Research Article

Design and Theoretical Analysis of Highly Negative Dispersion-Compensating Photonic Crystal Fibers with Multiple Zero-Dispersion Wavelengths

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Abstract

This paper presents a highly negative dispersion-compensating photonic crystal fiber (DC-PCF) with multiple zero dispersion wavelengths (ZDWs) within the telecommunication bands. The multiple ZDWs of the PCF may lead to high spectral densities than those of other PCFs with few ZDWs. The full-vectorial finite element method with a perfectly matched layer (PML) is used to investigate the optical properties of the PCFs. The numerical analysis shows that the proposed PCF, i.e., PCF (b), exhibits multiple ZDWs and also achieves a high negative chromatic dispersion of $-15089.0 \text{ps/nm}$ at $1.55 \mu\text{m}$ wavelength, with the multiple ZDWs occurring within the range from 0.8 to 2.0 $\mu\text{m}$ range. Other optical properties such as the confinement loss of 0.059 dB/km, the birefringence of $4.11 \times 10^{-1}$, the nonlinearity of $18.92 \text{W}^{-1}\text{km}^{-1}$, and a normalized frequency of 2.633 was also achieved at $1.55 \mu\text{m}$ wavelength. These characteristics make the PCF suitable for high-speed, long-distance optical communication systems, optical sensing, soliton pulse transmission, and polarization-maintaining applications.

1. Introduction

The discovery of a photonic crystal fiber (PCF) has presented numerous opportunities for optical communication systems than conventional fibers [1–3]. PCFs operate by guiding light based on the properties of photons. These guiding mechanisms of light in PCF can either be index-guiding photonics [4], photonic bandgap guiding [5], or a hybrid of the two guiding principles [6]. The index-guiding PCFs are a subclass of PCFs with a central solid core [7] surrounded by microstructured cladding. The core has a higher refractive index than that of the cladding. On the other hand, the photonic bandgap PCFs confine light based on the bandgap effects [5]. Each type of PCF has its unique strength; however, a hybrid of the two results in a PCF with flexible features [8]. Also, based on the flexible structure design, material use, and wide range of applications, PCF has recently received much more attention than conventional optical fibers [9–10]. Achieving the desired property in a PCF is often obtained by tailoring the geometrical properties of air holes in the cladding [11–13] and the optical properties of the dopant. These flexibilities give PCFs due advantage over the conventional fibers in applications such as optical communication [14], refractive index and temperature measurement [15], liquid chemical sensing [16, 17], gas sensing [18], biosensing [19–21], nonlinear optic devices, high-speed terahertz propagation of the optical signal to remote distances [22], coherent optical tomography [23], optical transmissions (lasers) [24], submarine communications [25], chromatic dispersion management [26, 27], reduction in confinement and bending losses [28, 29], high birefringence, high nonlinearity [30, 31], as well as single or multimode [27, 32, 33] operations of optical signals. Despite all these benefits that PCF offers, it is faced with challenges.
such as chromatic dispersion, confinement, and bending losses when propagating signals over long distances. To deal with these limitations, one can design a fiber with zero dispersion within the C-band in the telecommunication spectrum. One could also design a fiber with a high negative dispersion coefficient to counteract the effects of any dispersion that may arise in the optical link.

The authors in [30] reported a hexagonal-structure PCF with four rings using circular air holes around the core to achieve a negative chromatic dispersion of $-47.72\, \text{ps/} \mu\text{m}$, a birefringence of $2.02 \times 10^{-2}$, and a nonlinear coefficient of $40.68 \, \text{W}^{-1}\text{km}^{-1}$ at 1.55 $\mu\text{m}$ wavelength. The results achieved a high nonlinear coefficient. However, it recorded a low negative chromatic dispersion. Similarly, the authors in reference [34] proposed a PCF of dual-concentric design. They reported a $-78010 \, \text{ps/nm} \cdot \text{km}$ dispersion at a wavelength of 1.55 $\mu\text{m}$. However, the dual-core concentric nature of the PCF will pose fabrication challenges. Besides, their design did not report nonlinearity, an equally important property of PCFs for optical communication.

