Research Article

Laser Doppler Signal Denoising Based on Wavelet Packet Thresholding Method

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In laser Doppler velocimeter (LDV), calculation precision of Doppler shift is affected by noise contained in Doppler signal. In order to restrain the noise interference and improve the precision of signal processing, wavelet packet threshold denoising methods are proposed. Based on the analysis of Doppler signal, appropriate threshold function and decomposition layer number are selected. Heursure, sqtwolog, rigrsure, and minimaxi rules are adopted to get the thresholds. Processing results indicate that signal-to-noise ratio (SNR) and root mean square error (RMSE) of simulated signals with original SNR of 0 dB, 5 dB, and 10 dB in both low- and high-frequency ranges are significantly improved by wavelet packet threshold denoising. A double-beam and double-scattering LDV system is built in our laboratory. For measured signals obtained from the experimental system, the minimum relative error of denoised signal is only 0.079% (using minimaxi rule). The denoised waveforms of simulated and experimental signals are much more smooth and clear than that of original signals. Generally speaking, denoising effects of minimaxi and saqtwolog rules are better than those of heursure and rigrsure rules. As shown in the processing and analysis of simulated and experimental signals, denoising methods based on wavelet packet threshold have ability to depress the noise in laser Doppler signal and improve the precision of signal processing. Owing to its effectiveness and practicability, wavelet packet threshold denoising is a practical method for LDV signal processing.

1. Introduction

As having advantages as high sensitivity, high precision, wide measuring range, and noncontact measurement, the laser Doppler velocimeter (LDV) has widespread applications in industry and scientific research [1–6]. Research and design of LDV have drawn a lot of attention from researchers [7–11]. Double-beam and double-scattering optical structure adopted in our research is mainly used for the contactless velocity measurement of solid and fluid. For example, in high-speed mill, laser Doppler velocimeters are utilized to measure the accurate speed parameters of the rolling strip for the precise control of the strip rolling thickness, which has important significance for the production of high-quality rolling products. And in some scientific research studies, LDV is adopted for the velocity

measurements of fluid, gas fluid, and flame (such as exhaust flame of rocket engine). Applications of LDV put forward requirement for the real-time performance of signal processing. Meanwhile, for commercial applications such as high-speed mill and fluid velocity measurement, the cost of signal processing system is limited. Therefore, a complex denoising method is not applicable for LDV. The aim of our research is to find a practical, stable, reliable, and less-complicated noise reduction method for the laser Doppler velocimeter. As the LDV has a wide measuring range, from statics to hypersonics, the distribution range of Doppler frequency is wide. Meanwhile, because of the complexity of the surface morphology of tested solid and the change of diameter of measured particles in fluid, the broadening of the peak spectrum of Doppler signal often occurs in practical applications. The above reasons lead to the poor

performance of denoising methods based on frequency-domain analysis in LDV. The time-frequency analysis method is more applicable for LDV signal denoising.

Compared with wavelet denoising, wavelet packet denoising subdivides the high-frequency signal and can analyze the signal better. In this paper, the selection of parameters, such as the basis function, the number of decomposition layers, and the threshold determination method, which directly affect the noise reduction effect, is studied in detail through the analysis of experimental signal and the comparison of SNR and RMSE. And the optimal parameters are determined. The main novelty of the proposed work can be summarized as follows: (1) the study of the applicability of wavelet packet denoising method to LDV signal; (2) the construction of double-beam and double-scattering experiment system; (3) the optimization of the parameters of wavelet packet threshold denoising method applied in LDV signal. The study provides a practical and reliable method for LDV signal denoising in application.

2. Principle of Laser Doppler Velocimeter

In this experiment, the double-beam and double-scattering model in optical heterodyne detection mode was selected for the detection of Doppler shift. The Doppler shift in the double-beam and double-scattering model is independent of the scattered light and is only related to the directions of the two incident light beams. In an actual measurement, two beams W_1 and W_2 are incident on the surface of the moving object, forming a very small spot on the surface of the object measured. Both incident beams are scattered. The basic model is shown in Figure 1.

