Hindawi Publishing Corporation
International Journal of Optics
Volume 2012, Article ID 863875, 9 pages
doi:10.1155/2012/863875

*Research Article*

# Complex 3D Vortex Lattice Formation by Phase-Engineered Multiple Beam Interference

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Received 8 June 2011; Accepted 20 July 2011

Academic Editor: Shunichi Sato



We present the computational results on the formation of diverse complex 3D vortex lattices by a designed superposition of multiple plane waves. Special combinations of multiples of three noncoplanar plane waves with a designed relative phase shift between one another are perturbed by a nonsingular beam to generate various complex 3D vortex lattice structures. The formation of complex gyrating lattice structures carrying designed vortices by means of relatively phase-engineered plane waves is also computationally investigated. The generated structures are configured with both periodic as well as transversely quasicrystallographic basis, while these whirling complex lattices possess a long-range order of designed symmetry in a given plane. Various computational analytical tools are used to verify the presence of engineered geometry of vortices in these complex 3D vortex lattices.

## 1. Introduction

Optical wavefront dislocations occur naturally in an interference field during the superposition of three or more coherent plane waves [1–4]. An optical screw dislocation in a given plane, also called as optical vortex or phase singularity, is a point phase defect where the amplitude is zero and the phase is indeterminate [4]. The wavefront of the vortex beam is associated with the spiral flow of the electromagnetic energy. Vortices are thus points or lines of absolute darkness with undefined phase embedded within the spatial structure of the wave field. Regular wave function can have singular points only when the real as well as imaginary parts of the wave function vanishes simultaneously [3]. These two conditions define two surfaces in space whose intersection determines the position of vortex lines. However, the vanishing of the wave function is the necessary but not the sufficient condition for the existence of vortex. They exist only if the circulation of phase around the line, where the wave function vanishes, is different from zero. Diverse approaches are implemented to generate array of optical vortices, called as vortex lattices [1, 4–18]. Vortex lattices possess the property of orbital angular momentum due to spatial distribution of intensity and phase distribution of the optical field. We find these phenomenal structures and their dynamics also in other physical systems like Bose-Einstein condensates, super fluidics, and so forth [19]. Among various fabrication methods for 2D and 3D photonic lattice structures, the fabrication approach involving a designed combination of light fields dynamically sculpting the structures to control the light is proven to be versatile in terms of scalability, tunability, and large area fabrication flexibility in a single step [20–25]. It has also been shown that optical field patterns belonging to all 14 Bravais crystallographic lattice 3D structures can be formed by designed plane wave interference [26, 27]. By means of a spatial light modulator (SLM-) assisted approach, the basic multiple beam interference technique has been extended to reconfigurable single-step fabrication process for both 2D and 3D periodic as well as transversely quasicrystallographic photonic lattices [1, 4, 13–16, 24, 25].

Various periodic as well as transversely quasicrystallographic vortex lattices have been investigated in view of their applications especially in the field of nondiffracting beams [14]. Recently, we demonstrarted for the first time by a phase only spatial light modulator (SLM-) assisted reconfigurable optical phase engineering approach, 3D chiral lattices belonging to periodic as well as transversely quasicrystallographic basis which possessed spiraling vortices [13]. We

