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Research Article

Development of Optimal Tilt Angle Models of a Photovoltaic Module for Maximum Power Production: Ethiopia

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The power generated from the photovoltaic module is directly related to the magnitude of total incident solar radiation on the surface of the solar module. The total incident solar radiation depends on the location, tilt angle, and orientation of the solar module. In this paper, generic models were developed that determine the seasonal and annual optimal tilt angle of the Photovoltaic module at any location in Ethiopia without using meteorological data. Both isotropic and anisotropic diffuse solar radiation models were used to estimate monthly, seasonal, and annual optimal tilt angles. The monthly average daily global horizontal solar radiation for a total of 44 cities -32 for training and 12 for testing were obtained from the National Aeronautical and Space Administration database, and algorithms were developed and implemented using MATLAB and R programming software to obtain optimum tilt angle and regression models. The study showed that the developed model accurately estimates the optimal tilt angle with the minimum statistical validation errors. It is also found that 5.11% to 6.275% (isotropic) and 5.72% to 6.346% (anisotropic models) solar radiation energy is lost when using the yearly average fixed optimal tilt angle as compared with the monthly optimal tilt angle. The result of this study was also validated by comparing it with the previously published works, PVGIS and PVWatt online software. The graphical abstract is included in the supplementary file.

1. Introduction

Ethiopia is located in the Horn of Africa at coordinates of 9.1450°N and 40.4897°E with a population of more than 114 million people, of which 78.1% live in rural areas [1]. Currently, the country's installed grid power generation installed capacity is 4244 MW, almost entirely dependent on hydropower, which accounts for 89.9%. The remaining 7.6% and 2.5% are generated from wind and thermal sources, respectively [2]. According to the World Bank data in 2019 only 48.3% of the total population has access to electricity. From this 92.8% of the urban and 36.3% of the rural population is electrified [3]. To increase the electricity access rate, the Ethiopian government planned to increase the generation by constructing new power plants from hydropower, wind, solar, and geothermal energy sources. Among these, solar energy is one of the most environmentally friendly and promising sources of electrical power. The estimated exploitable potential ranges from 4 to 6 $kWh/m^2/day$ [4].

The amount of solar energy on the surface of the photovoltaic (PV) module is affected by global horizontal solar radiation, tilt angle, orientation of the solar panel, and ground reflectance [5]. The available solar radiation at a specific site depends on the location and time of the year. Measured total solar radiation at the horizontal and at the inclined surface are not available for different sites in Ethiopia. Therefore, it is necessary to calculate the solar radiation on the surface of the module for proper design and installation of the photovoltaic systems.

Maximum daily solar radiation can be obtained through the proper installation of the PV panel by optimizing the solar panel installation, tilt angle, and orientation. In general, the solar panel must be oriented toward the equator, which indicates that installations in the southern hemisphere must be oriented towards the north, and the location of Ethiopia being in the northern hemisphere, PV panels must be oriented towards the south [6]. On the other hand, the solar module installation tilt angle is influenced by the location and path of the sun. Therefore, the tilt angle can be set to one optimal value or seasonally varied to a different optimal tilt angle to obtain a maximum incident solar radiation on the surface of the solar module. Different authors have proposed the optimal tilt angle-latitude relation [7–9], as a rule of thumb formula to determine the optimal tilt angle for many locations. This method is not universally applicable and may not be an optimal tilt angle value for different locations, as the solar radiation not only varies with latitude but also varies with a change in elevation of the location. Therefore, the accurate optimum tilt angle for a certain region must be evaluated based on the location of solar radiation data.

The monthly average daily total solar radiation on the surface of the solar panel (tilted surface) is the sum of beam (direct) radiation, diffuse radiation, and reflected radiation. Several models are developed by different authors to estimate the solar radiation on the inclined surface. All models apply the same principle to estimate the reflected and beam radiation. However, they are different in determining the diffuse portion of the total solar radiation incident on the inclined surface. The diffuse radiation is assumed to be uniformly distributed over the skydome (isotropic), or it is assumed to be the sum of the circumsolar region and isotopically distributed portion (anisotropic) [10].

Many researchers use isotropic [11–16], anisotropic [17–19], or both isotropic and anisotropic models [5, 20–22] to predict total solar radiation on an inclined surface. Nicolás-Martín [23] proposed models that determine the annual optimal tilt angle without local meteorological data. Parameters such as latitude, albedo, and diffuse fraction were considered for the development of the optimal tilt angle models. A global optimal tilt angle regression model as a function of latitude was proposed with 2^o RMSE for latitudes of -50^o to 90^o. The results of this literature reveal a 1% energy loss with a 10^o variation of the tilt angle from the suggested optimal tilt angle values.

Kallioğlu et al. [24] proposed a regression model to obtain the optimal tilt angle for three provinces in Turkey. In this literature, the highest optimal tilt angle value is found in autumn and winter and the lowest is observed in spring and summer. For the three provinces of Turkey, the optimum tilt angle model is expressed as a function of the declination angle, while for the Northern hemisphere the optimal tilt angle is determined by taking the latitude as the main parameter. The accuracy of the proposed models is compared using the coefficient of determination (R^2) and the best model for provinces and the northern hemisphere obtained with R^2 values of 0.9979 and 1, respectively. The authors of this work assume that the diffuse radiation is uniformly distributed across the skydome (isotropic model only). The result showed that the productivity of the monthly optimal tilt angle is increased by 16.97%, 15.57%, and 15.02% in the three provinces of Turkey as compared to the annual.

Memon et al. [25] performed a case study at Sukkur IBA University Pakistan to determine the optimal tilt angle of a 1-MW photovoltaic system. The optimal tilt angle of the existing system, i.e. 15^o, is compared with the tilt angle varied between 0 and 90^o. The optimal tilt angle for the photovoltaic system at this location is found to be 29.5 degrees. The anisotropic model by Reindel et al. is used to determine the diffuse radiation. However, the proposed optimal tilt angle could be used only for the specified location but not for the locations that have different latitudes.

Benghanem [5], conducted a case study on determining of optimal tilt angle for Madinah, Saudi Arabia. The result of the study indicated that the annual optimal tilt angle (23.5^o) is nearly equal to the latitude of the study location. This fixed annual tilt angle resulted in an 8% energy loss reduction compared with the energy obtained by setting the solar panel at its monthly optimal tilt angle. Hailu et al. [20], conducted a study in Greater Toronto-Canada to obtain the optimum tilt angle and orientation of the solar panel using isotropic and anisotropic models. The study indicated that the optimum tilt angles were 37 to 44^o (isotropic models) and 46 to 47^o (anisotropic models) oriented west or east of due south. A 1% total solar radiation reduction was observed with a 15^o change in orientation west or east of due south. The effect of the orientation of the solar panel on the outlet temperature was also investigated.

Jamil et al. [16] conducted a case study in the Humid subtropical climatic region of India. In this study estimation of solar radiation and optimal tilt angle for Aligarh and New Delhi were observed. The annual optimum tilt angle for Aligarh and New Delhi was found to be 27.62^o and 27.95^o, respectively (close to the latitude of the respective location). The seasonal optimal tilt angle results in an energy loss of 1.16% for Aligarh and 1.18% for New Delhi. Furthermore, the annual optimal tilt angle results in an energy loss of 5.68% for Aligarh and 4.91% for New Delhi.

Hassan et al. [26] conducted a case study in Iraq. The optimum tilt angle has been determined for eight cities in Iraq using an anisotropic HDKR model. The optimization process is performed by using hourly experimental solar radiation data. The results demonstrate that the maximum solar radiation can be collected with the tilt angle from 0^o to 64^o. The optimum tilt angle values increased during winter and decrease during summer, marking the highest values in January and December and the lowest values in June and July for all cities.

In general, the optimum tilt angle of the solar module is one of the parameters that affect the output of the PV system. The summary of the reviewed literature work is given in Table 1. Most of the previous research works were done for specific locations because of the site-specific parameters used. As a result, the optimal tilt angle obtained in one location may not be an optimal value for another. In references [16, 23–26] to evaluate the diffuse radiation only isotropic or anisotropic models are used. In other works [5, 20, 25] numerical optimal tilt angle values are suggested for the specific study area. These values cannot be taken as optimal values for other locations of different latitudes and elevations. Additionally, there is limited data available on determining the optimal tilt angle and the corresponding total solar radiation on the surface of the solar module for different locations in Ethiopia that could justify the optimal design of the PV system.

Table 1: Summary of related previous works.

