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Instability Threshold of a Rigid Rotor-Tilting Pad Journal Bearings System

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The stability of a rigid rotor supported on radial tilting pad journal bearings is analysed. This study has been tackled both for small unbalance values by linearising the equations of motion, and also in the case where, because of the high unbalance value, the rotor axis describes orbits with an amplitude such that the system's non-linearity cannot be ignored. In both cases the system's stable operation maps have been obtained and verified through numerical integration of the differential equations of motion.

Keywords: Tilting pad journal bearings, rotor-bearing system, stability

INTRODUCTION

Tilting-pad journal bearings are widely used for supporting the rotors of turbomachines as they help prevent the occurrence of oil film instability phenomena that can be induced by high rotation speeds. Nevertheless, for the components in question, there are forms of instability that arise if the operating conditions, and the rotation speed in particular, exceed certain limits which should thus be regarded as thresholds of stable behaviour.

In the case of well-balanced rotors, the residual unbalance forces the journal to describe orbits that have a very small amplitude around the stationary equilibrium position and are synchronous with rotor speed. In these conditions, if the pads are arranged symmetrically to the direction of the static load and if the pads have a negligible mass compared to that of the journal, then the rotor bearing system is always stable.

If the pads have a mass that is not negligible compared to that of the rotor, a form of instability may arise whose threshold can be found by linearising the journal-pads system's equations of motion around the stationary equilibrium position.

The search for the real part of the eigenvalues of the non-conservative system with several degrees of freedom constituted by the journal, the pads and the linearised action of the oil film in the gaps, makes it possible to determine the stability threshold of the

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stationary equilibrium position for given geometrical and operating conditions.

The study is conducted on a linear system and the corresponding instability is thus indicated as "linear instability".

In the case of unbalanced rotors, the journal and the pads describe periodic motions with a non-negligible amplitude and the non-linearity of the system is thus important. Under these circumstances it will be necessary to investigate the stability, not of the stationary equilibrium position but of the motion (which is synchronous with the rotation speed) that the journal and the pads undergo as a result of the unbalance. In this case the instability arises even if the mass of the pads is negligible and it assumes the forms typical of systems with a non linear behaviour.

This instability is therefore indicated as "non linear instability" and it arises with the onset in the system motion of components having a frequency equal to an integer submultiple of that corresponding to the rotation speed, and with amplitudes that can be so large as to compromise the very operation of the system. The study of this form of instability is conducted using the methods of non linear analysis.

Hereafter both the linear and the non linear limit stability curves of the tilting pads-rotor bearing system are determined in an appropriate operating conditions plane. These curves are validated by verifications carried out by numerically integrating the non linear equations of motion.

The analysis is conducted with reference to a four-pad bearing but the illustrated method has a general validity and can thus be applied to the case of bearings with a different number of pads without encountering any difficulties of a conceptual nature.

EQUATIONS OF MOTION

The system analysed is made up of a journal and a bearing with four equal tilting pads arranged at equal distances around the circumference. Figure 1 shows a diagram of the bearing and indicates the main geometrical dimensions and the fixed reference frame whose origin lies in the centre of the bearing, and whose z-axis coincides with the axis of the bearing, and is oriented in the same direction as the angular speed. Under the assumption of rigid pads and no misalignment, the meatus thickness of the i-th pad assumes the following dimensionless expression:

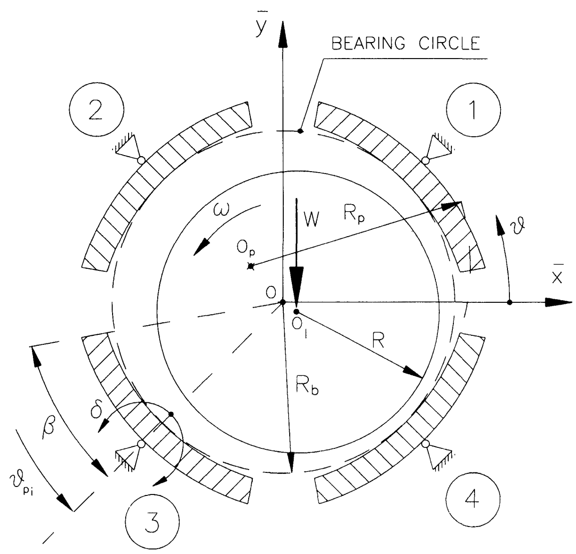

FIGURE 1 Bearing notation.

$$h_i = 1 - x\cos\vartheta - y\sin\vartheta + A_i\sin(\vartheta_{pi} - \vartheta) - m\cos(\vartheta_{pi} - \vartheta) \qquad (1)$$

Under the hypothesis of laminar and isothermal flow, the dimensionless form of the Reynolds equation can be written as:

$$\frac{\partial}{\partial\vartheta}\left(h^3\frac{\partial p}{\partial\vartheta}\right) + \left(\frac{R}{L}\right)^2\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right) = 6\frac{\partial h}{\partial\vartheta} + 12\frac{\partial h}{\partial\tau} \qquad (2)$$

The Reynolds equation can be integrated using a variety of different techniques; in the present paper in order to reduce the numerical calculation workload the pressure distribution is approximated by making [Adams, M. L., Payandeh, S. 1983—White et al., 1982]:

$$p(\vartheta, z) = p(\vartheta, 0)[1 - (2z)^2] \tag{3}$$

So, it is possible to write eq. (2) in the following one-dimensional form:

$$\frac{\partial}{\partial\vartheta}\left(h^3\frac{\partial p(\vartheta, 0)}{\partial\vartheta}\right) - 2\left(\frac{D}{L}\right)^2 h^3 p(\vartheta, 0) = 6\frac{\partial h}{\partial\vartheta} + 12\frac{\partial h}{\partial\tau} \tag{4}$$

Eq. (4), numerically integrated with the boundary conditions corresponding to null pressure values at the edge of the pad, provides the pressure distribution for each pad.

Eq. (3), clearly introduces an approximation in the pressure distribution compared to that which is determined by integrating the Reynolds equation in form (2). This approximation is excellent in stationary conditions for L/D ratio values between 0.25 and 1 and for not very high eccentricity values. In unstationary conditions and also for high eccentricity values, the approximation is still acceptable compared to the considerable reduction in the calculation workload that eq. (3) entails. In this connection, figure 2 provides a comparison between the pressure distribution on the pads calculated in unstationary conditions by performing a finite differences integration of (2) and (4). The components $f_x(x,y,\dot{x},\dot{y},A_i,\dot{A}_i)$ and $f_y(x,y,\dot{x},\dot{y},A_i,\dot{A}_i)$ of the dimensionless overall hydrodynamic force that the four pads exert on the journal are obtained by integrating the pressure on the pads' surface according to the following expression:

$$\begin{Bmatrix} f_x \\ f_y \end{Bmatrix} = -\sum_{i=1}^{4}\int_{\vartheta_{i1}}^{\vartheta_{i2}}\int_{-1/2}^{1/2} p(\vartheta, z)\begin{Bmatrix} \cos\vartheta \\ \sin\vartheta \end{Bmatrix} d\vartheta dz \tag{5}$$

Once the components f_x and f_y are known, the dynamic behaviour of the journal-pads system can be derived from the equations of motion:

$$M\ddot{x} = f_x(x, y, \dot{x}, \dot{y}, A_i, \dot{A}_i) + M\rho\cos\tau$$

$$M\ddot{y} = f_y(x, y, \dot{x}, \dot{y}, A_i, \dot{A}_i) - \frac{1}{\sigma} + M\rho\sin\tau$$

$$I\ddot{A}_i = T_i(x, y, \dot{x}, \dot{y}, A_i, \dot{A}_i) \tag{6}$$

where $T_i(x,y,\dot{x},\dot{y},A_i,\dot{A}_i)$ is the moment of the hydrodynamic force acting on the i-th pad with respect to the pivot of the same pad, and is equal to:

$$T_i = \int_{\vartheta_{i1}}^{\vartheta_{i2}}\int_{-1/2}^{1/2} p(\vartheta, z) sin(\vartheta_{pi} - \vartheta) d\vartheta dz \tag{7}$$

The direct numerical integration of the motion equations [Pagano et al. 1995] provides, for given geometrical and operating conditions, the trajectories of the orbital motion of the journal axis and the inclination of the pads around the pivots. This makes it possible to simulate the dynamic behavior of the system and thus to verify, case by case, the results of the stability analyses illustrated below. In the present paper, numerical integration has been performed using the fourth order Runge-Kutta method.