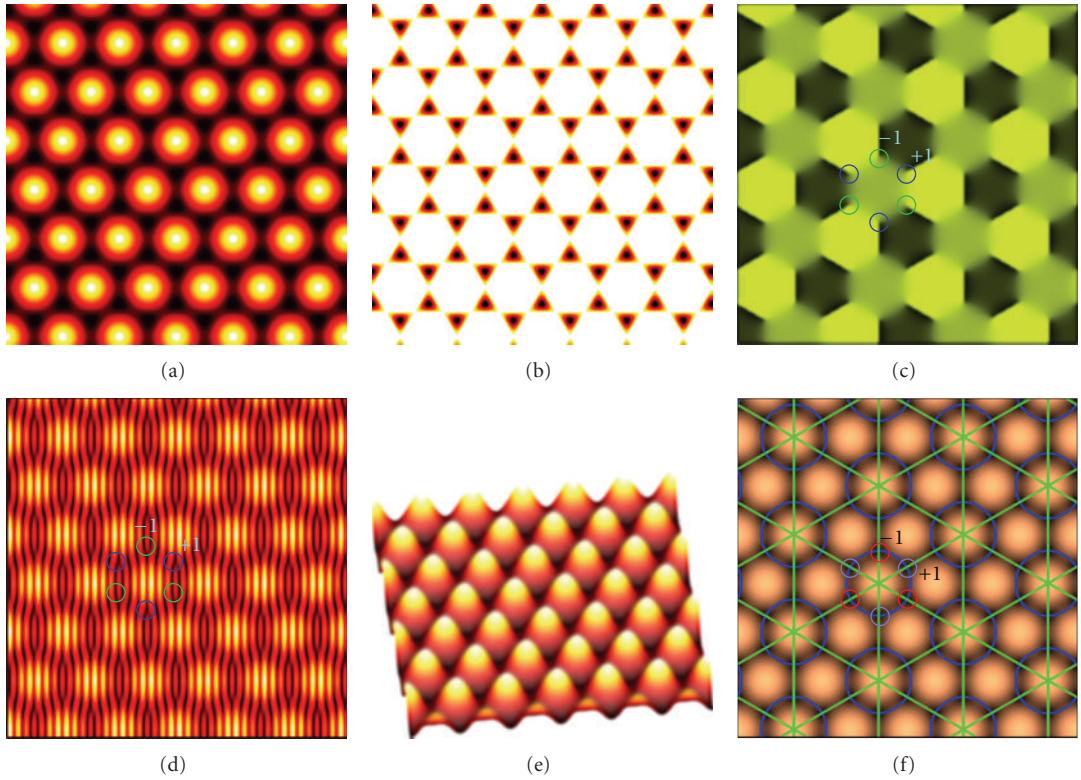


Figure 1: Analysis of three noncoplanar beam interference patterns possessing a distribution of single-charged vortices. (a) Intensity distribution in $x$-$y$ plane. (b) Intensity distribution with a lower threshold value in order to highlight the darker regions alone. (c) Phase profile. (d) The fork formation while the pattern is interfered with a plane wave launched at a large angle from the axis. (e) Intensity mesh plot. (f) Zero crossing plot (blue line: zeros of $\mathrm{Re}\,E(r)$ and green line: zeros of $\mathrm{Im}\,E(r)$).

envisage that the photonic structures with the spiral phase modulation will add a new degree of freedom and would open up the possibility of forming lattices with well-defined phase defects with the special property of orbital angular momentum [1]. In this paper, we present the formation and analysis of 3D vortex lattice structures by means of interbeam relative phase engineering of interfering plane waves. It has been observed that minimum three plane waves are required to form the lattice structure carrying optical vortices in case of multiple plane wave interference [5]. Here, we investigate the combinations of multiples of three plane waves having a designed relative phase between them to form novel 3D chiraling vortex lattices with a long-range order of designed symmetry in a given transverse plane. These real-time tunable spiraling vortex superlattice structures are envisaged to find novel applications in advanced multiple particle trapping/manipulation [1, 28], photonic cloaking [29], dense coded satellite communications and dynamic optical vortex meteorology [1, 30], and so forth.

## 2. Wave Design for Complex Vortex Lattices

It has been observed that in the interference field of three or more plane waves, the optical vortices are formed [5]. Recently, we had investigated the fabrication of 3D photonic chiral lattices [13]. In the following, we formulate the wave design of interfering beams for complex 3D spiraling lattices imbedded with tunable vortex clusters. As three plane waves being the minimum number of interfering plane waves for the resultant vortex-embedded irradiance field, in our wave design, we choose $n = 3$ plane waves as the basic combination of noncoplanar equidistant side beams. With respect to one another, we introduce an initial phase of $\varphi_{\mathrm{m}} = (2\pi/3) \times m$ to these beams, where $m$ is from 1 to 3. Then we define an integer "$p$" as a multiplying factor with $n$, leading to $q = n \times p$ number of axially equidistant noncoplanar plane waves, where $n = 3$. Moreover, their Fourier components in turn lie axially equidistantly on a circle [23]. The complex irradiance profile formed by the interference of these coherent beams is subsequently perturbed by means of an axially launched noncoplanar plane wave [4, 13]. Considering the multiples of $n = 3$, the overall irradiance profile of the interference pattern of $q + 1$ linearly polarized plane waves is given by [13, 25],

$$\mathrm{I}(\mathbf{r}) = \sum_{i=0}^{q} |\mathbf{E}_i|^2 + \sum_{i=0}^{q} \sum_{\substack{j=0 \\ j \neq i}}^{q} \mathbf{E}_i \mathbf{E}_j^* \cdot \exp\left[ i\left(\mathbf{k}_i - \mathbf{k}_j\right) \cdot \mathbf{r} + i\varphi_{ij} \right], \tag{1}$$

where $\mathbf{E}_{i,j}$, $\mathbf{k}_{i,j}$, $\mathbf{r}$, and $\varphi_{ij}$ are the complex amplitudes, the wave vectors, position vector, and the difference in initial offset phase of the interfering side beams, respectively. It