A signal’s power spectral density (PSD) is the distribution of optical energy in a communication system. High PSD is ideal for reducing fiber losses [35]. One way to achieve that is by designing PCF with many zero-crossing dispersion wavelengths (ZDW). The authors of reference [26] reported the design of a PCF with three ZDWs using circular air holes, while the authors of reference [27, 31, 36] proposed and designed PCFs with two zero-crossing dispersion wavelengths. Specifically, Amoah et al. [26] designed a PCF with three ZDWs and a dispersion of $-220.39 \, \text{ps/} \mu\text{m}$. However, the dual-core concentric nature of the PCF will pose fabrication challenges.

2. Design of the Proposed DC-PCF

In this investigation, silica-based index-guiding PCFs with a hexagonal arrangement of circular air holes in the claddings are designed. An inner diameter, $D_i = 9.8 \, \mu\text{m}$ and an outer diameter, $D_o = 11.8 \, \mu\text{m}$, is used throughout the three designs. Each cladding consists of an equilateral triangle of sides equivalent to the pitch ($\Lambda$). Detailed designs of each of the structures are illustrated below.

(i) In Figure 1(a), uniform air holes of diameter ($d_0$) = 1.5 $\mu\text{m}$ were used in this design without inserting tiny air holes in the cladding while the pitch($\Lambda$) was kept constant at 2.2 $\mu\text{m}$.

(ii) Similarly, in Figure 1(b), bigger air holes of diameter ($d_6$) = 1.5 $\mu\text{m}$ and a constant pitch ($\Lambda$) of 2.2 $\mu\text{m}$ were inserted. Furthermore, 10 tiny uniform air holes of diameter ($d_1$) = 0.75 $\mu\text{m}$ were inserted diagonally in between the bigger air holes in the cladding, as shown in Figure 1(b).

(iii) In Figure 1(c), bigger air holes of varying diameters $d_2 = 1.3$, $d_3 = 1.4$, $d_4 = 1.5$, $d_5 = 1.6$, and $d_6 = 1.7 \, \mu\text{m}$, respectively, were inserted from the inner ring towards the outer ring across the diagonals of the hexagonal structure. In addition, medium air holes of uniform diameter ($d_1$) were symmetrically inserted in the cladding. Furthermore, 6 tiny air holes of uniform diameter ($d_1$) = 0.75 $\mu\text{m}$ were inserted between the bigger air holes in the first ring to form a circular structure. The proposed designs were simulated with COMSOL Multiphysics 5.5.

### 2.1. Principle of Operation and Simulation of PCFs

The full-vector finite element method (FV-FEM) with a perfectly matched layer (PML) is used to analyze the optical characteristics of all PCFs. In order to absorb the evanescent waves in the PCF models, a circular anisotropic perfectly matched layer (C-PML) of 2.0 $\mu\text{m}$ was applied around the boundaries of each PCF to truncate and confine the optical waves in the core, as shown in Figures from 2(a) to 2(c). Each design has been subdivided into four computational regions. The fine-meshed element size was applied on each PCF’s given minimum element quality of 0.1895 and average element quality of 0.7555. The meshed-generated structures are shown in Figures from 3(a) to 3(c). For PCF geometry (a), 31150 domain elements and 2730 boundary elements were obtained. Figure (b) yielded 32038 domain elements and 3004 boundary elements. Similarly, Figure (c) has 56140 and 2784 domain and boundary elements, respectively. Maxwell’s equation [30] shown in equation (1) with the FV-FEM in PML is used to simulate the PCFs.

