The laser beams were scattered and attenuated when they propagate in fogs for laser communication, laser remote sensing detection. For different density and droplets distribution of fogs, the laser scatter and attenuation are different, the corresponding mechanism needs thorough investigation. The characteristics of laser beam scattering in different types of fogs are studied based on the droplet size characteristics of advection fog and radiation fog, the scattering coefficients of droplets with different laser wavelengths (0.86 \( \mu m \), 0.91 \( \mu m \), 1.06 \( \mu m \), 1.3015, and 10.6 \( \mu m \)) are calculated, the multi scattering of laser beam is studied by the Monte Carlo method, the propagation path and scattering direction of photons is analyzed, relations between asymmetry factor, albedo of fog droplets, and the visibility are presented, and the forward scattering intensity and the backward scattering intensity versus scattering angle are gotten and discussed.

1. Introduction

Fogs contain small water droplets, these droplets suspend in the air, and they attenuate laser beam; fogs have adverse effects on optical communication, lidar, remote sensing, etc. Many scientists have conducted researches in this field. Nebuloni derived relations between an extinction coefficient and visibility of fog for near, mid, and far-IR (infrared) light transmission of free-space optic communications; the extinction coefficients in the visible 0.55 \( \mu m \), the near IR 1.2 \( \mu m \), and the mid IR 3.7 \( \mu m \) are comparable to and roughly twice as much as that in the far-IR 10.6 \( \mu m \) when visibility is less than a few hundred meters [1]. Grabner and Kvicer analyzed the multiple scattering of light and the impulse response of optical channel in rain and fog for free-space optical links. Simulations show that optical attenuation of the multiple scattering due to rain is about two times lower than predicted by single scattering approach [5]. Mori and Marzano proposed an atmospheric water particles scattering model (a unified microphysically oriented atmospheric particle scattering model) and simulated the scattering effects on free-space optics links, scattering, absorption, and extinction coefficients and the asymmetry factor are numerically simulated for each particle...
class, and then parametrized with respect to particle water content, full rate, and visibility, spanning from visible to infrared wavelengths [6]. Wu et al. analyzed the changes of extinction coefficients and visibilities based on short and long time lidar data, and evaluated the free-space optical communication link for different wavelengths under the sea surface environment. The horizontal extinction coefficients and visibilities are obtained by inverting the collected data. On this basis, the free-space optical communication link availabilities of the 850 and 1550 nm links are evaluated during the observation period [7]. Al Naboulisi and Sizun presented results on fog attenuation at wavelengths 650, 850, and 950 nm; they have compared the experimental data with our fog attenuation models; the advection and the radiation one deduced from FASCODE (Fast Atmospheric Signature Codes). This comparison clearly shows that, for the three studied wavelengths, their models lead to the best fits for the measured data [8]. Shapiro used ultrashort (femtosecond duration) light pulses for line-of-sight free-space optical (FSO) communication through fog; their study show that scattering-induced multipath spread is less than the reciprocal of the scattering-induced doppler spread [9]. Su et al. investigates scintillation and fog attenuation effects for THz and IR signals by measuring bit error rates (BER), signal power, and phase front distortions [10]. Shah et al. established an engineering model of laser transmission in fog, and compared with other models, selected nine models including theoretical and empirical models, which were simulated in MATLAB, and the comparison results were discussed [11]. Wang et al utilized a large quantity of high-quality water vapor lidar data collected by a multifunction Raman lidar system, investigated the air mass trajectories, variations of water vapor over Xi’an. The results indicate that local airflow trajectories mainly affect water vapor transport below the boundary layer, and that these flows are closely related to the formation of fog and haze events in the Xi’an area [12]. Di et al. proposed a method to measure the aerosol and fog particle size distribution by optimizing scattering angle and wavelengths; seven wavelengths for extinction coefficients and five wavelengths for forward scattering coefficients were chosen for the retrieval of particle size distribution in the measurement [13]. Zeng et al. studied the propagation of linear and circular polarization in fog, and comparing their transmission characteristic with different condition, investigated the transmission performance of circular and linear polarization in variable foggy environments, exploring the impact of the detection range in particular [14]. Zhang et al. calculated on the reflection and transmission of light propagating through sea fog by dividing fog into different layers with different refractive indices. The scattering processes of the radiation in the polydisperse sea fog layer are traced in their improved Monte Carlo (MC) simulation. They provide more accurate calculations on the reflection and transmission when radiation propagates through poly-disperse sea fog media of two different refractive indices [15]. Vasseur and Gibbins developed an experimental method to infer the physical characteristics of fog from concurrent attenuation measurements at millimeter, infrared, and visible wavelengths; a gamma function comprised of three parameters is assumed for the drop size distribution. Simulations of fog attenuation based on the inferred drop size distribution are found in excellent agreement with the measurements [16]. Al Naboulisi et al. investigated laser system performance in the advection and convection fog in the 0.4 μm to 15 μm spectral zone, the author also proposes a fast transmission relations based on an exact Mie theory calculation valid in the 0.69 μm to 1.55 μm spectral bands [17].