LINEAR STABILITY ANALYSIS

For balanced rotors, i.e. rotors subjected only to residual unbalance, the motion of the journals and the pads is periodic and is limited to around the stationary equilibrium position that the system assumes for the static load alone. The above equilibrium position is determined by solving the following system of equations [Pagano et al., 1994]:

$$\begin{cases} f_x(x, y, 0, 0, A_i, 0) = 0 \\ f_y(x, y, 0, 0, A_i, 0) = \dfrac{1}{\sigma} \quad i = 1, .., 4 \\ T_i(x, y, 0, 0, A_i, 0) = 0 \end{cases} \tag{8}$$

which express the stationary equilibrium.

Figures 3 and 4 respectively report the eccentricity of the journal and the inclination of the pads in the equilibrium conditions as the Sommerfeld number varies. The stability of these positions can be studied by linearising the equations of motion (6), particularised for the case of a balanced rotor ($\rho = 0$),

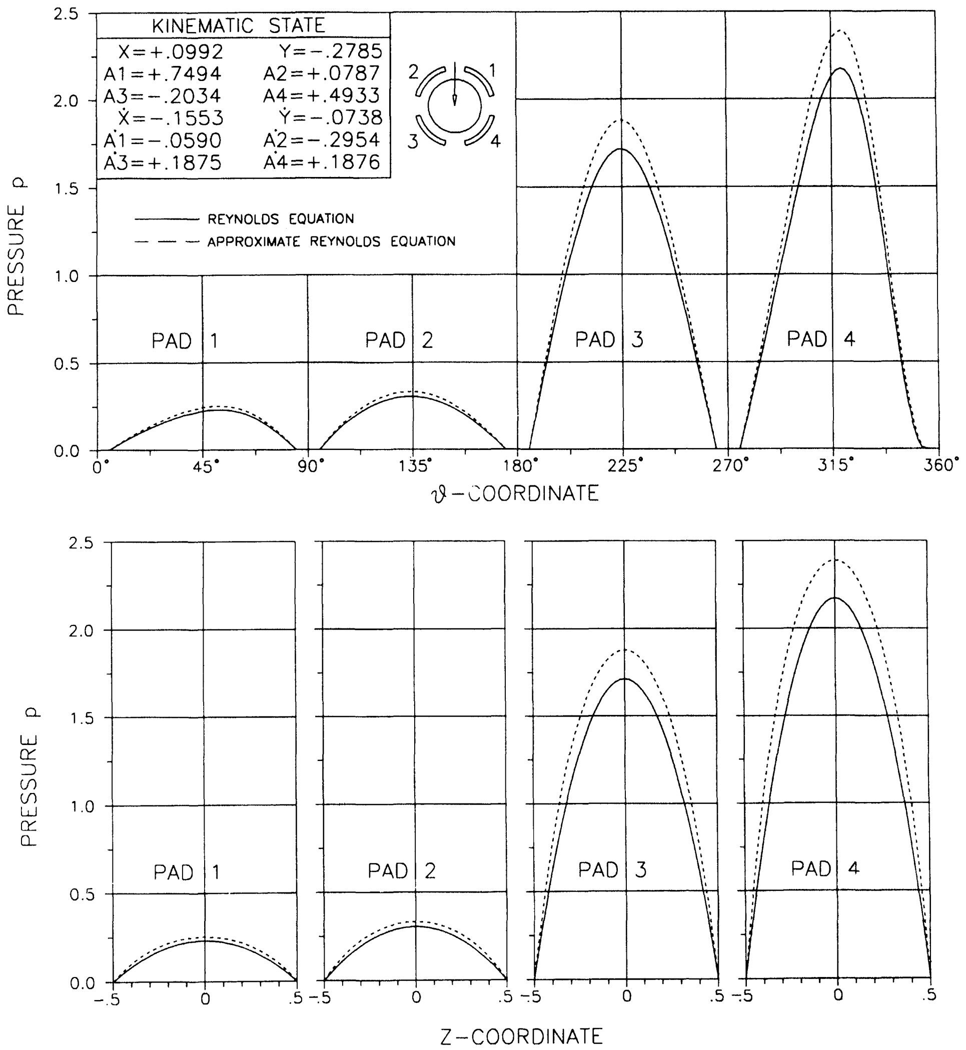

FIGURE 2 Circumferential and axial pressure distribution: comparison between the results obtained with the two-dimensional and the one-dimensional methods, for a given kinematic state and for: L/D = 0.7; m = 0.35; M = 20; I = 0.1, σ = 1.0.

through a Taylor series expansion of the hydrodynamic force and maintaining only the first order terms:

$$M\ddot{x} + b_{xx}\dot{x} + b_{xy}\dot{y} + k_{xx}x + k_{xy}y + \sum_{i=1}^{n} (k_{xAi}A_i + b_{xAi}\dot{A}_i) = 0 \qquad (9)$$

$$M\ddot{y} + b_{yx}\dot{x} + b_{yy}\dot{y} + k_{yx}x + k_{yy}y + \sum_{i=1}^{n} (k_{yAi}A_i + b_{yAi}\dot{A}_i) = 0$$

$$I\ddot{A}_i + k_{AAi}A_i + k_{Axi}x + k_{Ayi}y + b_{AAi}\dot{A}_i + b_{Axi}\dot{x} + b_{Ayi}\dot{y} = 0$$

after making:

$$k_{pq} = \left(\frac{\partial G_p}{\partial q}\right); \; b_{pq} = \left(\frac{\partial G_p}{\partial \dot{q}}\right) \quad \text{with:}$$

$$G = f_x, f_y, T_i \quad \text{and} \quad p, q = x, y, A_i$$

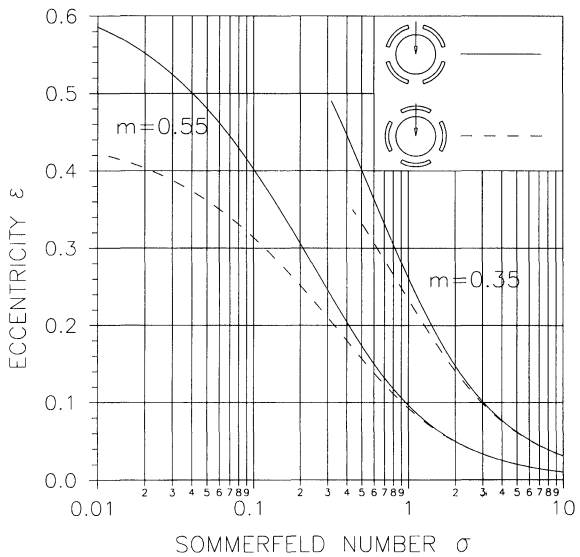

FIGURE 3 Eccentricity ratio versus Sommerfeld number.