$$\nabla \times (\left[ s^{-1}\right] \nabla \times E) - k_o^2 n^2 |s| E = 0,\tag{1}
$$

where $E$ is the electric field vector, $n$ is the refractive index of silica determined by the Sellmeier equation, $|s|$ is the PML matrix, $[s^{-1}]$ is the inverse matrix of $[s]$, $k_o = 2\pi/\lambda$ is the
wave number in a vacuum, and \( \lambda \) is the wavelength. The refractive index of silica material for our proposed PCF was determined using the Sellmeier equation [30] as shown by the following equation:

\[
n^2 = 1 + \frac{B_1 \lambda^2}{\lambda^2 - C_1} + \frac{B_2 \lambda^2}{\lambda^2 - C_2} + \frac{B_3 \lambda^2}{\lambda^2 - C_3},
\]

(2)

where \( \lambda \) is the wavelength in (\( \mu \)m), \( n \) is the refractive index of pure silica material, and \( B_{1,2,3} \) and \( C_{1,2,3} \) are the Sellmeier constants as shown in Table 1.

2.2. Optical Properties of PCFs. Chromatic dispersion is a major inhibiting factor for the transmission of optical signals. It causes the spread of signals in the fiber link, resulting in a high bit-error rate (BER). Dispersion may result from the waveguides, \( D_w(\lambda) \), or the optical property of the material, \( D_m(\lambda) \). Chromatic dispersion, \( D(\lambda) \), is the summation of the waveguide and material dispersion. Each of the components of dispersion is computed using the following equations [39]:

\[
D_m(\lambda) = -\frac{\lambda}{c} \frac{\partial^2 n_{\text{silica}}}{\partial \lambda^2},
\]

(3)

where \( \lambda \) is the wavelength and speed of light in free space and \( n_{\text{silica}} \) is the refractive index of silica material.

\[
D_w(\lambda) = -\frac{\lambda}{c} \frac{\partial [\text{Re}(nf)]_{\text{silica}(\lambda)=\text{const}}}{\partial \lambda^2},
\]

(4)

where \( c \) is the speed of light in free space, \( \text{Re}(n_{\text{eff}}) \) is the real value of the effective refractive index, and \( \lambda \) is the wavelength. The net dispersion is calculated as follows:

\[
D(\lambda) = D_m(\lambda) + D_w(\lambda) \frac{\rho_s}{nm.km}.
\]

(5)

The refractive index of the optical material is affected by the polarization of light. This effect results in the difference in refractive indexes of the orthogonally x-y polarized modes. Such an absolute difference is known as birefringence. A high birefringence value is necessary for designing PCF with sensing applications [40]. Birefringence is determined numerically by using the following equation [41]:

\[
B = \lambda \left( \frac{\beta_x - \beta_y}{2\pi} \right) \left| n_x - n_y \right|,
\]

(6)

where \( n_x \) and \( n_y \) are the refractive indices of the material’s two fundamental polarization modes. When light propagates through a transparent medium, some of its optical power may be lost due to absorption, bending, or leaking from the core due to cladding or due to an infinite number of air holes in the cladding. The associated optical energy thus will be leaked from the core to the cladding. This phenomenon constitutes confinement loss. Confinement loss is calculated using the imaginary part of the effective refractive index as shown by the following equation [33]:

\[
C_{\text{loss}} = \frac{40\pi \text{Im}(n_{\text{eff}}) dB/km}{In(10) \times \lambda} = 8.686 \times 2\pi \times \text{Im}(n_{\text{eff}}) \frac{dB}{m},
\]

(7)

where \( \lambda \) is the wavelength and \( \text{Im}(n_{\text{eff}}) \) is the imaginary part of the effective refractive index. The effective mode area is the area in optical fiber over which the electric field (E) is optimally distributed. The effective mode area is an essential parameter in analyzing fiber nonlinearity. A large effective mode area is essential in systems with high-bit data transmission. However, for nonlinear systems small effective mode is required. The effective mode area of the fiber is computed using the following equation [42]:

\[
A_{\text{eff}} = \left( \frac{\int_{-\infty}^{\infty} E^2 dx dy}{\int_{-\infty}^{\infty} E^4 dx dy} \right) (um^2).
\]