References	Model	Advantages	Disadvantages
[23]	$\beta_{opt} = -0.007021\phi^2 + 1.091\phi + 2.132$	(i) Latitude, diffuse fraction, and albedo parameters are considered (ii)anisotropic (peers model) was used	(i) Other isotropic and anisotropic models were not considered in the study (ii) this model is not accurately applied to other location
[24]	$0.8845\phi + 1.5908$	(i) the main parameters are declination angle and latitude (ii) simple for analysis due to the consideration of the isotropic model	(i) Seasonal variation of optimal tilt angle not considered (ii) the diffused solar radiation was assumed isotropic (only isotropic model is used)
[25]	The optimal tilt angle proposed is 29.5 degree	(i) the optimal tilt angle is obtained for the study site (ii) Reindel et al. model is used	(i) Other diffuse radiation evaluation techniques were not considered (ii) the result of this study can not be equally applied as optimal value for different locations
[5]	The optimal tilt angle is 23.5^o	(i) the annual optimal tilt angle found is nearly equal to the study site latitude	(i) this optimal tilt angle could not be equally applied to a different latitude locations
[20]	The optimal tilt angles are 37 to 44^o (isotropic models) and 46 to 47^o (anisotropic models)	(i) the study location optimal tilt angles were found using both isotropic and anisotropic models	(i) the result is only applicable to the study site, no general model is suggested for other locations (ii) no comparison of the result obtained by different models using statistical tools
[16]	Optimal tilt angles for Aligarh and New Delhi are 27.62^o and 27.95^o, respectively	(i) Annual optimal tilt angles of the study sites were determined (ii) A simplified isotropic model was used (iii) numerical values of the tilt angle were suggested	(i) Diffused radiation was assumed uniformly distributed (ii) can not be accurately applied to other locations
[26]	Linear, second-order, and third-order polynomial models are proposed	(i) the optimal tilt angle was obtained by using hourly experimental solar radiation data (ii) optimal tilt angles and the corresponding models were developed using an anisotropic model	(i) Other parameters such as latitude were not considered in the study

The main aims of this work are to determine the optimal tilt angle and to develop a generic optimal tilt angle model that accurately estimates the seasonal and yearly optimal tilt angle at any location in Ethiopia using different isotropic and anisotropic models without using the meteorological data.

The monthly average daily global, diffuse, and beam solar radiation on a horizontal surface for 44 cities in Ethiopia were determined. Based on the maximum solar radiation received on the surface of the photovoltaic system, the monthly, seasonal, and yearly optimal tilt angles were obtained using isotropic and anisotropic models of diffuse solar radiation. The generic seasonal and yearly optimal tilt angle models as a function of latitude have been developed that accurately estimate the optimum tilt angle at any location in Ethiopia. The accuracy to predict the proposed models using both isotropic and anisotropic models has been validated using statistical indices of RMSE, MBE, MAE, MAPE, R^2, previous works in the literature, and online software (PVGIS and PVWatt).

The novelty of this study is: (1) solar radiation at the surface of the photovoltaic module is evaluated and determined by considering isotropic and anisotropic diffuse radiation. (2) a new and generic optimal tilt angle model as a function of the latitude, using isotropic and anisotropic diffuse radiation is developed that accurately estimates the optimal tilt angle at any location in Ethiopia without using meteorological data. (3) The models are developed and compared to use in isotropic and anisotropic diffuse radiation and the best model which accurately predicts the optimal tilt angle is selected. This model is very useful for researchers and designers in the field of photovoltaic engineering to determine and apply the optimal tilt angle model by knowing only the location latitude without using meteorological data and the declination angle at any location.

The rest of the paper is organized as follows: Section 2 describes the materials and methodology used in the study, i.e. used data sets, solar radiation models, and procedures for the determination of the optimal tilt angle. The result

Figure 1: Study sites.

and discussion part of the paper is given in section 3 and finally, the conclusion is provided in section 4.

2. Materials and Methods

The solar tracker positions the solar module in the direction of the sun throughout the day to obtain the maximum solar radiation. However, this method has a high cost and requires a more complex system than the fixed tilt angle method. Therefore, it is necessary to determine the optimal tilt angle of the solar module which can be set to one optimal tilt angle value or that can be varied seasonally. Studies showed that the solar radiation on the surface of the panel varies with the change in the day of the year and latitude of the location. This phenomenon will be verified in this study. This section describes the approach used for determining the optimal tilt angle and the corresponding generic models to obtain the optimal tilt angle at any location in Ethiopia.

2.1. Data Set. To develop the generic optimal tilt angle models, 44 cities of Ethiopia are randomly selected and their monthly average daily global horizontal radiation was obtained from the National Aeronautical and Space Administration (NASA) database from 01/01/2010 to 31/12/2020 as shown in Figure 1. A sample monthly average daily total horizontal solar radiation data for Bahir Dar city is shown in Table 2. The monthly average extraterrestrial radiation, clearness index, and diffuse radiations are evaluated for each study site using the solar radiation model and is discussed in the following subsection.

2.2. Solar Radiation on a Horizontal Surface. The solar radiation above the earth's atmosphere is called extraterrestrial radiation. The daily extraterrestrial radiation on a horizontal surface can be calculated using equation (1) [7]

$$H_o = \frac{24G_{sc}}{\pi}\left(1 + 0.033\cos\frac{360n}{365}\right) \cdot \left(\cos(\phi)\cos(\delta)\cos(\omega_s) + \frac{\pi\omega_s}{180}\sin(\phi)\sin(\delta)\right) \quad (1)$$

Table 2: Geographical coordinates and monthly average daily global horizontal radiation data of Bahir Dar city.

Latitude (°)	Longitude (°)	Month	Global horizontal radiation, H (kWh/m²/day)
11.5742	37.3614	January	6.131
		February	6.585
		March	6.797
		April	6.860
		May	6.186
		June	5.846
		July	5.021
		August	4.998
		September	5.833
		October	6.186
		November	5.922
		December	5.975

Where ω_s is the sunset hour angle (degree) and computed as [7]:

$$\omega_s = \cos^{-1}(-\tan(\phi)\tan(\delta)) \quad (2)$$

The declination angle (δ) is the angle formed between the line joining the center of the earth and the sun and the equatorial plane. Its angle value changes over the year from 23.45° north to 23.45° south. The declination angle as a function of the day of the year (n), can be evaluated using the equation (3) [25]:

$$\delta = 23.45^o \sin\left(360^o\frac{284+n}{365}\right) \quad (3)$$

Only a portion of extraterrestrial radiation reaches the earth's horizontal surface, which is expressed using the clearness index (K_t). The monthly average daily clearness index is the ratio of the monthly average daily global radiation on a horizontal surface (H) to the monthly average daily global extraterrestrial radiation (H_o) and is evaluated as [11].

$$K_t = \frac{H}{H_o} \quad (4)$$

The diffused radiation (H_d) on the horizontal surface is a function of the clearness index and global horizontal radiation and can be determined using equations (5) and (6) [27].

For $\omega_s \le 81.4^\circ$ and $0.3 \le K_t \le 0.8$

$$H_d = H\left(1.391 - 3.560K_t + 4.189K_t^2 - 2.137K_t^3\right) \quad (5)$$

For $\omega_s > 81.4^\circ$ and $0.3 \le K_t \le 0.8$

$$H_d = H\left(1.311 - 3.022K_t + 3.427K_t^2 - 1.821K_t^3\right) \quad (6)$$

2.3. Total Solar Radiation on a Tilted Surface. The monthly average daily total solar radiation on the inclined surface of the solar panel is composed of (1) beam or direct radiation, (2) diffuse radiation, and (3) reflected radiation [20]. Thus for the tilted surface, the incident total monthly average daily total radiation, H_T (kWh/m^2/day) is obtained using equation (7) [17].

$$H_T = H_B + H_D + H_R \tag{7}$$

2.3.1. Beam Radiation (H_B). The total daily beam radiation on the tilted surface of the solar energy converter can be expressed as [12].

$$H_B = (H - H_d)R_b \tag{8}$$

Where R_b is the ratio between the monthly mean daily direct radiation on a horizontal surface to the inclined surface. R_b is defined as [7]:

$$R_b = \frac{\cos(\theta)}{\cos(\theta_z)} \tag{9}$$

The cosine value of incident angle, cos (θ) can be defined as a combination of different solar angles as follows [28].

$$\begin{aligned}\cos(\theta) = {} & \sin(\delta)\sin(\phi)\cos(\beta) - \sin(\delta)\cos(\phi)\sin(\beta)\cos(\gamma) \\ & + \cos(\delta)\cos(\phi)\cos(\beta)\cos(\omega) \\ & + \cos(\delta)\sin(\phi)\sin(\beta)\cos(\gamma)\cos(\omega) \\ & + \cos(\delta)\sin(\beta)\sin(\gamma)\sin(\omega)\end{aligned} \tag{10}$$

The cosine of the zenith angle (cos (θ_z)) is computed by setting the panel tilt angle to zero (β=0), as [29].

$$\cos(\theta_z) = \cos(\phi)\cos(\delta)\cos(\omega) + \sin(\phi)\sin(\delta) \tag{11}$$

Hence, for the south-facing surface where the azimuth angle value is zero (γ=0°), the equation (7) can be written as [30]:

$$R_b = \frac{\cos(\phi-\beta)\cos(\delta)\sin(\omega_{ss}) + (\pi/180)\omega_{ss}\sin(\phi-\beta)\sin(\delta)}{\cos(\phi)\cos(\delta)\sin(\omega_s) + (\pi/180)\omega_s\sin(\phi)\sin(\delta)} \tag{12}$$

Where ω_{ss} is the sunset hour angle for the tilted surface for the mean day of the month, obtained by equation (13) [30].

$$\omega_{ss} = \min\begin{cases}\cos^{-1}(-\tan(\phi)\tan(\delta)) \\ \cos^{-1}(-\tan(\phi-\beta)\tan(\delta))\end{cases} \tag{13}$$

2.3.2. Diffuse Radiation (H_D). Different studies have shown that there are several empirical models to determine diffuse radiation. The sky diffuse radiation on the inclined surface of the panel can be determined by equation (14) [24]:

$$H_D = H_d R_d \tag{14}$$

The ratio of the average daily diffuse solar radiation on the tilted surface to the diffuse radiation on the horizontal surface, (R_d), can be evaluated by using different isotropic (equations (15)–(18) and anisotropic (equations ((19) (20) (22) and (23)) empirical formulas as follows

Isotropic model: Isotropic models assume that the diffuse sky radiation is uniformly distributed over the skydome. Different researchers suggest different empirical formulas to calculate R_d, these are:

Liu and Jordan's model [31].

$$R_d = \frac{1+\cos(\beta)}{2} \tag{15}$$

Badescu model [13].

$$R_d = \frac{3+\cos(2\beta)}{4} \tag{16}$$

Tian et al. model [14].

$$R_d = 1 - \frac{\beta}{180} \tag{17}$$

Koronakis model [15].

$$R_d = \frac{1}{3}(2+\cos(\beta)) \tag{18}$$

Anisotropic model: The anisotropic model assumes that diffuse sky radiations are the sum of anisotropically distributed diffused radiation components in the circumsolar region (sky near the solar disc) and isotropically distributed diffuse components from the rest of the skydome.