The relationship between Doppler shift and velocity is as follows:

$$V = \frac{\lambda}{2 \sin \theta} \Delta f, \quad (1)$$

where Δf is the Doppler shift, θ is the angle between the incident beam and the intersection bisector, and λ is the laser wavelength. From equation (1), we know that Δf is a linear function of velocity V , so in the actual speed measurement, if we can accurately determine the Doppler shift, we are able to quickly calculate the corresponding speed.

3. Selection and Principle of Wavelet Packet Denoising Method

3.1. Selection of Wavelet Packet Denoising Method. Owing to the complexity of noise components of Doppler velocity measurement signals, the noise signal may be distributed in different time-frequency subspaces. The wavelet-based signal analysis is not able to re-decompose the signal in the high-frequency region, and the high-frequency noise rejection is not ideal. To deal with this shortcoming, wavelet packets are applied to process Doppler signals, which, compared with wavelet transforms, can further decompose the high-frequency components that are not subdivided in the multiresolution analysis

FIGURE 1: Double-beam and double-scattering model.

and can thus readily eliminate the noises in each spectrum range of the signal [12, 13].

Compared with other wavelet packet denoising methods such as modulus maxima reconstruction denoising based on signal singularity and spatial correlation denoising based on signal correlation, threshold denoising has the characteristics of low computation complexity, good denoising performance, and applicability to low signal-to-noise-ratio (SNR) signal processing. As the process of LDV signal requires a high real-time response, threshold denoising is used in our research.

Wavelet packet thresholding denoising is divided into three steps: wavelet packet decomposition, threshold quantization on the coefficients created by the decomposition, and signal reconstruction. The denoising performance depends on the following factors: selection of the wavelet packet basis, determination of the number of packet decomposition layers, and selection of the threshold function and the threshold estimation method.

3.2. Principle of Wavelet Packet Transform. In the multi-resolution analysis of orthogonal wavelet basis, the scale subspace and wavelet subspace are defined as V_j and W_j , respectively, where j is the scale factor ($j \in \mathbb{Z}$). The orthogonal decomposition of the Hilbert space $L^2(\mathbb{R})$ can be expressed as $V_{j-1} = V_j \cup W_j$. The two-scale equation of the scaling function $\phi(t)$ is [13]

$$\begin{cases} \phi(t) = \sqrt{2} \sum_{k \in \mathbb{Z}} h_k \phi(2t - k), \\ \Psi(t) = \sqrt{2} \sum_{k \in \mathbb{Z}} g_k \phi(2t - k), \end{cases} \quad (2)$$

where $\{h_n\}_{n \in \mathbb{Z}}$ and $\{g_n\}_{n \in \mathbb{Z}}$ are, respectively, the low- and high-pass representations of a set of conjugate image filters, $g_n = (-1)^n h_{1-n}$. We introduce new notations $\mu_0(t) = \phi(t)$ and $\mu_1(t) = \psi(t)$, and then, $\mu_0(t)$ and $\mu_1(t)$ satisfies the following equations:

$$\begin{cases} \mu_0(t) = \sqrt{2} \sum_{k \in \mathbb{Z}} h_k \mu_0(2t - k), \\ \mu_1(t) = \sqrt{2} \sum_{k \in \mathbb{Z}} g_k \mu_1(2t - k). \end{cases} \quad (3)$$

Using μ_0, μ_1, g , and h , a set of wavelet packet functions at a certain scale are defined as $\mu_n, n = 0, 1, 2, \dots$:

$$\begin{cases} \mu_{2n}(t) = \sqrt{2} \sum_{k \in \mathbb{Z}} h_k \mu_n(2t - k), \\ \mu_{2n+1}(t) = \sqrt{2} \sum_{k \in \mathbb{Z}} g_k \mu_n(2t - k). \end{cases} \quad (4)$$

The function μ_n is referred to as the wavelet packet determined by function $\mu_0 = \phi$. The discrete wavelet packet decomposition function is

$$\begin{cases} d_j^{2n}[k] = \sum_{l \in \mathbb{Z}} h_{l-2k} d_{j+1}^n[l], \\ d_j^{2n+1}[k] = \sum_{l \in \mathbb{Z}} g_{l-2k} d_{j+1}^n[l]. \end{cases} \quad (5)$$

