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Research Article

Sum Squeezing of the Field Amplitude in Frequency Upconversion Process

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Sum squeezing of the field amplitude is studied in the nondegenerate and degenerate frequency upconversion process under the short interaction time. It is shown that sum squeezing can be converted into normal squeezing via sum-frequency generation in the nondegenerate frequency upconversion process, while the amplitude-squared squeezing of the fundamental mode directly changed into the squeezing of the harmonic in the degenerate frequency upconversion process. All reachable conditions of uncorrelated modes for obtaining a sum squeezing in two modes and its dependence on the squeezing of individual field modes are investigated. It is found that the squeezed states are associated with large number of pump photons. It is also confirmed that the higher-order squeezing (sum squeezing) is directly associated with coupling of the field and interaction time.

1. Introduction

Over the past years, the squeezing [1–6] in quantized electromagnetic fields has received a great deal of attention because of its wide applications in many branches of science and technology, especially for low quantum fluctuations [7–9] having potential application in optical telecommunication [10], quantum cryptography [11, 12], and others. It is a consequence of uncertainty relations. A state is squeezed when the quantum fluctuation (amplitude noise or phase noise) in one variable is reduced below the symmetric limit at the expense of the increased quantum fluctuation in the conjugate variable such that the Heisenberg uncertainty relation is not violated. It has been focused on theoretical as well as experimental evidences of squeezed states in various nonlinear optical processes, such as harmonic generation [13–16], multiphoton processes [17–20], and Raman and hyper-Raman processes [21–25]. Hong and Mandel [26, 27] and further Hillery [28] have introduced the notion of higher-order squeezing of quantised electromagnetic field as generalization of the much discussed normal squeezing and followed by [29] for improving the performance of many optical devices. Squeezing and photon statistical effect of the field amplitude in optical parametric processes and in Raman and hyper-Raman scattering has been reported by Perina et al. [30] in which it is demonstrated that squeezing accompanies antibunching very often, but not always. In some cases, squeezing may occur and antibunching may not and vice versa. Kim and Yoon [31] have also studied higher-order sub-Poissonian statistics of light and pointed out the nonclassical measure of the higher-order sub-Poissonian photon statistics of the number state is half as same as that of the known lowest order. Recently, Prakash and Mishra [32, 33] have also reported the higher-order sub-Poissonian photon statistics and their use in detection of Hong and Mandel squeezing and amplitude-squared squeezing. Another type of seminal paper on higher-order squeezing, called sum and difference squeezing, was proposed by Hillery [34] for the two modes which are in fact the simplest versions of multimode higher-order squeezing. These concepts have been generalized to include three modes for sum and difference squeezing [35–37] as well as an arbitrary number of modes for sum and difference squeezing [38–40]. Furthermore, more recently, Prakash and Shukla [41],

Mukherjee et al. [42], and Mukherjee et al. [43] have also studied and reported about sum and difference squeezing and their detections in some nonlinear optical processes.

The objective of this paper is to study the sum squeezing in the nondegenerate and degenerate frequency upconversion process under the short-time scale based on a fully quantum approach. It is a higher-order squeezing of the radiation field to achieve significantly larger quantum noise reduction. Since higher-order squeezing is the higher powers of the field amplitude, which is directly associated with the large numbers of photons that make it possible to achieve significantly larger noise reduction than ordinary squeezing. This motivates us to study sum squeezing (higher-order) in the frequency upconversion process in the line of seminal paper [34]. The paper is organized as follows. Section 2 gives the definition of one- and two-mode higher-order squeezing. Sum squeezing of the field amplitude in the nondegenerate frequency upconversion process is investigated in Section 3. Detection of sum squeezing of two-mode field in this process is also studied in Section 3. In Section 4, sum squeezing of the field amplitude in the degenerate frequency upconversion process is studied and the relation between sum squeezing and amplitude-squared squeezing is established. Finally, we conclude the paper in Section 5.

2. Definition of One- and Two-Mode Higher-Order Squeezing

Suppose a single mode of radiation field having frequency ω_a with creation and annihilation operators $a^\dagger$ and a, respectively, and defining amplitude-squared squeezing in terms of operators Y_1 and Y_2 given by

$$Y_1 = \frac{1}{2}\left(A^2 + A^{\dagger 2}\right), \qquad Y_2 = \frac{1}{2i}\left(A^2 - A^{\dagger 2}\right), \tag{1}$$

where $A = a\exp(i\omega_a t)$ and $A^\dagger = a^\dagger \exp(-i\omega_a t)$ are slowly varying operators.

Equation (1) obeys the commutation relation

$$[Y_1, Y_2] = i\left(2N_A + 1\right), \tag{2}$$

where $A^\dagger A = N_A$ is the number operator.