(8)

The nonlinear coefficient [33] is computed with the following formula:

\[
y = \frac{2\pi}{\lambda} \left( \frac{n_2}{A_{\text{eff}}} \right).
\]

(9)
where $n_2$ is the coefficient of the refractive index of silica material, $2\pi/\lambda = k\theta$ is the wave number in free space, $\lambda$ is the wavelength, and $A_{\text{eff}}$ is the effective mode area. For this research study, $n_2$ is taken as $2.76 \times 10^{-20} \text{ m}^2 \text{ W}^{-1}$ [30].

### 3. Results and Discussion

The results in Figure 2 depict the confinement of the optical energy in the core of the three PCF structures when simulated at 1.55 $\mu$m wavelength. It can be justified that the optical energy is confined in the core in all three cases. The intensity of the optical energy can be deduced from the legend of the 2D plot $\lambda = 1.55 \mu$m. However, the results of X-Y polarized modes shown in Figure 4 depict that the PCF with geometry (b) and the proposed PCF gives clear polarized modes among the three PCF structures.

#### 3.1. Effects of Variation of the Diameter of Air Holes on the Effective Index of PCFs

Here, we analyse the effects of variation of the diameter of air holes on the effective index of the different designs, since the effective index plays a sensitive role in the analysis of all other characteristics of PCFs. We varied the diameter of the bigger air holes from 1.0 to 1.5 $\mu$m while keeping the pitch size constant at 2.2 $\mu$m. We simulated the designs at operating wavelengths of 1.3 and 1.55 $\mu$m. It is observed from Figure 5 that the effective refractive index is high when the diameter of the corresponding air holes is relatively smaller in all three cases. The modal indexes decrease monotonically as the diameter of the air holes increases across the wavelength from 1.3 to 1.55 $\mu$m. However, we investigated other characteristics while keeping the diameter of the air holes constant at 1.5 $\mu$m for all three designs.

Figure 6 shows a graph of the effective refractive index as a function of wavelength for the three PCF structures. We deduced a gradual decline in the effective refractive index from 1.2 to 2.0 $\mu$m as in the other three structures, which justifies the relation $n_{\text{eff}} = k/\lambda$. It is also worth noting that in PCF (b) and PCF (c), the effective refractive indexes decrease steeply as the wavelength increases. In contrast, PCF (a) results show the least decrement.

#### 3.2. Effects of Variation of the Diameter Air Holes on Dispersion

Figure 7 shows chromatic dispersion as a function of operating wavelength for geometric variations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Constant value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refractive index of the core</td>
<td>1.45</td>
<td>—</td>
</tr>
<tr>
<td>$B_1$</td>
<td>0.696166300</td>
<td>—</td>
</tr>
<tr>
<td>$C_1$</td>
<td>4.679148$e^{-3}$</td>
<td>—</td>
</tr>
<tr>
<td>$B_2$</td>
<td>0.407942600</td>
<td>—</td>
</tr>
<tr>
<td>$C_2$</td>
<td>1.3512063$e^{-2}$</td>
<td>—</td>
</tr>
<tr>
<td>$B_3$</td>
<td>0.897479400</td>
<td>—</td>
</tr>
<tr>
<td>$C_3$</td>
<td>97.9340025</td>
<td>—</td>
</tr>
</tbody>
</table>
of the number and diameter of bigger air holes ($d_0$) for the three PCF structures. As the diameter of air holes, $d_0$ varied from 1.0 to 1.5 $\mu$m, the dispersion coefficient decreased gradually at a 1.5 $\mu$m wavelength. At a geometry specified below the graph, PCF (b) produced high negative dispersion than the other two PCFs. Since we aim to design a high negative dispersion-compensating (DC-PCF) with multiple zero-crossing dispersion wavelengths, we included smaller air holes for further analysis.