Equations (9) are the linearised equations in the neighbourhood of the equilibrium position written in a reference originating in the point of stationary equilibrium.

The above position is stable if the real part of all the eigenvalues of system (9) are negative. Therefore, the limit stability conditions can be determined as the conditions for which an eigenvalue has a real part equal to zero as all the other eigenvalues have a negative real part. The imaginary part of the above eigenvalue will provide the oscillation frequency of the system at the instability threshold.

NON LINEAR STABILITY ANALYSIS

For unbalanced rotors, stability analysis is considerably more complex as the non linearity of the field of forces entails several solutions of the equations of motion (6) indicating the existence of several motions: periodic synchronous, subsynchronous, almost periodic, non periodic and even chaotic [Adams et al., 1994], for each of which the stability should be evaluated.

The present paper evaluates the stability of the synchronous motion that the journal and the pads undergo due to the unbalance.

For critical values of the operating parameters (M, σ, ρ), and thus of the rotation speed, the synchronous motion becomes unstable whereas a second order subsynchronous motion is stable [Brancati et al., 1995]. In these conditions, the journal describes an orbit characterised by a period that is double the one corresponding to the rotor's speed and by an amplitude greater than that of the synchronous motion, thus causing greater vibrations in the machine. As the speed increases, the period of motion is further doubled: then the system motion can become non periodic. In any case these motions are characterised by ever greater amplitudes and therefore, from a technical viewpoint, a "safe" operating threshold for the rotor bearing system coincides with the stability threshold of the synchronous solution.

The stability analysis proposed requires the synchronous solution of equations (6) to be determined in analytical form by particularising [Russo et al., 1993] (see Appendix A) a harmonic balance method [Nayfeh et al., 1979—Szemplinska-Stupnicka, 1990], which provides the law of motion in the form:

$$x(\tau) = x_0 + \sum_{j=1}^{k} [x_{cj}\cos(j\tau) + x_{sj}\sin(j\tau)]$$

$$y(\tau) = y_0 + \sum_{j=1}^{k} [y_{cj}\cos(j\tau) + y_{sj}\sin(j\tau)]$$

$$A_i(\tau) = A_{i0} + \sum_{j=1}^{k} [A_{icj}\cos(j\tau) + A_{isj}\sin(j\tau)] \tag{10}$$

Perturbing only the journal motion by means of the functions $\delta x(\tau)$ and $\delta y(\tau)$, and substituting the perturbed solution into the equations of motion (6) gives the following system of variational equations:

$$M\,\ddot{\delta x} - \left(\frac{\partial f_x}{\partial x}\right)_* \delta x - \left(\frac{\partial f_x}{\partial y}\right)_* \delta y - \left(\frac{\partial f_x}{\partial \dot{x}}\right)_* \dot{\delta x} - \left(\frac{\partial f_x}{\partial \dot{y}}\right)_* \dot{\delta y} = 0$$

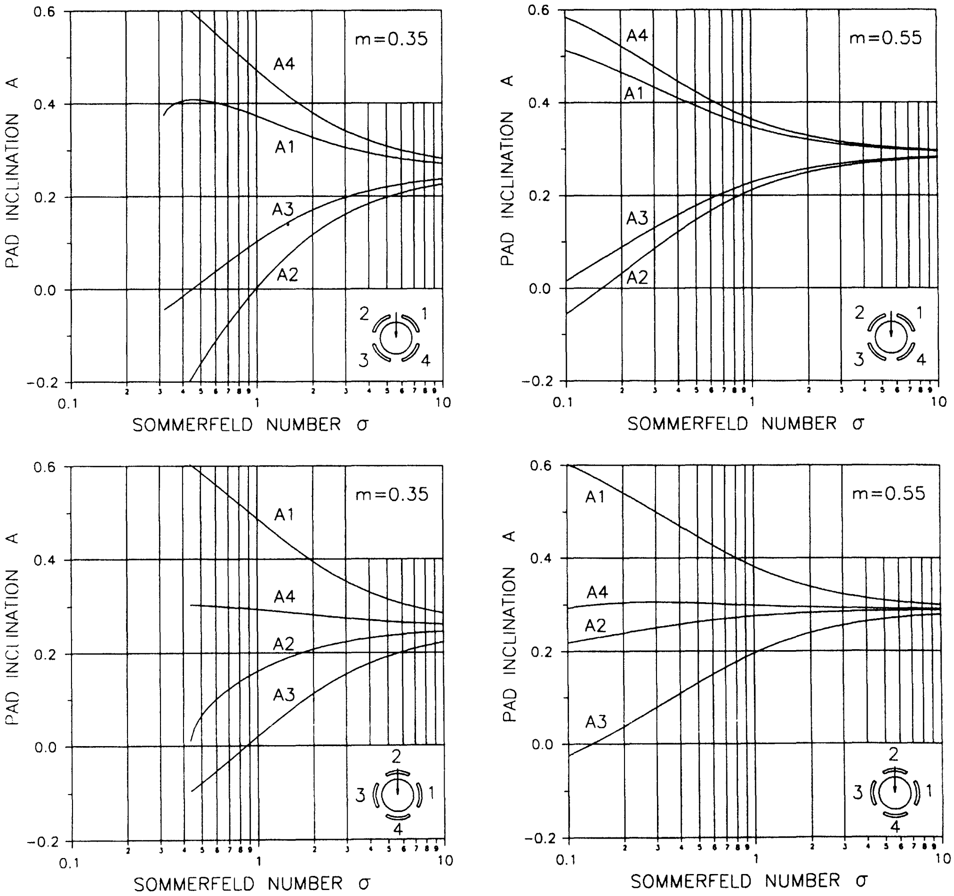

FIGURE 4 Pad inclination versus Sommerfeld number.

$$M\,\delta\ddot{y} - \left(\frac{\partial f_y}{\partial x}\right)_* \delta x - \left(\frac{\partial f_y}{\partial y}\right)_* \delta y - \left(\frac{\partial f_y}{\partial \dot{x}}\right)_* \delta\dot{x} - \left(\frac{\partial f_y}{\partial \dot{y}}\right)_* \delta\dot{y} = 0 \qquad (11)$$

obtained through a Taylor series development of f_x and f_y starting from solution (10) and retaining only the linear terms. The proposed procedure is clearly approximated considering that only the journal motion has been disturbed to provide, instead of six equations (i.e. the same number as there are in system (6)), a system of variational equations made up of only two equations.

This method clearly allows the calculation workload to be considerably reduced: the goodness of the results has been verified by integrating the complete system of the equations of motion (6) in the proximity of the stability threshold obtained using the above method.

