\right)^2 < 0, \forall y_1, y_2, y_3 \neq 0. \tag{59}$$

We proceed to make equation (50) (iv) negative definite. This can only be possible by letting

$$\begin{aligned} z_3 + \phi_4 - \dot{\phi}_3 &\triangleq -z_4, \\ \phi_4 &= -z_4 - z_3 + \dot{\phi}_3. \end{aligned} \tag{60}$$

From equation (38), we have

$$\begin{aligned} z_4 &= -6y_1^3y_2 + 3y_2 + 3y_3 + 6y_1y_2 + 2y_1y_3 + 2y_1y_2^2 + 8y_1^2y_2 + y_1^2y_3 \\ &\quad + y_1 + y_1^4y_2 + 3y_1^2 + 3y_1^3 + 3y_2^2 - 3y_1^4 - 11y_1^5 - 8y_1^6 + y_4, \\ \dot{\phi}_3 &= \frac{\partial\phi_3}{\partial y_1}\dot{y}_1 + \frac{\partial\phi_3}{\partial y_2}\dot{y}_2 + \frac{\partial\phi_3}{\partial y_3}\dot{y}_3. \end{aligned} \tag{61}$$

Making appropriate substitutions, we obtain

$$\begin{aligned} \dot{\phi}_3 &= 9y_1^2y_2^2 - 3y_3 - 3y_4 - y_2 + 2y_1^3y_2^2 - 6y_1y_2 - 6y_1y_3 - 2y_1y_4 - 8y_2y_3 - 18y_1y_2^2 - 18y_1^2y_2 - 10y_1^2y_3 - 4y_1^3y_2 - y_1^2y_4 + 6y_1^3y_3 \\ &\quad + 81y_1^4y_2 + 3y_1^4y_3 - 56y_1^5y_2 - 3y_1^6y_2 - y_1^2 - 5y_1^3 - 9y_2^2 - 3y_1^4 - 2y_2^3 + 21y_1^5 + 43y_1^6 - 103y_1^7 + 48y_1^8 - 6y_1y_2y_3, \\ \phi_4 &= 6y_1^3y_2 - 3y_2 - 3y_3 - 6y_1y_2 - 2y_1y_3 - 2y_1y_2^2 - 8y_1^2y_2 - y_1^2y_3 - y_1 - y_1^4y_2 - 3y_1^2 - 3y_1^3 - 3y_2^2 + 3y_1^4 + 11y_1^5 \\ &\quad + 8y_1^6 - y_4 - y_3 + 2y_1^4 - 2y_2 + 2y_1y_2 - y_1^2y_2 - 2y_1^2 - y_1^3 - 2y_1 + 9y_1^2y_2^2 - 3y_3 - 3y_4 - y_2 + 2y_1^3y_2^2 - 6y_1y_2 \\ &\quad - 6y_1y_3 - 2y_1y_4 - 8y_2y_3 - 18y_1y_2^2 - 18y_1^2y_2 - 10y_1^2y_3 - 4y_1^3y_2 - y_1^2y_4 + 6y_1^3y_3 + 81y_1^4y_2 + 3y_1^4y_3 - 56y_1^5y_2 \\ &\quad - 3y_1^6y_2 - y_1^2 - 5y_1^3 - 9y_2^2 - 3y_1^4 - 2y_2^3 + 21y_1^5 + 43y_1^6 - 103y_1^7 + 48y_1^8 - 6y_1y_2y_3. \end{aligned} \tag{62}$$

Simplifying, we get

$$\begin{aligned} \phi_4 &= 9y_1^2y_2^2 - 6y_2 - 7y_3 - 4y_4 - 3y_1 + 2y_1^3y_2^2 - 14y_1y_2 - 8y_1y_3 - 2y_1y_4 - 8y_2y_3 - 20y_1y_2^2 - 27y_1^2y_2 - 11y_1^2y_3 + 2y_1^3y_2 - y_1^2y_4 \\ &\quad + 6y_1^3y_3 + 82y_1^4y_2 + 3y_1^4y_3 - 56y_1^5y_2 - 3y_1^6y_2 - 6y_1^2 - 9y_1^3 - 12y_2^2 + 2y_1^4 - 2y_2^3 + 32y_1^5 + 35y_1^6 - 103y_1^7 + 48y_1^8 - 6y_1y_2y_3. \end{aligned} \tag{63}$$