3.3. Effects of Inclusion of Tiny Air Holes on Chromatic Dispersion. The results plotted in Figure 8 depicted chromatic dispersion for the PCFs when tiny air holes were inserted symmetrically in PCF (b) and PCF (c). To underpin the novelty of our proposed design, it is observed that with the insertion of tiny air holes in the cores, both PCF (b) and PCF (c) produced multiple ZDWs and high negative chromatic dispersions. In contrast, PCF (a) produced nearly all positive and ultraflattened dispersion over a wavelength from 800 to 1600 nm. Comparing PCF (b) and PCF (c), it was also observed that with the inclusion of 10 tiny air holes in the first ring, PCF (b) yielded a high dispersion of $-15089$ ps/nm-km and $-986.57$ ps/nm-km at 1.55 $\mu$m and 1.3 $\mu$m, respectively, than PCF (c). This characteristic gives PCF (b) an edge over PCF (a) and PCF (c) in dispersion compensation, underpinning the novelty of our design.

The results in Figure 9 show the confinement loss against the wavelength for PCF (a) and PCF (b) at a constant air hole size of 1.5 $\mu$m, a pitch size of $\Lambda = 2.2$, and an air filling ratio of $d_0/\Lambda = 0.68$. Figure 11 shows the nonlinearity coefficient as a function of operating wavelength for all the different PCF structures. It is observed that the nonlinearity decreases as the
wavelength increases for all three PCFs at constant air holes of \( d_0 = 1.5 \, \mu m \) and wavelength \( \lambda = 1.55 \, \mu m \) for the different structures of PCF.

Figure 12 shows the birefringence against the wavelength for all PCFs. As can be observed from the figure, birefringence is higher and increases sharply for the PCF with 10 tiny air holes. It, however, remains fairly constant in a zigzag manner in PCF (a) and PCF (c). At \( \lambda = 1.55 \, \mu m \), PCF (b) yielded a birefringence of 0.411, while PCF (a) and PCF (c) yielded a birefringence of 0.069 and 0.045, respectively. Clearly, PCF (a) and PCF (c) could not produce a high birefringent and hence will not be suitable for sensor applications.
The effective mode area, $A_{\text{eff}}$, is essential in analyzing the nonlinearity and confinement losses in PCF. Here, we plot a graph of $A_{\text{eff}}$ to variation in operating wavelength for all three PCF designs. A highly effective area is essential in reducing the confinement loss. However, it is inversely related to nonlinearity. In Figure 13, the effective area of the PCF (a) and PCF (b) increases monotonically with an increase in the wavelength while that of PCF (c) remains constant from 1.2 to 2.0 µm wavelengths. The graph depicts that PCF without the inclusion of tiny air holes yielded a relatively high effective mode area than the others at $d_0=1.5 \, \mu\text{m}$ and $\Lambda=2.2$.

The normalized frequency, also known as the V-number, as seen in Figure 14, decreases with an increase in the operating wavelength in the PCF with 10 tiny air holes in all three cases of variation in the geometric dimension of the PCF. It is worth noting that the V-number decreases linearly in PCF (b). The V-number is the character used to determine the number of modes of a PCF. When the V-number is less than 3.1415 throughout the entire length of the fiber, then such fiber is endlessly as single mode; otherwise, it is a multimode PCF. It observed that at $d_0=1.5$, $d_1=0.75$, and $\Lambda=2.2 \, \mu\text{m}$, the proposed PCF has a V-number of less than 3.1415 from 1.2 to 2.0 µm thus, exhibiting single-mode characteristics. The V-numbers at 1.3 µm and 1.55 µm are 2.912 and 2.633, respectively. However, when $d_0=1.3$, $d_1=0.7$, and $\Lambda=2.2 \, \mu\text{m}$, the PCF behaves as a multimode at lower wavelengths and moves towards a single mode as wavelength increases. Therefore, the proposed PCF can support both single-mode and multimode operations.