In equations (11) the asterisk denotes that the derivatives have been calculated around solution (10). These derivatives are time functions with a period equal to that of the solution and can, therefore, be substituted by their Fourier series development stopped at the first order terms. Setting:

$$\left(\frac{\partial G_p}{\partial q}\right)_* = k_{pq_0} + k_{pq_c}\cos\tau + k_{pq_s}\sin\tau = -\,\bar{k}_{pq}$$

$$\left(\frac{\partial G_p}{\partial \dot{q}}\right)_* = b_{pq_0} + b_{pq_c}\cos\tau + b_{pq_s}\sin\tau = -\,\bar{b}_{pq}$$

with: $G = fx, fy; p, q = x, y$

Equations (11) can be rewritten:

$$M\delta\ddot{x} + \bar{k}_{xx}\,\delta x + \bar{k}_{xy}\,\delta y + \bar{b}_{xx}\,\delta\dot{x} + \bar{b}_{xy}\,\delta\dot{y} = 0$$

$$M\,\delta\ddot{y} + \bar{k}_{yx}\,\delta x + \bar{k}_{yy}\,\delta y + \bar{b}_{yx}\,\delta\dot{x} + \bar{b}_{yy}\,\delta\dot{y} = 0 \qquad (12)$$

The variational equations in the form (12) are Hill type equations and the stability of their trivial solution coincides with the stability of solution (10), i.e. with the stability of the synchronous motion of the journal-pads system [Szemplinska-Stupnicka W. 1990]. Therefore, by searching for the limit conditions that make the trivial solution of equation (12) unstable (see Appendix B), it is possible to determine the stability threshold of the synchronous solution and, for the above reasons, of "safe" rotor operation.

LINEAR ANALYSIS RESULTS

For given values of the rotor mass parameter (Mσ), the limit stability curves of the stationary equilibrium position have been plotted in the plane σ-Iσ. In this plane the operating line [Holmes 1960], i.e. the locus of the points representing the operating conditions that have in common all the parameters except the rotor's speed, is a straight line with a slope of 2. The analysis has been conducted with reference to both the configuration with "load between pads" and "load on pads". Two different geometrical preload values have been considered; the angular extension of the pads γ is equal to 80° and the offset has been set at 0.5. Figure 5 shows an example of the curves obtained using the proposed method. Analysis of the results confirms how, in the case of the symmetrical arrangement of the load with respect to the pads, the system is always stable if the moment of the pads' forces of inertia is negligible compared to the moment of the hydrodynamic force and, thus, if the pads' moment of inertia can be considered null.

The curves reported present a characteristic "jump" with a consequential sudden variation in the threshold value of the Iσ parameter. The jump is due to the fact that as the Sommerfield number decreases, the journal's stationary equilibrium position shifts downwards: after a certain value of σ the upper pads are unloaded and system operation can be compared to that of a journal supported on a bearing with two tilting pads [Pagano et al. 1994]. The left-hand branch of the limit stability curves are thus characterised by this feature.

The characteristic trend of the limit stability curves has been analysed using the numerical simulation program described. With reference to the operating conditions identified by the operating points reported in the σ-Iσ plane in figure 6, the system response for given initial conditions of the kinematics state is illustrated.

Points A and B, below the limit stability curve, are points of stable operation: the motion of the pads and the orbit described by the journal axis tend towards the stationary equilibrium position. Points C and D, on the other hand, are points of unstable operation and the motion of the journal axis and that of the pads tend to increase. The oscillographs of points C and D have been obtained by using, as initial conditions, the stationary equilibrium positions in the operating conditions represented by points A and B, respectively.

The results described not only confirm the validity of the stability curve but also show how, for points A and C, all the pads participate in the system motion while, for points B and D (left-hand branch of the curve) the upper pads are unloaded and therefore do not track the journal motion. In the case illustrated in figure 6, pad 2 assumes the maximum inclination envisaged by the program while pad 1, which is subjected to a light load, tilts with a frequency that is different from that of the journal.

In these conditions, the upper pads may be subject to violent sub-synchronous self-excited vibrations [Adams et al. 1983], and the resulting collisions between pad and journal destroy the pads' babbitt metal.

In order to load the upper pads in the bearing, the preload value has to be increased. With reference to figure 7, it can be noted that an initial increase in the preload (from m = 0.35 to m = 0.55) causes the upper pads to be loaded but it also takes the system

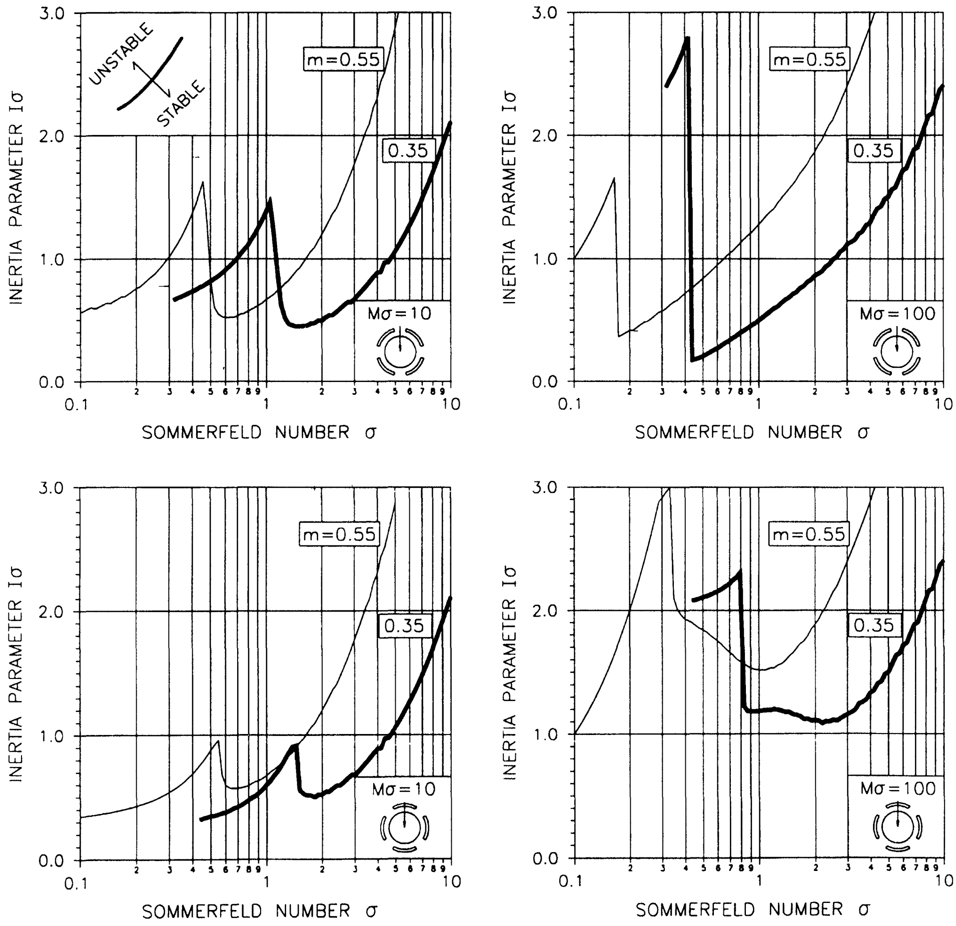

FIGURE 5 Limit stability curves for the linearised system (L/D = 0.7).

into an unstable operating condition as point E is above the limit curve obtained for the new preload value. The same figure points out that for the system to be stable, the geometrical preload value m has to be further incremented (from m = 0.55 to m = 0.65), so that the operating point E is below the right-hand branch of the corresponding limit stability curve.

Numerous other simulations of the same type, as those illustrated, have been performed in points near the limit stability curves, confirming their validity even for bearings with "load on pads".

NON LINEAR ANALYSIS RESULTS

In the case of the unbalanced rotor, the analysis of the stability of system motion coincides with the study of the stability of the synchronous solution of the equations of motion which must first be determined in analytical form.

The analytical form of the solution is Eq. (10), and figure 8 compares a synchronous analytical solution (dashed line) obtained by making k = 4, with one that was instead obtained by numerical simulation.