The visibility is an important parameter of fog, which can be measured directly and quickly; however, the correlation between fog droplet parameter and the visibility and the scattering parameters of beams changing with fog’s visibility for different beams’ wavelengths have not been reported by now. The current laser transmission model is simple, for laser with different wavelength, it is urgent to study scattering character of fog particles and the multiple scattering of low visibility fog.

In this paper, we propose on the correlation between visibility and fog droplet parameter, such as distribution, droplet extinction coefficient, and asymmetry factor of different types of fogs. Based on this, using Monte Carlo method, for infrared laser beams with different wavelength, the multi scattering of laser beam in fogs is studied. The coefficients versus visibility and scattering intensity distribution versus scattering angle are studied in detail.

### 2. The Particle Size Distribution of Fog

The fog particles are small water droplets. The particle size distribution function is given as follows [8, 14]:

\[
\ell(r) = a^2 e^{-br},
\]

where \( n \) is the number of droplets per unit volume and unit radius and \( r \) is the droplet radius. Other parameters are determined by the size distribution of fog.

Visibility of fog is related to the optical attenuation \( \gamma \), which is expressed as follows [18]:

\[
V = \frac{3.912}{\gamma}.
\]

The optical attenuation \( \gamma \) is as follows [16]:

\[
\gamma = 4.343 \cdot 10^{-3} \int_0^\infty Q_e(r) \pi r^2 n(r) dr dB/km,
\]

where \( Q_e(r) \) is the optical extinction efficiency. The dimensions of fog droplets are much greater the visible optical wavelength, so \( Q_e(r) \) of fog droplets can be consider as 2 [17]. \( \gamma \) can be written as follows [16]:

\[
\gamma = 4.343 \cdot 10^{-3} \int_0^\infty 2 \pi r^2 \cdot a^2 e^{-br} dr
\]

\[
= \frac{8.686 \cdot 10^{-3} \pi a^4! dB}{b^3} km^{-1}
\]

Liquid water content is expressed as follows[5]:

\[
W = \frac{4}{3} \pi \rho \int_0^\infty r^3 n(r) dr,
\]

where \( \rho \sim 10^6 \) g/m³.

With equation (1), \( W \) is rewritten as follows:
\[
W = 10^6 \frac{4\pi}{3} \int_0^\infty ar^2 e^{-br}dr = \frac{4\pi a}{3b^6} \times 10^6. \quad (6)
\]

By equations (2)-(6), \(a\) and \(b\) is expressed by \(V\) and \(W\) as follows:
\[
a = \frac{9.781}{V^6 W^5} 10^{15}, \quad b = \frac{1.304}{VW} 10^4. \quad (7)
\]

The droplet concentration can be obtained as follows:
\[
N = \int_0^\infty n(r)dr = \frac{2a}{b^3} = \frac{8.222}{V^3 W^2} \cdot 10^3. \quad (8)
\]

The model radius corresponds to the peak value of droplet size distribution curve as follows:
\[
r_o = \frac{2}{b} = 1.534 \cdot 10^{-4} V W. \quad (9)
\]

The average radius of droplet size distribution is as follows:
\[
\bar{r} = \frac{1}{N} \int_0^\infty rn(r)dr = \frac{3}{2} \frac{r_0}{b} = 2.301 \cdot 10^{-4} V W. \quad (10)
\]

According to different regions, fog can be divided into two types, advection fog and radiation fog. For advection fog, relations between the liquid water content \(W\) and the \(V\) is shown as follows[19]:
\[
W = (18.35V)^{-1.43} = 0.0156V^{-1.43}\left(\frac{g}{m^2}\right). \quad (11)
\]

For radiation fog, it is shown as follows:
\[
W = (42.0V)^{-1.54} = 0.0316V^{-1.54}\left(\frac{g}{m^2}\right). \quad (12)
\]