The discrete wavelet packet reconstruction function is

$$d_{j+1}^n[k] = \sum_{l \in \mathbb{Z}} h_{k-2l} d_j^{2n}[l] + \sum_{l \in \mathbb{Z}} g_{k-2l} d_j^{2n+1}[l]. \quad (6)$$

3.3. Wavelet Packet Global Threshold Denoising. The application of the wavelet packet function to signal denoising mainly has two methods: one is wavelet packet multithreshold denoising, and the other is wavelet packet global threshold denoising. The main difference between them is that wavelet packet multithreshold denoising is used to threshold the decomposition coefficients, the principle of which is to select an appropriate threshold for each wavelet packet decomposition coefficient for threshold quantization, while wavelet packet global threshold denoising is used to select an appropriate threshold for all wavelet packet decomposition coefficients for denoising processing. Although wavelet packet multithreshold denoising is a more accurate denoising method, the computation in this method is high in complexity and is slow in speed. For the characteristics of Doppler speed measurement signal (a stringent real-time requirement), we choose the wavelet packet global threshold denoising method with faster computation speed.

3.4. Selection of Threshold Function. The difference in thresholding functions reflects the difference of coefficient processing rules; the two commonly used thresholding functions are the soft thresholding function and hard thresholding function. The hard thresholding function is [13]

$$\bar{w}_{i,j} = \begin{cases} w_{j,k}, & |w_{j,k}| \geq \lambda, \\ 0, & |w_{j,k}| < \lambda, \end{cases} \quad (7)$$

and the soft thresholding function is

$$\bar{w}_{i,j} = \begin{cases} \text{sign}(w_{j,k}) (|w_{j,k}| - \lambda), & |w_{j,k}| \geq \lambda, \\ 0, & |w_{j,k}| < \lambda, \end{cases} \quad (8)$$

where $w_{j,k}$ and $\bar{w}_{i,j}$ are the wavelet coefficients before and after the denoising process, respectively; λ is the defined threshold; and the two thresholding functions are used to remove the small wavelet coefficients and to shrink or to

retain the large wavelet coefficients. On the basis of hard thresholding, soft thresholding shrinks the continuous points to zero on the declining boundaries, which can effectively avoid the discontinuity, making the reconstructed signals smoother. However, in the soft thresholding method, $w_{j,k}$ and $\bar{w}_{i,j}$ always have a constant deviation and lead to loss of certain high-frequency information. However, the high-frequency information in the Doppler speed measurement signals contains less useful signals, so the soft thresholding function is used in this experiment.

3.5. Estimation Method of Threshold. There are usually four threshold estimation methods available: heuristic thresholding (heursure), fixed thresholding (sqtwolog), unbiased risk estimation thresholding (rigrsure), and minimaxi thresholding (minimaxi) [13]. The rules for the minimaxi thresholding and the rigrsure thresholding are conservative. When a very small amount of the high-frequency information of the noisy signal exists in the noise range, these two thresholdings are very useful and able to extract the weak signals. The sqtwolog and heursure rules are more complete and effective in denoising; however, they tend to mistake useful high-frequency signals as noise and remove them. The four threshold rules are as follows [14].

① Heuristic threshold (heursure):

$$\begin{aligned} c &= \sqrt{\frac{1}{N} \left(\frac{\ln N}{\ln 2} \right)^3}, \\ e &= \frac{\sum_{i=1}^N |S_i|^2 - N}{N}, \end{aligned} \quad (9)$$

where, N is the length of signal S_i . If $e < c$, a fixed threshold (sqtwolog) should be selected. Otherwise, the smaller threshold obtained from the fixed threshold (sqtwolog) and the heuristic threshold (heursure) should be selected by this criterion.

② Fixed threshold (sqtwolog):

$$\lambda = \sqrt{2 \log(N)}, \quad (10)$$

where N is the length of signal sequence S_i .

