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Research Article

Tight Focusing of Partially Coherent Vortex Beams

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Tight focusing of partially polarized vortex beams has been studied. Compact form of the coherence matrix has been derived for polarized vortex beams. Effects of topological charge and polarization distribution of the incident beam on intensity distribution, degree of polarization, and coherence have been investigated.

1. Introduction

Polarization distribution of tightly focused optical beams has drawn considerable interest in recent years and recent advances in the field of nanophotonics have contributed much to the rapid progress of the field [1–7]. Tightly focused structure of the optical field possesses three-dimensional structure, and x-, y-, and z-polarization components are involved in shaping the focal structure [1]. Role of longitudinal polarization component becomes significant in shaping the focused structure in contrast to low NA-focusing system where contribution of z component is negligible in comparison to that of transverse polarization components [1]. Input polarization of the beam becomes a dominating factor apart from complex amplitude in shaping the focused structure. For example, optical vortex produces doughnut structure in the low NA focusing, whereas existence of the doughnut in tightly focused structure of vortex beam depends on its topological charge as well as on its polarization distribution [6, 8–12].

A vortex beam has a helical phase structure with a point of undefined phase in the heart of the surface, and this point is referred to as "phase singularity." Accumulated phase variation around the singularity is an integral multiple of 2π and is referred to as "topological charge" [13]. Doughnut structure is useful in several applications ranging from optical trapping, microscopy to lithography [13–15]. Scattered vortex beam from the sample or object provides important information about the sample, and this sensing property of the optical beam is named as "singular microscopy" [16]. Helical phase structure in association with polarization distribution is also used to generate optical beams with nonuniform polarization distributions [17–19]. Optical beams with non-uniform polarization distribution are a solution of the wave equation in the cylindrical coordinates, and this family of optical beams is referred cylindrically polarized (CV) beams [19]. Azimuthally and radially polarized beams are examples of CV beams, and have been exploited in shaping the focal spot in recent years due to their unique characteristics and applications [20–23].

The azimuthally polarized nonvortex beam possesses a sharp doughnut structure and less susceptible to azimuthal aberration in comparison to circularly polarized unit-charged vortex beam [11]. However, doughnut structure with on-axis intensity null ceases to exist for the unit-charged azimuthally polarized vortex beam due to contribution from the longitudinal polarization component [11]. Focal spot may be extremely small when the incident beam possesses radial polarization distribution and such kind of beam has been used for various purposes. Therefore, the state of polarization in the focal region of an optical beam decides its shape and size. Polarization distribution of focused singular beam can play an important role in singular microscopy due to significant role of the scattered light in sensing the object.

Investigations on polarization and coherence properties of the randomly fluctuating field have also been area of interest from many years [24–28]. A fair amount of literature is available on the coherence matrix to deal with such situation. Modification has also been made in the coherence matrix to take consideration of the vectorial nature and

3D characteristics of the tightly focused beam [29–31]. Detailed study on the tight focusing of uniformly partially polarized radiation has been carried out in the past [30]. Such investigations for partially coherent vortex beams have been carried out recently [32, 33]. With increasing importance of polarized vortex beams in several applications and in physical optics, it is also important to carry out such investigations. In this paper, we have investigated effect of coherence and polarization on the intensity distribution, degree of polarization, and coherence in the focal region of a high numerical aperture systems.

2. Theory

Complex field distribution of incident quasimonochromatic beam in the focal region of a high NA aplanatic lens is given [1] as

$$E(r,z) = \left(-i\frac{A}{\lambda f}\right)\int_0^{\alpha}\int_0^{2\pi} A_2(\theta)e^{im\phi}P(\theta,\varphi), \times \begin{bmatrix} E_{xo} \\ E_{yo} \end{bmatrix} e^{ik\vec{s}\cdot\vec{r}}\sin\theta d\varphi\, d\theta, \tag{1}$$

where A is related to the optical system parameters and λ is wavelength of light in the medium with refractive index n in the focal region. θ is the focusing angle, ϕ is azimuthal angle on the incident plane, α is maximum angle of convergence ($\alpha = \theta_{\max}$), and the numerical aperture NA $= n\sin\vartheta$. P represents the polarization matrix distribution at the exit pupil, and $A_2(\vartheta)$ is apodization factor and is equal to $\cos^{1/2}\vartheta$ for an aplanatic system.