By making appropriate substitutions in equation (50) (iv), we have

$$\begin{aligned} z_4\left[z_3 + \phi_4 - \dot{\phi}_3\right] &= -\left[-6y_1^3y_2 + 3y_2 + 3y_3 + 6y_1y_2 + 2y_1y_3 + 2y_1y_2^2 + 8y_1^2y_2 + y_1^2y_3 + y_1 + y_1^4y_2 + 3y_1^2 + 3y_1^3 + 3y_2^2 - 3y_1^4 \right. \\ &\quad \left. -11y_1^5 - 8y_1^6 + y_4\right]^2 < 0, \quad \forall y_1, y_2, y_3, y_4 \neq 0. \end{aligned} \tag{64}$$

We consider (50) (v), i.e., $z_5[z_4 + u - \dot{\phi}_4]$. In order for this to be negative definite, we let

$$\begin{aligned} z_4 + u - \dot{\phi}_4 &\triangleq -z_5, \\ u &= -z_5 - z_4 + \dot{\phi}_4. \end{aligned} \tag{65}$$

From equation (44), we get

$$z_5 = y_5 - \phi_4. \tag{66}$$

Making appropriate substitutions, we obtain

$$\begin{aligned} z_5 &= 3y_1 + 6y_2 + 7y_3 + 4y_4 + y_5 - 9y_1^2y_2^2 - 2y_1^3y_2^2 + 14y_1y_2 + 8y_1y_3 + 2y_1y_4 + 8y_2y_3 + 20y_1y_2^2 + 27y_1^2y_2 + 11y_1^2y_3 - 2y_1^3y_2 \\ &\quad + y_1^2y_4 - 6y_1^3y_3 - 82y_1^4y_2 - 3y_1^4y_3 + 56y_1^5y_2 + 3y_1^6y_2 + 6y_1^2 + 9y_1^3 + 12y_2^2 - 2y_1^4 + 2y_2^3 - 32y_1^5 - 35y_1^6 + 103y_1^7 - 48y_1^8 + 6y_1y_2y_3, \\ \dot{\phi}_4 &= \frac{\partial\phi_3}{\partial y_1}\dot{y}_1 + \frac{\partial\phi_3}{\partial y_2}\dot{y}_2 + \frac{\partial\phi_3}{\partial y_3}\dot{y}_3 + \frac{\partial\phi_4}{\partial y_4}\dot{y}_4, \\ \dot{\phi}_4 &= 332y_1^3y_2^2 - 6y_3 - 7y_4 - 4y_5 - 49y_1^2y_2^2 - 3y_2 - 271y_1^4y_2^2 - 20y_1^5y_2^2 - 12y_1y_2 - 14y_1y_3 - 8y_1y_4 - 32y_2y_3 - 2y_1y_5 - 10y_2y_4 \\ &\quad - 62y_1y_2^2 - 47y_1^2y_2 + 12y_1y_2^3 - 6y_1y_3^2 - 35y_1^2y_3 - 46y_1^3y_2 - 13y_1^2y_4 - 12y_1^3y_3 - 12y_2^2y_3 + 193y_1^4y_2 - y_1^2y_5 + 6y_1^3y_4 \\ &\quad + 122y_1^4y_3 + 534y_1^5y_2 + 5y_1^4y_4 - 62y_1^5y_3 - 1247y_1^6y_2 - 15y_1^6y_3 + 590y_1^7y_2 + 15y_1^8y_2 - 3y_1^2 - 9y_1^3 - 21y_2^2 - 15y_1^4 - 28y_2^3 \\ &\quad - 8y_3^2 + 35y_1^5 + 152y_1^6 + 50y_1^7 - 931y_1^8 + 1105y_1^9 - 384y_1^{10} + 22y_1^2y_2y_3 + 16y_1^3y_2y_3 - 62y_1y_2y_3 - 8y_1y_2y_4. \end{aligned} \tag{67}$$