Reindl et al. model [22].

$$R_d = \frac{H_b}{H_o}R_b + \left(1-\frac{H_b}{H_o}\right)\left(\frac{1+\cos(\beta)}{2}\right)\left(1+\sqrt{\left(\frac{H_b}{H}\right)}\sin^3\left(\frac{\beta}{2}\right)\right) \tag{19}$$

Hay model [10].

$$R_d = \frac{H_b}{H_o}R_b + \left(1-\frac{H_b}{H_o}\right)\left(\frac{1+\cos(\beta)}{2}\right) \tag{20}$$

The daily beam radiation incident on the horizontal surface is the difference between the global radiation on the horizontal surface and the diffuse radiation on the horizontal surface, as given in equation (21) [27].

$$H_b = H - H_d \tag{21}$$

Skarveit and Olseth model [18].

$$R_d = \frac{H_b}{H_o}R_b + \Omega\cos(\beta) + \left(1-\frac{H_b}{H_o}-\Omega\right)\left(\frac{1+\cos(\beta)}{2}\right) \tag{22}$$

Where $\Omega = \{\mathrm{Max}[0, (0.3 - 2(H_b/H_o))]\}$

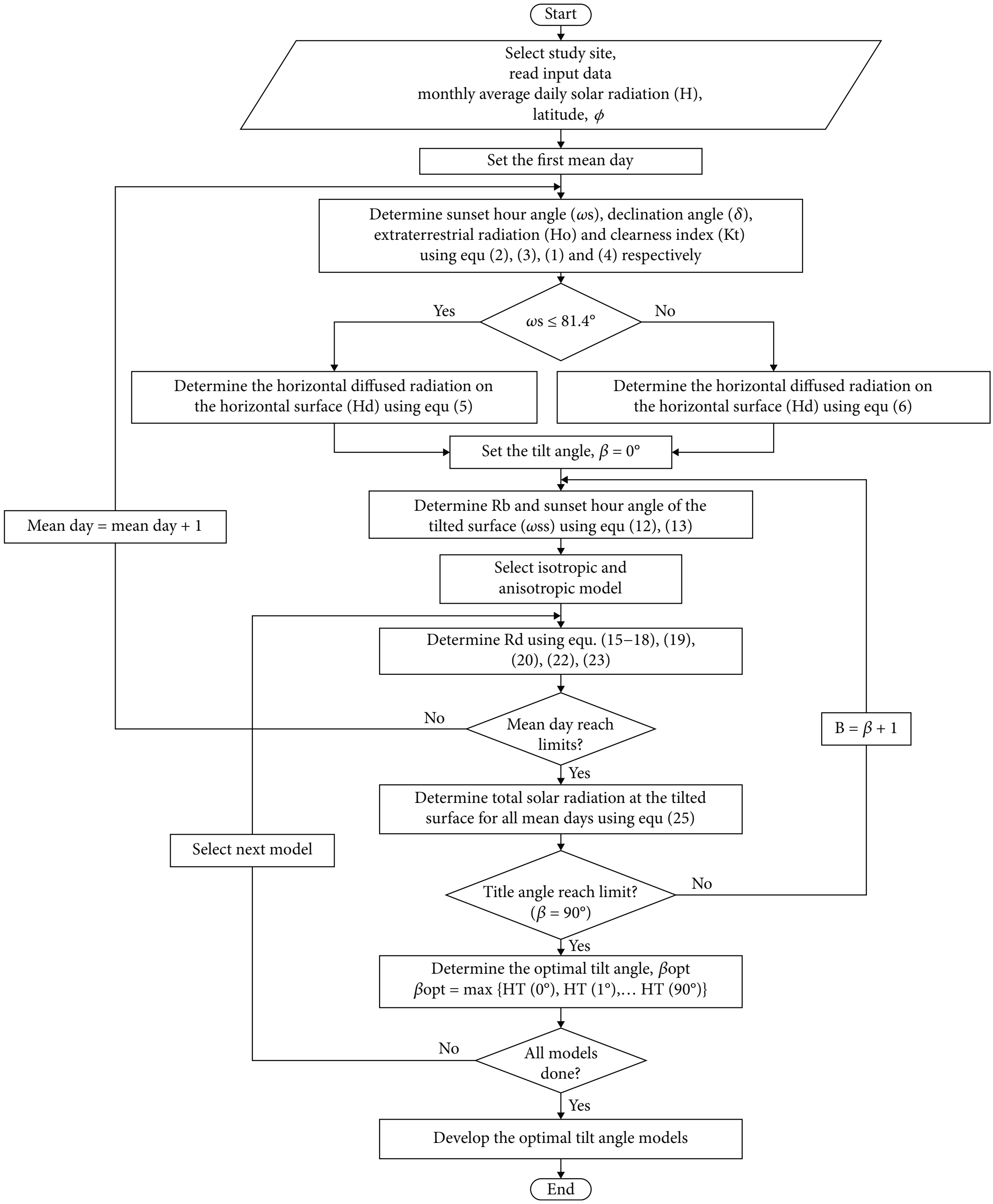

Figure 2: Flow chart for the procedure in determining the optimal tilt angle model.

Steven and Unsworth model [19].

$$R_d = 0.51R_b + \left(\frac{1+\cos(\beta)}{2}\right) - \frac{1.74}{1.26\pi} \cdot \left(\sin(\beta) - \left(\frac{\beta\pi}{180}\right)\cos(\beta) - \pi\sin^2\left(\frac{\beta}{2}\right)\right) \quad (23)$$

2.3.3. Reflected Radiation (H_R). The daily ground reflected radiation can be estimated using equation (24) [7].

$$H_R = H\rho\frac{1-\cos\beta}{2} \quad (24)$$

Where ρ is the albedo or ground reflectance. Its value

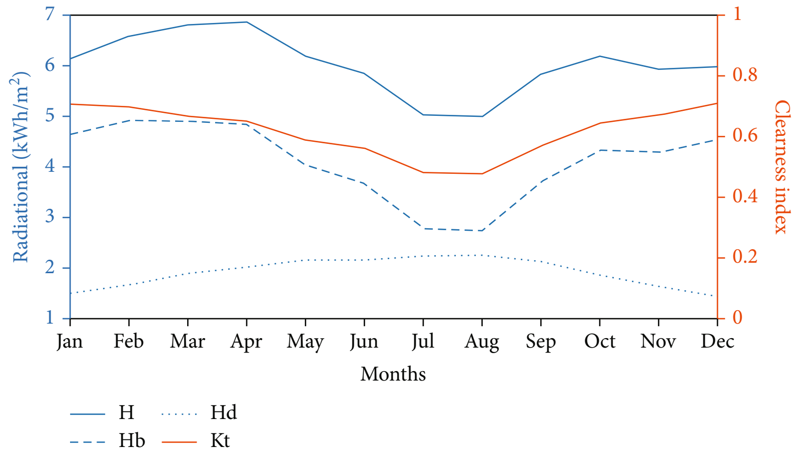

Figure 3: Global horizontal radiation (H), beam radiation (Hb), diffuse radiation (Hd), and clearness index (Kt) of Bahir Dar city.

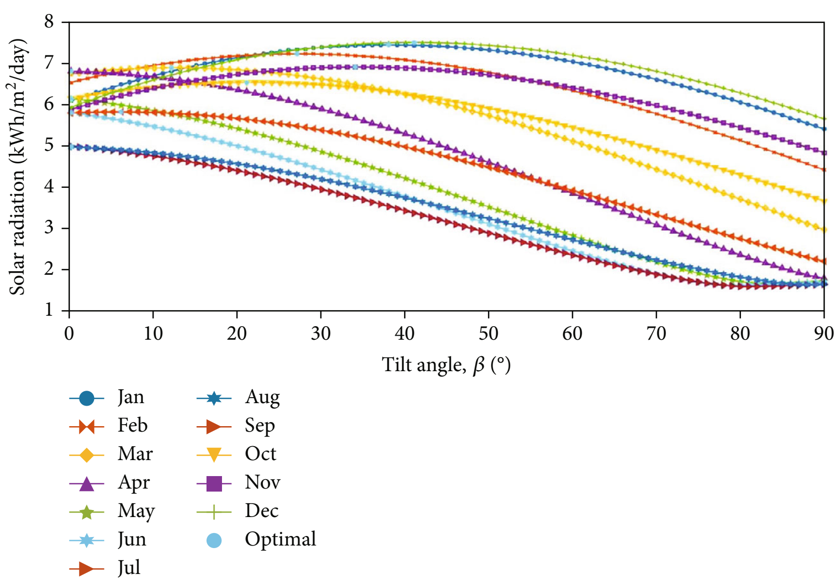

Figure 4: Monthly average daily solar radiation availability and an optimal value of a tilted surface using the isotropic model of equation (16), Bahir Dar city.

varies from 0.2 for a snow-free surface and 0.6 with fresh snow [32]. In this study, it is taken to be 0.2 [12].