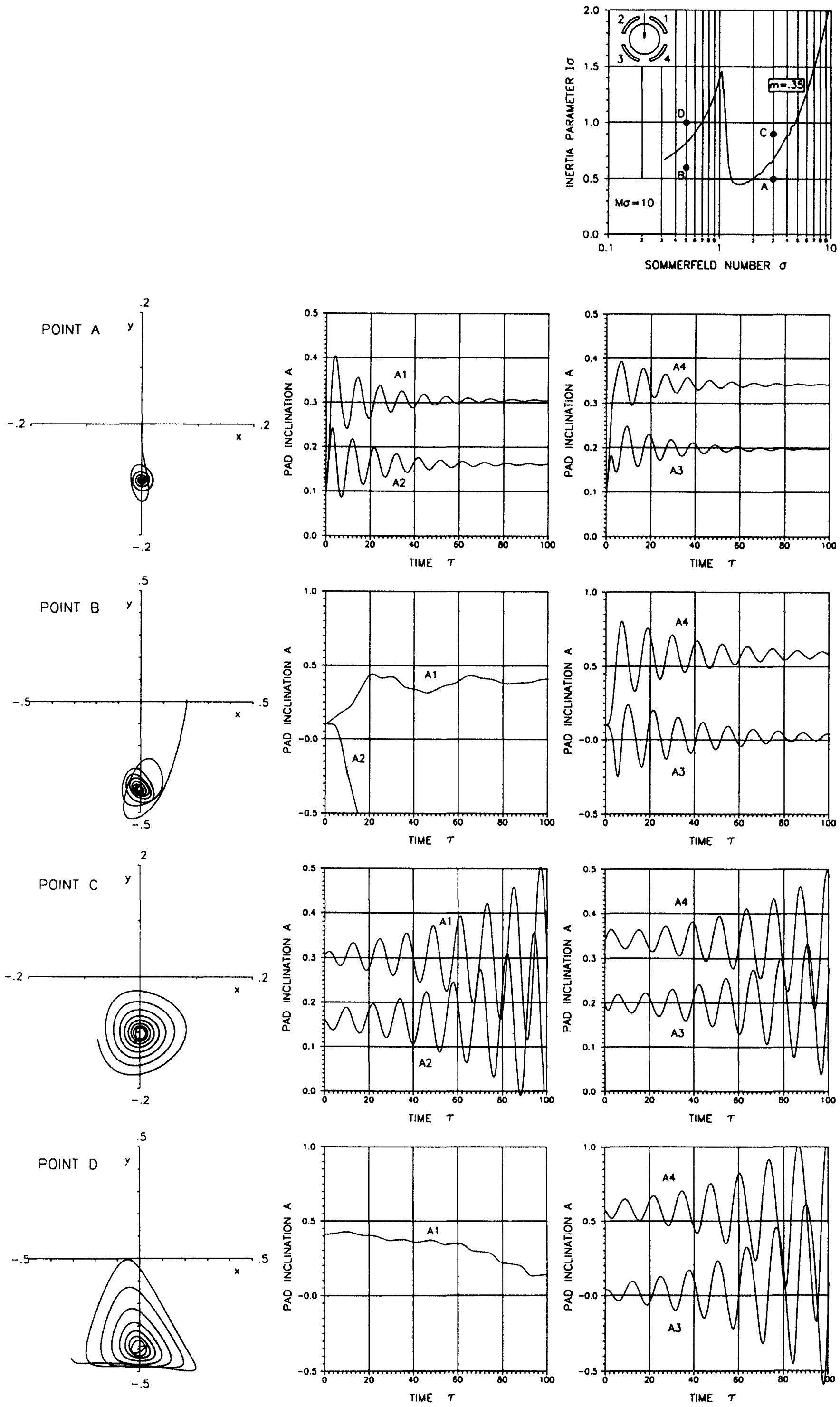

FIGURE 6 Journal orbits and pad oscillations for different operational conditions and: $\rho = 0$; $L/D = 0.7$; $m = 0.35$.

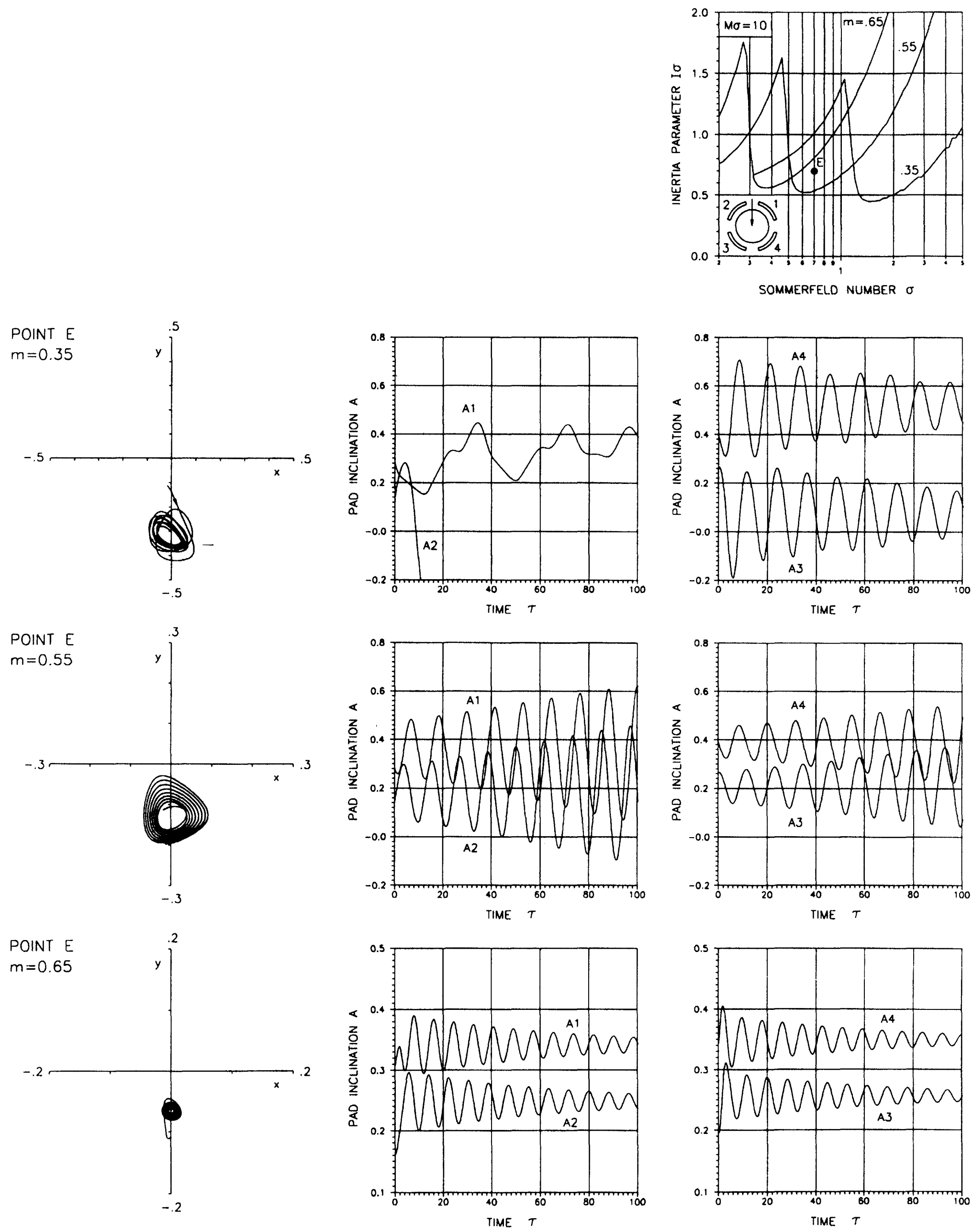

FIGURE 7 Journal orbits and pad oscillations for different geometrical preload values and: $\rho = 0$; $\sigma = 0.7$; $I\sigma = 0.7$; $L/D = 0.7$.