For advection fog, droplet size distribution is calculated from equations (1), (7), (11), and (12), as follows:
\[
n_a, n(r) = 1.059 \cdot 10^7 V^{1.15} r^2 e^{-0.8359V^{0.43} r}
\]
\[
= 3.73 \cdot 10^7 W^{-0.804} r^2 e^{-0.2392W^{-0.301} r}. \quad (13)
\]

For radiation fog, droplet size distribution is as follows:
\[
n_a, n(r) = 3.104 \cdot 10^{10} V^{1.7} r^2 e^{-4.122V^{0.54} r}
\]
\[
= 5.400 \cdot 10^7 W^{-1.104} r^2 e^{-0.5477W^{-0.351} r}. \quad (14)
\]

### 3. Scattering Coefficient of Fog Droplets

Using Mie scattering theory, the extinction coefficient \(Q_e\), scattering coefficient \(Q_s\), absorption coefficient \(Q_a\), asymmetry factor \(g\), and single albedo \(w_0\) of a single particle can be calculated as follows [20]:

\[
Q_e = \frac{2}{\alpha_1} \sum_{n=1}^{\infty} (2n+1) \Re \left( a_n + b_n \right),
\]

\[
Q_s = \frac{2}{\alpha_1} \sum_{n=1}^{\infty} (2n+1) \left( |a_n|^2 + |b_n|^2 \right),
\]

\[
Q_a = Q_e - Q_s,
\]

\[
w_0 = \frac{Q_e}{Q_s},
\]

\[
g = \frac{4}{\alpha_1 Q_s} \sum_{n=1}^{\infty} \left\{ \frac{n(n+1)}{n+1} \Re \left( a_n a_{n+1}^* + b_n b_{n+1}^* + \frac{2n+1}{n(n+1)} \Re (a_n b_n^*) \right) \right\}.
\]

\[
\langle \sigma_a \rangle = \frac{\pi}{N} \int_0^\infty r^2 Q_a(r)n(r)dr.
\]

The average absorption coefficient is as follows:
\[
\langle \sigma_a \rangle = \frac{\pi}{N} \int_0^\infty r^2 Q_a(r)n(r)dr.
\]

The droplet concentration is as follows:
\[
N = \int_0^\infty n(r)dr.
\]

The average single albedo is as follows:
\[ \langle w_0 \rangle = \frac{\langle \sigma_i \rangle}{\langle \sigma_e \rangle} \quad (20) \]

The average asymmetry factor is as follows:
\[ \langle g \rangle = \frac{\int_0^\infty r^2 Q_s(r) g(r) n(r) dr}{\int_0^\infty r^2 Q_s(r) n(r) dr} \quad (21) \]

With the size distribution of the fog, we can calculate the average coefficients of fogs.


The Monte Carlo method can be employed for the complex case of light scattering [15, 22, 23], the incident photon and the particles of the medium are regarded as the dispersive particles interacting with each other. There are scattering and absorption during the transmission of photons in the random medium. In the multiple scattering processes, it can be described by Markov process. The transmission history of photons is expressed as a phase space point \( s = (r, \mu, \varphi) \), and the particle transport equation can be expressed as follows [22]:
\[ I(s) = I_{t_i}(s) + \int I(s') K(s' \rightarrow s) ds'. \quad (22) \]

Obviously,
\[ \int I_{t_i}(s) ds = \int_0^\infty e^{-r} dr \int \frac{\delta (\mu - 1) \delta (\varphi)}{4\pi} d\mu d\varphi = 1. \quad (23) \]

Equation (22) has the Neumann series solution as follows:
\[ P(s_0) \geq P(s_0 s_1) \geq \cdots \geq P(s_0 s_1 \cdots s_{m-1} s_m) > 0, \quad (27) \]
\[ P(s_0 s_1 \cdots s_{m-1} s_m) = P(s_0) P(s_1|s_0) P(s_2|s_1) \cdots P(s|s_0 s_1 \cdots s_{m-1}) = P(s_0) P(s_1|s_2) P(s_2|s_1) \cdots P(s_m|s_{m-1}). \quad (28) \]

\[ L = -\ln \frac{\xi_i}{Q_e} \quad (29) \]

When scattering occurs, a new propagation direction must be specified, the H-G function is an appropriate phase function for fog particle scattering, the probability density function corresponding to normalized phase function is as follows [23]:
\[ P(\nu) = \frac{(1 - g^2)(1 + g^2 - 2g \cos \nu)^{-3/2}}{4\pi}. \quad (30) \]

The scattering angle in the particle local coordinate system is \( \nu \) and \( g \) is the asymmetry factor; the photon scattering direction is as follows:
\[
\cos \nu = \frac{(1 + g^2) - (1 - g^2)^2}{2g(1 - 2g\xi_3)} \quad \text{for } g \neq 0,
\]
\[
\cos \nu = 2\xi_3 - 1, \quad g = 0.
\]

(31)

By using the propagation distance \( L \) and scattering angle \( \nu \), the scattering direction and position of photons can be determined.

