In-Line and Off-Axis Hybrid Digital Holographic Imaging Method Based on Off-Axis Hologram Constraints

Fengpeng Wang, Feifan Fan, Deren Yuan, and Xinghua Wang

School of Physics and Electronic Information, Gannan Normal University, Ganzhou 341000, Jiangxi, China

Correspondence should be addressed to Fengpeng Wang; wangfengpeng@163.com

Received 6 June 2022; Revised 27 October 2022; Accepted 15 November 2022; Published 25 November 2022

Academic Editor: Paramasivam Senthilkumaran

Copyright © 2022 Fengpeng Wang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

We put forward a novel hybrid iterative algorithm to improve the imaging quality of digital holography. An off-axis hologram is added to the iteration process via interference and inverse interference process and becomes part of the constraints. A frequency domain filter varying with the number of iterations is used to improve the competitive advantage of low frequency information in the early iterations, while retaining the high frequency information. In practical applications, an additional iterative process is used after averaging filtering to suppress the influence of the imperfect consistency between the reconstructed reference wave and the actual reference wave. Numerical simulations and experiments show that image reconstruction may be significantly improved compared to the conventional method.

1. Introduction

Digital holography (DH) uses a digital image sensor (CCD or CMOS) to record the interference pattern (hologram) between the object wave and a reference wave, and then reconstruct numerically the image starting from digitized holograms [1]. DH offers several advantages, including high precision and digital focus without the need of contact with the object. Currently, DH is widely used in microscopic imaging [2–4], 3-D measurement [5–7], particle tracing, and sizing [8–10].

There are two main types of DH: in-line DH and off-axis DH, and the difference lies in the angle between the object wave and the reference wave. Off-axis DH may use a band-pass filter to separate the spectrum of the real image, the twin image, and the zero-order image to obtain information about the real image. However, recording distance is limited by the numerical aperture (NA) of the system. The spatial bandwidth product (SBP) is not fully exploited, and the filtering process leads to a loss of high-frequency information, which in turn reduces the quality of the reconstructed image [11].

To improve the resolution of off-axis DH. He et al. [12] suggested a reconstruction algorithm (WDH) based on the wavefront coding, which combines phase recovery technology (PR) [13] and off-axis DH, using a reference wave as an encoding wave, and employs a band-pass filter with increased bandwidth to retrieve spectrum information. This method improves the resolution of off-axis DH, but requires high accuracy in collimation and reconstruction of the reference, which generally leads to a reduced field-of-view (FOV).

In-line DH can fully exploits the SBP of the image sensor to obtain a larger numerical aperture (NA), and its theoretical resolution is higher than that of off-axis DH. However, the zero-order, twin, and real images of in-line DH are overlapped. Therefore, the traditional in-line DH generally uses phase shift method [14, 15] to eliminate the twin image, which itself requires multiple wavelengths or multiple exposures, and in turn, high accuracy requirement of the experimental device, making it challenging in practical applications. An iterative algorithm can be used to eliminate twin images in Gabor in-line DH [16, 17], but it is only suitable for sparse objects, weak phase fluctuations, or finite support domain.
Some studies have reported that more information about the objects can be effectively obtained by using a hybrid approach which merges the advantages of in-line DH and off-axis DH to improve the imaging quality of DH. Orzó [18] suggested extract the low-frequency part of the complex amplitude of the object wave from the off-axis digital holographic reconstruction image, and replace it in the in-line DH iterative process to improve the reconstruction speed and eliminate the twin image. However, when the object shows strong phase fluctuations, the iterative process cannot converge. Wang et al. [19, 20] suggested a new hybrid method (IOHDH), the approximate phase distribution of the object wave from the off-axis hologram by using the constrained optimization algorithm [21], and the amplitude information of the in-line hologram as the initial value of the iterative process. However, the convergence is affected by the phase-stagnation problem due to insufficient constraints. In conclusion, the previously proposed iterative algorithm for DH could improve the performance of traditional DH, but the actual performance depends on the accuracy of initial guess, or the results of traditional off-axis DH.