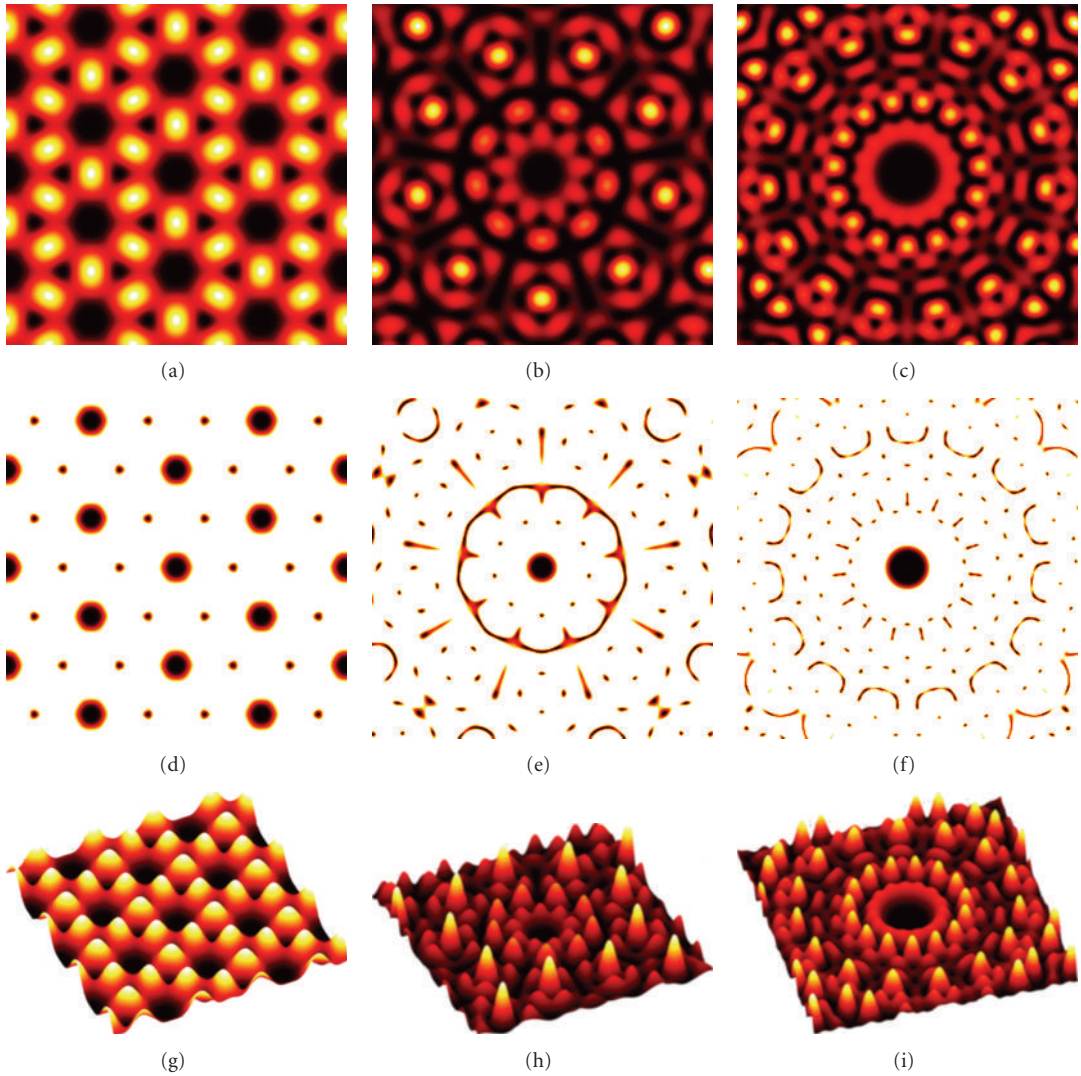


Figure 2: Simulation of complex 2D vortex lattice intensity profile possessing a distribution of higher-order topological charged as well as single charged vortices. Taking the multiples of relatively phase-shifted three beams, the numbers of interfering beams involved are, respectively 6 beams (first column), 9 beams (second column), and 15 beams (third column). (a)–(c) Intensity distribution in $x$-$y$ plane. (d)–(f) Intensity distribution with a lower threshold value in order to high light the darker regions alone. (g)–(i) Intensity mesh plots.

has been observed that interference between an L-G beam and a plane wave transforms the azimuthal phase variation into an azimuthal intensity variation [28], and in accordance with the "$p$" number of $2\pi$ phase variations, "$p$" spiral arms could be formed. This is analogous to the unfolding of a vortex of higher-order topological charge "$p$" into "$p$" singly charged vortices with identical sign in the presence of a nonsingular perturbing beam [4]. So in fact, the presence of the perturbing beam shifts the position of the generated vortices in the interference field. Similar behaviour is also applicable in the case of a distribution of vortices under consideration [31, 32]. Moreover, it has also been observed that a distribution of vortices of small localized core function carrying unit charge of identical sign in a host Gaussian beam envelop begin to rotate as a rigid pattern [32]. As given in equation (1), the dynamically generated interference pattern formed by means of the interference of the designed relatively phase engineered plane waves possesses a designed distribution of vortices within complex spiraling vortex lattices. In the presence of the axially launched perturbing optical field, these $q + 1$ plane waves with specific beam geometries interfere to sculpt complex 3D photonic lattices embedded with vortices.

## 3. 2D Vortex Lattices: Folded Phase Dislocations

In this section, we analyse our results on complex 2D vortex lattices formed by following the wave design given above. In a 2D vortex lattice structure, the transverse irradiance profile of the interference pattern remains invariant along the $z$-direction. So we consider, in this section, the interference of only "$q$" axially equidistant side beams in the absence of a