The polarization distribution matrix $P(\theta,\phi)$ at the exit pupil plane is written [9] as

$$P(\theta,\phi) = \begin{bmatrix} a[\cos\theta\cos^2\phi + \sin^2\phi] \\ \quad +b[\cos\theta\sin\phi\cos\phi - \sin\phi\cos\phi] \\ a[\cos\theta\cos\phi\sin\phi - \sin\phi\cos\phi] \\ \quad +b[\cos\theta\sin^2\phi + \cos^2\phi] \\ -a\sin\theta\cos\phi - b\sin\theta\sin\phi \end{bmatrix}, \tag{2}$$

where a and b are the strengths of the x-, and y-polarized incident beams, respectively. Strength factors are position dependent in the case of nonuniformly polarized or CV beams and constant in the case of uniformly polarized beam.

The second-order coherence properties of the vector field in the focal region of a high NA system can be studied using the coherence matrix. Compact form of the 3×3 coherence-polarization matrix can be written using 2×2CP matrix of the incident field, and given [30] as

$$W(r_1,r_2,z) = M^*(r_1,z)W_oM^T(r_2,z), \tag{3}$$

where W is 3×3 CP matrix, and element of this matrix is written as $W_{\alpha\beta}(r_1,r_2,z) = \langle E_\alpha{}^*(r_1,z)E_\beta(r_2,z)\rangle$. $\alpha,\beta = x, y, z$ and r_1,r_2 are position coordinates in the observation plane. Angle bracket and asterisk denote ensemble average and complex conjugate, respectively. M is 3×2 matrix and T represents transpose of the matrix. Complete form of matrix depends on the input polarization distribution.

The CP matrix of the incident field W_o takes on the form

$$W_o(r_{1o},r_{2o}) = \begin{bmatrix} W_{xxo} & |\mu_{xyo}|\left[W_{xxo}W_{yyo}\right]^{1/2}\exp(i\beta_o) \\ |\mu_{xyo}|\left[W_{xxo}W_{yyo}\right]^{1/2}\exp(-i\beta_o) & W_{yyo} \end{bmatrix}, \tag{4}$$

where W_{xxo} and W_{yyo} are the intensities of the x and y components; respectively, and $|\mu_{xyo}|$ and β_o are the magnitude and phase of their complex correlation coefficients. $W_{pqo}(r_{1o},r_{2o}) = \langle E^*{}_p(r_{1o})E_q(r_{2o})\rangle$ is the CP matrix of order 2×2 at the input plane, and $p,q = x,y$.

Degree of coherence and degree of polarization are the important parameters in addition to intensity distribution for the evaluation of effect of partial coherence and partial polarization. These parameters at a point r in the focal plane of high NA system are given [30] as

$$I(r_P,r_P,z) = W_{xx}(r_P,r_P,z) + W_{yy}(r_P,r_P,z) + W_{zz}(r_P,r_P,z), \tag{5}$$

$$ptP_3^2(r_P,r_P,z) = \frac{3}{2}\left[\frac{tr[W^2(r_P,r_P,z)]}{tr^2[W(r_P,r_P,z)]} - \frac{1}{3}\right], \tag{6}$$

$$\mu_{ij}(r_P,r_P,z) = \frac{W_{ij}(r_P,r_P,z)}{\left[W_i(r_P,r_P,z)W_j(r_P,r_P,z)\right]^{1/2}}, \tag{7}$$

where I, μ, and P are, respectively, intensity distribution, degree of coherence, and degree of polarization in the focal region at point r.

Strength factors of x- and y-polarized component of the incident beam are independent of the position coordinate and beam referred to as uniformly or homogeneously polarized beam. Complex field distribution in the focal region for uniformly polarized beam can be obtained by substituting proper strength factor into (2) and subsequently using (1). Using trigonometric identities Matrix M for homogeneously or uniformly polarized vortex beam can be written [10, 11] as

$$M = \begin{bmatrix} \left(I_m e^{im\phi_P} - 0.5\left\{I_{m-2}e^{i(m-2)\phi_P} + I_{m+2}e^{i(m+2)\phi_P}\right\}\right) & 0.5i\left(-I_{m-2}e^{i(m-2)\phi_P} + I_{m+2}e^{i(m+2)\phi_P}\right) \\ 0.5i\left(I_{m+2}e^{i(m+2)\phi_P} - I_{m-2}e^{i(m-2)\phi_P}\right) & \left(I_m e^{im\phi_P} + 0.5\left\{I_{m-2}e^{i(m-2)\phi_P} + I_{m+2}e^{i(m+2)\phi_P}\right\}\right) \\ -\left(I_{m-1}e^{i(m-1)\phi_P} + I_{m+1}e^{i(m+1)\phi_P}\right) & i\left(-I_{m-1}e^{i(m-1)\phi_P} + I_{m+1}e^{i(m+1)\phi_P}\right) \end{bmatrix},$$