5. Calculation Results

Based on previous theories, the following calculations are proposed and advection fog and radiation fog are considered. The fog spectrum distribution is gamma distribution, as shown in equations (1)–(8). The value of visibility is between 0.01 km and 10 km. The laser wavelengths are 0.86 \( \mu m \), 0.91 \( \mu m \), 1.06 \( \mu m \), 1.3015, and 10.6 \( \mu m \), respectively. Firstly, we calculate the extinction coefficient, albedo, asymmetry factor of fog droplets versus visibility, then we use these parameters in the calculation of Monte Carlo method, the forward scattering and backscattering of laser light are calculated.

Figure 2 shows the average extinction coefficient of advection fog and radiation fog versus visibility at the light wavelength of 0.86 \( \mu m \), 0.91 \( \mu m \), 1.06 \( \mu m \), 1.3015 \( \mu m \), and 10.6 \( \mu m \). As shown in Figure 2, the average extinction coefficient decreases as the visibility increases, and the average extinction coefficient of advection fog is bigger than that of radiation fog under the same visibility. The average extinction coefficient fog is basically the same under the wavelength of 0.86 \( \mu m \), 0.91 \( \mu m \), 1.06 \( \mu m \), and 1.315 \( \mu m \). For wavelength of 10.6 \( \mu m \), the average extinction coefficient is smaller than that of other wavelengths, and decreases quickly as visibility increases.
According to equations (16) and (19),

\[
\langle \sigma \rangle = \frac{\pi}{\lambda} \int_0^\infty \frac{r^2 Q_e (r) n (r) dr}{\int_0^\infty n (r) dr},
\]

(32)

\(\langle \sigma \rangle\) mainly depends on \(Q_e\) and \(r\). The droplet particle radius is much bigger than the wavelength of the incident light wave, so the change of \(Q_e\) is small, \(Q_e\) is close to 2. Therefore, the change trend of \(\langle \sigma \rangle\) is mainly determined by \(r^2\) in equation (16). From equations (9)–(11), it is shown that with the increase of \(V\), \(r_0\) decreases, \(\tau\) decreases, correspondingly \(r^2\) decreases, so \(\langle \sigma \rangle\) decreases. According to equation (15), \(Q_e\) contains parameter \(2/\alpha_0^2\), \(\alpha_0 = 2\pi r_0 \lambda\), \(\lambda\) is wavelength, the wavelength of 10.6 \(\mu m\) is about 10 times bigger than other wavelengths (0.86 \(\mu m\), 0.91 \(\mu m\), 1.06 \(\mu m\), and 1.315 \(\mu m\)), and its \(Q_e\) is calculated to be less than 2, which is smaller than that of other wavelengths, and with the same visibility, \(\langle \sigma \rangle\) of 10.6 \(\mu m\) is smaller than that of other wavelengths.

Figure 3 shows the average albedo of advection fog and radiation fog versus visibility. It can be seen that the average albedo of wavelengths of 0.86 \(\mu m\), 0.91 \(\mu m\), 1.06 \(\mu m\), and 1.315 \(\mu m\) in the two kinds of fog is close to 1. The average albedo of wavelength of 10.6 \(\mu m\) decreases as the visibility increases, it is obviously different from other wavelengths.

Study on the refractive index of droplet of fog is as follows:

\(n = n' + in''\),

(33)

where \(n'\) is the real part of the refractive index that represent the scattering capacity of the particle \(Q_e\) and \(n''\) is the complex part that represents the absorption by the particle \(Q_e\). The extinction of the particle includes scattering and absorption \(Q_e = Q_s + Q_a\), as shown in Table 1, the refractive index is related to the wavelength of the incident wave.

For the wavelength of incident wave (0.86 \(\mu m\), 0.91 \(\mu m\), 1.06 \(\mu m\), and 1.315 \(\mu m\)), the complex part of its refractive index is small, \(Q_a\) is small. According to equations (16)–(18), \(\langle \sigma \rangle\) can be ignored, \(\langle \sigma \rangle\) is close to \(\langle \sigma \rangle\), and by equation (20), \(\langle w_0 \rangle\) is approximately equal to 1. For the incident wave with wavelength of 10.6 \(\mu m\). The complex part of refractive index is relatively big, \(\langle \sigma \rangle\) is relatively big. The particle average size decreases with the increase of visibility, and particle size parameter \(\alpha_1 = 2\pi r_0 \lambda\) is close to 1 and \(\langle \sigma \rangle\) decreases rapidly, but \(\langle \sigma \rangle\) changes slowly, so \(\langle w_0 \rangle\) decrease as visibility increase.