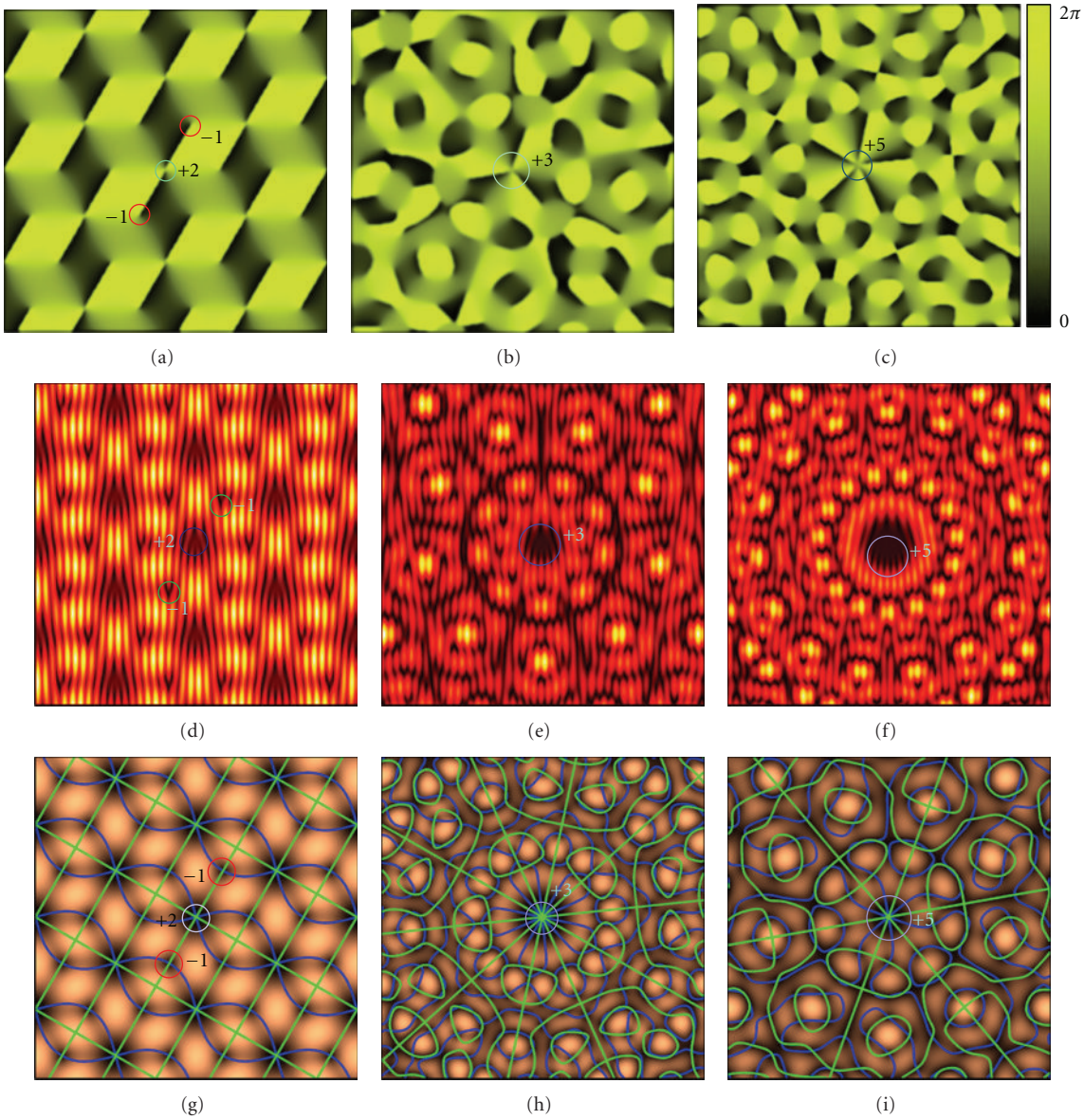


Figure 3: Computational analysis of complex 2D vortex lattices. Taking the multiples of relatively phase-shifted three beams, the numbers of interfering beams involved are, respectively, 6 beams (first column), 9 beams (second column), and 15 beams (third column). (a)–(c) Phase profile. (d)–(f) The fork formation while the pattern is interfered with a plane wave launched at a large angle from the axis. (g)–(i) Zero crossing plot (blue line: zeros of $\mathrm{Re}\,E(r)$ and green line: zeros of $\mathrm{Im}\,E(r)$). Circle at the center in cyan color highlights the higher-order vortex.

perturbing central beam. We call such vortices as "folded" [4], as in the absence of an additional nonsingular perturbing beam, they remain in their original higher-order topological charged state without being unfolded into singly charged vortices of identical sign. In Figures 1–3, there are detailed analyses of various 2D vortex lattices for varying values of "$q$." For $p = 1$, the lattice structure has a triangular basis embedded with vortex structure of honey comb lattice. The computed phase patterns for respective lattice structures show the $2\pi$ phase variations leading to phase dislocations as given in Figure 1(c) and Figures 3(a)–3(c). And these phase profiles clearly indicate the presence of vortex distribution. As given in Figure 1(d) and Figures 3(d)–3(f), as the lattice forming beams are interfered with an additional plane wave, launched with a large angle with respect to the axis, this leads to the formation of forks as the clear classical signature of the positions of the phase dislocations.

Further, the direction of the generated fork structure indicates the relative sign of the topological charge. Considering the resultant complex amplitude $E(r)$ by the superposition of beams, the optical vortex occurs at the intersection lines of the surfaces given by zeros of $\mathrm{Re}\,E(r)$ and $\mathrm{Im}\,E(r)$ [3]. These lines are plotted for the case of an $x$-$y$ plane of complex amplitude for the unperturbed structure in