In this paper, we present a novel iterative scheme for in-line and off-axis hybrid holography based on off-axis constraints (IOHDH-OC). In particular, the off-axis hologram in our scheme is used in the iterative process to strengthen the rigid constraints at the recording plane. The remainder of the paper is organized as follows: in Section 2, the improved retrieval scheme is described in details. In Sections 3 and 4, we present and compare the results obtained by different methods on the basis of numerical simulated and experimental data. Section 5 closes the paper with some concluding remarks.

2. Principle

A typical DH setup is schematically depicted in Figure 1. The object wave at the $\langle x_0, y_0 \rangle$ plane is denoted by $O(x_0, y_0)$. According to the angular spectrum approach, the complex field of the object wave in the $(x, y)$ plane can be written as follows

$$U(x, y) = \text{FFT}^{-1}\{\text{FFT}[O(x_0, y_0)]h(u, v, L)\}.$$  (1)

where FFT and FFT$^{-1}$ denote the Fast Fourier Transform and inverse Fast Fourier Transform, respectively, and $h(u, v, L) = \exp\{j(2\pi/\lambda)Ly[1 - \lambda^2(u^2 + v^2)] \}$ is the transfer function at the wavelength $\lambda$. $L$ is the recording distance and $u$ and $v$ are the spatial frequency components. The in-line hologram in the $(x, y)$ plane is given by $I_O = |U(x, y)|^2$. The plane reference wave $R(x, y)$ interferes with the object wave $U(x, y)$ in the recording plane. Then, the off-axis hologram can be expressed as follows:

$$I_H = |U|^2 + |R|^2 + U^*R + UR^*,$$  (2)

where $^*$ denotes the complex conjugation. For simplicity, we omit the coordinates $(x, y)$ in equation (2) and in the following formulas.

In the first step of the reconstruction, the in-line hologram in the presence of the sample $I_O$, the background in-line hologram without the sample $I_{O-S}$, the off-axis hologram with the sample $I_H$ and the reference wave intensity $I_R$ are recorded, respectively. Then, they are normalized in terms of the positive absorption prior [16]. The normalized in-line hologram $I_{OI}$ is given by $I_O/I_{O-S}$, the normalized off-axis hologram $I_{HI}$ by $I_H/I_{O-S}$, and the normalized reference wave intensity $I_{RI}$ by $I_R/I_{O-S}$. The reference wave at the recording plane may be written as [12].

$$R(x, y) = \sqrt{I_{RI}(x, y)}e^{jk_0(x\cos\alpha + y\cos\beta)},$$  (3)

where $\alpha$ and $\beta$ can be obtained from spectrum of $I_{HI}$, and the wave vector is given by $k_0 = 2\pi/\lambda$. The complex amplitude at the recording plane can be written as $U = \sqrt{I_{OI}}e^{j\Phi_0}$, where the initial phase $\Phi_0$ may be set to 0 without loss of generality.

In order to accurately reconstruct the complex amplitude of the object wave from the hologram, an iterative algorithm, as shown in Figure 2 can be used. The different steps of the algorithm may be summarized as follows:

1. Calculate the frequency spectrum of $U_n$ and then impose a hole support constraint with diameter based on nth iteration of the first iteration process according to the following formula:

$$F_H(u, v) = F_n(u, v) \times S_n(u, v) = \text{FFT}[U_n(x, y)] \times S_n(u, v),$$  (4)

where $S_n(u, v)$ represents a binary hole function with the actual object wave spectrum at the center of the circle and a radius $r_n = r_0 + \epsilon n$. The value of $S_n(u, v)$ inside of the circle is 1 (and 0 outside), $r_0$ is the initial radius that may safely set to 1%~10% of the number of pixels on any axis, $\epsilon$ can be set to 0.1%~5% of the number pixels on any axis, and $n$ is the iteration number. This constraint can be viewed as a varying low-frequency filter providing a competitive advantage to original image during the early iterations, since twin images correspond to high-frequency components of the spectrum. When the radius $r$ is larger than half of the number of pixels in the $x$ or $y$ axis, the constraint no longer has a restrictive effect, i.e., it does not cause information loss.
(2) The revised object wave of the object $U_n$ is obtained by inverse Fourier transform and back-propagation to the object plane is simulated by the angular spectrum approach. The obtained complex field distribution at the object plane is denoted by $O_n$.