Therefore, the total solar radiation on the tilted surface can be evaluated as [33]:

$$H_T = (H - H_d)R_b + H\rho\frac{1 - \cos(\beta)}{2} + H_d R_d \tag{25}$$

2.4. Statical Methods of Model Evaluation. To evaluate the accuracy of the proposed models' several statistical evaluation indices are used. These include the root mean square error (RMSE), mean bias error (MBE), mean absolute error (MAE), mean absolute percentage error (MAPE), and coefficient of determination (R^2) which are defined by equation (26), respectively [34–37].

$$\begin{aligned}
\mathrm{RMSE} &= \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\beta_{cal,i} - \beta_{predt,i}\right)^2} \\
\mathrm{MBE} &= \frac{1}{n}\sum_{i=1}^{n}\left(\beta_{cal,i} - \beta_{predt,i}\right) \\
\mathrm{MAE} &= \frac{1}{n}\sum_{i=1}^{n}\left|\beta_{cal,i} - \beta_{predt,i}\right| \\
\mathrm{MAPE} &= \frac{1}{n}\sum_{i=1}^{n}\left\{\left|\frac{\beta_{cal,i} - \beta_{predt,i}}{\beta_{cal,i}}\right|\right\} \mathrm{x}100\% \\
R^2 &= 1 - \frac{\sum_{i=1}^{n}\left(\beta_{cal,i} - \beta_{predt,i}\right)^2}{\sum_{i=1}^{n}\left(\beta_{cal,i}\right)^2}
\end{aligned} \tag{26}$$

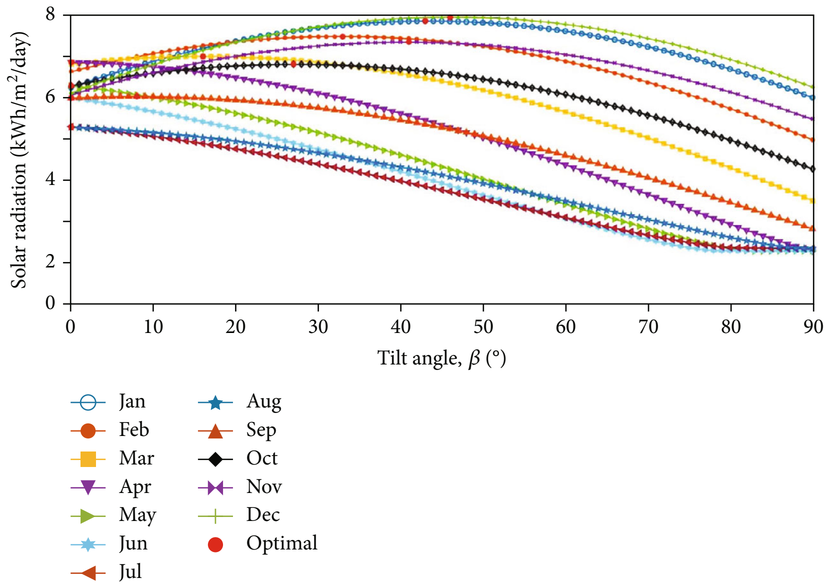

Figure 5: Monthly average daily solar radiation availability and an optimal value of a tilted surface using an anisotropic model of equation (19), Bahir Dar.

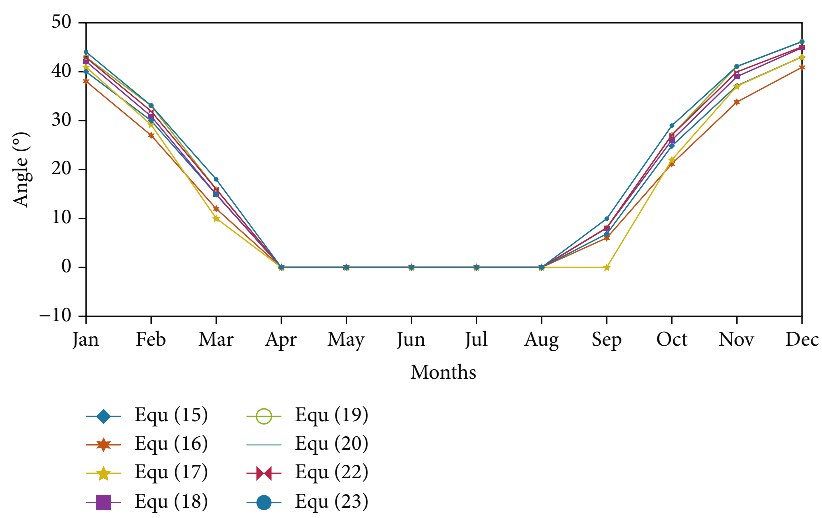

Figure 6: Monthly optimal tilt angle evaluated by isotropic and anisotropic models for Bahir Dar city.

2.5. Procedure for the Determination of the Optimal Tilt Angle. A MATLAB code was developed to obtain the optimal tilt angle for a solar module facing due south. The diffuse solar radiation and the monthly average daily total solar radiation on the inclined surface of all study sites were determined using both isotropic and anisotropic models. The angle varies from 0^o to 90^o with 1^o step and the maximum solar radiation incident on the tilted surface is determined. The optimal tilt angles were determined by finding the angle for which the total solar radiation on the surface of the solar module was maximum.

The optimal tilt angle generic models have been developed that estimate the seasonal and yearly optimal tilt angle at any location in Ethiopia using the data obtained from 32 study locations using R 4.1.2. statistical programming software. The capability of the developed models to estimate the seasonal and yearly optimal tilt angle for any location was tested using the data obtained from 12 study sites and their accuracy were validated using RMSE, MBE, MAE, MAPE, and R^2 statistical indices. The summary of the workflow is given in Figure 2.

3. Results and Discussion

The determination of the optimal tilt angle is done for all study sites, and general seasonal and annual optimal tilt angle models have been developed that work for any location in Ethiopia. However, to minimize the paper size the detailed result is described for one study site, Bahir Dar city, that can be applied similarly to others too.

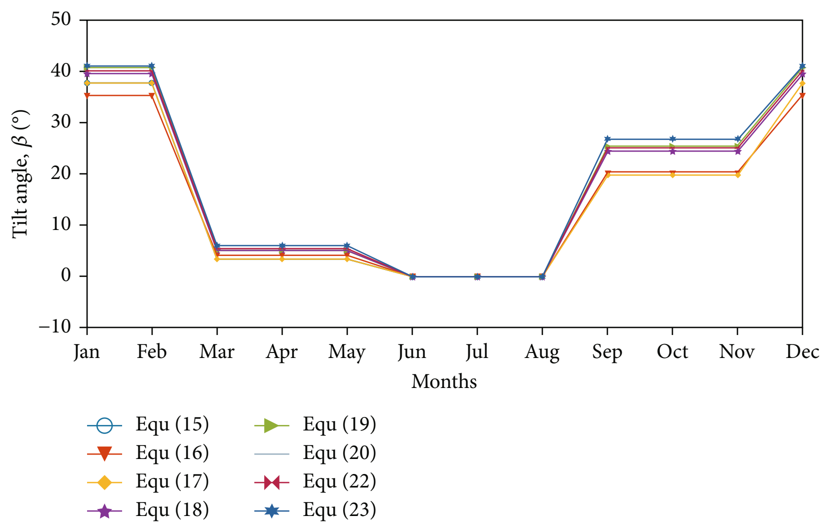

FIGURE 7: Seasonal optimal tilt angle evaluated by isotropic and anisotropic models for Bahir Dar city.

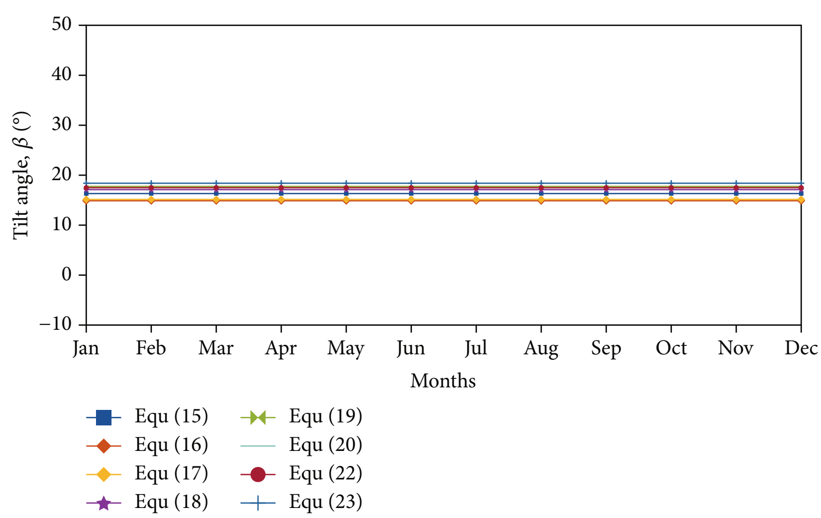

FIGURE 8: Yearly optimal tilt angle evaluated by isotropic and anisotropic model for Bahir Dar city.