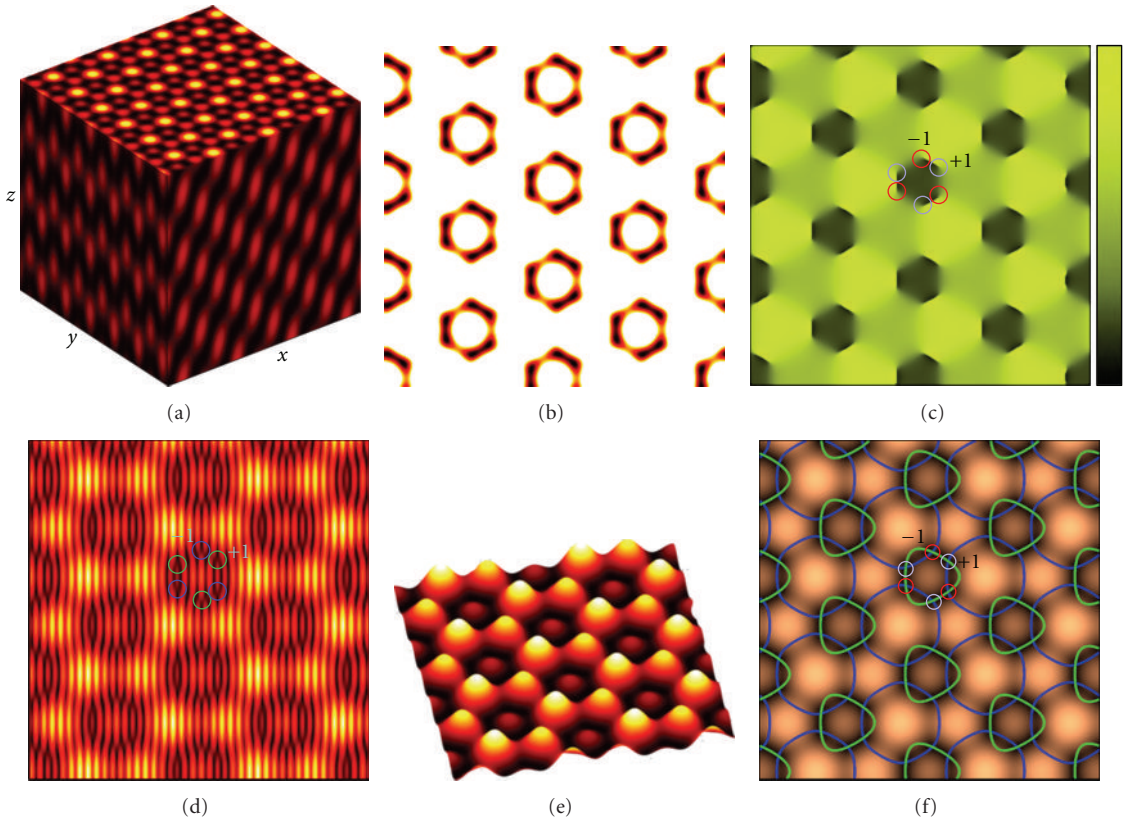


Figure 4: Analysis of vortices in a 3D photonic lattice formation by 3 + 1 noncoplanar beam interference. (a) 3D intensity distribution. (b) Intensity distribution of $x$-$y$ plane with a lower threshold value in order to highlight the darker regions alone. (c) Phase profile. (d) The fork formation while the pattern is interfered with a plane wave launched at a large angle from the axis. (e) Intensity mesh plot. (f) Zero crossing plot (blue line: zeros of $\mathrm{Re}\,E(r)$ and green line: zeros of $\mathrm{Im}\,E(r)$).

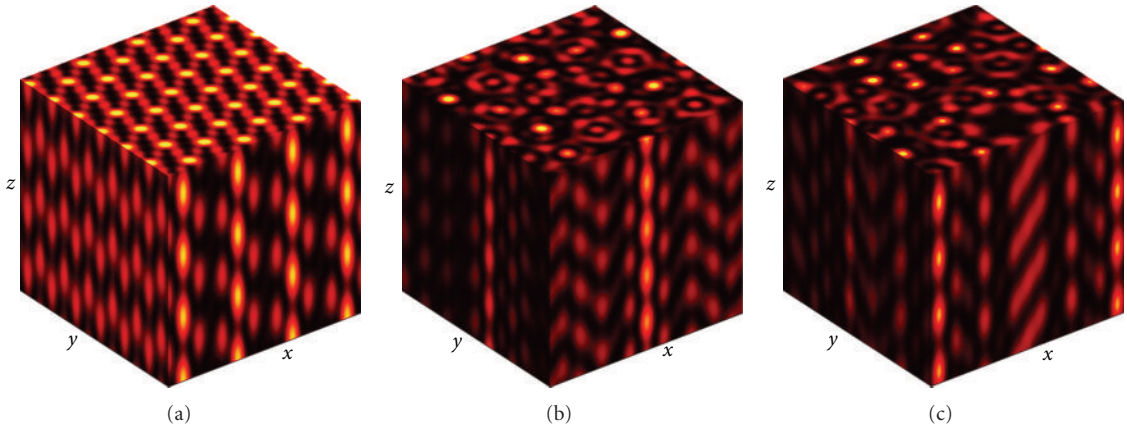


Figure 5: Computer simulations of intensity distributions of complex 3D photonic vortex lattices. Taking the multiples of relatively phase-shifted three beams, where the numbers of interfering beams involved are, respectively, (a) 6 + 1 beams, (b) 9 + 1 beams, and (c) 15 + 1 beams.