Relation (2) leads to the uncertainty relation ($\hbar = 1$):

$$\Delta Y_1 \Delta Y_2 \geq \left\langle N_A + \frac{1}{2}\right\rangle. \tag{3}$$

Equation (3) exists amplitude-squared squeezing if it follows the condition:

$$\left(\Delta Y_j\right)^2 < \left\langle N_A + \frac{1}{2}\right\rangle, \tag{4}$$

where $j = 1$ or 2 and ΔY_1 and ΔY_2 are the uncertainties in the quadrature operators Y_1 and Y_2, respectively.

A quantum state is amplitude-squared squeezed in the Y_1 direction if $(\Delta Y_1)^2 < \langle N_A + (1/2)\rangle$ and is amplitude-squared squeezed in the Y_2 direction if $(\Delta Y_2)^2 < \langle N_A + (1/2)\rangle$.

Now, for two modes having frequency ω_a and ω_b with creation (annihilation) operators $a^\dagger(a)$ and $b^\dagger(b)$, let us introduce two operators which correspond to real and imaginary parts of the product of the field amplitudes as

$$W_1 = \frac{1}{2}\left(AB + A^\dagger B^\dagger\right), \tag{5}$$

$$W_2 = \frac{1}{2i}\left(AB - A^\dagger B^\dagger\right), \tag{6}$$

where $A = a\exp(i\omega_a t)$ and $B = b\exp(i\omega_b t)$ are slowly varying operators.

Equations (5) and (6) follow the commutation relation

$$[W_1 W_2] = \frac{i}{2}\left(N_A + N_B + 1\right), \tag{7}$$

and satisfy the uncertainty relation ($\hbar = 1$):

$$\Delta W_1 \, \Delta W_2 \geq \frac{1}{4}\langle N_A + N_B + 1\rangle, \tag{8}$$

where $N_A = A^\dagger A$ and $N_B = B^\dagger B$ are the number operators.

Sum squeezing in the W_j direction exists if

$$\left(\Delta W_j\right)^2 < \frac{1}{4}\langle N_A + N_B + 1\rangle, \tag{9}$$

where $j = 1$ or 2 and ΔW_1 and ΔW_2 are the uncertainties in the quadrature operators W_1 and W_2, respectively.

A state is sum squeezed in the W_1 direction if $(\Delta W_1)^2 < (1/4)\langle N_A + N_B + 1\rangle$ and is sum squeezed in the W_2 direction if $(\Delta W_2)^2 < (1/4)\langle N_A + N_B + 1\rangle$.

3. Sum Squeezing in the Nondegenerate Frequency Upconversion Process

Frequency upconversion, shown in Figure 1, is a three-wave interaction nonlinear optical phenomenon, in which two input photons (a signal and a pump photon) at different frequencies, ω_a and ω_b, annihilate and another photon at their sum frequency, ω_c, is simultaneously generated in the nonlinear optical medium.

It serves as basic building blocks for the implementation of quantum optical experiments. By using this technique, near-infrared light can be converted to light in the visible or near-visible range and therefore detected by commercially available visible detectors with high efficiency and low noise. It can be realized in nonlinear crystals, but the relatively low conversion efficiency requires a high-power laser or a resonant cavity [44]. This model is chosen to make a realistic one, and our theoretical discussions hold for all similar models.

The Hamiltonian of this process may be written as ($\hbar = 1$)

$$H = \omega_a a^\dagger a + \omega_b b^\dagger b + \omega_c c^\dagger c + g\left(abc^\dagger + a^\dagger b^\dagger c\right), \tag{10}$$

where $a^\dagger(a)$, $b^\dagger(b)$, and $c^\dagger(c)$ are the creation (annihilation) operators of the A, B, and C modes, respectively, and g is the coupling constant between the two modes of the order of 10^2–10^4 per second and is proportional to the nonlinear susceptibility of the medium as well as the complex

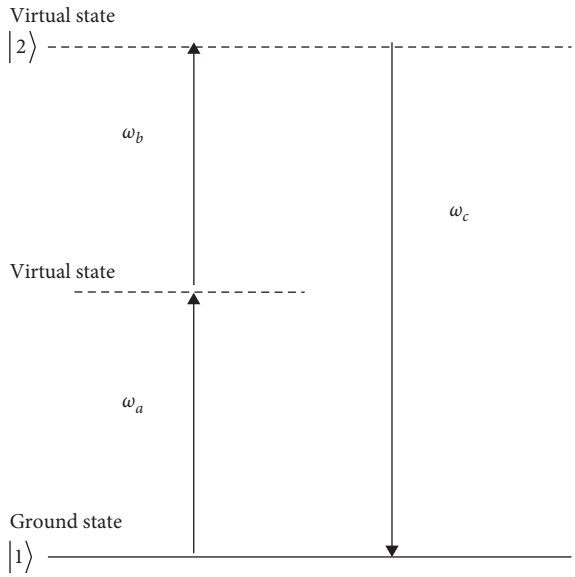

Figure 1: Energy level diagram of the frequency upconversion process.

amplitude of the pump field [4, 30, 45]. However, to take care of complex g, we have used $|g|^2$ in the place of g^2 [4].