3.1. Daily Total Solar Radiation. The monthly average daily global, diffuse, and beam horizontal radiation and clearness index of Bahir Dar city are shown in Figure 3. As indicated in the figure, the monthly average daily global horizontal solar radiation varies between 4.998 - 6.860kWh/m^2/day, with the minimum and maximum clearness index of 0.477 and 0.707, respectively. The beam and diffuse components which account for 75.6% and 24.4%, respectively, of the global horizontal solar radiation are also depicted in the figure.

The optimum monthly average daily total solar radiation on the surface of the module with the monthly optimal tilt angle was evaluated using 4 isotropic and 4 anisotropic models. The isotropic equation (17) model provides the minimum average total solar radiation at the optimum tilted surface and the maximum total solar radiation is obtained using an anisotropic model of equation (23). Anisotropic models of equations (20) and (22) provide the same result solar radiation and optimum tilt angle.

The monthly optimum tilt angle using both isotropic and anisotropic models is 0^o for April-August. For the other months, using the isotropic models, the optimal tilt angles are varied between 0^o (in September) of equation (16) and 45^o (in December) of equation (17). On the other hand, using anisotropic models, it varied between 8^o (September) and 46^o (December).

Figures 4 and 5 show the average daily total solar radiation and the optimum points on a south-facing inclined surface obtained using isotropic (equation (16)) and anisotropic (equation (19)) models with different tilt angle values. These models were selected since equations (16) and (19) preferred to estimate the total incident solar radiation on the inclined surface with the smallest statistical errors among all other models [10, 20].

The monthly, seasonal, and yearly optimal tilt angles obtained by using isotropic and anisotropic models are shown in Figures 6–8, respectively. As indicated in Figure 7, the seasonal optimal tilt angle using isotropic

Table 3: The seasonal and yearly optimal tilt angle of 44 cities in Ethiopia using the isotropic model, equation (16).

Cities	Winter	Spring	Summer	Autumn	Annual
Adama	32.33	3.33	0	17.67	13.33
Addis Ababa	33	3.33	0	18.33	13.67
Adigrat	37	4.67	0	23	16.17
Arbaminch	29.33	2.33	0	15	11.67
Asela	31.67	3	0	16	12.67
Asosa	33.33	3.67	0	17.67	13.67
Assaita	34	4	0	20.67	14.67
Awash	32.67	3.33	0	17.33	13.33
Awassa	31.33	3	0	16	12.58
Axum	37.67	5	0	23	16.42
Bahir Dar	35.33	4	0	20.33	14.92
Butajira	32.33	3	0	17.33	13.17
Debre Birhan	33.33	3.67	0	18.67	13.92
Debre Markose	33.67	3.67	0	19	14.08
Degehbur	32.67	3.33	0	17.67	13.42
Dek Island	36	4.33	0	21	15.33
Dessie	34	3.67	0	19.67	14.33
Dilla	30	2.67	0	15	11.92
Diredawa	33.33	3.67	0	18.67	13.92
Dubti	34.33	4	0	20.67	14.75
Filtu	29	2.33	0	13.33	11.17
Finoteselam	34.33	3.67	0	19.33	14.33
Gambela	29.67	3	0	15.67	12.08
Gebreguracha	33.33	3.67	0	18.67	13.92
Geladi	31	3	0	16.33	12.58
Ginir	31	3	0	15.67	12.42
Godie	30	2.67	0	15	11.92
Gonder	36.33	4.33	0	21.33	15.5
Harar	33.33	3.67	0	18.33	13.83
Hosaena	30.33	3	0	16.33	12.42
Jima	29.33	2.67	0	16	12
Jinka	28	2.33	0	14	11.08
Kebri Dahir	31	3	0	15.67	12.42
Logia	33.67	4	0	20.67	14.58
Mega	27.67	2	0	12	10.42
Mekelle	36.67	4.67	0	22.33	15.92
Metekle	34.33	4	0	19	14.33
Metema	36.67	4.67	0	21.33	15.67
Mizan Teferi	28	2.33	0	14.67	11.25
Nekemt	32.33	3.33	0	17.67	13.33
Semera	34.33	4	0	21	14.83
Shashemene	31.33	3	0	16.33	12.67
Welyta Sodo	30.33	2.67	0	15.33	12.08
Ziway	32.33	3	0	17	13.08

Table 4: The seasonal and yearly optimal tilt angle of 44 cities in Ethiopia using the anisotropic model, equation (19).

Cities	Winter	Spring	Summer	Autumn	Annual
Adama	38	4.33	0	22.33	16.17
Addis Ababa	38.33	4.33	0	22.67	16.33
Adigrat	43	6.33	0	28	19.33
Arbaminch	35.33	3.33	0	19.33	14.50
Asela	37.33	4	0	21.33	15.67
Asosa	39.33	4.67	0	23	16.75
Assaita	40.33	5.33	0	25.33	17.75
Awash	38.33	4.33	0	22.33	16.25
Awassa	36.33	3.67	0	20.33	15.08
Axum	43	6.33	0	28	19.33
Bahir Dar	40.67	5.33	0	25.33	17.83
Butajira	37.33	4	0	21.67	15.75
Debre Birhan	39	4.67	0	23.33	16.75
Debre Markose	39.33	5	0	24	17.08
Degehbur	37.67	4.33	0	22.33	16.08
Dek Island	41	5.33	0	25.67	18
Dessie	40.33	5	0	25	17.58
Dilla	35.67	3.33	0	19.67	14.67
Diredawa	39	4.67	0	23.33	16.75
Dubti	40.33	5.33	0	25.67	17.83
Filtu	34.67	3	0	18.33	14
Finoteselam	40	5	0	24.33	17.33
Gambela	37	4	0	21.33	15.58
Gebreguracha	39.33	4.67	0	23.33	16.83
Geladi	36.33	3.67	0	20.67	15.17
Ginir	36.33	3.67	0	20.67	15.17
Godie	36.33	3.67	0	20.67	15.17
Gonder	41.67	5.67	0	26.33	18.42
Harar	38.33	4.67	0	23.33	16.58
Hosaena	36.67	4	0	21.33	15.50
Jima	36.33	3.67	0	21	15.25
Jinka	34.67	3.33	0	18.67	14.17
Kebri Dahir	36.33	3.67	0	20.33	15.08
Logia	40.33	5.33	0	25.67	17.83
Mega	33.33	2.67	0	17.33	13.33
Mekelle	42.33	6	0	27.33	18.92
Metekle	40	5	0	24.33	17.33
Metema	42	6	0	26.33	18.58
Mizan Teferi	35.67	3.67	0	20	14.83
Nekemt	38.33	4.33	0	22.33	16.25
Semera	40.33	5.33	0	25.67	17.83
Shashemene	36.33	3.67	0	21	15.25
Welyta Sodo	36.33	3.67	0	20.33	15.08
Ziway	37.33	4	0	21.33	15.67

models were estimated at 35.33^o (equation (16)) to 39.33^o (equation (18)), 3.33^o (equation (17)) to 5^o (equation ((15) and (16)), 0^o, and 19.67^o (equation (17)) to 24.33^o (equation (18)) for winter, spring, summer, and autumn, respectively. Using an anisotropic model, the seasonal optimal tilt angles were found to be between 40^o to 41^o, 5.33^o to 6^o, 0^o, and 25^o

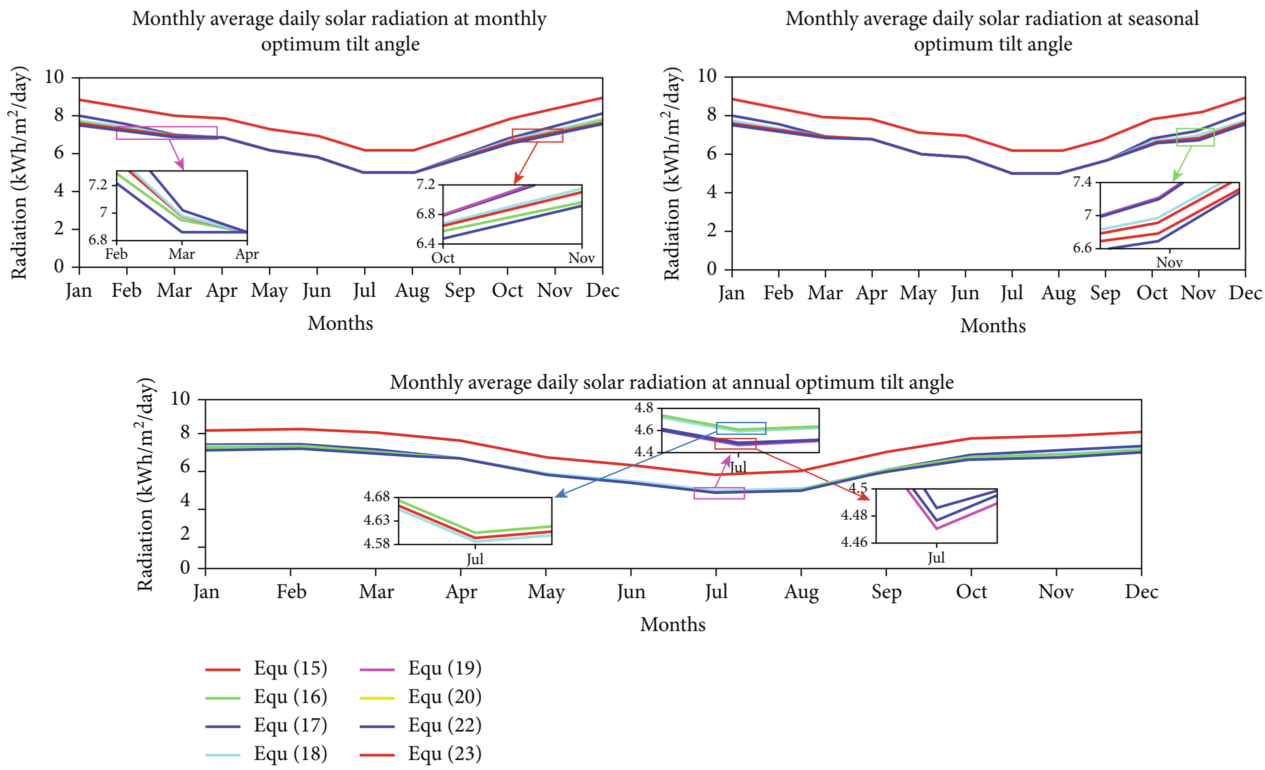

Figure 9: Monthly average daily solar radiation at monthly, seasonal, and yearly optimum tilt angle using the isotropic and anisotropic models, Bahir Dar city.