(3) Since the hologram has been normalized, the amplitude outside the range of 0 to 1 is due to the twin images [16] and the positive absorption constraint can be used to eliminate them. After locating the pixels with $|O_n| > 1$, the value is modified as $|O_n| = 1$.

(4) $O_n$ is propagated forward to the recording plane by the angular spectrum approach, and the revised complex amplitude $U'_n$ at the recording plane is obtained.

(5) Calculate interference waves $H'_n = U'_n + R$ using the reference wave $R$.

(6) Use the off-axis hologram $I_{HI}$ and intensity constraints to correct interference waves $HO_n'' = \sqrt{1 + \text{angle}(HO_n^*)}$.

(7) Remove the reference wave $R$ and obtain the revised complex amplitudes of the object wave at the recording plane as $U_n'' = H''_n - R$. This step is referred to as the inverse interference process.

(8) Apply the in-line hologram $I_{OI}$ to update the complex amplitude distribution and obtain the revised diffractive wave $U_{n+1} = \sqrt{I_{OI} e^{\text{angle}(U_n)}}$, $U_{n+1}$ is also the initial guess at the $(n + 1)^{th}$ iteration.

(9) Repeat steps (1)–(8) until $O_n$ convergences.

In practical applications, the reference wave may not be a strictly plane wave and the intensity may not be completely uniform. Therefore, there may be some errors in the reconstructed reference wave calculated by equation (3). We can use an additional iterative process summarized below to suppress or attenuate the effect of reference wave errors.

(10) An averaging filter $G$ may be selected to suppress the carrier frequency component into the solution. The averaging filter with a spatial extent of approximately equal to half the carrier-fringe period (corresponding to the second harmonic of the carrier fringe) is sufficient for this purpose. The complex-amplitude field $U'_n$ after smoothing at the recording plane is obtained by $G \otimes U_n$, where $\otimes$ denotes convolution.

(11) The in-line hologram is used to update the complex amplitude $U_t$ at the recording plane, where $t$ represents the $t^{th}$ iteration of the second iteration process.

(12) The updated complex amplitude $U'_t$ at the recording plane is propagated back to the object plane by the spectral propagation method, and the complex amplitude distribution $O_t$ at the object plane is obtained.

(13) Repeat step (3) to obtain a new complex amplitude distribution at the object plane $O'_t$.

(14) The updated complex amplitude $O'_t$ at the object plane is propagated forward to the recording plane.
by the angle spectrum propagation method, and the estimated complex amplitude $U_{r+1}$ of object wave is obtained.

(15) Repeat steps (11)–(14) until convergence. The output complex amplitude $O_{t}$ is the reconstructed image of the object.

Unlike the previous in-line and off-axis hybrid algorithms, our method does not require the reconstruction of the off-axis hologram and thus preserves the integrity of the reconstructed image information. At the same time, further rigid constraints are introduced through three steps. The main advantages of our approach may be summarized as follows: First, the complete recurrence of interference and inverse interference processes between $R(x, y)$ and $U(x, y)$ in step (5) and step (7) lead to fast convergence. Second, the low-frequency varying filter provides a competitive advantage to real image during the early iterations. When the radius $r_n$ is larger than half of the number of pixels in the $x$ or $y$ axis, the constraint will no longer have restrictive effect, i.e., it does not cause information loss. Last but not least, the averaging filtering and the additional iterative process between the recording plane and the object plane may be used to correct the errors in the reconstructed reference wave that may occur in practical applications.

3. Numerical Simulations

In order to assess the effectiveness of the method proposed in this article, computer simulations have been carried out. The simulated object is a resolution plate of $1000 \times 1000$ pixels, shown in Figure 3(a). The sampling interval is 3.91. The amplitude transmittance of the white background part of the object is 1, the phase is 0 rad, the amplitude transmittance of the black part is 0.2, and the phase is $-2$ rad. The wavelength of the wave is 532 nm, the recording distance is 8.1 cm, and the incident angle of the reference wave is 22 mrad. The simulated in-line and off-axis holograms are shown in Figures 3(b) and 3(c), respectively.