Using the interaction Hamiltonian of equation (10) in the coupled Heisenberg equation of motion,

$$\dot{A}(t) = \frac{\partial A(t)}{\partial t} + i[H, A(t)], \quad \hbar = 1, \tag{11}$$

where the dot denotes time derivative.

Equation (11) leads to

$$\begin{aligned} \dot{A} &= -igB^{\dagger}C, \\ \dot{B} &= -igA^{\dagger}C, \\ \dot{C} &= -igAB, \end{aligned} \tag{12}$$

where A, B, and C are slowly varying operators, which are defined by $A = a\exp(i\omega_a t)$, $B = b\exp(i\omega_b t)$, and $C = c\exp(i\omega_c t)$, with the relation $\omega_a + \omega_b = \omega_c$. The operators $A(t)$ and $A^{\dagger}(t)$ induce a slower dependence on time as compared to fast variation during the interaction between modes.

The system evolution during a short period of time is practically relevant because the actual interaction is in fact very short. Hence, the interaction time is taken to be short, of the order of 10^{-10} s, and a nanosecond or picosecond pulse laser can also be used as the pump field for time resolved measurements [4, 44]. For real physical situation in the short-time scale $gt \ll 1$ ($gt \sim 10^{-6}$), the expectation value of mean pump photon numbers is very large ($\langle A^{\dagger}A\rangle = |\alpha|^2 \gg 1$) and it is possible to obtain much simpler approximate analytical formulas describing the variances [4].

Using Taylor's expansion on $A(t)$ as $A(t) = A(0) + t\dot{A}(0) + (t^2/2!)\ddot{A}(0) + \ldots\ldots$ and keeping terms up to second order in t, we have

$$A(t) = A + t\left(-igB^{\dagger}C\right) + \frac{t^2}{2!}\left[-ig\left(\dot{B}^{\dagger}C + B^{\dagger}\dot{C}\right)\right], \tag{13}$$

where $\dot{A} = -igB^{\dagger}C$ and $\ddot{A}(0) = [-ig(\dot{B}^{\dagger}C + B^{\dagger}\dot{C})]$ and after simplification, we get

$$\begin{aligned} A(t) &= A + t\left(-igB^{\dagger}C\right) + \frac{t^2}{2!}\left[-ig\left(igAC^{\dagger}C\right) + B^{\dagger}(-igAB)\right] \\ \text{or } A(t) &= A + t\left(-igB^{\dagger}C\right) + \frac{g^2t^2}{2!}\left[C^{\dagger}C - B^{\dagger}B\right]A \\ \text{or } A(t) &= A - igtB^{\dagger}C + \frac{1}{2}|g|^2t^2(N_C - N_B)A, \end{aligned} \tag{14}$$

where $C^{\dagger}C = N_C$ and $B^{\dagger}B = N_B$.

$$A^{\dagger}(t) = A^{\dagger} + igtB^{\dagger}C + \frac{1}{2}|g|^2t^2(N_C - N_B)A^{\dagger}. \tag{15}$$

Similarly,

$$B(t) = B - igtA^{\dagger}C + \frac{1}{2}|g|^2t^2(N_C - N_A)B^{\dagger}, \tag{16}$$

$$B^{\dagger}(t) = B^{\dagger} - igtAC^{\dagger} + \frac{1}{2}|g|^2t^2(N_C - N_A)B. \tag{17}$$

Also,

$$C(t) = C - igtAB - \frac{1}{2}|g|^2t^2(N_A + N_B + 1)C, \tag{18}$$

$$C^{\dagger}(t) = C^{\dagger} - igtA^{\dagger}B^{\dagger} - \frac{1}{2}|g|^2t^2(N_A + N_B + 1)C^{\dagger}. \tag{19}$$

Let us examine squeezing in the C mode, and we define two general quadrature components:

$$X_{1C}(t) = \frac{1}{2}\left[C(t) + C^{\dagger}(t)\right], \tag{20}$$

$$X_{1C}(t) = \frac{1}{2}\left[C(t) - C^{\dagger}(t)\right]. \tag{21}$$

Using equations (18) and (19) in equations (20) and (21), we obtain

$$X_{1C}(t) = X_{1C} + |g|tW_2 - \frac{1}{2}|g|^2t^2(N_A + N_B + 1)X_{1C}, \tag{22}$$

$$X_{2C}(t) = X_{2C} - |g|tW_1 - \frac{1}{2}|g|^2t^2(N_A + N_B + 1)X_{2C}. \tag{23}$$