Making appropriate substitutions, we obtain

$$\begin{aligned} u &= 334y_1^3y_2^2 - 12y_2 - 16y_3 - 12y_4 - 5y_5 - 40y_1^2y_2^2 - 4y_1 - 271y_1^4y_2^2 - 20y_1^5y_2^2 - 32y_1y_2 - 24y_1y_3 - 10y_1y_4 - 40y_2y_3 - 2y_1y_5 \\ &\quad - 10y_2y_4 - 84y_1y_2^2 - 82y_1^2y_2 + 12y_1y_2^3 - 6y_1y_3^2 - 47y_1^2y_3 - 38y_1^3y_2 - 14y_1^2y_4 - 6y_1^3y_3 - 12y_2^2y_3 + 276y_1^4y_2 - y_1^2y_5 + 6y_1^3y_4 \\ &\quad + 125y_1^4y_3 + 478y_1^5y_2 + 5y_1^4y_4 - 62y_1^5y_3 - 1250y_1^6y_2 - 15y_1^6y_3 + 590y_1^7y_2 + 15y_1^8y_2 - 12y_1^2 - 21y_1^3 - 36y_2^2 - 10y_1^4 \\ &\quad - 30y_2^3 - 8y_3^2 + 78y_1^5 + 179y_1^6 - 53y_1^7 - 883y_1^8 + 1105y_1^9 - 384y_1^{10} + 22y_1^2y_2y_3 + 16y_1^3y_2y_3 - 68y_1y_2y_3 - 8y_1y_2y_4. \end{aligned} \tag{68}$$

From equation (50) (v), we obtain

$$\begin{aligned} z_5\left[z_4 + u - \dot{\phi}_4\right] &= -\begin{bmatrix} 3y_1 + 6y_2 + 7y_3 + 4y_4 + y_5 - 9y_1^2y_2^2 - 2y_1^3y_2^2 + 14y_1y_2 + 8y_1y_3 + 2y_1y_4 + 8y_2y_3 + 20y_1y_2^2 + 27y_1^2y_2 + 11y_1^2y_3 - 2y_1^3y_2 + y_1^2y_4 - 6y_1^3y_3 - 82y_1^4y_2 \\ -3y_1^4y_3 + 56y_1^5y_2 + 3y_1^6y_2 + 6y_1^2 + 9y_1^3 + 12y_2^2 - 2y_1^4 + 2y_2^3 - 32y_1^5 - 35y_1^6 + 103y_1^7 - 48y_1^8 + 6y_1y_2y_3 \end{bmatrix}^2 \\ &< 0, \quad \forall y_1, y_2, y_3, y_4, y_5 \neq 0. \end{aligned} \tag{69}$$