Table 1: The refractive index of fog droplet related to the wavelength of the incident light.

<table>
<thead>
<tr>
<th>Wavelength ((\mu m))</th>
<th>Real part</th>
<th>Complex part</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.86</td>
<td>1.329</td>
<td>2.93 \times 10^{-7}</td>
</tr>
<tr>
<td>0.91</td>
<td>1.323</td>
<td>5.70 \times 10^{-7}</td>
</tr>
<tr>
<td>1.06</td>
<td>1.326</td>
<td>2.89 \times 10^{-6}</td>
</tr>
<tr>
<td>1.315</td>
<td>1.316</td>
<td>1.639 \times 10^{-5}</td>
</tr>
<tr>
<td>10.6</td>
<td>1.178</td>
<td>0.071</td>
</tr>
</tbody>
</table>

Table 1 shows the refractive index of fog droplet related to the wavelength of the incident light. When visibility is 0.1km, the particle asymmetry factor \(g = 0.75\), and \(w_0 = 0.9\) is albedo. As shown in Figure 5(a), the forward scattering intensity increases when albedo increases, when the scattering angle is greater than 20°, the scattering intensity is relatively small, indicating that the forward scattering beam is relatively concentrated. As can be seen from Figure 5(b), the backward scattering intensity increases when single albedo increases, however, compared with the forward scattering intensity, the backward scatter intensity is much smaller. The backscattering intensity decreases when the scattering angle is larger than 60°, which indicates that the backscattering beams are more dispersed.

In Figure 6, the relationship between scattering intensity and scattering angle is calculated, particle albedo \(w_0 = 0.9\). Figure 6(a) show that, the forward scattering increases when the asymmetry factor increases. The scattering intensity decreases when the scattering angle increases. It is shown in Figure 6(b), the backscattering intensity decreases when the asymmetry factor increases. At the same time, when the scattering angle is less than 50°, the scattering intensity changes little, and decreases with the increase of the scattering angle when the scattering angle is bigger than 50°. And the backward scatter intensity is much smaller than the forward scattering intensity.

Figure 7 shows the backscattering intensity versus scattering angle at several typical wavelengths when the visibility is 0.6 km. It can be seen from the Figure 7(a), for advection fog, when the scattering angle is less than 40°, the scattering intensity oscillates rapidly, and there is little difference in the scattering intensity curves for wavelengths of 0.86 \(\mu m\), 0.91 \(\mu m\), and 1.06 \(\mu m\). Figure 7(b) show that, for advection fog, the scattering intensity curves of 0.86 \(\mu m\), 0.91 \(\mu m\), and 1.06 \(\mu m\) of radiation fog are almost same. Comparing the two figures, it can be seen that the backscattering intensity of radiation fog is bigger than that of advection fog, and the backward scattering intensity of 10.6 \(\mu m\) is much smaller than that of other wavelengths, that means the forward transmittance of 10.6 \(\mu m\) is much bigger than that of other wavelengths.
Based on the size distribution of fog, the scattering coefficient is calculated by Mie theory. The average extinction coefficient, albedo, and asymmetry factor of fogs versus visibility for the laser at wavelengths of 0.86 μm, 0.91 μm, 1.06 μm, 1.315 μm, and 10.6 μm are discussed. The scattering coefficients are similar for the laser at the wavelength of 0.86 μm, 0.91 μm, and 1.315 μm, but the coefficients of the laser at the wavelength of 10.6 μm are smaller. By the Monte Carlo method, correlations between of asymmetry factor, albedo, and scattering intensity are calculated. The forward and backward scattering intensity increases when the albedo increases. When the asymmetry factor increases, the forward scattering intensity increases, but the backscattering intensity decreases. The backward scattering intensity of radiation fog is bigger than advection fog for same visibility, and the backward scattering intensity of 10.6 μm is much smaller than that of other wavelengths. Compared with other studies, our model considers more parameters for the Monte Carlo method and can precisely study beam propagation in fogs. This study can be applied to laser
Figure 5: The scattering intensity versus scattering angle for different albedo.

Figure 6: The scattering intensity versus scattering angle for different asymmetry factors.
communication, laser remote sensing detection, and lidar detection in fog.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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