In the figure the solution is reported in the form of the orbital motion of the journal axis, with components $x(\tau)$ and $y(\tau)$, and of the oscillations of the pads; the trajectories obtained by numerical integration do not include the initial transient. As can be seen from the comparison, the approximated analytical solution with four harmonics, determined using the described method, is excellent.

The systematic study of the synchronous solutions thus determined made it possible to plot, in the oper-

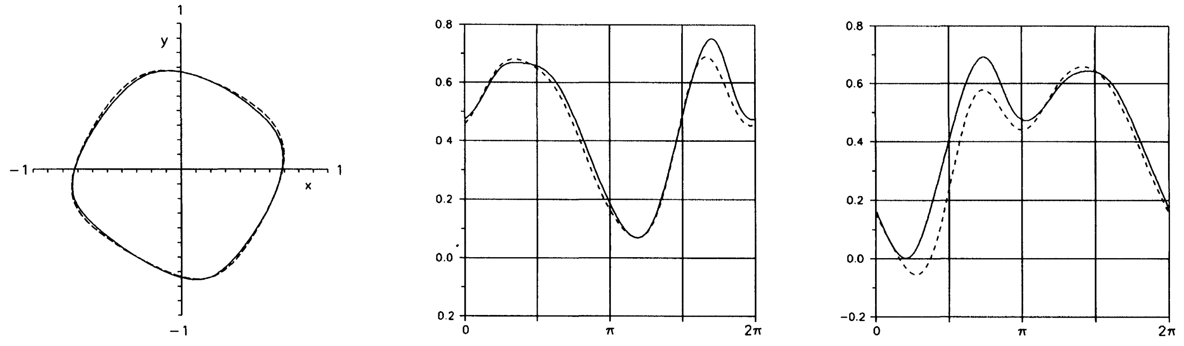

FIGURE 8 Comparison between calculated and predicted trajectories.

ating conditions plane σ-Mσ, the limit stability curves of the above solutions and therefore to delimit the journal's stable field of operation. For given geometrical and operating conditions, the intersection of the operating line, which also in this plane is a straight line with a slope 2, with the limit stability curve indicates the threshold value for the speed above which motion is no longer synchronous but, initially, has a period that is doubled and a greater amplitude. Above this threshold it is possible to find solutions with a further doubled period, non periodic solutions and even chaotic motions. Clearly, from an engineering viewpoint, the limit curve determined using the proposed method is the most useful as it makes it possible to identify the operating speed at which the first form of instability arises.

Figure 9 shows, as an example, one of the curves obtained for an unbalance value ρ = 0.3 and for a geometrical preload m = 0.35.

Figure 10 shows the verification for the above curve carried out by numerically integrating the system's equations of motion for values of the parameters corresponding to the points indicated on the map. As can be seen in the figure, in the operating conditions relative to points (A, C) the system exhibits a synchronous behaviour, highlighted by the presence of one time-marker in the orbits; in the operating conditions identified by points above the threshold (B, D), the orbits are described with a period that is double that of the rotor's speed, as indicated by the presence of two time-markers per orbit.

CONCLUSIONS

This paper has analysed the stability of the system made up of a rigid rotor on radial bearings with four tilting pads, and has proposed a method for determining the stability limit curves both in the case of a balanced rotor, and hence for a system with linear behaviour, and in the case of an unbalanced rotor for which the non linearity of the field of forces assumes considerable importance.

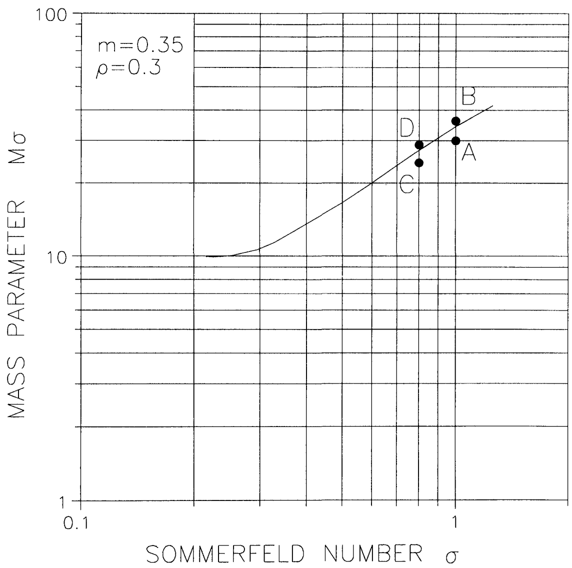

FIGURE 9 Limit stability curve (L/D = 0.7).

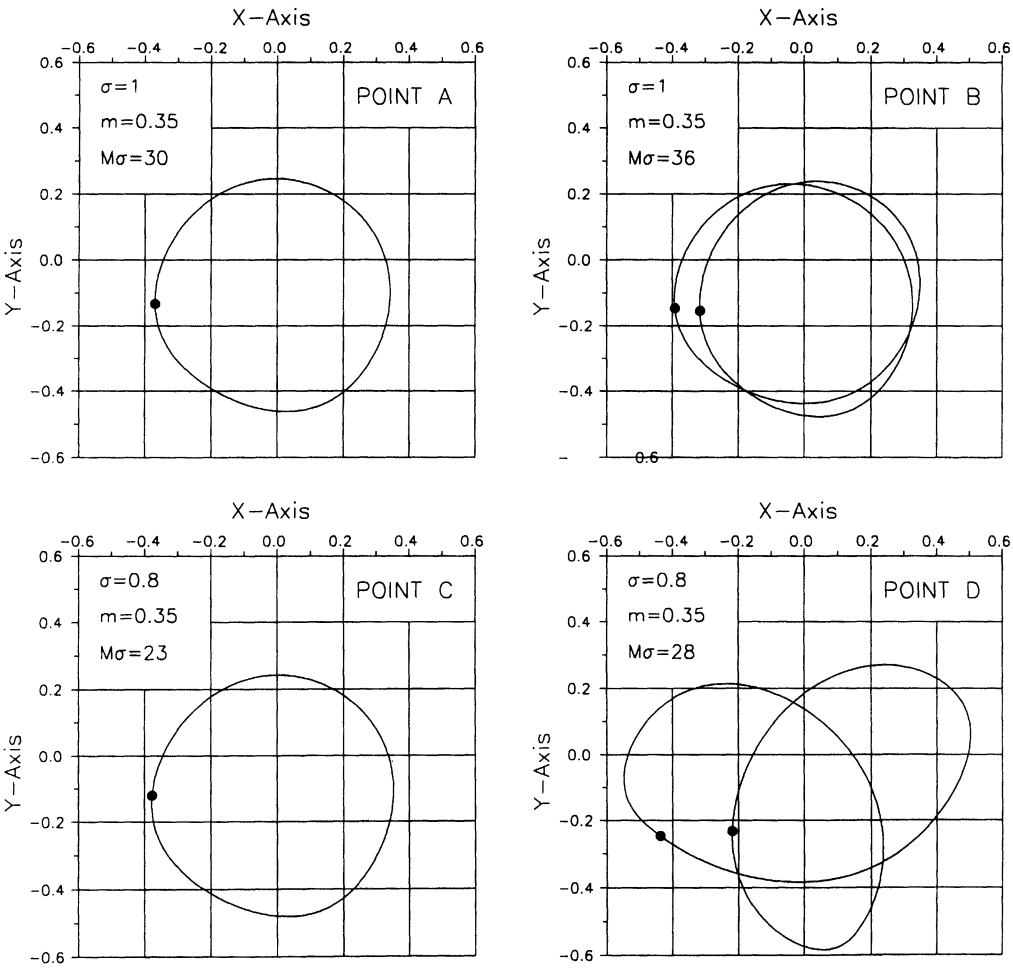

FIGURE 10 Validation of the limit stability curve.