③ Unbiased risk estimation threshold (rigrsure):

Each element of the signal S_i is taken as an absolute value and then sorted from small to large. After that, each element is squared to obtain a new sequence:

$$f(k) = (\text{sort}(|S_i|))^2, \quad k = 0, 1, 2, \dots, N-1. \quad (11)$$

If the threshold is the square root of the k th element of $f(k)$, as shown in equation (12), then

$$\lambda_k = \sqrt{f(k)}. \quad (12)$$

The risk generated by the threshold is

$$\text{Risk}(k) = \left[N - 2k + \frac{\sum_{i=1}^k f(i) + (N-k)f(N-k)}{N} \right]. \quad (13)$$

In the risk curve $\text{Risk}(k)$, the value of k corresponding to the minimum risk point is denoted as $k_{\min}$. And the threshold of rigrsure rule is defined as

$$\lambda = \sqrt{f(k_{\min})}. \quad (14)$$

④ Minimaxi threshold (minimaxi):

$$\lambda = \begin{cases} 0.3936 + 0.1829 \left(\frac{\ln N}{\ln 2} \right), & N > 32, \\ 0, & N \leq 32. \end{cases} \quad (15)$$

4. Wavelet Packet Denoising of Simulated Signals

4.1. Simulation of Laser Doppler Signal. In order to verify the effectiveness of wavelet packet threshold denoising, simulated laser Doppler signals were processed by heursure, sqtwolog, rigrsure, and minimaxi rules separately. The ideal laser Doppler signal is a continuous Gauss-distributed sinusoidal waveform. The Gauss-distributed base is caused by intensity variety of incidence light. And the sinusoidal signal modulated on base signal is caused by the light interference on the surface of test object. As the actual signal contains high intensity noises, including stray light noise, shot noise, and Johnson noise, Gaussian white noise was selected as the synthetic noise. A sequence of simulated LDV signal with SNR of 0 dB is shown in Figure 2.

4.2. Selection of Decomposition Layer Number. As the wavelet packet function and the number of decomposition layers will affect the performance of wavelet packet thresholding denoising, it is important to choose the optimal wavelet packet function and the optimal number of the wavelet packet decomposition layers. In this experiment, the sym8 function in the symN function system of the wavelet packet function is used because the symN function is an approximate symmetric wavelet packet function, and its waveform is similar to the waveform of the Doppler signal. In addition, the sym8 function is the wavelet packet function with the highest symmetry and the highest similarity in the symN function system [15]. In the process of selecting the number of the wavelet packet decomposition layers, we use the sym8 wavelet packet function and thresholding for the denoising of noise-added simulation signals and determine the number of the decomposition layers by analyzing and comparing the denoising effect. Taking sqtwolog rule, for example, ten groups of signals with original SNR of 0 dB are denoised by one to five layer decomposition. Table 1 shows the average SNR of each layer. The calculation formula of SNR is shown in the following equation [14]:

FIGURE 2: Simulated signal with SNR of 0 dB.

TABLE 1: SNR comparison of different layers.

Layer number	1	2	3	4	5
SNR	3.41	6.70	8.92	11.94	9.36

$$\text{SNR} = 10 \lg \frac{P_s}{P_n}, \quad (16)$$

where P_s is the power of the original signal and P_n is the power of noise. The denoising effect is proportional to SNR [16].

Figure 3 shows the denoised waveforms of the signal given by Figure 2. As shown in Figure 3, compared with other three denoised signals, four-layer decomposition denoised signal has the clearest and smoothest waveform.

It can be concluded from Table 1 that in the denoising process performed on the noise-added simulation signals, as the number of decomposition layers increases, the denoising effect improves. However, after the number of decomposition layers reaches four layers, the increasing of the denoising effect is not obvious and meanwhile the computation complexity is increased. By performing multiple denoising comparisons on the noise-added simulation signals, it was decided to use four-layer decomposition for wavelet packet decomposition.

4.3. Processing of Simulated Signals. Wavelet threshold denoising of heursure, sqtwolog, rigrsure, and minimaxi rules was utilized to process simulated signals in both low-frequency range (100 kHz~1000 kHz, corresponding to low speed) and high-frequency range (100 MHz~9000 MHz, corresponding to high speed). The sampling frequency was 5 MHz in low-frequency range and 5 GHz in high-frequency range. The simulated signals were interfered by Gaussian-distributed white noise with SNR of 0 dB, 5 dB, and 10 dB. SNR comparisons of four threshold rules in low- and high-frequency ranges are shown in Figures 4 and 5.