At $t = 0$, for uncorrelated modes, we get

$$\left[\Delta X_{1C}(t)\right]^2 = (\Delta X_{1C})^2 + |g|^2t^2 \cdot \left[(\Delta W_2)^2 - \langle N_A + N_B + 1\rangle(\Delta X_{1C})\right]^2, \tag{24}$$

$$\left[\Delta X_{2C}(t)\right]^2 = (\Delta X_{2C})^2 + |g|^2t^2 \cdot \left[(\Delta W_1)^2 - \langle N_A + N_B + 1\rangle(\Delta X_{2C})\right]^2. \tag{25}$$

If initially the C mode is in a coherent state, then

$$(\Delta X_{1C})^2 = (\Delta X_{2C})^2 = \frac{1}{4}, \tag{26}$$

and equations (24) and (25) reduce to

$$[\Delta X_{1C}(t)]^2 - \frac{1}{4} = |g|^2 t^2 \left[(\Delta W_2)^2 - \frac{1}{4} \langle N_A + N_B + 1 \rangle \right], \tag{27}$$

$$[\Delta X_{2C}(t)]^2 - \frac{1}{4} = |g|^2 t^2 \left[(\Delta W_1)^2 - \frac{1}{4} \langle N_A + N_B + 1 \rangle \right]. \tag{28}$$

Equations (27) and (28) establish the relation between sum squeezing and normal squeezing in the frequency upconversion process. We find that if the input state is sum squeezed in the W_2 or W_1 direction, then normal squeezing will occur in the X_{1C} or X_{2C} direction, respectively. This result suggests a method for detection of nonclassical properties of radiation in the frequency upconversion process.

We plot a graph (Figures 2 and 3) between left-hand side of equations (27) and (28) say S_S and S_S', respectively, versus $|gt|^2$ with typical values $(\Delta W_2)^2 = (\Delta W_1)^2 = (1/4)$ so that it could satisfy equation (9).

The steady fall of the curves infers that the sum squeezing exists and responses nonlinearly to the number of pump photons. It shows that when $|\alpha|^2$ increases, the degree of sum squeezing also increases, i.e., S_S is getting more negative. This confirms that the squeezed states are associated with large number of pump photons. It also confirms that the higher-order squeezing (sum squeezing) is directly associated with the coupling of the field and interaction time. Hence, optimum squeezing can be realized in short-time scale.

Comparing Figures 2 and 3, we inferred that the depth of nonclassicality is increasing with an increase of $|\beta|^2$. Hence, it is inferred that a higher multiphoton absorption process is suitable for generation of optimum squeezed light.

It is also of interest to study sum squeezing in the C mode as a function of time; we define the quadrature operators as follows:

$$W_{1C}(t) = \frac{1}{2} \left[A(t)B(t) + A^\dagger(t)B^\dagger(t) \right], \tag{29}$$

$$W_{2C}(t) = \frac{1}{2i} \left[A(t)B(t) - A^\dagger(t)B^\dagger(t) \right]. \tag{30}$$

Under short-time approximation, we keep terms up to first order in "gt" in Taylor's expansion to get

$$A(t) = A(0) + t\dot{A}(0) + \ldots\ldots, \tag{31}$$

$$B(t) = B(0) + t\dot{B}(0) + \ldots\ldots. \tag{32}$$

Using equations (31) and (32) in equations (14)–(17) gives

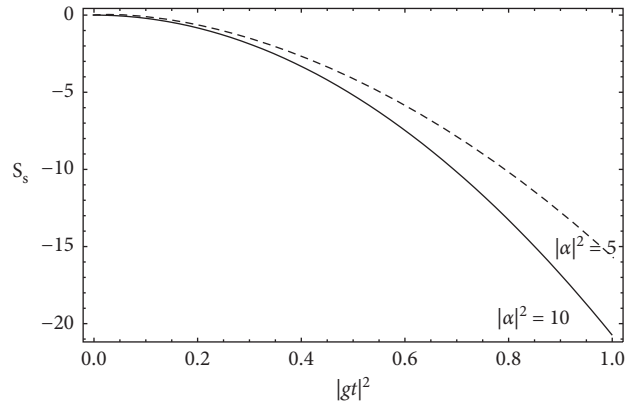

Figure 2: Variation of the sum squeezing S_S with $|gt|^2$ in the nondegenerate frequency upconversion process (when $\langle N_B \rangle = |\beta|^2 = 10$).