Figures 3(g)–3(i). By tuning the value of "$p$," we could design the lattice structures embedded with higher-order vortices, where, in the present case, the higher-order topological charge is directly proportional to the integer multiples of the relatively phase shifted three plane waves. This is being verified in Figures 2 and 3. Unlike the normal transversely quasicrystallographic photonic lattice formation or photonic vortex lattice formation, in the case of this complex vortex lattice where $p = 5$, we get a variant of 15-fold transversely quasicrystallographic structure with a topological charge of charge 5 at the center. We have used five-fold multiple of three beams leading to a 15-beam configurations to realize

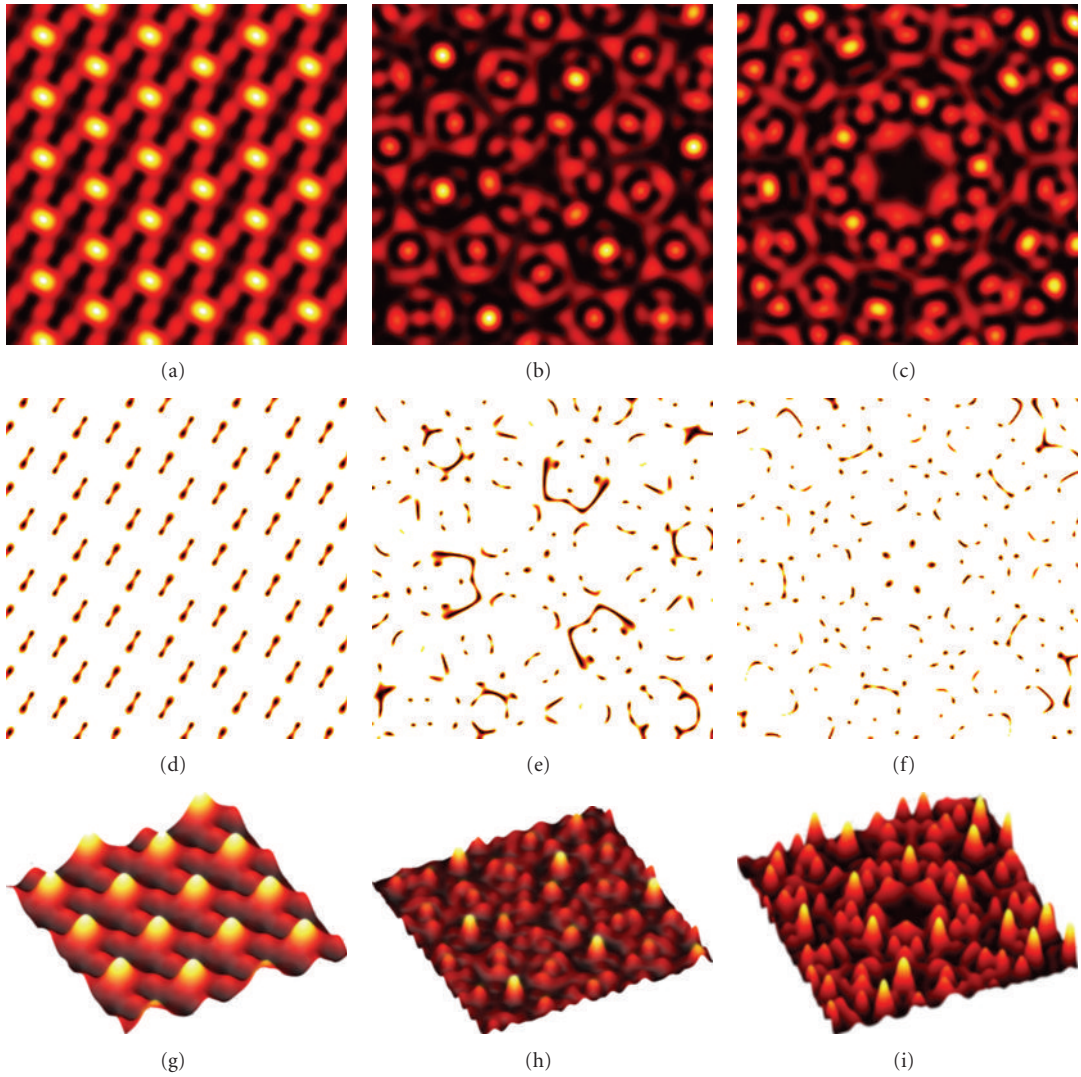


Figure 6: Computational analysis of complex 3D complex vortex lattices. Taking the multiples of relatively phase-shifted three beams, where the numbers of interfering beams involved are, respectively, 6 + 1 beams (first column), 9 + 1 beams (second column), and 15 + 1 beams (third column). (a)–(c) Intensity distribution in $x$-$y$ plane. (d)–(f) Intensity distribution with a lower threshold value in order to highlight the darker regions alone. (g)–(i) Intensity mesh plots.

this. These complex 2D vortex lattices could be further configured as spiraling vortex lattices in which photonic lattices embedded with vortex spirals could be formed.