Figures 3(d) and 3(e) are the amplitude and phase reconstructions by using the IOHDH method [20] with 80 iterations. Errors in the reconstruction are apparent, especially in phase image. In the simulation of the proposed IOHDH-OC, the initial parameters of the algorithm are $r_0=60$ pixel and $\varepsilon=10.4$ pixel Results are shown in Figures 3(f) and 3(g) after 80 iterations. It can be seen that there is no interference of twin image in the reconstructed image, and the reconstructed image has a good resolution. In order to provide a quantized comparisons, the profiles along vertical direction indicated by the red dashed lines in Figures 3(f) and 3(i) are also illustrated by Figures 3(e) and 3(h), respectively. The reconstructed phase image is clearly improved compared with the IOHDH method.

We use the RMSE (Root Mean Square Error) to evaluate the convergence of the algorithm, namely,

$$\text{RMSE} = \sqrt{\frac{1}{M \times N} \sum_{x=1}^{M} \sum_{y=1}^{N} [O(x, y) - O_n(x, y)]^2},$$

where $\sum$ is the summation sign, $M$ and $N$ are the number of pixels in the $x$ and $y$ directions of the original image and the reproduced image, $O(x, y)$ is the original image and $O_n(x, y)$ is the reconstructed image. Figure 4(a) is the RMSE of the reconstructed amplitude image as a function of the numbers of iterations, and Figure 4(b) is the RMSE of the reconstructed phase image. As it can be seen, the RMSE of IOHDH decreases slowly with the number of iterations, and stops decreasing after about 20 iterations. In addition, the convergence of IOHDH suffers from phase-stagnation problem due to insufficient constraints. The RMSE of our IOHDH-OC method declines rapidly during the first 10 iterations, and it nearly vanishes after 60 iterations. The maximum number of iterations $n$ is chosen have $r_n = r_0 + \varepsilon n$ slightly larger than half of the number of pixels in any direction of the image sensor.

The influence of the error of the measured plane reference wave on the quality of the constructed image has been also investigated by simulation. The amplitude of the plane reference wave is 1. For an amplitude and angle offset errors of the plane reference wave of 0.1 and 0.1 mrad respectively, the simulated results are shown in Figure 5. Without the additional iterative process, the results of reconstruction after 70 iterations are shown in Figure 5. There is a lot of streak-like high-frequency noise in the reconstructed image, especially in amplitude image. The quality of the reconstructed image is much lower than that of the IOHDH with the same number of iterations, as shown in Figure 5. After 10 additional iterations, the reconstructed images are shown in Figure 5. Compared with the reconstructed images obtained by IOHDH, the reconstructed amplitude images obtained by IOHDH-OC have less background noise. In the phase image, the contrast between the background phase and the delayed phase may clearly see, and the numbers in the resolution plate are clearer. In order to provide a quantitative comparisons, the profiles along vertical direction indicated by the red dashed lines in Figures 3(c), 3(f), and 3(i) are also illustrated by Figures 3(b), 3(e), and 3(h), respectively.

4. Experiments

Our experimental setup is the Mach-Zehnder interferometer shown in Figure 6. The laser beam has wavelength 532 nm and after passing through a polarizer (P) is collimated by a beam expander (BE). Then, it is split in two beams by a polarizing beam splitter (PBS). The polarization direction of the transmitted beam is parallel to the incident wave. The beam passes through the object and then it is sent to the image sensor (CCD) digital camera by a beam splitter (BS) as the object wave. The reflected beam passes through a half-wave plate (HWP), making polarization direction parallel to the object wave, and then it is reflected to the image sensor of
Figure 3: Comparison between of the performance of IOHDH and IOHDH-OC. (a) Object. (b) In-line hologram. (c) Off-axis hologram. (d) Reconstructed amplitude image obtained by using the IOHDH. (e) Reconstructed phase image obtained by using the IOHDH. (g) Reconstructed amplitude image obtained by using steps (1)–(9) of the IOHDH-OC. (h) Reconstructed phase image obtained by using step (1)–(9) of the IOHDH-OC. (f) and (i) are the profiles along the black dashed lines in (e) and (h), respectively.