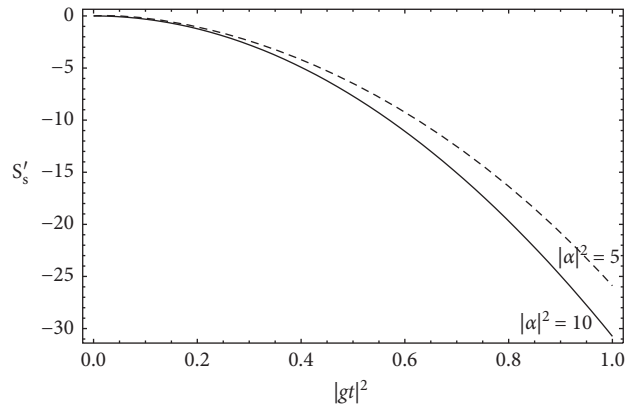

Figure 3: Variation of the sum squeezing S_S' with $|gt|^2$ in the nondegenerate frequency upconversion process (when $\langle N_B \rangle = |\beta|^2 = 20$).

$$A(t) = A - igtB^\dagger C, \tag{33}$$

$$A^\dagger(t) = A^\dagger + igtBC^\dagger, \tag{34}$$

$$B(t) = B - igtA^\dagger B, \tag{35}$$

$$B^\dagger(t) = B^\dagger + igtAB^\dagger. \tag{36}$$

Using equations (33)–(36) in equation (29), we find

$$W_{1C}(t) = \frac{1}{2} \left[\left(AB + A^\dagger B^\dagger \right) - igt \left\{ B^\dagger B \left(C - C^\dagger \right) + A^\dagger A \left(B - B^\dagger \right) \right\} \right]. \tag{37}$$

As the wave function of the system starts as a coherent state at $t = 0$ and evolves as a squeezed state at a later time [46], we use the initial coherent state.

Now, initially we consider a quantum state as a product of coherent states $|\alpha\rangle$ and $|\beta\rangle$ for the pump modes A and B, respectively, and $|\gamma\rangle$ for the sum-frequency mode C, i.e.,

$$|\psi\rangle = |\alpha\rangle_A |\beta\rangle_B |\gamma\rangle_C. \tag{38}$$

Using equation (38) in equation (37), we obtain

$$\left\langle \psi \left| W_{1C}^2 \right| \psi \right\rangle = \frac{1}{4}\Big[\alpha^2\beta^2 + \alpha^{*2}\beta^{*2} + 2|\alpha|^2|\beta|^2 + |\alpha|^2 + |\beta|^2 + 1 - igt\Big(2\alpha\beta^*|\beta|^2\gamma - 2\alpha\beta^*|\beta|^2\gamma^* + 2\alpha|\alpha|^2\beta^2 - 2\alpha|\alpha|^2|\beta|^2 + 2\alpha^*\beta^*\gamma|\beta|^2 - 2\alpha^*\beta^*\gamma^*|\beta|^2 + 2\alpha^*|\beta|^2|\alpha|^2 - 2\alpha^*\beta^{*2}|\alpha|^2 + \alpha\beta\gamma - \alpha\beta\gamma^* + \alpha\beta^2 - \alpha^*\beta^{*2} - \alpha|\beta|^2 + \alpha^*|\beta|^2 - \alpha|\alpha|^2 + \alpha^*|\alpha|^2 + \alpha^*\beta^*\gamma - \alpha^*\beta^*\gamma^* - \alpha + \alpha^*\Big)\Big], \tag{39}$$

$$\langle \psi \left| W_{1C} \right| \psi \rangle^2 = \frac{1}{4}\Big[\alpha^2\beta^2 + \alpha^{*2}\beta^{*2} + 2|\alpha|^2|\beta|^2 - igt\Big(2\alpha\beta^*|\beta|^2\gamma - 2\alpha\beta^*|\beta|^2\gamma^* + 2\alpha|\alpha|^2\beta^2 - 2\alpha|\alpha|^2|\beta|^2 + 2\alpha^*\beta^*\gamma|\beta|^2 - 2\alpha^*\beta^*\gamma^*|\beta|^2 + 2\alpha^*|\beta|^2|\alpha|^2 - 2\alpha^*\beta^{*2}|\alpha|^2\Big)\Big]. \tag{40}$$

$$\begin{aligned}\left[\Delta W_{1C}(t)\right]^2 &= \langle W_{1C}^2(t)\rangle - \langle W_{1C}(t)\rangle^2 \\ &= \frac{1}{4}\Big[|\alpha|^2 + |\beta|^2 + 1 - igt\Big(\alpha\beta\gamma - \alpha\beta\gamma^* + \alpha\beta^2 - \alpha^*\beta^{*2} - \alpha|\beta|^2 + \alpha^*|\beta|^2 - \alpha|\alpha|^2 + \alpha^*|\alpha|^2 + \alpha^*\beta^*\gamma \\ &\quad - \alpha^*\beta^*\gamma^* - \alpha + \alpha^*\Big)\Big].\end{aligned} \tag{41}$$

The numbers of photons are

$$N_A(t) = A^\dagger(t)A(t) = N_A + igt\left(ABC^\dagger - A^\dagger B^\dagger C\right), \tag{42}$$

$$N_B(t) = B^\dagger(t)B(t) = N_B + igt\left(ABC^\dagger - A^\dagger B^\dagger C\right). \tag{43}$$