$$I_m(\nu, u) = \left(\frac{1}{2}\right)2\pi i^m \int_0^\alpha \cos^{1/2}\theta(1+\cos\theta)J_m\left[\frac{\nu}{\sin\alpha}\sin\theta\right]\exp\left[i\frac{u}{\sin^2\alpha}\cos\theta\right]\sin\theta\, d\theta, \tag{8}$$

$$I_{m\pm1}(\nu, u) = \left(\frac{1}{2}\right)2\pi i^{(m\pm1)} \int_0^\alpha \cos^{1/2}\theta J_{m\pm1}\left[\frac{\nu}{\sin\alpha}\sin\theta\right]\exp\left[i\frac{u}{\sin^2\alpha}\cos\theta\right]\sin^2\theta\, d\theta,$$

$$I_{m\pm2}(\nu, u) = \left(\frac{1}{2}\right)2\pi i^{(m\pm2)} \int_0^\alpha \cos^{1/2}\theta\ (1-\cos\theta)J_{m\pm2}\left[\frac{\nu}{\sin\alpha}\sin\theta\right]\exp\left[i\frac{u}{\sin^2\alpha}\cos\theta\right]\sin\theta\, d\theta.$$

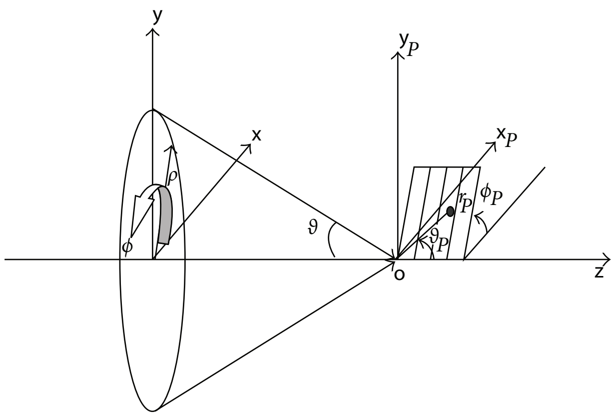

Figure 1: Focusing geometry of optical beams.

Here $\nu = kr_P \sin\theta_P \sin\alpha, u = kr_P \cos\theta_P \sin^2\alpha$ and $J_m(\cdot)$ is Bessel Function.

In the case of nonvortex beam ($m = 0$), (7) corresponds to the matrix M for uniformly partially polarized beam [30]. In our study, we have assumed incident field as completely coherent and only focused on the partially polarized beam.

3. Results

Intensity distribution, degree of polarization, and coherence in the focal region of an aplanatic system can be obtained using (5)–(7) with proper polarization matrix for different input cases for input coherence matrix given by (3). We have presented results at the focal plane ($u = 0$) of an air aplanatic lens with NA $= 0.9$. Results were evaluated for homogeneously polarized [30] and completely unpolarized [31] cases of nonvortex beam compared, and good agreement is found between both.

Results of intensity distribution and degree of polarization for polarized vortex beams are shown in Figure 2 and for two values of topological charge with $\mu_o = 0.5$ and $\mu_o = 0.8$ for $\beta = \pi/2$. Figures 2(a) and 2(b) represent intensity distribution for vortex beam with $m = 1$, whereas intensity distribution for beam with $m = 2$ is shown in Figures 2(c) and 2(d). Intensity possesses circular symmetry with lowest value at the center, and it decreases even to zero for fully correlated case. Size of low-intensity region increases with an increase in the topological charge. The distribution of degree of polarization as shown in second row of Figure 2 shows circular symmetry with rings with fully polarized at the center, and degree of polarization fluctuates around center. Figures 2(e)–2(h) represent change in degree of polarization due to change in topological charge and coherence. Intensity profiles of vortex beams with $m = 0, 1$, and 2 in the focal plane are shown by curves a, b, and c in Figure 3 for $\mu = 0.5$ and $\beta = \pi/2$. Intensity value at the focal point decreased with an increase in the topological charge and also decreases with an increase in the correlation coefficient. Figure 4 shows distribution of degree of polarization for non-vortex ($m = 0$) and vortex beams with $m = 1$ and 2. Degree of polarization for vortex beams possesses unit value at the center, and its variation around the center is sharp for unit-charged beam. However non-vortex beam possesses unit value away from the center. Degree of coherence for unit-charged vortex beam with $\beta = \pi/2$ and two values of correlation coefficients are shown in Figure 5 in the focal plane along $\phi = \pi/4$. Curves a and b in Figure 5 represent profile of μ_{xy} and μ_{xz} for $\mu_o = 0.5$, whereas curves c and d show results for $\mu_o = 0.8$. Results of μ_{yz} matche with results of μ_{xz} for both cases.