## 4. Complex 3D Spiraling Vortex Lattices: Unfolded Phase Dislocations

In order to generate complex 3D vortex lattices, we perturb the interference field with an axially launched plane wave as per the wave design formula for $q + 1$ interfering beams given in (1). In the simplest case, by keeping $p = 1$, four plane wave interference including the axially launched central beam forms a 3D photonic lattice [23, 24] as depicted in Figure 4. As given in Figures 5, 6, and 7, with various analytical tools, the perturbing central beam "unfolds" the higher-order vortices into singly charged vortices of identical sign and shifts them away from their respective higher-order vortex core centers [4, 32]. As shown in Figure 5, various 3D vortex lattices could be realized for varying values of $p$. In the spiraling vortex 3D photonic lattices as we have designed, a variant of 15-fold transversely quasicrystallographic vortex lattice could be unfolded into a variant of spiraling vortex transversely photonic quasicrystallographic lattice with a 5-fold symmetry distribution of bright lattice points spiraling with respect to the central axis. The identical sign of the unfolded single-charged vortices at the center is also verified by the direction of the multiple forks formed as given in Figures 8(a)–8(c). The complex spiral arms are formed while the generated photonic vortex lattice forming wave is superposed with a spherical wave (Figures 8(d)–8(f)). The interference intensity distribution of different planes along the $z$-direction clearly depicts the spiraling vortex lattice as given in Figure 9. As we move from one plane to another for varying values of "$Z$" of a Cartesian coordinate

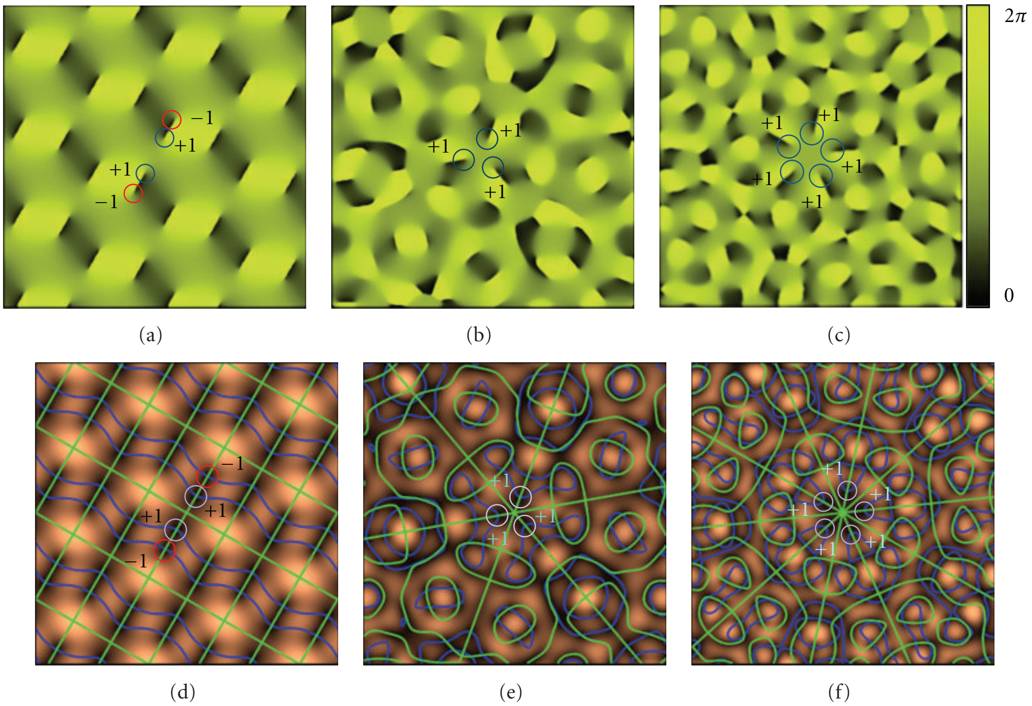

Figure 7: Computational analysis of complex 3D vortex lattices. Taking the multiples of relatively phase-shifted three beams, the numbers of interfering beams involved are, respectively, 6 beams (first column), 9 beams (second column), and 15 beams (third column). (a)–(c) Phase profile. (d)–(f) Zero crossing plot (blue line: zeros of Re $E(r)$ and green line: zeros of Im $E(r)$). Under perturbation, the unfolding and shifting of the vortices are clearly visible and circled to identify the position.