Table 2 shows our proposed PCF’s point of zero-crossing dispersion wavelengths. The literature has established that multiple ZDWs give PCFs a high power spectral density (H-PSD), making our proposed PCF a good candidate for chromatic dispersion compensation, applicable for high-speed optical communication systems. Table 3 shows the performance comparison between the PCFs reviewed and our proposed PCF at 1.55 µm. Our proposed PCF achieved a relatively high negative chromatic dispersion and multiple ZDWs within a wide range of wavelengths.
**Table 3:** Comparison of the PCF designs with the proposed PCF with dispersion at 1.55 μm.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Nature of air holes</th>
<th>Dispersion (ps/nm-km)</th>
<th>Number of ZDW</th>
<th>Wavelength range (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[6]</td>
<td>All circular air holes</td>
<td>−47.72</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>[18]</td>
<td>All circular air holes</td>
<td>−366.5</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>[24]</td>
<td>All circular air holes</td>
<td>−2012</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>[30]</td>
<td>All circular air holes</td>
<td>−1672</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>[21]</td>
<td>All circular air holes</td>
<td>−220.39</td>
<td>3</td>
<td>1.53 – 1.8</td>
</tr>
<tr>
<td>[27]</td>
<td>All circular air holes</td>
<td>—</td>
<td>2</td>
<td>0.81 – 1.55</td>
</tr>
<tr>
<td>[38]</td>
<td>—</td>
<td>—</td>
<td>3</td>
<td>0.771 – 1.014</td>
</tr>
<tr>
<td>[37]</td>
<td>All circular air holes</td>
<td>—</td>
<td>3</td>
<td>1.2 – 2.2</td>
</tr>
<tr>
<td>[43]</td>
<td>All circular air holes</td>
<td>−2357.54</td>
<td>—</td>
<td>1.34 – 1.58</td>
</tr>
<tr>
<td>[44]</td>
<td>All circular air holes</td>
<td>−571.68 to −1889</td>
<td>—</td>
<td>1.34 – 1.64</td>
</tr>
<tr>
<td>[45]</td>
<td>All circular air holes</td>
<td>−1732.10</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>[Proposed]</td>
<td>All circular air holes with the insertion of tiny air holes within the first ring</td>
<td>−15089.0</td>
<td>6</td>
<td>0.8 – 2.0</td>
</tr>
</tbody>
</table>

The symbol — indicates that the research study did not consider that property is not available.
4. Conclusion

A novel silica-based dispersion-compensating photonic crystal fiber (DC-PCF) with multiple zero dispersion wavelengths (ZDWs) within the telecommunication bands was investigated and proposed. The multiple ZDWs of the PCF may lead to high spectral densities than those of other PCFs with few ZDWs. Using the full-vectorial finite element method with an anisotropic perfectly matched layer (PML), the numerical analysis shows that the proposed PCF, i.e., PCF (b), achieved a high negative chromatic dispersion (~15089.0 ps/nm-μm) at λ = 1.55 μm which is higher than the results in [43–45]. Besides, the multiple zero-crossing dispersion wavelengths occurring within the wavelength range from 0.8 to 2.0 μm give the proposed PCF a high power spectral density (HPSD) making it suitable for dispersion compensation. Other optical properties such as the confinement loss of 0.059 dB/km, the birefringence of 4.11 ×10−3, the nonlinearity of 18.92 W−1 km−1, and a normalized frequency of 2.633 was also achieved at 1.55 μm wavelength. These characteristics make the PCF suitable for high-speed, long-distance optical communication systems, optical sensing, soliton pulse transmission, and polarization-maintaining applications.

Data Availability

The supplementary data used to support the findings of the study can be obtained from the corresponding author via e-mail upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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