4. Conclusions

Investigations on the tightly focused vortex beams have been carried out using the vectorial diffraction integral. Contribution of different polarization components in the focal region according to the incident beam polarization and vortex characteristics have direct impact on the intensity distribution, degree of polarization, and degree of coherence in the focal region.

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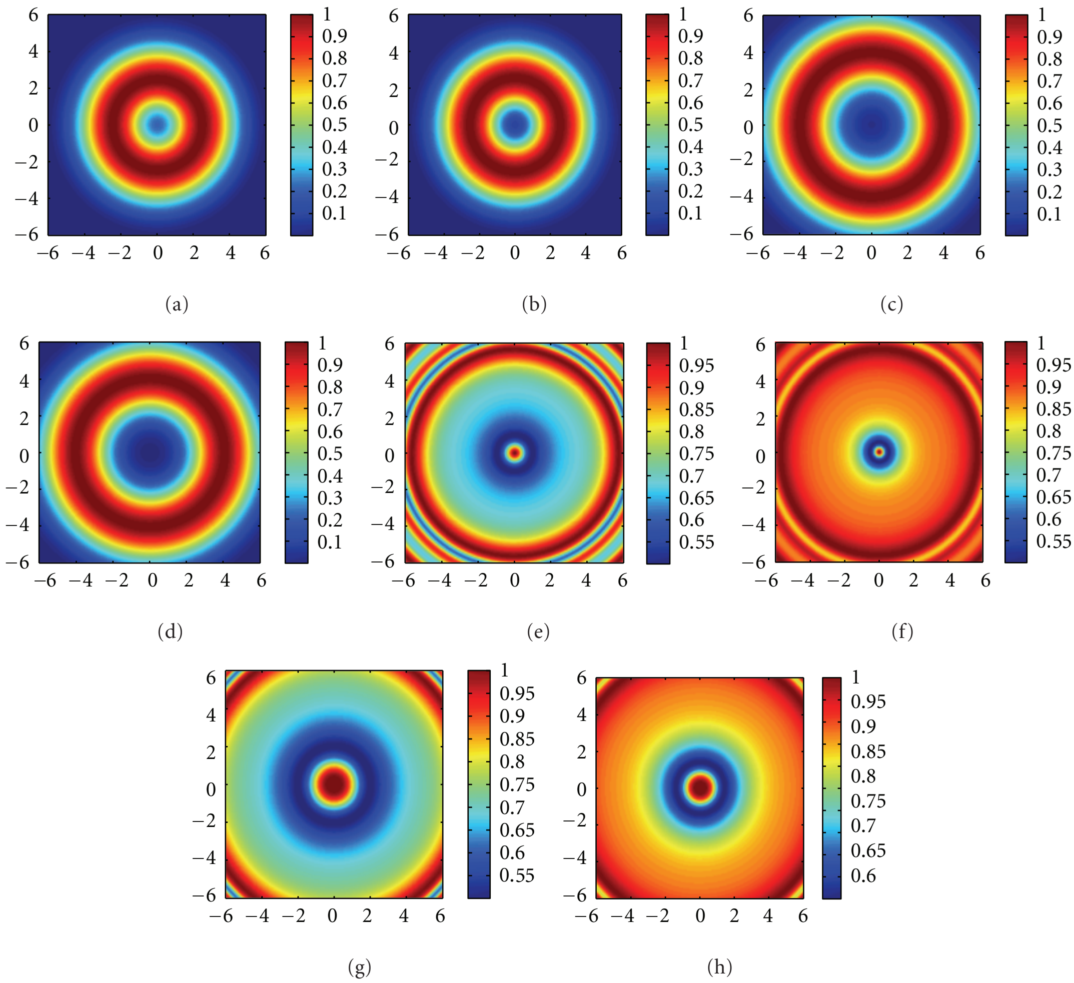

FIGURE 2: Intensity distribution of uniformly polarized vortex beams at the focal plane of an air aplantic lens with NA = 0.9: with $m = 1$ and $\beta = \pi/2$ for (a) $\mu_o = 0.5$ (b) $\mu_o = 0.8$; for $m = 2$ and $\beta = \pi/2$ with (c) $\mu_o = 0.5$ (d) $\mu_o = 0.8$; degree of polarization for $m = 1$ and $\beta = \pi/2$ for (e) $\mu_o = 0.5$ (f) $\mu_o = 0.8$; for $m = 2$ and $\beta = \pi/2$ with (g) $\mu_o = 0.5$ (h) $\mu_o = 0.8$.