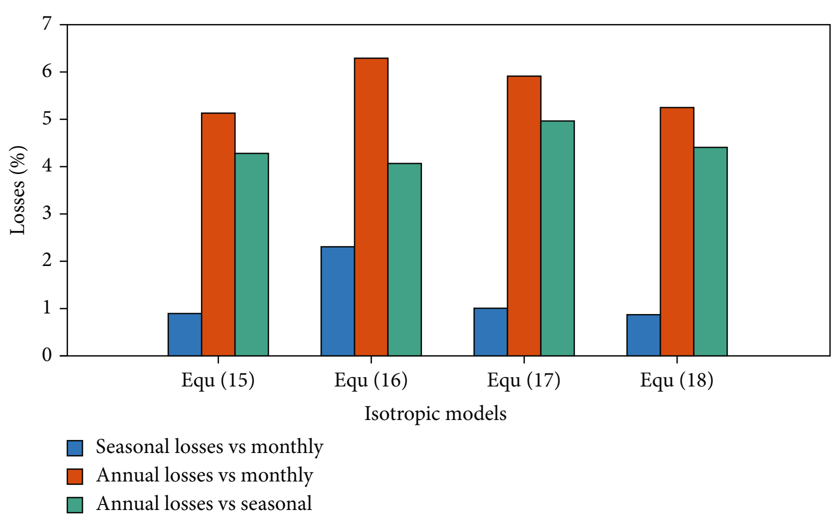

Figure 10: Solar radiation seasonal losses, annual losses relative to monthly and annual losses relative to seasonal (%).

to 26.67° for winter, spring, summer, and autumn, respectively. The anisotropic model gives less variation in tilt angle value than isotropic models. For both models, the minimum is 0° in summer and the maximum is in winter which varies between 35.33° to 41°.

The yearly optimal tilt angles were evaluated and found to be between 14.92° to 17.17° using the isotropic models and 17.58° to 18.42° using the anisotropic model, given in Figure 8. The variation using the anisotropic model is very small as compared to the isotropic model. The detailed seasonal and annual optimal tilt angles of 32 and 12 study locations in Ethiopia used to develop and test the models are given in Tables 3 and 4, respectively.

The monthly average daily solar radiation at monthly, seasonal, and yearly optimal tilt angles are shown in Figure 9. The monthly average total solar radiation resulting

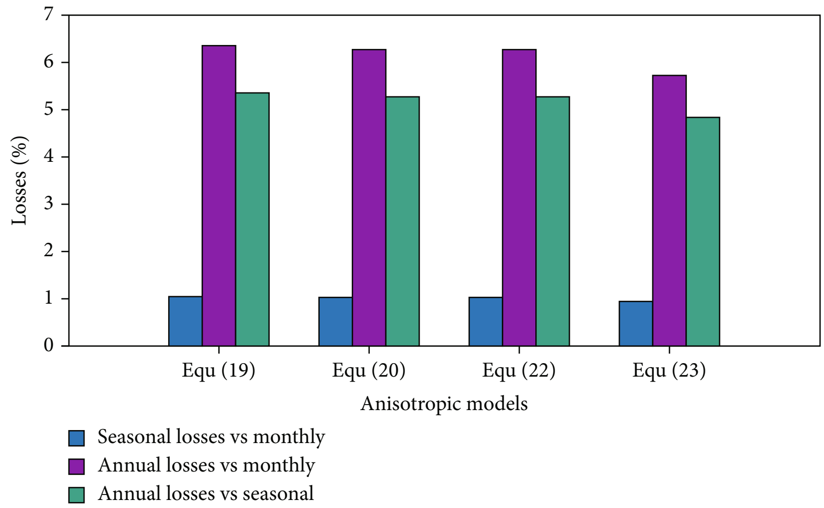

Figure 11: Solar radiation seasonal losses, annual losses relative to monthly and annual losses relative to seasonal (%).

Table 5: Developed isotropic and anisotropic seasonal and annual optimal tilt angle models.

Time	Isotropic model			
	Equ (15)	Equ (16)	Equ (17)	Equ (18)
Winter	$\beta_{opt} = 0.956\,\phi + 26.474$	$\beta_{opt} = 1.047\phi + 22.7561$	$\beta_{opt} = 1.1280\phi + 24.1350$	$\beta_{opt} = 0.961\phi + 28.056$
Spring	$\beta_{opt} = 0.340\phi + 0.911$	$\beta_{opt} = 0.297\phi + 0.645$	$\beta_{opt} = 0.4980\phi - 2.6410$	$\beta_{opt} = 0.347\phi + 1.117$
Summer	$\beta_{opt} = 0$	$\beta_{opt} = 0$	$\beta_{opt} = 0$	$\beta_{opt} = 0$
Autumn	$\beta_{opt} = 0.998\phi + 11.332$	$\beta_{opt} = 1.004\phi + 8.636$	$\beta_{opt} = 1.009\phi + 7.958$	$\beta_{opt} = 0.996\phi + 12.674$
Annual	$\beta_{opt} = 0.585\phi + 9.566$	$\beta_{opt} = 0.597\phi + 7.909$	$\beta_{opt} = 0.657\phi + 7.439$	$\beta_{opt} = 0.587\phi + 10.356$
	Anisotropic model			
	Equ (19)	Equ (20)	Equ (22)	Equ (23)
Winter	$\beta_{opt} = 0.965\,\phi + 29.426$	$\beta_{opt} = 0.977\,\phi + 28.528$	$\beta_{opt} = 0.977\,\phi + 28.528$	$\beta_{opt} = 0.876\,\phi + 30.818$
Spring	$\beta_{opt} = 0.366\,\phi + 1.088$	$\beta_{opt} = 0.370\,\phi + 0.978$	$\beta_{opt} = 0.370\phi + 0.978$	$\beta_{opt} = 0.363\,\phi + 1.708$
Summer	$\beta_{opt} = 0$	$\beta_{opt} = 0$	$\beta_{opt} = 0$	$\beta_{opt} = 0$
Autumn	$\beta_{opt} = 1.030\,\phi + 13.282$	$\beta_{opt} = 1.051\,\phi + 12.517$	$\beta_{opt} = 1.051\,\phi + 12.517$	$\beta_{opt} = 0.959\,\phi + 15.338$
Annual	$\beta_{opt} = 0.602\,\phi + 10.826$	$\beta_{opt} = 0.614\,\phi + 10.363$	$\beta_{opt} = 0.614\,\phi + 10.363$	$\beta_{opt} = 0.562\,\phi + 11.842$

from the monthly optimal tilt angle is higher than the seasonal and the solar radiation with the fixed yearly optimal tilt angle is the lowest. As indicated in the figure, the variation in total solar radiation due to monthly and seasonal optimal tilt angles is very small. However, there is a significant change in total solar radiation when using a seasonal tilt angle than a yearly optimal tilt angle (fixed). In all cases, the estimated total solar radiation at the optimal tilt angle with the anisotropic equation (23) results in the highest value than other models.

3.2. Losses in Solar Radiation. The percentage seasonal and yearly total solar radiation losses of an inclined surface compared to the solar radiation at the monthly optimal tilt angle were evaluated using equation (27) [16].

$$H_{losses}\,(\%) = \left(\frac{H_T\left(\beta = \beta_{opt,M}\right) - H_T\left(\beta = \beta_{opt,SY}\right)}{H_T\left(\beta = \beta_{opt,M}\right)}\right) x100 \tag{27}$$

The seasonal and yearly percentage total solar radiation losses on a tilted surface for Bahir Dar using isotropic and anisotropic models are shown in Figures 10 and 11, respectively. The minimum and maximum seasonal losses obtained by the isotropic model are 0.872% and 2.30%, respectively. Using an anisotropic model, the losses are between 0.941% to 1.053%. The yearly optimal tilt angle has a loss of 5.11% to 6.275% using the isotropic model and 5.72% to 6.346% using the anisotropic model, which shows a