Using equation (38), we obtain

$$\frac{1}{4}\langle N_A(t) + N_B(t) + 1\rangle = \frac{1}{4}\left[|\alpha|^2 + |\beta|^2 + 1 + 2igt\left(\alpha\beta\gamma^* - \alpha^*\beta^*\gamma\right)\right]. \tag{44}$$

Subtraction of equation (44) from equation (41) yields

$$\begin{aligned}&\left[\Delta W_{1C}(t)\right]^2 - \frac{1}{4}\langle N_A(t) + N_B(t) + 1\rangle \\ &= \frac{-igt}{4}\left[\left(\alpha\beta - \alpha^*\beta^*\right)\left(\gamma + \gamma^*\right) + \left(\alpha\beta^2 - \alpha^*\beta^{*2}\right) + \left(|\alpha|^2 + |\beta|^2 + 1\right)\left(\alpha^* - \alpha\right)\right] \\ &= \frac{gt}{2}\left[2|\alpha\beta\gamma|\sin(\theta_1 + \theta_2)\cos\theta_3 + \left|\alpha\beta^2\right|\sin(\theta_1 + 2\theta_2) - \left(|\alpha|^2 + |\beta|^2 + 1\right)|\alpha|\sin\theta_1\right],\end{aligned} \tag{45}$$

where $\alpha = |\alpha|\exp(i\theta_1)$, $\beta = |\beta|\exp(i\theta_2)$, and $\gamma = |\gamma|\exp(i\theta_3)$.

Equation (45) means that squeezing of W_{1C} will occur whenever θ_1 and $\theta_2 < 0$ and $\theta_3 > 0$. It suffices to choose $\theta_1 = \theta_2 = -(\pi/3)$ and $\theta_3 = (\pi/2)$, and the square bracket becomes nonnegative.

Let us now study the dependence of sum squeezing for two-mode states on squeezing of individual modes in which the modes are uncorrelated, i.e., mode in a coherent state at $t = 0$.

We define for two-mode sum squeezing as [34]

$$W_{C\varphi} = \frac{1}{2}\left(e^{i\varphi}A^{\dagger}B^{\dagger} + e^{-i\varphi}AB\right). \tag{46}$$

The squeezed state exists if $\Delta W_{C\varphi} < (1/2)$ for some φ. Using equation (46), we obtain

$$\begin{aligned}\langle W_{C\varphi}^2\rangle &= \frac{1}{4}\langle e^{2i\varphi}\left(A^{\dagger}B^{\dagger}\right)^2 + A^{\dagger}B^{\dagger}AB + ABA^{\dagger}B^{\dagger} + e^{-2i\varphi}(AB)^2\rangle \\ &= \frac{1}{4}\langle e^{2i\varphi}\left(A^{\dagger}B^{\dagger}\right)^2 + 2B^{\dagger}BA^{\dagger}A + A^{\dagger}A + B^{\dagger}B + 1 + e^{-2i\varphi}(AB)^2\rangle,\end{aligned} \tag{47}$$

$$\langle W_{C\varphi}\rangle^2 = \frac{1}{4}\left[\langle e^{2i\varphi}\langle A^{\dagger}B^{\dagger}\rangle^2 + 2\langle A^{\dagger}B^{\dagger}\rangle\langle AB\rangle + e^{-2i\varphi}\langle AB\rangle^2\rangle\right]. \tag{48}$$

Hence, the variance of field is

$$\begin{aligned}\left[\Delta W_{C\varphi}\right]^2 &= \langle W_{C\varphi}^2\rangle - \langle W_{C\varphi}\rangle^2 \\ &= \frac{1}{4}\left\{e^{2i\varphi}\left[\langle\left(A^{\dagger}B^{\dagger}\right)^2\rangle - \langle A^{\dagger}B^{\dagger}\rangle^2\right] + 2\langle B^{\dagger}BA^{\dagger}A\rangle + \langle B^{\dagger}B\rangle + \langle A^{\dagger}A\rangle + 1\right. \\ &\quad \left. - 2\langle A^{\dagger}B^{\dagger}\rangle\langle AB\rangle + e^{-2i\varphi}\left[\langle(AB)^2\rangle - \langle AB\rangle^2\right]\right\}.\end{aligned} \tag{49}$$

Using equation (9) in equation (49), we find

$$\begin{aligned}\left[\Delta W_{C\varphi}\right]^2 - \frac{1}{4}\langle N_A + N_B + 1\rangle &= \frac{1}{4}\left\{e^{2i\varphi}\left[\langle\left(A^{\dagger}B^{\dagger}\right)^2\rangle - \langle\left(A^{\dagger}B^{\dagger}\right)\rangle^2\right] + 2\langle B^{\dagger}BA^{\dagger}A\rangle\right. \\ &\quad \left. - 2\langle A^{\dagger}B^{\dagger}\rangle\langle AB\rangle + e^{-2i\varphi}\left[\langle(AB)^2\rangle - \langle AB\rangle^2\right]\right\}.\end{aligned} \tag{50}$$