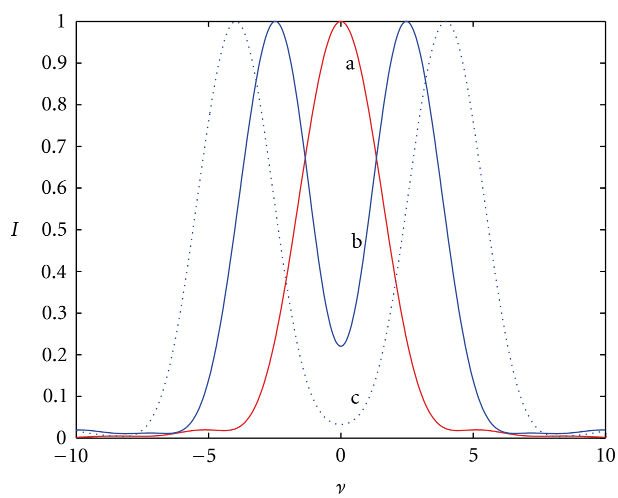

FIGURE 3: Intensity profile of uniformly polarized vortex beams with NA = 0.9, $\mu_o = 0.5$, $\beta = \pi/2$, for m (a) 0, (b) 1, and (c) 2.

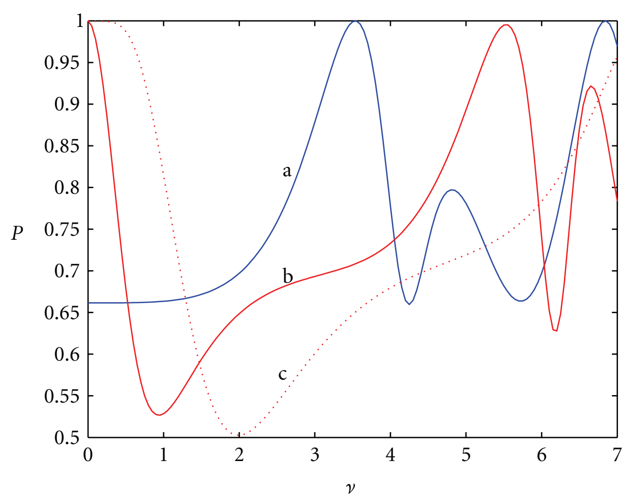

FIGURE 4: Degree of polarization along $\pi/4$ in focal plane of an aplanatic lens with NA = 0.9 of uniformly polarized vortex beams $\mu_o = 0.5$, $\beta = \pi/2$, for m (a) 0 (b) 1, and (c) 2.

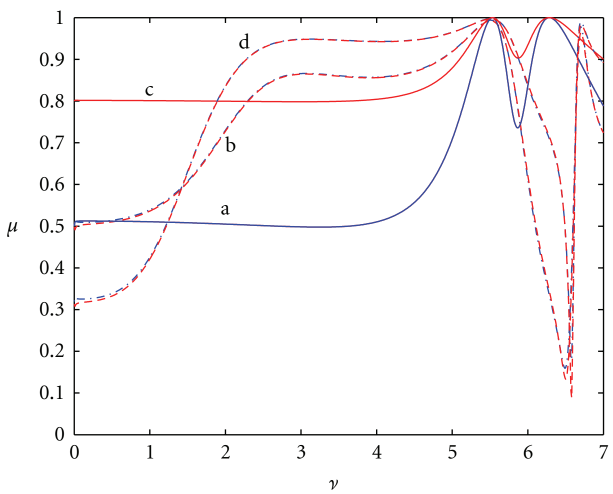

Figure 5: Degree of coherence along $\pi/4$ in focal plane of aplanatic lens with NA $= 0.9$ for uniformly polarized vortex beams with $m = 1$ $\mu_o = 0.5$, $\beta = \pi/2$; (a) μ_{xy} (b) μ_{xz}; for $\mu_o = 0.8$ (c) μ_{xy} (d) μ_{xz}.

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