As shown in Figures 4 and 5, all four kinds of threshold rules have significantly improved the SNRs of simulated signals in both low- and high-frequency bands. The maximum difference between denoised signal and original signal is 12.52 dB (1000 kHz signal with an original SNR of 0 dB denoised by minimaxi rule). Taking SNR as an indicator, the denoising effects of minimaxi and sqtwolog rules are more obvious than that of heursure and rigrsure rules.

Along with SNR, root mean square error (RSME) is another parameter for the evaluating of denoising effect. The formula of RMSE is as follows [14]:

FIGURE 3: Denoising effects of different decomposition layers. (a) One-layer decomposition denoising. (b) Two-layer decomposition denoising. (c) Three-layer decomposition denoising. (d) Four-layer decomposition denoising.

FIGURE 4: SNR comparison of four threshold rules in low-frequency range. (a) SNR of original signal is 0 dB. (b) SNR of original signal is 5 dB. (c) SNR of original signal is 10 dB.

FIGURE 5: SNR comparison of four threshold rules in high-frequency range. (a) SNR of original signal is 0 dB. (b) SNR of original signal is 5 dB. (c) SNR of original signal is 10 dB.

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_N [x(n) - x^s(n)]^2}, \quad (17)$$

where $x(n)$ is the original signal, $x^s(n)$ is the filtered signal, and N is the length of the signal. The smaller the RMSE is, the better the denoising effect is. The comparisons of RMSE of signals in both low- and high-frequency bands are shown in Figures 6 and 7.

As shown in Figures 6 and 7, all four kinds of threshold rules have satisfactory RMSEs for simulated signals in both low- and high-frequency bands. Taking RMSE as an indicator, the denoising effects of minimaxi and sqtwolog rules are better than that of heursure and rigrsure rules.

In Figure 8, the waveform of an original simulated signal (SNR = 0 dB) is shown. The denoised signals using four kinds of threshold rules are shown in Figure 9.

As shown in Figure 9, the noise contained in original signal can be reduced significantly by all four threshold rules.

Compared with original signal shown in Figure 8, waveforms of the denoised signals are more smooth and clear. Generally speaking, minimaxi and sqtwolog rules have better performance than heursure and rigrsure rules.

5. Wavelet Packet Denoising of Experimental Signals

An experimental LDV system with double-beam and double-scattering structure was built in our laboratory. The structure of this experimental system is shown in Figure 10. An optical fiber splitter (TN632R5F2, Thorlabs) was utilized to equally divide the output laser into two beams. The two beams were emitted from two GRIN single-mode fiber collimators (50-630-FC, Thorlabs) parallelly to a focusing lens and focused on the surface of a 150 mm diameter metal disk driven by using a DC motor. The light scattered from metal disk surface was collected and focused on another

FIGURE 6: RMSE comparison of four threshold rules in low-frequency range. (a) SNR of original signal is 0 dB. (b) SNR of original signal is 5 dB. (c) SNR of original signal is 10 dB.

FIGURE 7: Continued.

FIGURE 7: RMSE comparison of four threshold rules in high-frequency range. (a) SNR of original signal is 0 dB. (b) SNR of original signal is 5 dB. (c) SNR of original signal is 10 dB.

FIGURE 8: Original signal with SNR = 0 dB.

FIGURE 9: Denoising effects of four threshold rules. (a) Heursure threshold rule denoising. (b) Sqtwolog threshold rule denoising. (c) Rigrsure threshold rule denoising. (d) Minimaxi threshold rule denoising.

collimator by collecting lens. And then, the light was transmitted to APD detector through an optical fiber. Three collimators were fixed in an aluminium support.