A state is squeezed if the term in brackets becomes negative. This term is smallest when

$$\arg\left[\langle(AB)^2\rangle - \langle AB\rangle^2\right] - 2\varphi = \pi. \tag{51}$$

If φ satisfies (51), then

$$\begin{aligned}\left[\Delta W_{C\varphi}\right]^2 - \frac{1}{4}\langle N_A + N_B + 1\rangle &= \frac{1}{2}\left[\langle N_A N_B\rangle - |\langle AB\rangle|^2\right. \\ &\quad \left. - \left|\langle(AB)^2\rangle - \langle AB\rangle^2\right|\right].\end{aligned} \tag{52}$$

Therefore, a state is sum squeezed if and only if

$$\left|\langle(AB)^2\rangle - \langle AB\rangle^2\right| > \langle(N_A N_B)\rangle - |\langle(AB)\rangle|^2. \tag{53}$$

If the modes are uncorrelated, then equation (53) becomes

$$\left|\langle A^2\rangle\langle B^2\rangle - \langle A\rangle^2\langle B\rangle^2\right| > \langle N_A\rangle\langle N_B\rangle - |\langle A\rangle\langle B\rangle|^2. \tag{54}$$

Let us consider the first case. If the modes are uncorrelated, i.e., there is no linear relationship between A and B or $<A> = <B> = 0$ and neither the A nor the B mode is squeezed [34], then

$$\left|\langle A^2\rangle - \langle A\rangle^2\right| \le \langle N_A\rangle - |\langle A\rangle|^2, \tag{55}$$

$$\left|\langle B^2\rangle - \langle B\rangle^2\right| \le \langle N_B\rangle - |\langle B\rangle|^2. \tag{56}$$

Furthermore, if none of the A and B modes are squeezed, i.e., at coherent state, none of the pairs can be sum squeezed [34], i.e.,

$$\left|\langle A^2\rangle\langle B^2\rangle - \langle A\rangle^2\langle B\rangle^2\right| \le \langle N_A\rangle\langle N_B\rangle - |\langle A\rangle\langle B\rangle|^2. \tag{57}$$

Comparing equations (54) and (57), we find that the A and B modes are not sum squeezed.

In the second case, if the A mode is squeezed and the B mode is in a coherent state of amplitude β, then we have

$$\left|\langle A^2\rangle\langle B^2\rangle - \langle A\rangle^2\langle B\rangle^2\right| = |\beta|^2\left|\left(\langle A^2\rangle - \langle A\rangle^2\right)\right| > |\beta|^2\left(\langle N_A\rangle - |\langle A\rangle|^2\right), \quad (58)$$

where $\langle N_B\rangle = |\beta|^2$ for coherent states and the inequality in equation (54) is fulfilled; hence, state is sum squeezed.

In the third case, if the B mode is squeezed and the A mode is in a coherent state of amplitude α, we then have

$$\left|\langle A^2\rangle\langle B^2\rangle - \langle A\rangle^2\langle B\rangle^2\right| = |\alpha|^2\left|\left(\langle B^2\rangle - \langle B\rangle^2\right)\right| > |\alpha|^2\left(\langle N_B\rangle - |\langle B\rangle|^2\right), \quad (59)$$

where $\langle N_A\rangle = |\alpha|^2$ and the inequality in equation (54) is satisfied hence the A and B modes are sum squeezed.

Finally, if A and B modes are squeezed, then we have

$$\left|\langle A^2\rangle\langle B^2\rangle - \langle A\rangle^2\langle B\rangle^2\right| > \langle N_A\rangle\langle N_B\rangle - |\langle A\rangle\langle B\rangle|^2. \quad (60)$$

This satisfies condition (54) and hence the state is sum squeezed.

4. Sum Squeezing in Degenerate Frequency Upconversion Process

If all the two pump modes are of the same frequency ($\omega_a = \omega_b$), the process reduces to second harmonic generation described by the Hamiltonian H' given by

$$H' = \omega_a a^\dagger a + \omega_c c^\dagger c + g\left(a^2 c^\dagger + a^{\dagger 2} c\right). \quad (61)$$

Equation (15) leads to coupled Heisenberg equations of motion:

$$\dot{A} = -2igA^\dagger C, \qquad \dot{C} = -igA^2. \quad (62)$$

Using Taylor's expansion upto second order in gt, we obtain

$$C(t) = C - igtA^2 - g^2t^2\left(2A^\dagger A + 1\right)C, \quad (63)$$

$$C^\dagger(t) = C + igtA^{\dagger 2} - g^2t^2\left(2A^\dagger A + 1\right)C^\dagger. \quad (64)$$