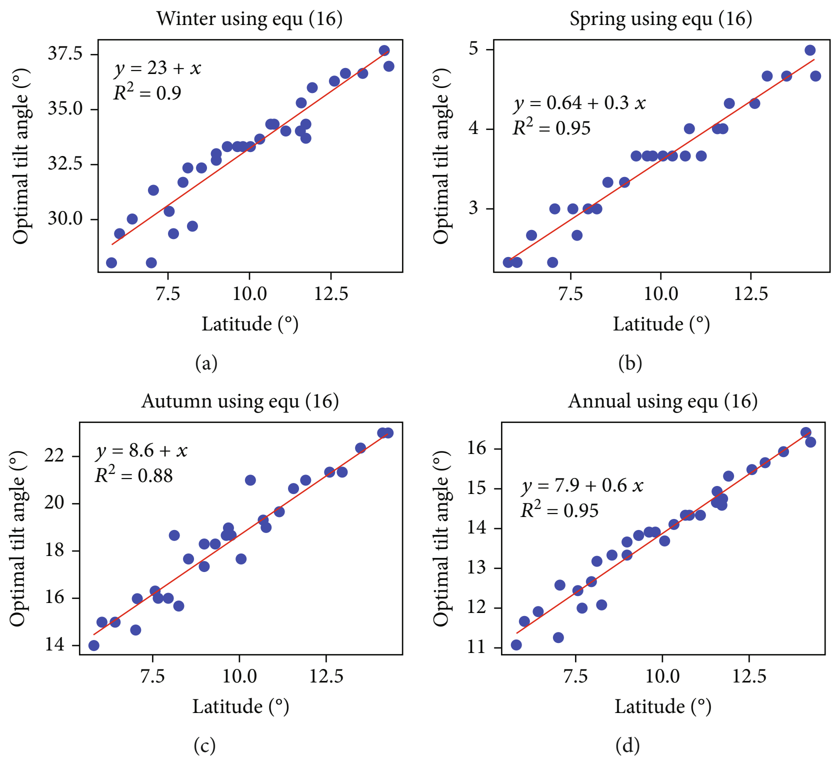

Figure 12: Regression model for seasonal and annual optimal tilt angle using Equation (16).

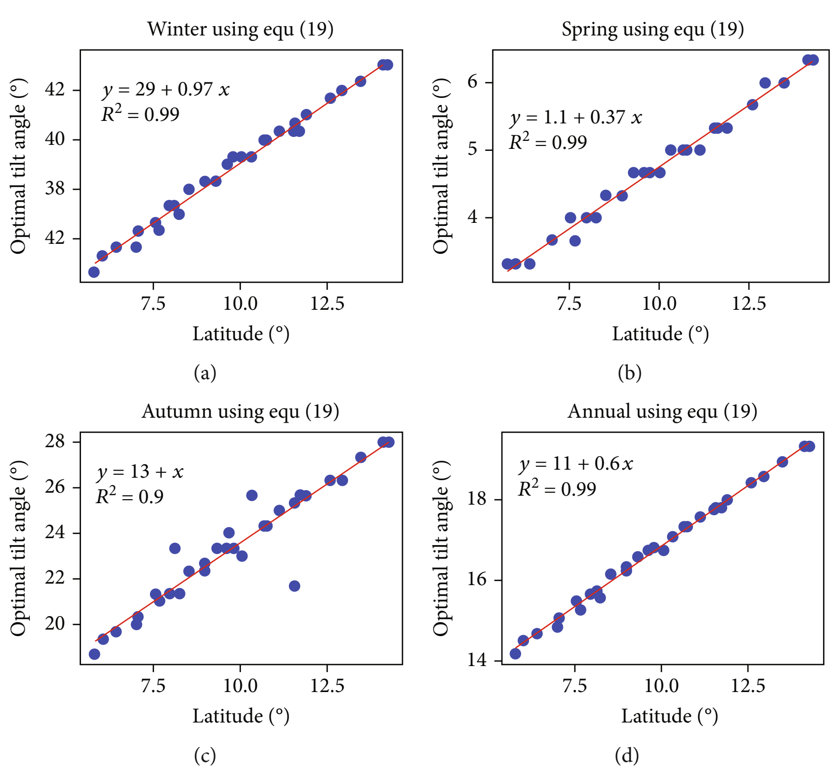

Figure 13: Regression model for seasonal and annual optimal tilt angle using Equation (19).

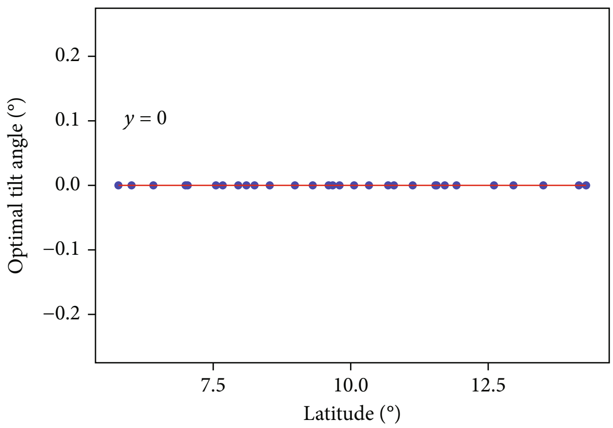

Figure 14: Regression model for summer optimal tilt angle using Equations ((15)–(20)), (22), and (23).

significant loss when using a fixed annual optimal tilt angle than the optimal tilt angle that varied seasonally.

The percentage gain of total solar radiation on a tilted surface compared to the horizontal surface is calculated by equation (28) [38]:

$$H_{\text{gains}}\,(\%) = \left(\frac{H_T\left(\beta = \beta_{opt,\text{MSY}}\right) - H_T(\beta = 0)}{H_T(\beta = 0)}\right) x100 \quad (28)$$

Using equation (28) the monthly, seasonal, and annual average total solar radiation gain of a tilted surface was evaluated for both isotropic and anisotropic models. For Bahir Dar city, the monthly average gain varies from 6.785% to 8.925% using isotropic and from 10.169% to 26.991% using the anisotropic model. The seasonal gains using the isotropic models vary between 5.723% to 7.532% and between 9.025% and 25.796% using the anisotropic model. Similarly, the yearly gain using isotropic and anisotropic models varied between 0.481% to 2.804% and 3.268% to 19.725%, respectively. From this result, the anisotropic model gives a better result than the isotropic model for monthly, seasonal, and yearly gains.

3.3. Developed Optimal Tilt Angle Models. In this study, the general models to predict the seasonal and annual optimal tilt angles were developed based on the linear regression model, shown in Table 5. Of the total data, 73% of the data (32 cities) were used to obtain the model and 27% of the data (12 cities) were used to test/evaluate the model. These models accurately predict the seasonal and annual optimal tilt angle of the solar panel for any city by using the location's latitude. The result from the developed model is in close agreement with the result obtained in [26]. The scatter plots with the regression developed models for winter, spring, autumn, and annual using equations (16) and (19) are given in Figures 12 and 13, respectively.

The optimal tilt angle of the study city is evaluated for the summer season using the isotropic and anisotropic models. For all isotropic and anisotropic models at this season, the optimal tilt angle is obtained to be zero. Its scatters plot of the developed regression model for both isotropic and anisotropic cases is shown in Figure 14.

3.3.1. Validation of the Results. Based on the analysis of accuracy testing tools, the proposed models are evaluated and the results are shown in Tables 6 and 7. As indicated in Tables 6 and 7, the smallest statistical error indicates that the developed model can accurately estimate both the seasonal and annual optimal tilt angle for any location in Ethiopia. The isotropic model equation (18) and from anisotropic model equations (19) and (23) have smaller errors, which is in agreement with the literature [10, 20]. The direction of the developed model to the actual value is determined by MBE. The anisotropic model of equation (23) has a negative MBE which indicates that the predicted values are overestimated than the actual values. The coefficient of determination (R^2) indicates to what extent the predicted values are close to the actual. The minimum R^2 is 0.8836 (88.36%) obtained for equation (17) and R^2 of nearly 1 (100%) is obtained by an anisotropic model of equations (19) and (23), which indicates that the anisotropic model of equations (19) and (23) predicts more accurately than isotropic models.

3.3.2. The Developed Model in Comparison with Literature and Other Software. The results of this study were also compared with the optimal tilt angle obtained from the published works and online software. The annual optimal tilt angle (β_{opt}) of this study location is determined from the correlation obtained from different literature. PVGIS and PVWatt are online software used to size PV systems at an optimized tilt angle. The annual optimal tilt angle comparison of the sample study site (Bahir Dar city) by using literature, online software, and this study is given in Table 8.

Yunus Khan et al. [33] proposed a latitude-based correlation of the optimal tilt angle model. The annual optimal tilt angle obtained from this literature is 13.48$^{\text{o}}$ which has a minor deviation of 1.34$^{\text{o}}$ to 4.87$^{\text{o}}$ compared to this study. A similar relation has been given in the literature by Darhmaoui and Lahjouji [40]. Wessley et al. [39] developed an optimal tilt angle correlation model to determine the Indian cities' optimal tilt angle. A small optimal tilt angle deviation of 0$^{\text{o}}$ to 3.53$^{\text{o}}$ has been observed between this literature and the current study. The current study also compared with the model proposed in the literature by Nicolas-Martín et al. [23] and a maximum of 4.53$^{\text{o}}$ deviations of the optimal tilt angle has been observed. However, the optimal tilt angle model proposed by Duffie and Beckman [7] results in a minimum of 8.22$^{\text{o}}$ tilt angle deviation as compared to this study. This is due to the models proposed in all literature are not equally applicable for different locations.

The results of this study were also validated by comparing with the online PV sizing software such as PVGIS and PVWatt. This software generates a very close optimal tilt angle to the current study and the observed minimum and maximum deviation with the study are 0.15 to 2.18 and 0.34 to 2.35 for PVGIS and PVWatt, respectively. These comparisons also validate the performance of the proposed system in determining the optimal tilt angle.

Table 6: Statistical errors of the developed equation using isotropic and anisotropic.