From equation (48), we have

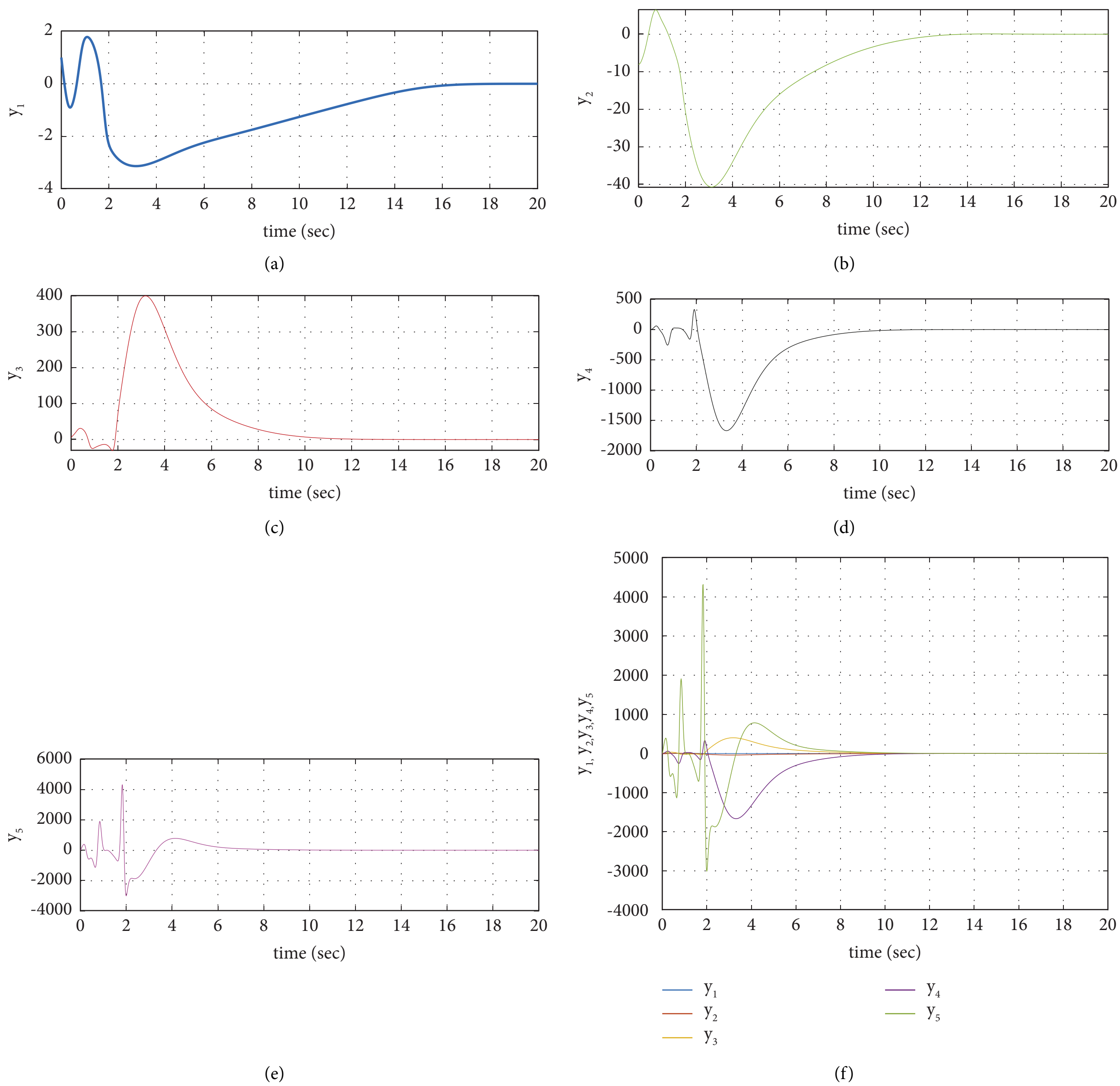

Figure 3: Graphs depicting each system's time history for nonlinear systems (23)–(27) of the backstepping controlled states (a) $y_1(t)$ for $y_1(0) = (1)$, (b) $y_2(t)$ for $y_2(0) = -8$, (c) $y_3(t)$ for $y_3(0) = 6$, (d) $y_4(t)$ for $y_4(0) = -1$, (e) $y_5(t)$ for $y_5(0) = 5$, and (f) $y_1(t), y_2(t), y_3(t), y_4(t), y_5(t)$ for $y_1(0), y_2(0), y_3(0), y_4(0), y_5(0) = (1, -8, 6, 1, 5)$.