The curves have been validated by the verifications carried out through numerical integration of the system's equations of motion.

The results of linear analysis, which consists of determining limit stability curves, confirm the stability of the stationary equilibrium position of the journal-pads system for negligible pad inertia values compared to the journal mass.

For non-negligible pad inertia values, the system may become unstable and, in this case, it abandons the stationary equilibrium position and evolves in large amplitude trajectories that are incompatible with bearing operation. The system can be stabilised by incrementing the preload value. In the case of low Sommerfeld number values, incrementing the preload initially produces a condition of stability with the upper pads unloaded, followed by an unstable system condition with all the pads loaded and, finally, a stable system condition with all the pads loaded.

The results of non linear analysis, which consists of elaborating a method for determining the limit stability curves of synchronous motion, a few examples of which are provided, confirm how in the presence of significant unbalance values there are operating conditions for which system motion generally has a large amplitude and is characterised by a fundamental frequency equal to a submultiple of the one corresponding to the rotor's rotation speed. The proposed method makes it possible to identify, in the plane of operating conditions, the limit curve that, for a given system, marks the passage from synchronous motion to motion with a fundamental frequency equal to half that of the synchronous one.

APPENDIX A

The analytical solution of the non linear system (6), which is necessarily approximated, is put in the form (10) in the text:

$$x(\tau) = x_0 + \sum_{j=1}^{k} [x_{cj}\cos(j\tau) + x_{sj}\sin(j\tau)]$$

$$y(\tau) = y_0 + \sum_{j=1}^{k} [y_{cj}\cos(j\tau) + y_{sj}\sin(j\tau)]$$

$$A_i(\tau) = A_{i0} + \sum_{j=1}^{k} [A_{icj}\cos(j\tau) + A_{isj}\sin(j\tau)] \qquad (10)$$

The expressions of the functions f_x, f_y and T_j which appear in the equations of motion, if the solution is periodic, are also periodic functions of the same period; they can thus be developed in Fourier series stopping the series development at the terms of the same order as that of the above solutions:

$$f_x(\tau) = f_{x0} + \sum_{j=1}^{k} [f_{xcj}\cos(j\tau) + f_{xsj}\sin(j\tau)]$$

$$f_y(\tau) = f_{y0} + \sum_{j=1}^{k} [f_{ycj}\cos(j\tau) + f_{ysj}\sin(j\tau)] \qquad (1A)$$

$$T_i(\tau) = T_{i0} + \sum_{j=1}^{k} [T_{icj}\cos(j\tau) + T_{isj}\sin(j\tau)]$$

with:

$$f_{x0} = \frac{1}{2\pi}\int_0^{2\pi} f_x d\tau;$$

$$f_{xcj} = \frac{1}{\pi}\int_0^{2\pi} f_x \cos(j\tau) d\tau;$$

$$f_{xsj} = \frac{1}{\pi}\int_0^{2\pi} f_x \sin(j\tau) d\tau; \ldots.etc.$$

By substituting (10) and (1A) into (6) gives the following expressions:

$$M\sum_{j=1}^{k} [j^2(x_{cj}\cos(j\tau) + x_{sj}\sin(j\tau))] + f_{x0} +$$
$$\sum_{j=1}^{k} [f_{xcj}\cos(j\tau) + f_{xsj}\sin(j\tau)] + M\rho\cos(\tau) = 0$$

$$M\sum_{j=1}^{k} [j^2(y_{cj}\cos(j\tau) + y_{sj}\sin(j\tau))] + f_{y0} +$$
$$\sum_{j=1}^{k} [f_{ycj}\cos(j\tau) + f_{ysj}\sin(j\tau)] - \frac{1}{\sigma} + M\rho\sin(\tau) = 0$$

$$I\sum_{j=1}^{k} [j^2(A_{icj}\cos(j\tau) + A_{isj}\sin(j\tau))] + T_{i0} +$$
$$\sum_{j=1}^{k} [T_{icj}\cos(j\tau) + T_{isj}\sin(j\tau)] = 0 \qquad (2A)$$

For each of Eqs. (2A), by grouping the terms in sine and cosine of the same argument and making them separately equal to zero, gives an algebraic system of equations in the unknowns made up of the coefficients of Eqs. (9). Therefore, by solving the above system (in the paper the Newton-Raphson method has been adopted), the analytical expression of the law of motion is obtained.

APPENDIX B

Hill's system of equations (11), at the instability threshold of the trivial solution allows the periodic solution:

$$\delta x(\tau) = \delta x_0 + \sum_{j=1}^{L} \delta x_{cj}\cos\left(\frac{j\tau}{2}\right) + \delta x_{sj}\sin\left(\frac{j\tau}{2}\right) = \overline{\delta x}$$

$$\delta y(\tau) = \delta y_0 + \sum_{j=1}^{L} \delta y_{cj}\cos\left(\frac{j\tau}{2}\right) + \delta y_{sj}\sin\left(\frac{j\tau}{2}\right) = \overline{\delta y} \qquad (1B)$$

Imposing equations (1B) as the approximated solution of equations (11) gives the following expressions for the residuals, indicated symbolically with $V_x(\delta a)$ and $V_y(\delta a)$:

$$V_x(\delta a) = M\,\overline{\ddot{\delta x}} + \bar{k}_{xx}\,\overline{\delta x} + \bar{k}_{xy}\,\overline{\delta y} + \textstyle\sum_i \bar{k}_{xA_i}\,\overline{\delta A_i}$$
$$+ \bar{b}_{xx}\,\overline{\dot{\delta x}} + \bar{b}_{xy}\,\overline{\dot{\delta y}} + \textstyle\sum_i \bar{b}_{xA_i}\,\overline{\dot{\delta A}}_i$$

$$V_y(\delta a) = M\,\overline{\ddot{\delta y}} + \bar{k}_{yx}\,\overline{\delta x} + \bar{k}_{yy}\,\overline{\delta y} + \textstyle\sum_i \bar{k}_{yA_i}\,\overline{\delta A_i}$$
$$+ \bar{b}_{yx}\,\overline{\dot{\delta x}} + \bar{b}_{yy}\overline{\dot{\delta y}} + \textstyle\sum_i \bar{b}_{yA_i}\cdot\overline{\dot{\delta A}}_i \tag{2B}$$

The symbol δa indicates the vector of the 4L + 2 unknowns made up of the Fourier coefficients in (1B):

$$\delta a = (\delta x_o, \delta x_{cj}, \delta x_{sj}, \delta y_o, \delta y_{cj}, \delta y_{sj}); \quad j = 1 \ldots L$$

Applying the harmonic balance method provides an algebraic system of 4L + 2 equations. Using T to indicate the period of the synchronous solution (10), the above system can be concisely written:

$$U_{x0} = \int_0^{2T} V_x\,(\delta a)\,d\tau = 0$$

$$U_{xcj} = \int_0^{2T} V_x\,(\delta a)\cos\left(\frac{j\tau}{2}\right)d\tau = 0$$

$$U_{xsj} = \int_0^{2T} V_x\,(\delta a)\sin\left(\frac{j\tau}{2}\right)d\tau = 0$$

$$U_{y0} = \int_0^{2T} V_y\,(\delta a)\,d\tau = 0$$

$$U_{ycj} = \int_0^{2T} V_y\,(\delta a)\cos\left(\frac{j\tau}{2}\right)d\tau = 0$$

$$U_{ysj} = \int_0^{2T} V_y\,(\delta a)\sin\left(\frac{j\tau}{2}\right)d\tau = 0 \tag{3B}$$

with j = 1.....L.