Time	Isotropic models									
	RMSE	MBE	MAE	MAPE	R^2	RMSE	MBE	MAE	MAPE	R^2
			Equ (15)					Equ (16)		
Winter	0.855	0.170	0.657	2.00	0.9994	0.926	0.731	0.859	2.77	0.9991
Spring	0.107	0.068	0.073	2.07	0.9991	0.205	0.145	0.173	6.04	0.9953
Autumn	0.354	0.068	0.319	1.73	0.9996	0.475	0.161	0.423	2.70	0.9991
Annual	0.204	0.172	0.172	1.24	0.9998	0.368	0.290	0.310	2.47	0.9991
			Equ (17)					Equ (18)		
Winter	0.821	0.653	0.758	2.31	0.9994	0.368	0.279	0.308	0.87	0.9999
Spring	0.572	0.473	0.489	8.02	0.8836	0.132	0.123	0.123	3.41	0.9988
Autumn	0.628	0.168	0.517	3.56	0.9984	0.252	0.044	0.194	0.94	0.9998
Annual	0.370	0.258	0.298	2.35	0.9991	0.170	0.136	0.154	1.05	0.9999

Table 7: Performance evaluation of the anisotropic developed optimal tilt angle model using statistical indices.

Time	Anisotropic models									
	RMSE	MBE	MAE	MAPE	R^2	RMSE	MBE	MAE	MAPE	R^2
			Equ (19)					Equ (20)		
Winter	0.259	0.125	0.213	0.58	1.000	0.459	0.328	0.405	1.12	0.9998
Spring	0.092	0.035	0.073	1.96	0.9994	0.117	0.089	0.096	2.84	0.9990
Autumn	0.257	0.082	0.212	1.00	0.9998	0.237	0.136	0.178	0.87	0.9999
Annual	0.139	0.094	0.119	0.78	0.9999	0.290	0.256	0.263	1.79	0.9996
			Equ (22)					Equ (23)		
Winter	0.459	0.328	0.405	1.12	0.9998	0.094	-0.033	0.080	0.21	1.000
Spring	0.117	0.089	0.096	2.84	0.9990	0.109	-0.010	0.086	2.24	0.9994
Autumn	0.237	0.136	0.178	0.87	0.9999	0.203	-0.098	0.171	0.80	0.9999
Annual	0.222	0.178	0.198	1.34	0.9998	0.047	-0.003	0.040	0.25	1.000

Table 8: Annual optimal tilt angle of Bahir Dar city by this study, literature, and online software.

Literature	Model	β_{opt}
Yunus Khan et al. [32]	$\beta_{opt} = 1.3793 + \phi(1.2011 + \phi(-0.014404 + \phi 0.000080509))$	13.48
Duffie and Beckman [7]	$\beta_{opt} = (\phi + 15) \pm 15$	26.57
Wessley et al. [39]	$\beta_{opt} = 0.823\phi + 8.8274$	18.35
Darhmaoui and Lahjouji [40]	$\beta_{opt} = 1.25351\phi - 0.00728944\phi^2$	13.53
Nicolas-Martín et al. [23]	$\beta_{opt} = -0.007021\phi^2 + 1.091\phi + 2.132$	13.82
PVGIS [41]	Online software	17
PVWatt [42]	Online software	16
This study	$\beta_{opt} = 0.585\phi + 9.566$ Equ (15)	16.34
	$\beta_{opt} = 0.597\phi + 7.909$ Equ (16)	14.82
	$\beta_{opt} = 0.657\phi + 7.439$ Equ (17)	15.04
	$\beta_{opt} = 0.587\phi + 10.356$ Equ (18)	17.15
	$\beta_{opt} = 0.602\phi + 10.826$ Equ (19)	17.79
	$\beta_{opt} = 0.614\phi + 10.363$ Equ (20)	17.47
	$\beta_{opt} = 0.614\phi + 10.363$ Equ (22)	17.47
	$\beta_{opt} = 0.562\phi + 11.842$ Equ (23)	18.35

Therefore, the result of this study gives an optimal tilt angle, and the developed model can be used in real applications of photovoltaic engineering to install the PV module at any location in Ethiopia. Installing the photovoltaic module with the suggested optimal tilt angle helps to obtain the maximum solar radiation on the surface of the module, hence gives the maximum energy output.

4. Conclusion

In this study, the monthly, seasonal, and yearly optimal tilt angle of the solar module to harvest maximum incident solar radiation was determined for 44 cities of Ethiopia using isotropic and anisotropic models. Four isotropic and four anisotropic models were used to predict the diffuse solar radiation and the optimal tilt angle. The most important findings of this study are summarized as follows:

(i) Four isotropic and four anisotropic models were taken into consideration and their accuracy in predicting the optimal tilt angle is validated. The result revealed that the anisotropic models of Reindle et al., Steven and Unsworth predict the lower statistical errors

(ii) The yearly optimal tilt angle varied between 14.92° to 17.17° and 17.58° to 18.42° using isotropic and anisotropic models, respectively (as obtained for Bahir Dar). The anisotropic model gives a more consistent optimal tilt angle than the isotropic model

(iii) The seasonal varied optimal tilt angles are obtained between 35.33° to 39.33° in winter, 3.33° to 5° in spring, 0° in summer, and 19.67° to 24.33° in autumn using the isotropic model. The optimal tilt angle obtained using anisotropic models are 40° to 41°, 5.33° to 6°, 0°, and 25° to 26.67° for winter, spring, summer, and autumn, respectively. The minimum optimal tilt angle is 0° found in summer and the maximum is 35° to 41°, found in winter

(iv) Significant solar radiation gains are obtained when the solar module operates at a seasonal optimal tilt angle than a fixed (annual) optimal tilt angle. The seasonal gains of total solar radiation were 5.723% to 7.532% and between 9.025% to 25.796% using isotropic and anisotropic models, respectively. This shows that the anisotropic models have more gain as compared to the isotropic model

(v) A generic seasonal and annual optimal tilt angle model as a function of latitude was developed that estimates the optimal tilt angle accurately at any location in Ethiopia without using meteorological data. The accuracy of the developed model was evaluated using RMSE, MBE, MAE, MAPE, and R^2. The model developed using anisotropically distributed diffused radiation results in a minim error, which is in agreement with the previous works in the literature. The model developed using Reindl et al. anisotropic model shows a smaller error of 0.139 (RMSE), 0.094 (MBE), 0.119 (MAE), and 0.78 (MAE) RMSE with a 0.9999 coefficient of determination (R^2).

The result of this study is verified using statistical indices, previous literature works, and online software. Online software has very close results with this work as those software use the local information to determine the optimal tilt angle. In this study, the developed models can determine the optimal tilt angle by taking only the latitude of the study site. However, this work can be further enhanced by considering other parameters in determining the optimal tilt angle models such as elevation and real-time measured solar radiation data.

Nomenclature

Latin Symbols, Subscripts, and Superscripts.

G_{sc}: Solar constant (1.367 kW/m^2)
H: The monthly average daily global horizontal radiation (kWh/m^2/day)
H_B: Monthly average daily total beam radiation on the tilted surface (kWh/m^2/day)
H_b: The monthly average daily beam radiation on the horizontal surface
H_D: Monthly average daily total diffuse radiation on the tilted surface (kWh/m^2/day)
H_d: Monthly average daily total diffuse radiation on the horizontal surface (kWh/m^2/day)
H_{gain}: Total radiation gain (%)
H_{losses}: Total solar radiation losses on the inclined surface (%)
H_o: Daily extraterrestrial radiation on the horizontal surface (kWh/m^2/day)
H_R: Monthly average daily total ground reflected radiation (kWh/m^2/day)
H_T: Monthly average daily total radiation on the tilted surface (kWh/m^2/day)
K_t: Daily clearness index
n: The nth day of the year
R_d: The ratio between the monthly daily diffuse radiation on the tilted surface to the horizontal surface.

Abbreviations

HDKR: Hay, Davies, Klucher and Reindl model
MAE: Mean absolute error
MAPE: Mean Absolute Percentage Error
MATLAB: Matrix Laboratory
MBE: Mean Bias Error
MW: Mega Watt
NASA: National Aeronautics and Space Administration
PV: Photovoltaic
PVGIS: Photovoltaic Geographical Information System
RMSE: Root Mean Square Error.

Greek Symbols

β: Tilt angle (degree)
$\beta_{opt,M}$: Monthly optimal tilt angle (degree)

β_{predt}: Predicted optimal tilt angle (degree)
β_{cal}: Calculated optimal tilt angle (degree)
$\beta_{opt,SY}$: Seasonal or yearly optimal tilt angle (degree)
$\beta_{opt,MSY}$: Monthly, seasonal, or yearly optimal tilt angle (degree)
γ: Azimuth angle (degree)
δ: Declination angle (degree)
θ: Solar radiation incident angle (degree)
θ_z: Zenith angle (degree)
π: Pi
ρ: Ground reflectance (albedo)
ϕ: Latitude (degree)
ω: Hour angle (degree)
ω_s: Sunset hour angle (degree)
ω_{ss}: Sunset hour angle of the tilted surface (degree).

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper

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No funding is received to carry on this research.

Supplementary Materials

The monthly average daily horizontal solar radiation and latitude data were obtained for the study sites and the maximum solar radiation and the corresponding optimal tilt angle were obtained using proposed algorithms using MATLAB software. Both isotropic and anisotropic models were applied to evaluate the optimal tilt angle. R-programming software was used to obtain the generic optimal tilt angle model that can be applied for any location without using the metrological data. *(Supplementary Materials)*

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