$$
\begin{aligned}
V(y_1, z_2, z_3, z_4, z_5) ={}& \frac{1}{2}y_1^2 + \frac{1}{2}\left[y_2 + y_1 + y_1^2\right]^2 + \frac{1}{2}\left[y_3 - 2y_1^4 + 2y_2 + 2y_1y_2 + y_1^2y_2 + 2y_1^2 + y_1^3 + 2y_1\right]^2 \\
&+ \frac{1}{2}\left[-6y_1^3y_2 + 3y_2 + 3y_3 + 6y_1y_2 + 2y_1y_3 + 2y_1y_2^2 + 8y_1^2y_2 + y_1^2y_3 + y_1 + y_1^4y_2 + 3y_1^2 + 3y_1^3 + 3y_2^2 - 3y_1^4 - 11y_1^5 - 8y_1^6 + y_4\right]^2 \\
&+ \frac{1}{2}\begin{bmatrix} 3y_1 + 6y_2 + 7y_3 + 4y_4 + y_5 - 9y_1^2y_2^2 - 2y_1^3y_2^2 + 14y_1y_2 + 8y_1y_3 + 2y_1y_4 + 8y_2y_3 + 20y_1y_2^2 + 27y_1^2y_2 + 11y_1^2y_3 - 2y_1^3y_2 + y_1^2y_4 \\ -6y_1^3y_3 - 82y_1^4y_2 - 3y_1^4y_3 + 56y_1^5y_2 + 3y_1^6y_2 + 6y_1^2 + 9y_1^3 + 12y_2^2 - 2y_1^4 + 2y_2^3 - 32y_1^5 - 35y_1^6 + 103y_1^7 - 48y_1^8 + 6y_1y_2y_3 \end{bmatrix}^2 \\
&> 0, \quad \forall y_1, y_2, y_3, y_4, y_5 \neq 0, \\
\dot{V} ={}& -y_1^2 - z_2^2 - z_3^2 - z_4^2 - z_5^2 < 0, \\
\dot{V}(y_1, z_2, z_3, z_4, z_5) ={}& -y_1^2 - \left[y_2 + y_1 + y_1^2\right]^2 - \left[y_3 - 2y_1^4 + 2y_2 + 2y_1y_2 + y_1^2 + 2y_1^2 + y_1^3 + 2y_1\right]^2 \\
&- \left[-6y_1^3y_2 + 3y_2 + 3y_3 + 6y_1y_2 + 2y_1y_3 + 2y_1y_2^2 + 8y_1^2y_2 + y_1^2y_3 + y_1 + y_1^4y_2 + 3y_1^2 + 3y_1^3 + 3y_2^2 - 3y_1^4 - 11y_1^5 - 8y_1^6 + y_4\right]^2 \\
&- \left[-6y_1^3y_2 + 3y_2 + 3y_3 + 6y_1y_2 + 2y_1y_3 + 2y_1y_2^2 + 8y_1^2y_2 + y_1^2y_3 + y_1 + y_1^4y_2 + 3y_1^2 + 3y_1^3 + 3y_2^2 - 3y_1^4 - 11y_1^5 - 8y_1^6 + y_4\right]^2 \\
&- \begin{bmatrix} 3y_1 + 6y_2 + 7y_3 + 4y_4 + y_5 - 9y_1^2y_2^2 - 2y_1^3y_2^2 + 14y_1y_2 + 8y_1y_3 + 2y_1y_4 + 8y_2y_3 + 20y_1y_2^2 + 27y_1^2y_2 + 11y_1^2y_3 - 2y_1^3y_2 + y_1^2y_4 \\ -6y_1^3y_3 - 82y_1^4y_2 - 3y_1^4y_3 + 56y_1^5y_2 + 3y_1^6y_2 + 6y_1^2 + 9y_1^3 + 12y_2^2 - 2y_1^4 + 2y_2^3 - 32y_1^5 - 35y_1^6 + 103y_1^7 - 48y_1^8 + 6y_1y_2y_3 \end{bmatrix}^2 \\
&< 0, \quad \forall y_1, y_2, y_3, y_4, y_5 \neq 0.
\end{aligned} \tag{70}
$$

Since theorem (8) is satisfied, systems (23)–(27) are globally asymptotically stable.

4. Simulation Results

The plots were carried out using MATLAB. From the plots (as shown in Figure 3), it can be seen that all states converge to zero as time approaches infinity.

5. Conclusion

In this paper, we present a backstepping design-based control strategy for a strict feedback nonlinear system that includes a controller and a Lyapunov function. To cancel the nonlinear effects and achieve asymptotic stability, the backstepping controller iteratively applies Lyapunov functions at each integrator level. The given system was plotted with our control in MATLAB to verify the behavior of the system; from the graph, it was seen that all states converge to zero from any initial point as time tends to infinity, which guarantees the global asymptotic stability of the equilibrium point at the origin. This research work can also be extended from general backstepping to adaptive backstepping in the case of uncertainties.

Data Availability

The data used to support the findings of this study are available on request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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