Using equations (20) and (21), this gives

$$X_{1C}(t) = X_{1C} + gtY_2 - 4g^2t^2\left(N_A + \frac{1}{2}\right)X_{1C}, \quad (65)$$

$$X_{2C}(t) = X_{2C} - gtY_1 - 4g^2t^2\left(N_A + \frac{1}{2}\right)X_{2C}. \quad (66)$$

If the C mode is initially in a coherent state at $t = 0$, then we obtain

$$\left[\Delta X_{1C}(t)\right]^2 - \frac{1}{4} = |gt|^2\left[\left(\Delta Y_2\right)^2 - \langle N_A + \frac{1}{2}\rangle\right], \quad (67)$$

$$\left[\Delta X_{2C}(t)\right]^2 - \frac{1}{4} = |gt|^2\left[\left(\Delta Y_1\right)^2 - \langle N_A + \frac{1}{2}\rangle\right]. \quad (68)$$

From equations (67) and (68), we infer that X_{1C} becomes squeezed if Y_2 is squeezed and X_{2C} is squeezed if Y_1 is squeezed. In other way, the C mode is squeezed in the X_{1C} direction if the fundamental modes are sum squeezed in the Y_2 direction and the C mode is squeezed in the X_{2C} direction if the fundamental modes are sum squeezed in the Y_1 direction. These equations show that the amplitude-squared squeezing of the fundamental can be turned directly into the squeezing of the harmonic in the degenerate frequency upconversion process. This result suggests a way to detect higher-order squeezing in this process.

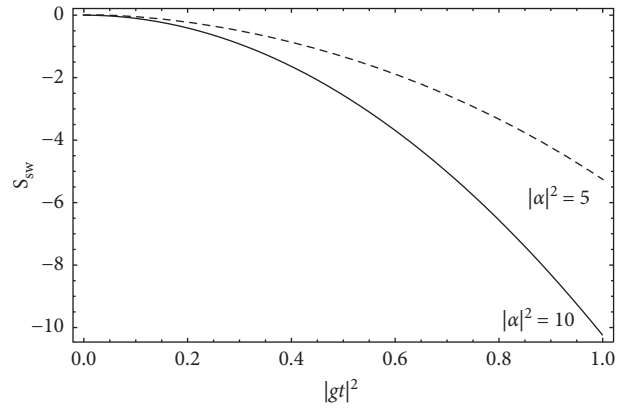

Figure 4: Variation of the sum squeezing S_{SW} with $|gt|^2$ in the degenerate frequency upconversion process.

We plot a graph (Figure 4) between left-hand side of equation (67) or (68) say S_{SW} and $|gt|^2$ with typical values $(\Delta Y_2)^2 = (\Delta Y_1)^2 = (1/4)$ so that it could satisfy equation (4).

Figure 4 shows that the plot responses nonlinearly to increase photon numbers. This confirms that the squeezed states are associated with large number of pump photons. It also confirms that the higher-order squeezing (sum squeezing) is directly associated with the coupling of the field and interaction time. Hence, optimum squeezing can be realized in short-time scale.

5. Conclusions

In this paper, we have concluded that sum squeezing can be turned into normal squeezing via sum-frequency generation in the nondegenerate frequency upconversion process. It is also established that the amplitude-squared (higher-order) squeezing of the fundamental can be converted directly into the squeezing of the harmonic in the degenerate frequency upconversion process. These findings suggest a method for generation and detection of higher-order squeezing in the frequency upconversion process. It is observed that the sum squeezing will occur conditional to the case of first-order coupling, while to the case of second-order coupling gives unconditional sum squeezing in nondegenerate as well as in the degenerate frequency upconversion process.

We have examined all possible conditions for an uncorrelated two-mode state. If both modes are not squeezed, then the state is not sum squeezed. If one mode is squeezed and the second one is in a coherent state, then the state is sum squeezed. Finally, if both modes are squeezed, then the state may or may not be sum squeezed in the nondegenerate frequency upconversion process. The steady fall of the curves infers that the sum squeezing exists and responses

nonlinearly to the number of pump photons. It shows that the squeezed states are associated with large number of pump photons. It also confirms that the higher-order squeezing (sum squeezing) is directly associated with the coupling of the field and interaction time. Hence, optimum squeezing can be realized in short-time scale. It is found that the depth of nonclassicality is increasing with an increase of the number of pump photons. Hence, it is inferred that a higher multiphoton absorption process is suitable for generation of optimum squeezed light.

These findings suggest and may help in selecting a suitable process to generate optimum squeezing of the radiation and further can be useful as a resource to improve high-quality optical telecommunication [47]. The results obtained in this paper are of interest for new experiments on the study of nonlinear optical processes in dielectric media using ultrashort intense laser pulses as exciting radiation, and the effects of damping and decoherence as well as higher-order time terms could be investigated.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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