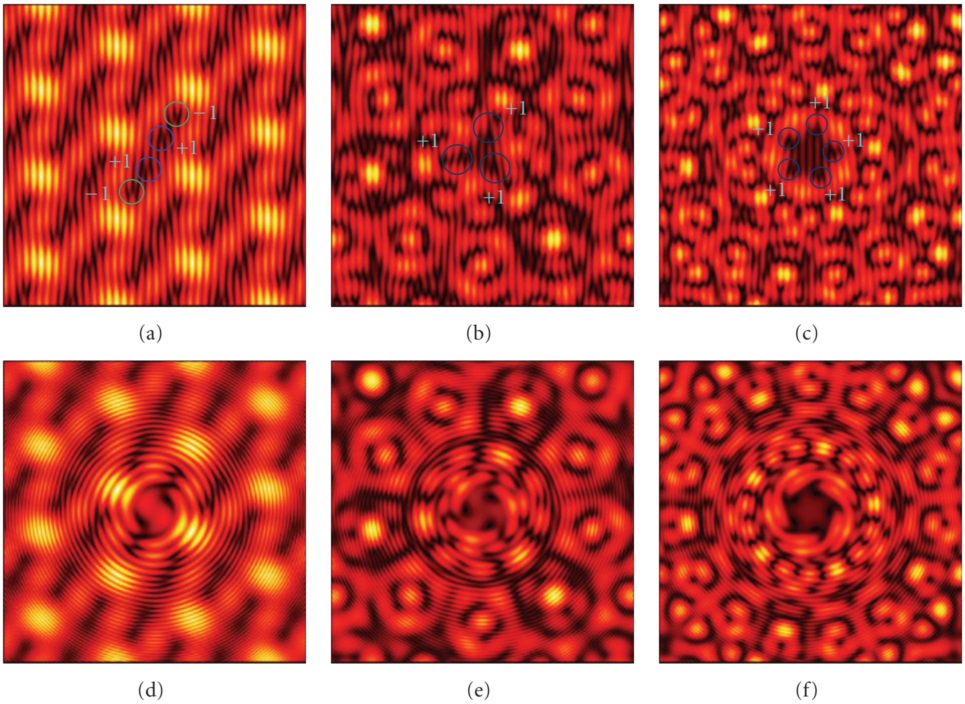

Figure 8: Analysis of presence of singularity in complex 3D vortex lattices. Taking the multiples of relatively phase-shifted three beams, the numbers of interfering beams involved are, respectively, 6 beams (first column), 9 beams (second column), and 15 beams (third column). (a)–(c) The fork formation, while the pattern is interfered with a plane wave launched at a large angle from the axis. (d)–(e) The spiral formation while the pattern is interfered with a spherical wave. The points of singularities are clearly visible as the signature of the presence of vortex distribution in the designed symmetry.

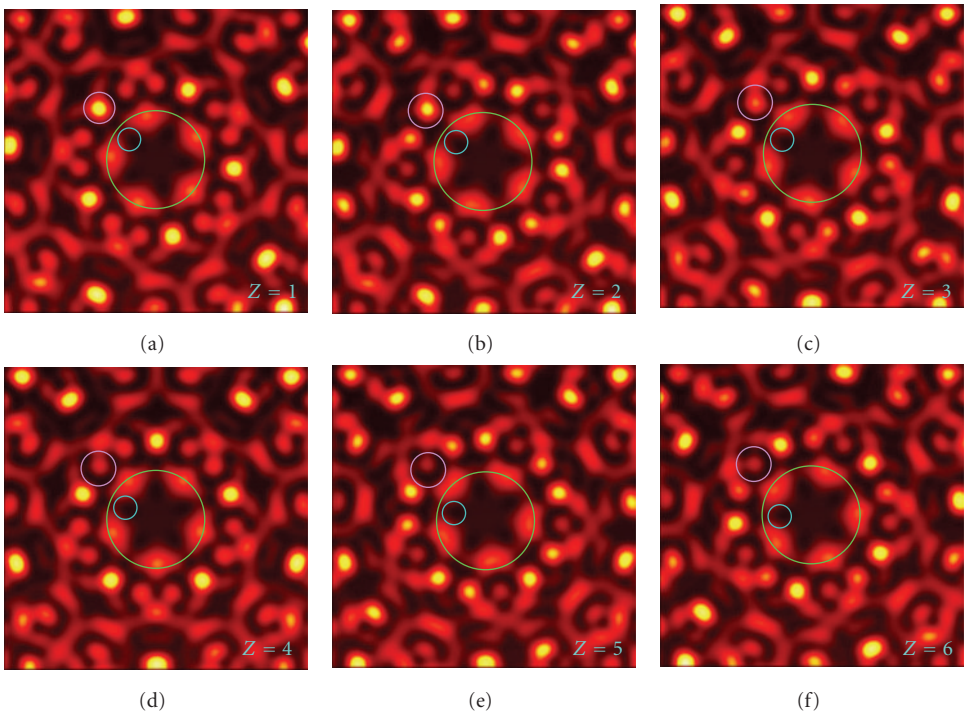


Figure 9: Computational analysis of spiraling complex 3D vortex lattice formed in the presence of a perturbing beam and a fivefold multiple of relatively phase-shifted three beams leading to 15 + 1 noncoplanar multiple beam interference. (a)–(f) The intensity distributions of $x$-$y$ planes while the value of "$Z$" is varied along the direction of propagation. The circled region (cyan) visualizes the spiraling vortex distribution, and the circled region (magenta) depicts how the energy gets coupled to the adjacent region, while the complex vortex lattice structure makes a whirling movement along the direction of propagation.

system, the adjacent dark lattice points become bright as the lattice axially spirals. The spiraling vortices embedded in a centrally-intertwined 3D vortex lattice (Figure 5(c)) are depicted by slicing different $x$-$y$ planes as given in Figure 9. By tuning the values of $p$, we could even design higher-order transverse rotational symmetry spiraling 3D vortex lattice structures.

## 5. Conclusion

We have presented the formation of diverse complex 3D vortex lattices by a designed superposition of multiple plane waves. By means of multiples of relatively phase-engineered three plane waves, we have formed complex structures with designed phase dislocations. These complex 3D lattice structures carrying even spiraling vortices are computationally investigated with various computational analytical tools. These complex lattices embedded with higher-order topological loci of darkness in folded as well as unfolded forms are envisaged to find applications in various fields of photonics in all-optical high-density information processing, real-time tunable adaptive nulling interferometry, ultra-high-resolution microscopy [1], and so forth.

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