System (3B) allows solutions different from the trivial one if the determinant of the coefficients is equal to zero, and thus the limit stability curve of the synchronous solution, for a given value of the geometrical conditions and of the unbalance, is determined as the locus of points in the σ-Mσ plane where the above determinant is equal to zero. In the present paper L has been made equal to 6.

NOMENCLATURE

- A_j = $\delta R/C_p$ = pad inclination;
- b_{ij} = damping coefficient;
- C_p = radial pad clearance = $R_p - R$;
- C_b = radial bearing clearance = $R_b - R$;
- e = journal eccentricity from bearing centre;
- E = rotor unbalance;
- $\bar{I}$ = mass moment of inertia of pad around axial axis;
- I = $\bar{I}\,\omega^2/RW\sigma$ = dimensionless mass moment of inertia of pad;
- k_{ij} = stiffness coefficient;
- R_b = bearing radius;
- R = journal radius;
- R_p = pad radius;
- F_i = hydrodynamic force component;
- f_i = $F_i/\sigma W$ dimensionless hydrodynamic force component;
- $\bar{h}$ = oil film thickness;
- h = $\bar{h}/C_p$ = dimensionless oil film thickness;
- L = bearing length;
- m = geometrical preload of pad = $(R_p - R_b)/(R_p - R) = 1 - C_b/C_p$;
- $\bar{M}$ = half rotor mass;
- M = $\bar{M}C_p\omega^2/W\sigma$ = dimensionless half rotor mass;
- $\bar{p}$ = oil pressure;
- p = $\bar{p}/\mu\omega\,(R/C_p)^2$ = dimensionless oil pressure;
- t = time;
- $\bar{T}_i$ = moment of the pressure force on the i^{th} pad with respect to its pivot;
- T_i = $\bar{T}_i/R\sigma W$ = dimensionless moment on the i^{th} pad;

W = externally applied load;
$\bar{x},\bar{y},\bar{z}$ = coordinates;
x = $\bar{x}/C_p$, $y = \bar{y}/C_p$, $z = \bar{z}/L$ dimensionless coordinates;
β = angular amplitude of pads;
δ = pad deflection angle;
ϵ = e/C_p = bearing eccentricity ratio;
ϑ = circumferential coordinate;
ϑ_{pi} = circumferential coordinate of the i^{th} pivot;
μ = dynamic oil viscosity;
ν = frequency;
ν = $\bar{\nu}2\pi/\omega$ = dimensionless frequency;
ρ = E/C_p = dimensionless rotor unbalance;
τ = ωt = dimensionless time;
ω = shaft angular speed;
σ = $(\mu\omega RL/W)(R/C_p)^2$ Sommerfeld number;
$(\cdot)$ = dimensionless time derivative.

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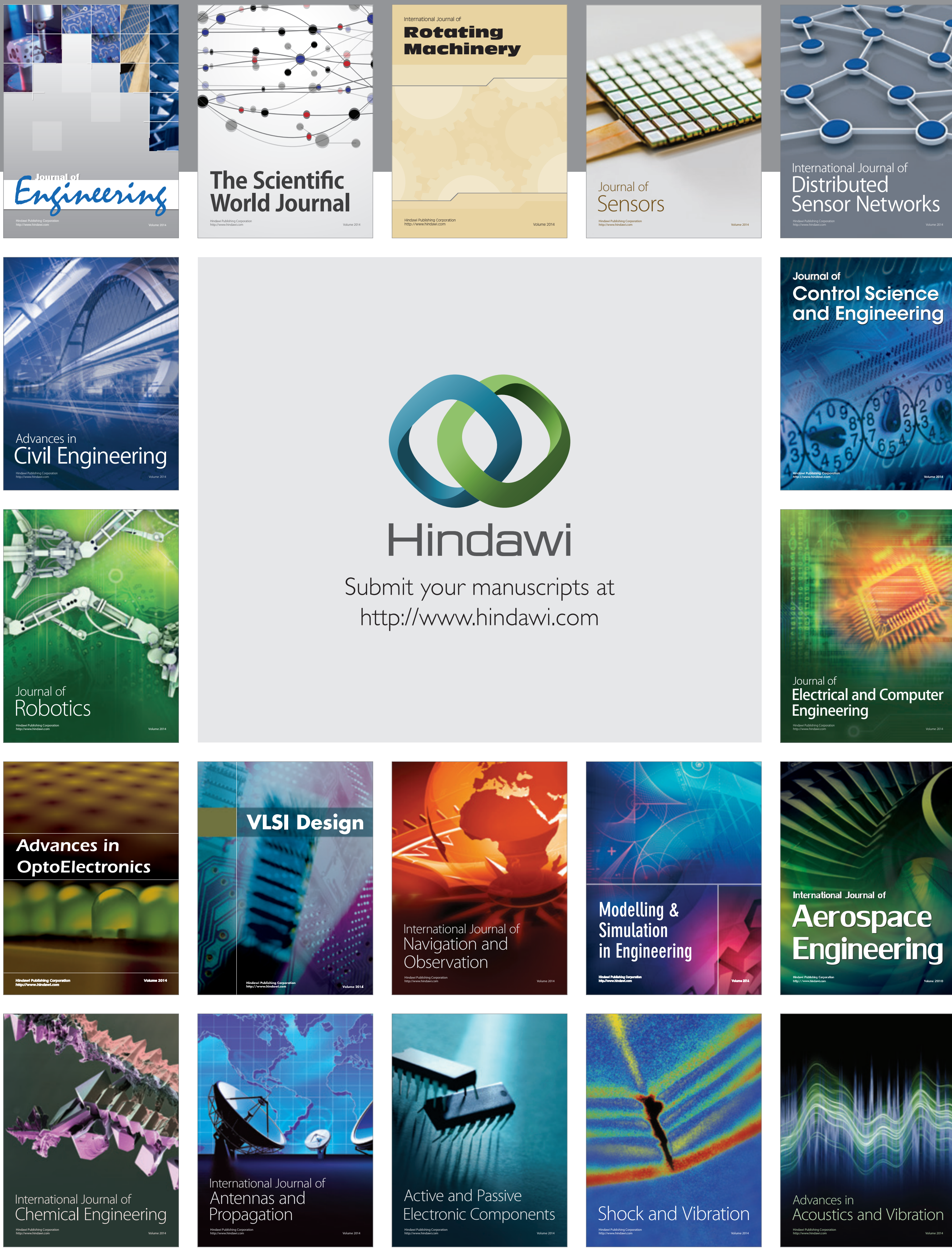
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