The He-Ne laser source (HNLS008R-EC, Thorlabs) used in our experiments has 0.8 mW output power and 632.8 nm wavelength. The APD detector (APD 430A/M, Thorlabs) has 400–1000 nm detection wavelength range, 8–80 μm optical power, and 400 MHz output bandwidth. The rotating rate of

the metal disk was measured by using a photoelectric encoder (UCD-IPH00-L100-ARW, Posital). And the linear velocity of the disk was calculated out by rotating rate and disk diameter. Half angle of two incident beams was set as 17° . The experimental LDV system is shown in Figure 11.

Ten groups of experimental signals were obtained from the LDV system shown in Figure 11. And then, these signals were processed by wavelet packet threshold denoising using

FIGURE 10: Structure of the experimental LDV system.

FIGURE 11: Experimental LDV system.

heursure, sqtwolog, rigrsure, and minimaxi rules separately. The wavelet packet function is sym8. The decomposition scale is 4. The sampling frequency was set as 10 MHz. After the threshold denoising, 1024 points fast Fourier transform (FFT) was adopted to get the Doppler frequency shift Δf . Then, equation (1) was used to calculate the rotating linear velocity of metal disk. Table 2 shows the accuracy comparison of four kinds of threshold rule applied on ten experimental signals.

As shown in Table 2, the threshold denoising methods have ability to provide satisfactory process effects. Overallly speaking, minimaxi and sqtwolog rules have higher precision. The minimum relative error is 0.079% (No. 8 signal using minimaxi rule). Taking a set of signal with a calibrated speed of 0.2705 m/s, for example, the signal waveform is shown in Figure 12. And the denoised signals using four threshold rules are shown in Figure 13.

As shown in Figure 13, the noise interferences were reduced significantly by four threshold rules. Minimaxi and sqtwolog rules denoised signal has more smooth and clear waveform than other denoised signals. Threshold denoising methods based on wavelet packet are able to depress the noise in experimental signal and are satisfactory for practical applications. In circumstance of Intel Core i3-3240 (CPU

TABLE 2: Processing accuracy of experimental signal.

No.	Reference speed (m/s)	Error (m/s)			
		Heursure	Sqtwolog	Rigrsure	Minimaxi
1	0.2705	0.0007	0.0006	0.0010	0.0003
2	0.3070	0.0007	0.0005	0.0011	0.0004
3	0.3852	0.0007	0.0006	0.0011	0.0003
4	0.4315	0.0008	0.0007	0.0012	0.0004
5	0.4861	0.0008	0.0006	0.0011	0.0005
6	0.5307	0.0008	0.0008	0.0013	0.0005
7	0.5725	0.0009	0.0007	0.0013	0.0006
8	0.6356	0.0010	0.0008	0.0012	0.0005
9	0.7012	0.0010	0.0009	0.0014	0.0006
10	0.7358	0.0009	0.0008	0.0013	0.0007

FIGURE 12: Experimental signal.

FIGURE 13: Denoised experimental signal. (a) Heursure threshold rule denoising. (b) Sqtwolog threshold rule denoising. (c) Rigrsure threshold rule denoising. (d) Minimaxi threshold rule denoising.

frequency of 3.4 GHz, memory of 4 GB), the average uptime of above 10 signal sequences is 0.00010 s. Taking the tracking filtering method adopted in [17] as a comparison, the average uptime of above 10 signal sequences is 0.00052 s. The passband width of tracking filter is set as 100 kHz, the same as [17]. It can be seen that the proposed method has a higher operation speed, which is important for high-speed measurement.

6. Conclusion

A signal denoising method based on wavelet packet is proposed in this research. Wavelet packet threshold denoising using heursure, sqtwolog, rigrsure, and minimaxi rules is used to process both simulated and measured signals. Features such as signal waveform, SNR, and RMSE of simulated signals with different original SNRs in low- and high-frequency ranges are significantly improved after the denoising. Furthermore, the validity of proposed methods on measured signals is proved by the denoising results of signals obtained from double-beam and double-scattering experimental system built-in laboratory. In general, the performances of threshold denoising using minimaxi and sqtwolog rules are better than those of heursure and rigrsure rules. As indicated in above research studies, the wavelet packet threshold denoising methods are effective and applicable for LDV signal.

Data Availability

The data used to support the findings of this study have not been made available because the data also form critical part of ongoing study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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