Figure 4: The reconstructed image error as a function of the number of iterations. (a) RMSE of the reconstructed amplitude image. (b) RMSE of the reconstructed phase image.
the digital camera by a beam splitter (BS) as the reference wave. The distance from the object to the recording plane is $L = 81\, \text{mm}$. The pixel size of the CCD is 3.91 $\mu\text{m}$.

As a first example, the USAF resolution plate was used as the object to be measured. 1000 × 1000 pixels holograms were used in the experiment. Results are shown in Figure 7.

The final results, by using the IOHDH-OC, are shown in Figures 7(e) and 7(f), to be compared with the results of IHODH shown in Figures 7(c) and 7(d). The improvement obtained by our method is apparent.

To further verify the phase contrast imaging effect of IOHDH-OC, we apply it to the wings of Grain Blue

**Figure 5**: Simulation results of reconstructed plane wave with errors. (a) Reconstructed amplitude image obtained by using step (1)–(9) of the IOHDH-OC. (b) Reconstructed phase image obtained by using step (1)–(9) of the IOHDH-OC. (d) Reconstructed amplitude image obtained by using the IOHDH. (e) Reconstructed phase image obtained by using the IOHDH. (g) Reconstructed amplitude image obtained by using the IOHDH-OC. (h) Reconstructed phase image obtained by using the IOHDH-OC. (c), (f), and (i) are the profiles along the black dashed lines in (b), (e), and (h) respectively.

**Figure 6**: Schematic diagram of the experimental setup.
Dragonfly. The pictures of the wings of Grain Blue Dragonfly are taken by a digital camera (SONY NEX-7) with a 1-magnification macro lens (SONY SEL30M35), as shown in Figure 8(a). It can be seen from the figure that the wings of dragonfly have a rather complex structure. The recorded in-line hologram is shown in Figure 8(b), and the off-axis hologram is shown in Figure 8(c). The intensity and phase images obtained by traditional in-line-and-off-axis hybrid digital holography (IOHDH) are shown in Figures 8(d) and 8(e). Figures 8(f) and 8(g) are the reconstruction obtained by our method. As can be seen, the quality of the intensity image is comparable to that of IOHDH, and our method can effectively restore the phase information, e.g., the vein pattern is clearer. For a better comparison, the reconstruction of section of dotted line in Figures 8(e) and 8(g) has been carried out, and results are shown in Figure 8(h). As it is apparent from the plot, our method retains more high-frequency information and provides more accurate phase information.
Figure 8: Experiment on the wings of grain blue dragonfly. (a) Common images of lacewing flies. (b) In-line hologram. (c) Off-axis hologram. (d) Amplitude reconstruction image of IOHDH. (e) Phase reconstruction image of IOHDH. (f) Amplitude reconstruction image of IOHDH-OC. (g) Phase reconstruction image of IOHDH-OC. (h) Phase profile curves.
5. Conclusions

We have proposed and demonstrated a hybrid iterative algorithm for in-line and off-axis holography based on off-axis hologram constraints. Off-axis hologram and range-changing frequency domain filter are exploited to improve the iterative process, and combined with the mean filtering technique and an additional iterative process to suppress the effects of errors in the reconstruction of the reference wave. Numerical simulations with or without errors in the reference wave have been completed, as well as experimental measurements using a USAF resolution plate and dragonfly wings. Reconstruction results clearly show that our method leads to improved reconstructed image compared to conventional method. In particular, our method shows clear advantages in the reconstruction of phase images even in the presence of speckle noise and initial errors. In conclusion, our proposed method allows one to obtain more accurate reconstructed images compared with the conventional in-line and off-axis hybrid DH imaging method and pave the way for a widespread application of DH to complex objects.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was financially supported by the Natural Science Foundation of China (NSFC) (61965002), the Science and Technology Program of Jiangxi Province (20192BBG70006), the Graduate Innovation Special Fundation of Jiangxi Province, China (YC2021-S738).

References


